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AN EMPIRICAL FORMULA FOR THE CRITICAL SHEAR STRESS OF CURVED SHEETS

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ADVANCE RESTRICTED REPORT

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SUMMARY

Tests were made to determine the critical shear stress of curved sheets. The empirical formula derived from these tests is applicable to panels with a ratio of radius to thickness of 300 or greater, a central angle of 1 radian or less, and a ratio of arc length to axial length not greater than 1. In some panels with faulty workmanship the critical shear stresses were found to be much lower than predicted by the formula. The critical shear stress decreased with repeated loading, but no general laws were found for determining the amount of decrease.

INTRODUCTION

A knowledge of the buckling stress of curved sheet under shear is of considerable importance in aircraft structural design. For complete circular cylinders, the problem has been attacked theoretically and experimentally by a number of authors. For a panel that constitutes only a part of the circumference, the published theory appears to be limited to papers by Leggett (reference 1) and by Kromm (reference 2), which give approximate solutions for a panel very long in the axial direction. Previous to the publication of references 1 and 2, Wagner had proposed a formula (reference 3) in which the buckling stress appears as the sum of a term expressing the effect of curvature and a term expressing the buckling strength of a flat plate. This formula was modified slightly in reference 4 by

adding a term correcting the flat-plate term for finite aspect ratio. An analysis of miscellaneous published and unpublished test data to determine the coefficient for the curvature term was also given in reference 4. The test data showed a large amount of scatter for reasons that could not be determined from the published evidence. The present paper gives the results of a systematic series of tests undertaken to obtain a more reliable formula than heretofore available.

SYMBOLS

a	length of panel in axial direction, inches
b	length of panel in circumferential direction, inches
E	Young's modulus of elasticity, psi
K_1	coefficient of curvature term in proposed formula for critical shear stress
K_2	coefficient of flat-plate term in proposed formula for critical shear stress
R	radius of curvature of plate, inches
P_{cr}	critical buckling load, pounds
t	thickness of plate, inch
τ_{cr}	critical shear stress, psi
τ_{cr_1}	critical shear stress for first loading, psi
τ_{cr_n}	critical shear stress for nth loading, psi

TEST SPECIMENS AND APPARATUS

The test panels were made of 24S-T aluminum alloy. Two identical panels formed opposite sides of a torsion box (fig. 1). Pure shear was produced in the panels by subjecting the box to torsion in the setup shown in

figure 2. The box rotated about knife-edge supports attached to the end bulkheads; the line of support was the center line of the box.

It is very difficult to realize in practice a simple edge support or a clamped edge support. Only the edge conditions normally existing in actual structures with stiffeners riveted to the edges of the sheet, and not the theoretical edge conditions, were reproduced in the tests. The longitudinal stiffeners were steel angles riveted to the outside of the sheet a short distance from the edges of the box (fig. 1). The transverse stiffeners were the flanged edges of the bulkheads. The test section proper of the panel lay between the longitudinal steel angles and bulkheads B and E. The panel ends between bulkheads A and B, or between E and F, served as cushion bays to smooth out irregularities of stress distribution caused by the nearness of the loaded end bulkheads. In a similar manner, the strips of sheet lying between the steel angles and the adjacent edges of the box helped to isolate the test section from possible disturbing effects of the edges.

The thicknesses, radii of curvature, and aspect ratios a/b of the curved test panels are given in table 1. In addition, flat panels of 0.040-inch thickness and aspect ratios of 1 and 3 were built. Aspect ratios of 1 (square panels) were obtained by riveting the panels to each bulkhead; aspect ratios of 3 were obtained by riveting the panels only to bulkheads A, B, E, and F. The panels with an aspect ratio of 3 were actually resting on the intermediate bulkheads, but these bulkheads were believed to exert only a negligible influence on the buckling stress.

The curvature of each panel was checked by means of a dial gage indicating to 0.0001 inch the rise between two points 4 inches apart. A straightedge was used to check for sagging between bulkheads, and a careful visual check was made for surface irregularities such as dimples around rivets or flat spots near the longitudinal stiffeners. These checks indicated that

some panels had very serious imperfections. These panels with faulty workmanship were tested, but the results were not considered in establishing the formula for critical shear stress. In order to ensure the same curvature at all points, the panels had to be preformed accurately before they were riveted to the side walls of the box.

TEST PROCEDURE

Each accepted test box comprised either two identical test panels with an aspect ratio of 3, or six identical panels with an aspect ratio of 1. Tuckerman strain gages of 2-inch gage length were placed in the centers of all panels of each box at right angles to the expected direction of the buckle. The box was then loaded in small increments to a load somewhat beyond that necessary to produce buckling of the sheet. The strains read were plotted against load and the point at which the strain-load plot departed from the initial straight line was taken as indicating the buckling load. The torque corresponding to the buckling loads was then used to compute the critical shear stress for the sheet. Two typical plots for this method of determining the buckling load are shown in figure 3. On one panel with the lowest radius-thickness ratio tested (specimen 12-1-40, table 1), buckling occurred with a snap-diaphragm action; the stress at which this action occurred was taken to be the buckling stress. The longitudinal angles remained straight after buckling occurred and were therefore assumed to be adequate as far as buckling resistance was concerned.

In order to obtain some information on the effect of repeated loading, a number of boxes were loaded 50 to 60 times. Buckling stresses were determined on the first, second, and third loadings and thereafter at intervals of 10 loadings.

RESULTS AND DISCUSSION

Derivation of formula for critical shear stress.- The formula for critical shear stress proposed by Wagner as modified in reference 4 is

$$\tau_{cr} = K_1 E\frac{t}{R} + K_2 E\left(\frac{t}{b}\right)^2\left[1 + 0.8\left(\frac{b}{a}\right)^2\right] \qquad (1)$$

The first term in this equation expresses the effect of curvature; the second term expresses the buckling strength of a flat plate. Theoretical solutions for the flat plate have been obtained for plates of various aspect ratios (references 5 and 6). The second term of equation (1) represents an attempt to combine the results of all these theoretical calculations into a single simple expression. If Poisson's ratio is taken as 0.316 for aluminum alloy, the theoretical value of the constant K_2 is 4.89 for simply supported edges and 8.20 for clamped edges.

The critical shear stresses for the test panels having zero curvature and aspect ratios of 1 and 3 corresponded to values of K_2 of 6.79 and 5.96, respectively, which are averages of all the individual panels in each test box. These values are reasonably consistent with each other and fall about halfway between the theoretical values for simply supported and for clamped edges. These results appear plausible for riveted edges and are in line with the well-established fact that the experimental buckling stress of flat plates under shear is in good agreement with the theory if the tests are carefully made. The results of the flat-plate tests may, therefore, be considered as justifying the strain-gage method of determining the buckling stress as well as establishing the coefficient K_2 for the riveted-edge condition.

The test results for the curved plates were evaluated with the aid of formula (1). For simplicity, the experimental values of the coefficient K_2 obtained from the flat plates of aspect ratios 1 and 3 were averaged, although the individual values might have been used with a negligible change in the final formula. With the average coefficient $K_2 = 6.38$, the flat-plate term of equation (1) was calculated for each specimen and subtracted from the experimental critical stress τ_{cr} to obtain the curvature term in equation (1). The values of K_1 calculated from the curvature terms are plotted in figure 4 against the radius-thickness ratio. The points are for those specimens that were considered to be of good workmanship. In spite of this fact and the fact that each point represents either an average of

six panels when the aspect ratio is 1 or an average of two panels when the aspect ratio is 3, the points scatter considerably. Within the accuracy defined by the width of the scatter band, however, the coefficient K_1 is independent of R/t for $\frac{R}{t} > 600$; for $\frac{R}{t} < 600$, the coefficient K_1 increases rapidly as R/t decreases.

In the region where K_1 is reasonably constant, the numerical value is about 0.115, or slightly higher than the tentative value of 0.1 given in reference 4. Within the range and the accuracy of these tests, K_1 appears to be independent of the aspect ratio of the plate. In analytical form, the value of K_1 may be given as

$$K_1 = 0.1\left[1.15 + 45\left(\frac{100t}{R}\right)^4\right] \tag{2}$$

This formula should not be extrapolated to values of $\frac{R}{t} < 300$, because the curve is extremely steep in this region. For $\frac{R}{t} > 300$, the buckling stress of a curved-sheet panel that is bounded by riveted-on stiffeners may therefore be expressed by

$$\tau_{cr} = 0.1E\frac{t}{R}\left[1.15 + 45\left(\frac{100t}{R}\right)^4\right] + 6.4E\left(\frac{t}{b}\right)^2\left[1 + 0.8\left(\frac{b}{a}\right)^2\right] \tag{3}$$

For $\frac{R}{t} > 600$, equation (3) reduces to

$$\tau_{cr} = 0.115E\frac{t}{R} + 6.4E\left(\frac{t}{b}\right)^2\left[1 + 0.8\left(\frac{b}{a}\right)^2\right] \tag{4}$$

to a degree of accuracy appreciably better than that of the test results. Because these formulas are empirical, they should be applied only to panels having dimensions falling within the test range, that is, to panels having an arc length no greater than the axial length and a central angle less than 1 radian.

Discussion of formula.- A comparison of the calculated critical stresses based on formula (3) with the average experimental stresses (table 1) shows that the error ranges from about -9 percent to about 15 percent. An idea of the scatter among identical panels may be obtained from figure 5, which shows the average coefficient for each test box as well as the maximum and minimum values obtained for any one individual panel. This scatter is caused partly by uncontrollable differences in the panels, partly by the uncertainty in the determination of the buckling stresses.

The effect of poor workmanship on the panels is shown graphically by figure 6, which is identical with figure 4 except that the test points for specimens with faulty workmanship have been added. For two specimens having radius-thickness ratios larger than 1000, the effect of faulty workmanship was sufficient nearly to eliminate the strengthening effect of curvature. Other panels with radius-thickness ratios larger and smaller than 1000 showed that the buckling stress as predicted by formula (3) may be materially decreased by faulty workmanship.

A comparison of the experimental results with Kromm's formula of reference 2

$$\tau_{cr} = 1.67E\frac{t}{b}\sqrt{\frac{t}{R}} \tag{5}$$

is shown in figure 7. This formula is applicable to infinitely long panels for which $\frac{b}{t}\sqrt{\frac{t}{R}} > 4.3$. It is obvious from the figure that Kromm's formula is very conservative even for the panels with an aspect ratio of 3, which may be considered as panels of infinite length.

No comparison was made with Leggett's formula (reference 1) because the proportions of the test panels of the present investigation were outside the range for which results are given in reference 1.

Reference 7 gives test results obtained on a series of complete cylinders subdivided into panels

by rings and longitudinal stiffeners. The buckling stresses were determined from torque-twist plots and from observations of sudden changes in load while the cylinders were being twisted. Table 2 shows that the observed buckling stresses taken from reference 7 exceed those predicted by formula (3) by amounts varying from 8 to 54 percent, with an average difference of 31 percent.

Reference 8 describes tests of curved-web beams. In the course of these tests, the critical shear stresses of individual panels were determined by visual observation. Comparison of the stresses given in table 3 shows that the experimental stresses exceed those predicted by formula (3) by amounts ranging from 5 to 79 percent, with an average difference of 37 percent. The experimental stresses given in table 3 are not taken directly from reference 8 but are the averages for the panels adjacent to the neutral axis of each beam; the other panels were excluded from the average because their critical stresses were changed by the presence of tension or compression stresses.

The methods of determining critical stresses used in references 7 and 8 are probably less sensitive than the methods of the present investigation. This difference may be responsible for the fact that the experimental buckling stresses of references 7 and 8 average higher than those of the present investigation.

Effect of repeated loading.- The effect of repeated loading on the buckling stresses is shown by the curves of figure 8. The first few loadings decreased the buckling stress appreciably; additional loadings generally caused a small but continued decrease, although some curves appear to level off. No permanent set was noticed visually except in one panel having $\frac{R}{t} = 300$, but presumably yielding had taken place in localized regions in the other panels even though the average shear stress for all panels was below the proportional limit. No general laws relating quantitatively the effect of repeated loading to the properties of the specimen were found - possibly because there was not sufficiently close control over such factors as quality of workmanship, initial tension in the sheet, and the amount by which the buckling load was exceeded.

CONCLUSIONS

Tests to determine the critical shear stress of curved sheets indicated that:

1. The buckling stress of a curved-sheet panel that is bounded by riveted-on stiffeners may be expressed, if $\frac{R}{t} > 300$, by the formula

$$\tau_{cr} = 0.1E\frac{t}{R}\left[1.15 + 45\left(\frac{100t}{R}\right)^4\right] + 6.4E\left(\frac{t}{b}\right)^2\left[1 + 0.8\left(\frac{b}{a}\right)^2\right]$$

or, if $\frac{R}{t} > 600$, by

$$\tau_{cr} = 0.115E\frac{t}{R} + 6.4E\left(\frac{t}{b}\right)^2\left[1 + 0.8\left(\frac{b}{a}\right)^2\right]$$

provided the arc length is not greater than the axial length and the central angle defining the arc length is less than 1 radian. In these formulas

τ_{cr} critical shear stress, psi

E Young's modulus of elasticity, psi

t thickness of plate, inch

R radius of curvature of plate, inches

b length of panel in circumferential direction, inches

a length of panel in axial direction, inches

2. The buckling stress of a curved panel as predicted by the foregoing formulas may be materially decreased by faulty workmanship.

3. Repeated loading appreciably decreased the critical stress for the first few loads; additional loadings generally caused a small but continued decrease.

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TABLE 1

DIMENSIONS OF SPECIMENS AND CRITICAL SHEAR STRESSES

Specimen (a)	t (in.)	R (in.)	$\frac{R}{t}$	$\frac{a}{b}$	τ_{cr} (psi)		Error $\frac{\text{Exp. - Calc.}}{\text{Calc.}}$ (percent)
					Calculated (formula (3))	Experimental	
12-1-40	0.0403	11.93	297	1.00	7558	[b]7900	4.5
12-3-40	.0415	12.61	304	3.01	6743	6660	-2.2
24-3-40	.0403	24.07	597	3.00	2940	3045	3.5
36-1-40	.0398	34.66	871	1.00	2738	2880	5.2
36-3-40	.0390	36.41	933	2.99	2095	1915	-8.6
48-1-40	.0393	44.25	1125	1.00	2380	2185	-8.1
48-3-40	.0392	45.32	1155	2.98	1827	1840	.8
12-1-32	.0321	12.01	371	1.00	4800	5510	14.6
12-3-32	.0321	12.08	376	2.99	4387	4440	1.3
12-1-20	.0209	11.21	537	1.00	2756	2585	-6.2
12-3-20	.0207	11.91	575	2.99	2406	2238	-6.8
24-1-20	.0206	22.52	1092	1.00	1472	1605	9.0
24-3-20	.0203	23.25	1145	3.00	1274	1366	7.2
36-1-20	.0207	31.41	1518	1.01	1162	1092	-6.2
36-3-20	.0207	33.59	1622	2.96	967	1099	13.7
48-3-20	.0207	42.49	2050	2.99	813	876	7.5

[a]First number is the nominal radius, the second number is the nominal aspect ratio, and the third number is the nominal sheet thickness in thousandths of an inch.

[b]Stress at which snap-diaphragm action occurred.

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TABLE 2

DIMENSIONS OF SPECIMENS AND CRITICAL SHEAR STRESSES

[From tests of reference 7]

Specimen	t (in.)	R (in.)	$\frac{R}{t}$	$\frac{a}{b}$	τ_{cr} (psi)		Error $\frac{\text{Exp. - Calc.}}{\text{Calc.}}$ (percent)
					Calculated (formula (3))	Experimental (reference 7)	
19	0.020	15.04	753	1.14	1951	2110	8
20	.020	15.04	753	2.29	2569	2870	12
15	.0195	15.04	772	1.49	2638	3370	28
15	.0195	15.04	772	1.53	2626	3590	42
21	.0205	15.04	734	1.53	2821	4120	46
21	.0205	15.04	734	1.14	3053	4740	54

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TABLE 3

DIMENSIONS OF SPECIMENS AND CRITICAL SHEAR STRESSES

[From tests of reference 8]

Specimen	t (in.)	R (in.)	$\frac{R}{t}$	$\frac{a}{b}$	τ_{cr} (psi)		Error $\frac{\text{Exp. - Calc.}}{\text{Calc.}}$ (percent)
					Calculated (formula (3))	Experimental (reference 8)	
2	0.0145	15.04	1035	1.05	1616	2260	39
3	.0143	15.04	1048	1.05	1584	2830	79
4	.0385	15.04	390	1.05	6770	8570	27
5	.0394	15.04	381	1.05	7090	8690	23
6	.0154	15.04	975	1.91	1565	2160	38
7	.0395	15.04	380	1.05	7100	7450	5
8	.0150	15.04	1000	1.91	1519	2120	40
10	.0154	15.04	975	1.91	1565	2240	43

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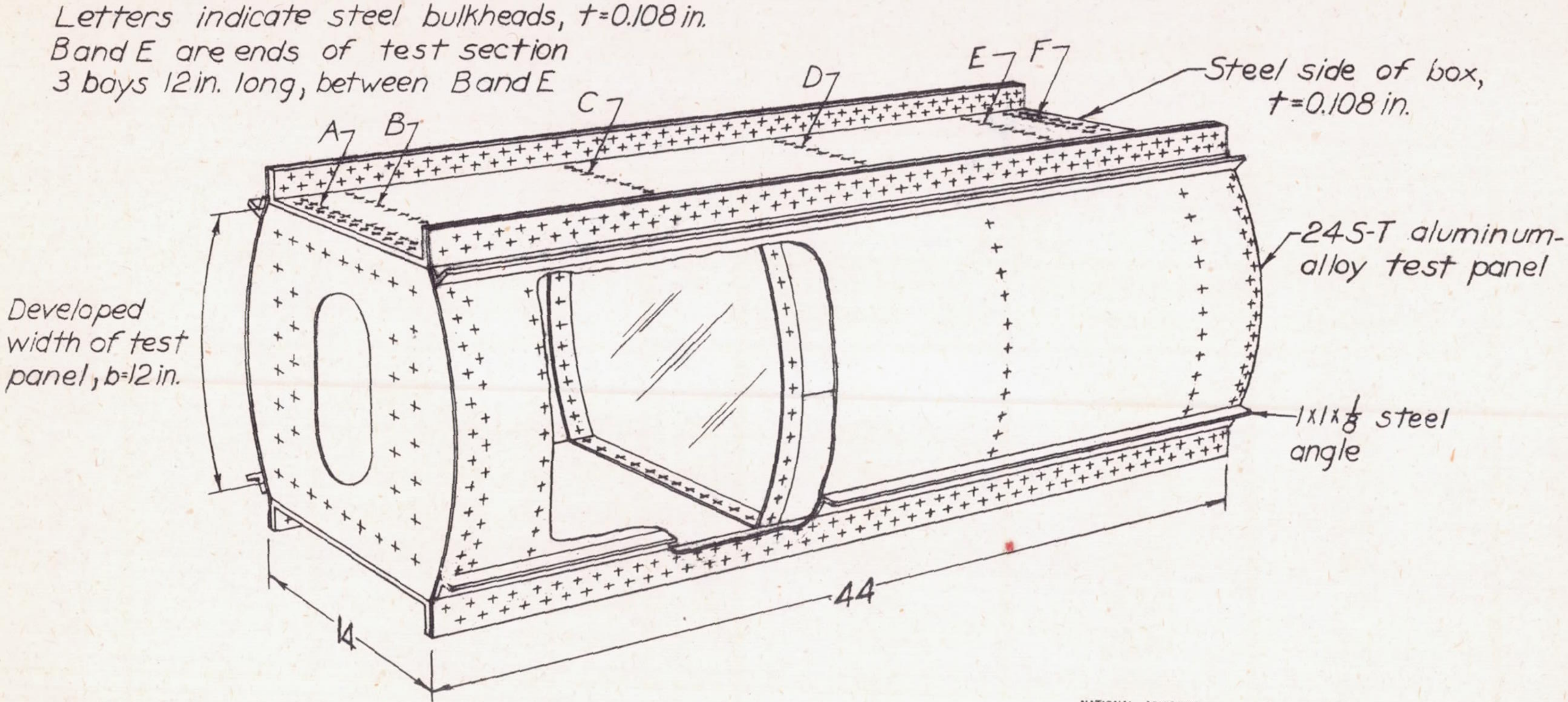

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Figure 1. - Typical torsion box used for buckling tests.

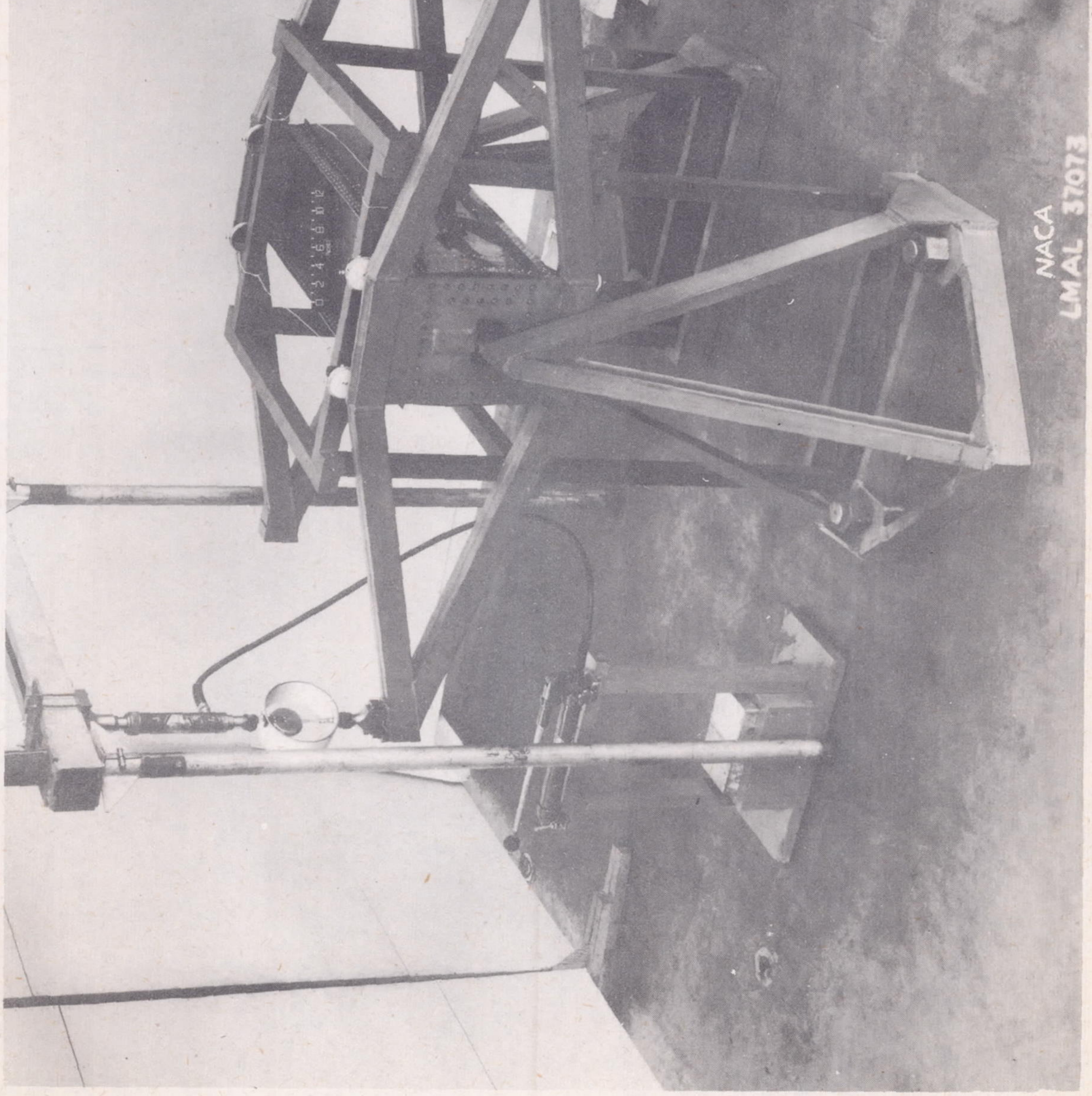

Figure 2.- Setup for torsion tests.

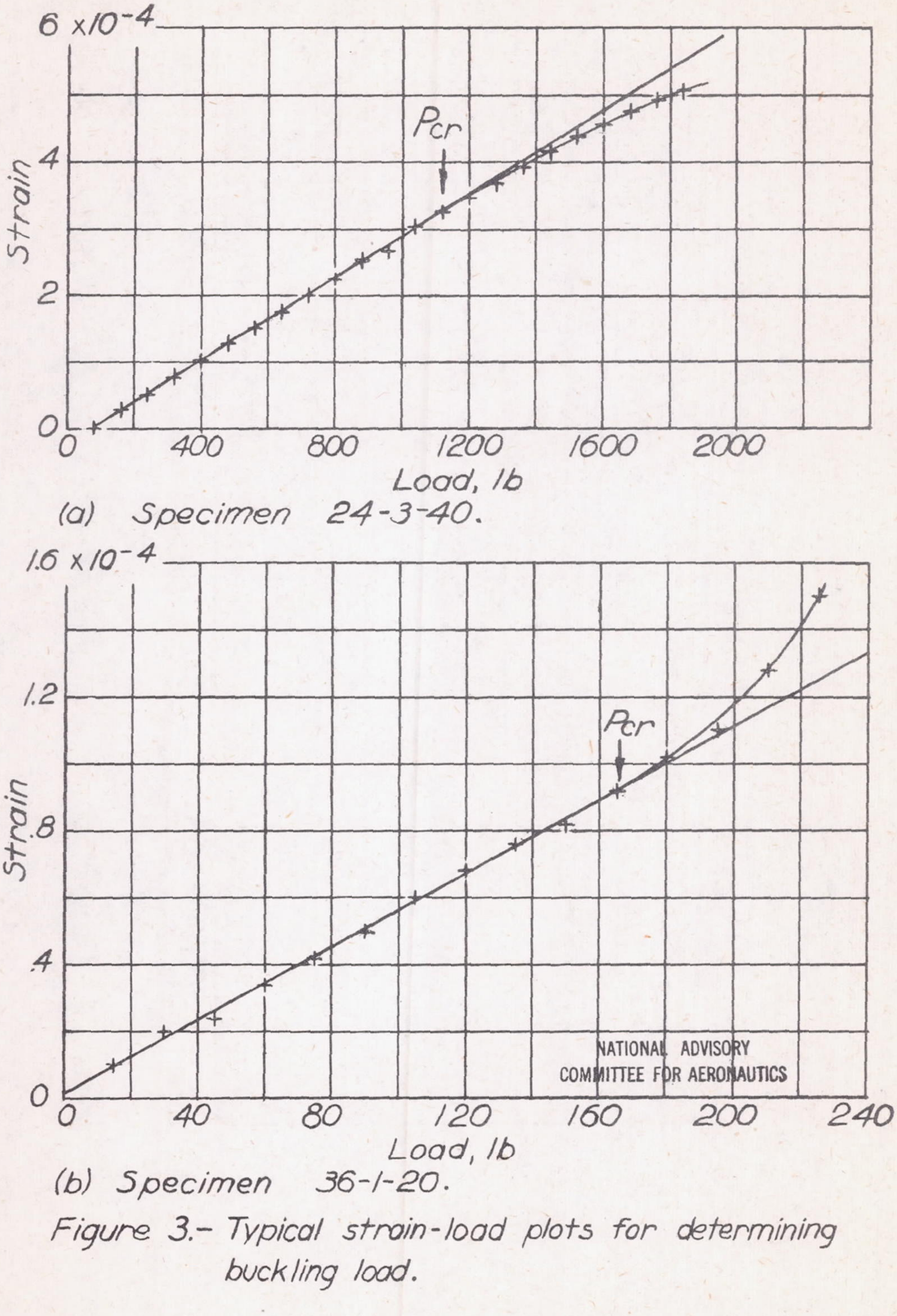

Figure 3.- Typical strain-load plots for determining buckling load.

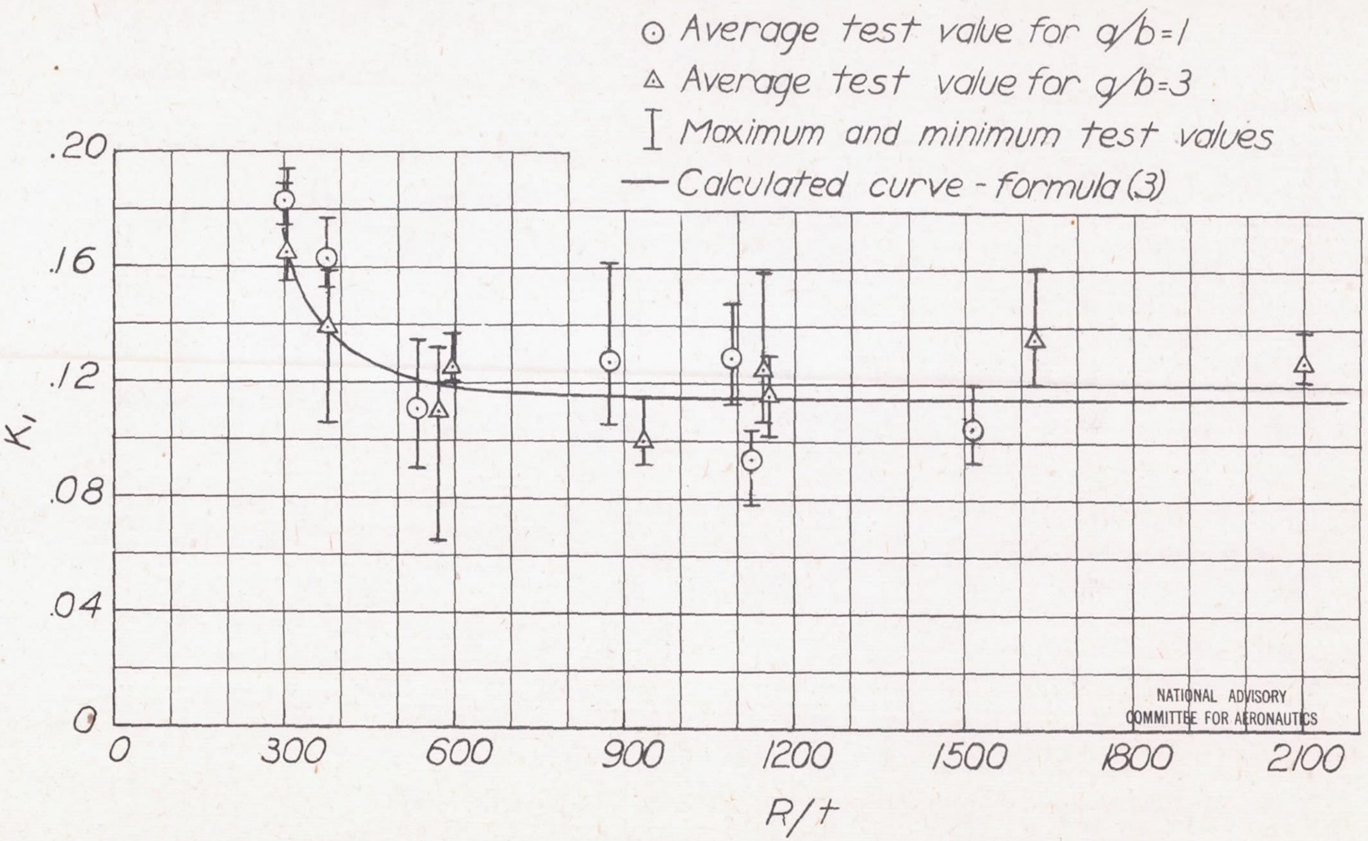

Figure 5.- Maximum and minimum test values of K_1 for identical panels.

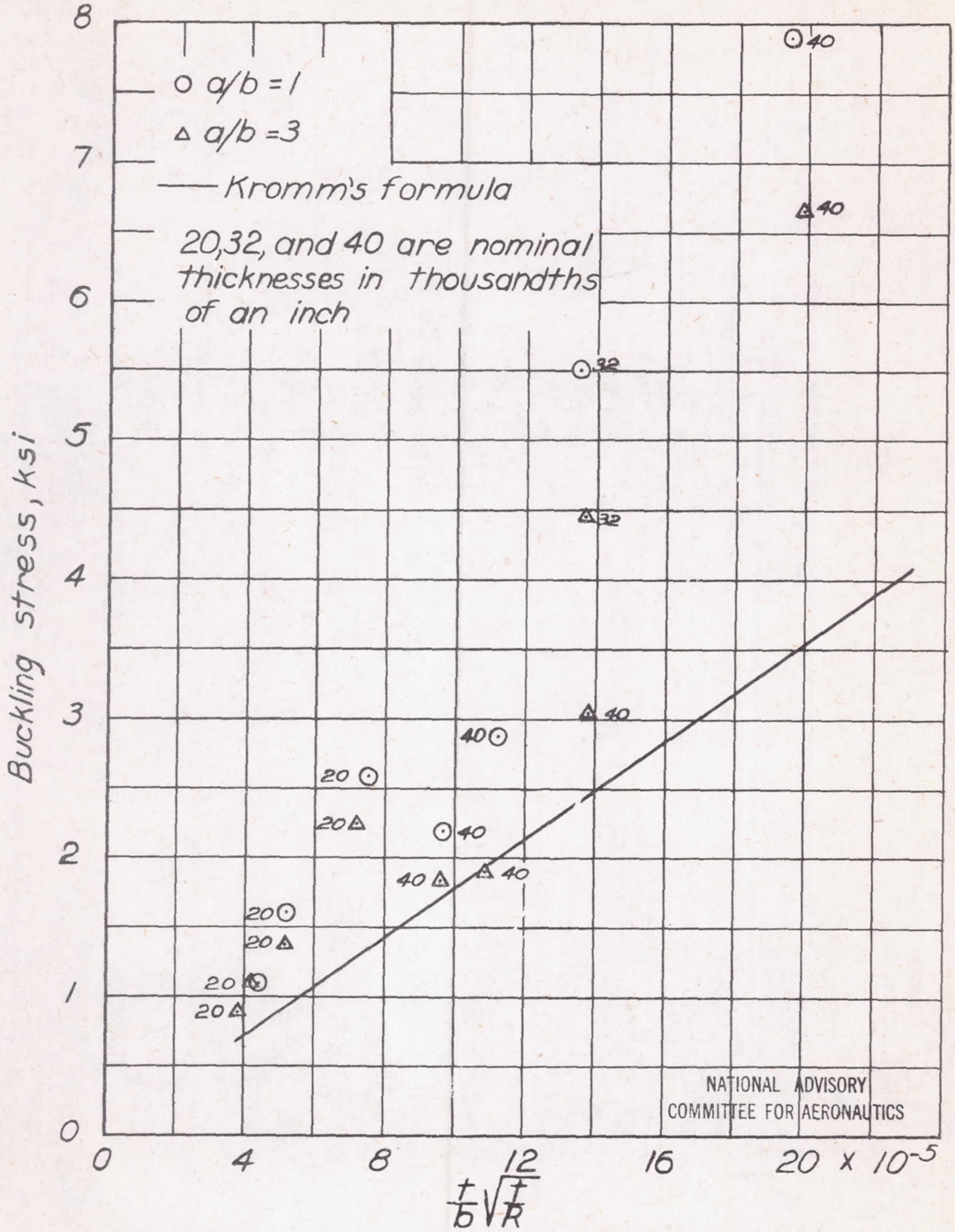

Figure 7.- Comparison of experimental buckling stresses with buckling stresses computed by Kromm's formula (reference 2).