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ESTIMATION OF STICK-FIXED NEUTRAL POINTS OF AIRPLANES

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

~~CONFIDENTIAL BULLETIN~~

ESTIMATION OF STICK-FIXED NEUTRAL POINTS OF AIRPLANES

By Maurice D. White

SUMMARY

A method is given for calculating the stick-fixed neutral point of an airplane with propeller windmilling, flaps neutral, and landing gear retracted. This method differs from those formerly used principally in the procedure for estimating the effect of the windmilling propeller. Comparison of the neutral points predicted by this method with neutral points obtained in flight tests indicates good agreement at low lift coefficients. The methods presented, in conjunction with the results given in NACA CB No. L4H01 "Effect of Power on the Stick-Fixed Neutral Points of Several Single-Engine Monoplanes as Determined in Flight," should be useful in estimating the stick-fixed neutral points of new designs for all flight conditions.

INTRODUCTION

Since the publication of reference 1, in which a method was presented for predicting the static longitudinal stability of airplanes, additional flight data on the longitudinal stability of airplanes have become available. Attempts to correlate these longitudinal-stability data with the longitudinal-stability data computed on the basis of reference 1 indicated that the methods of reference 1 were inadequate when applied to unconventional designs. A more rational method has therefore been developed for computing the longitudinal stability of airplanes in terms of the stick-fixed neutral point, which yields results in good agreement with flight results. This method differs from the method of reference 1 chiefly in the procedure for estimating the effects on longitudinal stability of the windmilling propeller and of the fuselage and nacelles, although the differences in change in longitudinal stability due to the fuselage

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and nacelles as computed by the two methods is generally small for conventional designs. The methods given may also be adapted readily to the estimation of the effect on neutral point of changes in configuration of existing designs.

### SYMBOLS

- $a_0$  two-dimensional lift-curve slope, per degree
- $w$  width of fuselage or nacelle at selected longitudinal station, feet
- $c$  wing chord, feet
- $\bar{c}$  mean aerodynamic chord, feet
- $D$  propeller diameter, feet
- $q$  dynamic pressure, pounds per square foot  $\left(\frac{1}{2}\rho V^2\right)$
- $r$  factor used in correcting lift-curve slope for effect of end plates
- $\rho$  air density, slugs per cubic foot
- $V$  velocity, feet per second
- $S$  gross area, including section through fuselage, square feet
- $M$  pitching moment, foot-pounds
- $x$  distance along longitudinal axis, feet
- $x_1$  distance from wing leading edge to middle of fuselage section, feet
- $\bar{x}_1$  distance from wing leading edge to front of fuselage section directly ahead of wing leading edge, feet
- $x_0$  neutral-point location, percent  $\bar{c}$
- $l_t$  longitudinal distance from center of gravity to quarter-chord point of mean chord of horizontal tail, feet

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- $l_p$  longitudinal distance from center of gravity to propeller plane
- $\beta$  angle of inclination of air flow relative to longitudinal axis, radians
- $\alpha$  angle of attack relative to free stream, radians
- $\epsilon$  angle of downwash, radians
- $C_L$  lift coefficient  $\left(\frac{\text{Lift}}{qS}\right)$
- $C_m$  pitching-moment coefficient  $\left(\frac{M}{qSc}\right)$
- $C_N$  normal-force coefficient  $\left(\frac{\text{Normal force}}{qS}\right)$
- A aspect ratio
- $T_c$  thrust disk-loading coefficient  $\left(\frac{T}{\rho V^2 D^2}\right)$
- T thrust, pounds
- N number of propellers, nacelles, or fuselages
- Subscripts:
- w wing
- f fuselage
- nac nacelle
- t horizontal tail
- p propeller
- $p_1$  propeller normal force
- $p_2$  downwash due to propeller
- L.E. wing leading edge
- mid midchord point of local chord
- T.E. wing trailing edge

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COMPUTATION OF NEUTRAL POINT

The neutral point of an airplane is defined as the center-of-gravity location at which the slope of the curve of airplane pitching-moment coefficient against lift coefficient  $dC_m/dC_L$  is zero. This airplane pitching-moment slope is the resultant of the pitching-moment slopes contributed by the various parts of the airplane. In order to predict the neutral point of an airplane, the estimated values of  $\partial C_m/\partial C_L$  due to the various parts of the airplane may be combined at each of several center-of-gravity locations and the neutral point may then be established as the center-of-gravity location at which the resultant value of  $dC_m/dC_L$  is zero. A procedure equivalent to this procedure for determining the neutral point is illustrated in figure 1, where the values of  $\partial C_m/\partial C_L$  for the various parts of the airplane are plotted against center-of-gravity location. The center-of-gravity location at which the total of the positive values of  $\partial C_m/\partial C_L$  is equal to the negative of the total of the negative values of  $\partial C_m/\partial C_L$  - that is, at which  $dC_m/dC_L$  is equal to zero - is the neutral point.

Detailed procedures for calculating the values of  $\partial C_m/\partial C_L$  due to the principal parts of the airplane - wing, fuselage, nacelles, horizontal tail, and propellers - are given herein. The methods apply best at low lift coefficients where the airplane drag coefficient may be considered not to vary with lift coefficient, the parameters involved in the calculations are most nearly linear, and the effects of air-flow separation are at a minimum. The airplane was assumed to operate with propeller windmilling, flaps neutral, and landing gear retracted. Negative changes in pitching-moment slope are stabilizing and positive changes are destabilizing.

The values of  $\partial C_m/\partial C_L$  for the individual parts computed by the following methods are based on the values of  $dC_L/da$  for the wing alone. In order to determine the actual value of  $\partial C_m/\partial C_L$  based on the total lift of the airplane the values of  $\partial C_m/\partial C_L$  computed by the present methods should be multiplied by the ratio

$$\frac{dC_L/d\alpha}{\frac{dC_L}{d\alpha} + \frac{dC_{N_t}}{d\alpha_t} \frac{s_t}{s_w} \frac{q_t}{q} \left[ \left( 1 - \frac{d\epsilon}{d\alpha_t} \right) - \frac{\left( \frac{dC_N}{d\alpha} \right)_p \left( \frac{d\beta}{d\alpha} \right)_p}{4} \right] + \frac{N_{\pi}}{4} \frac{\left( \frac{dC_N}{d\alpha} \right)_p \left( \frac{d\beta}{d\alpha} \right)_p D^2}{s_w}}$$

This correction does not affect the location of the neutral point as determined by the present methods because at that center-of-gravity location the resultant value of  $dC_m/dC_L$  of zero would be unaffected by the correction.

The methods described in the present report are effective for predicting stick-fixed neutral points at low lift coefficients with propeller windmilling, flaps neutral, and landing gear retracted. Inasmuch as the stick-fixed neutral point generally remains fixed or moves back with increasing lift coefficient in this condition of flight (reference 2), the results obtained from the present methods will be conservative for the higher lift coefficients.

The effects of power on the neutral-point location cannot at the present time be predicted by methods comparable to those given herein. By the use of data given in reference 2, however, it should be possible to make reasonable preliminary estimates of the shift in stick-fixed neutral point due to power.

A knowledge of the elevator hinge-moment characteristics is essential for determining the shift in neutral point due to freeing the elevator.

Pitching-moment slope due to wing.- At a given center-of-gravity location the value of the pitching-moment slope due to the wing with flaps neutral  $(\partial C_m / \partial C_L)_w$  is numerically equal to the distance between the center of gravity and the wing aerodynamic center expressed as a fraction of the mean aerodynamic chord. For the present purposes it has been found satisfactory to consider that the wing aerodynamic center is located at a percentage of the mean aerodynamic chord equal to the average of the percentages of the chords at which the aerodynamic centers of the root and tip airfoil sections are located. For center-of-gravity locations behind the wing aerodynamic center,  $(\partial C_m / \partial C_L)_w$  is positive.

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Pitching-moment slope due to fuselage and to engine nacelles.- The value of  $\partial C_m / \partial C_L$  due to the fuselage or to the nacelles may be expressed by

$$\left( \frac{\partial C_m}{\partial C_L} \right)_{f, nac} = \frac{\frac{1}{q} \left( \frac{dM}{da} \right)_{f, nac}}{S_w c \frac{dC_L}{da}} \quad (1)$$

The value of  $\frac{1}{q} \left( \frac{dM}{da} \right)_{f, nac}$  is calculated directly by the methods of reference 3. According to this method the value of  $\frac{1}{q} \left( \frac{dM}{da} \right)_{f, nac}$  may be computed from

$$\frac{1}{q} \left( \frac{dM}{da} \right)_{f, nac} = N \frac{\pi}{2} \int w^2 \frac{d\beta}{da} dx \quad (2)$$

where the integral is taken along the entire length of the fuselage or nacelle.

It is convenient in evaluating equation (2) to divide the fuselage or nacelle into finite sections. The factor  $\frac{\pi}{2} w^2 \frac{d\beta}{da} \Delta x$  is then calculated for each section with average values of  $w$  and  $d\beta/da$  used for each section and, finally, the values of  $\frac{\pi}{2} w^2 \frac{d\beta}{da} \Delta x$  for all the sections are totaled.

The factor  $d\beta/da$  represents the variation with angle of attack of air-flow angle relative to the X-axis of the fuselage or nacelle; in the calculation of  $d\beta/da$ , it is assumed that fuselage - or nacelle - interference effects may be ignored. Between the wing leading edge and trailing edge, where the flow follows the wing surface,  $d\beta/da$  is considered to be zero. Behind the wing trailing edge the value of  $d\beta/da$  is assumed to vary linearly from zero at the wing trailing edge to  $\left( 1 - \frac{d\epsilon}{da} \right)_t$  at the tail.

The discrepancy between the assumed linear variation and the actual variation of  $d\beta/da$  behind the wing will normally be of little importance because, for conventional

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fuselage arrangements, this portion of the computed fuselage moment is only about 10 percent of the entire fuselage moment.

Ahead of the wing, values of  $d\beta/da$  are greater than 1.0 because of the upwash induced by the wing; the variation of  $d\beta/da$  with distance from the wing leading edge is given in figure 2, which is taken directly from reference 3. Figure 2(b) is used for all sections except the section directly ahead of the wing leading edge where values of  $d\beta/da$  rise sharply as the leading edge is approached. For this region an average integrated value of  $d\beta/da$ ,

defined as  $\overline{\frac{d\beta}{da}} = \frac{1}{\bar{x}_1} \int_0^{\bar{x}_1} \frac{d\beta}{da} dx$  and given by figure 2(a), is

used. In determining  $\overline{d\beta/da}$  in figure 2(a) the value of  $\bar{x}_1$  used in calculating the abscissa is the distance from the wing leading edge to the front of the fuselage section; in determining  $d\beta/da$  in figure 2(b) the value of  $x_1$  used in calculating the abscissa is the distance from the wing leading edge to the middle of the fuselage section considered.

The curves in figure 2 correspond to a wing lift-curve slope of 4.5 per radian. In order to correct for other values of the lift-curve slope the values given by the curves are increased or reduced in direct ratio to the lift-curve slope.

An additional factor accounting for the effect of the fuselage or nacelle on the wing becomes important when the width of the fuselage or nacelle is not uniform along the wing chord. According to reference 3 this factor is computed by

$$\frac{1}{q} \frac{dM}{da} = N \frac{\pi}{16} (w_{L.E.} + 2w_{mid} - 3w_{T.E.}) c^2 \quad (3)$$

When the plan form of the body shows local protuberances in width directly ahead of the wing leading edge the application of equations (2) and (3) is believed to result in exaggerated values of the destabilizing effect. For such cases, reduction of the increment due to the protuberance, (obtained by calculating the factors with and without the protuberance) by one-half appears to result in more nearly correct values. This correction,

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which is based on very scant data, is subject to modification as more data are obtained.

Curves for estimating the wing lift-curve slope  $dC_L/d\alpha$  as a function of aspect ratio are given in figure 3. These curves were obtained from the expression

$$\frac{dC_L}{d\alpha} = \frac{a_0 \times 57.3}{1 + \frac{a_0 \times 57.3}{\pi A}}$$

and at low aspect ratios were corrected on the basis of data given in reference 4. The correction for the effect of vertical tails mounted as end plates (fig. 4) was also taken from reference 4.

The values of  $\partial C_m / \partial C_L$  due to the fuselages and nacelles are always positive.

Pitching-moment slope due to horizontal tail.— The value of the pitching-moment slope due to the horizontal tail  $(\partial C_m / \partial C_L)_t$  is calculated by the expression

$$\left( \frac{\partial C_m}{\partial C_L} \right)_t = - \frac{\frac{q_t}{q} \frac{dC_{Nt}}{d\alpha_t} S_t l_t \left( 1 - \frac{d\epsilon}{d\alpha} \right)_t}{S_w \bar{c} \frac{dC_L}{d\alpha}} \quad (4)$$

The various terms in equation (4) are calculated as follows:

$\frac{q_t}{q}$  A value of 0.9 is used for the condition of windmilling propeller.

$\frac{dC_{Nt}}{d\alpha_t}$  Curves for the evaluation of  $\frac{dC_{Nt}}{d\alpha_t}$  are given in figures 3 and 4.

$\frac{d\epsilon}{d\alpha}$  Values of  $\epsilon$  at the tail may be estimated from the charts of reference 5. A shorter empirical method for determining  $d\epsilon/d\alpha$  at the tail, based on these charts, is given in reference 1.

For conventional tail locations the value of  $\partial C_m / \partial C_L$  due to the horizontal tail is negative.

Pitching-moment slope due to normal force on windmilling propeller and to downwash increment resulting from this normal force. - The values of  $\partial C_m / \partial C_L$  due to the propeller are considered to arise from two sources - the normal force on the inclined propeller and the resulting increment of downwash at the tail. The value of  $\partial C_m / \partial C_L$  due to the normal force on the propeller is given by

$$(1) \left( \frac{\partial C_m}{\partial C_L} \right)_{p1} = \frac{N \pi}{L} \frac{\left( \frac{dC_N}{d\alpha} \right)_p \left( \frac{d\beta}{d\alpha} \right)_p \frac{1}{2} \rho D^2}{S_w \bar{c} \frac{dC_L}{d\alpha}} \quad (5)$$

The terms in equation (5) are calculated as follows:

$$\left( \frac{dC_N}{d\alpha} \right)_p$$

Reference 6 gives methods for estimating the

value of  $\left( \frac{dC_N}{d\alpha} \right)_p$  that should be used when the geometric and operating characteristics of the propeller are known. (In reference 6  $\left( \frac{dC_N}{d\alpha} \right)_p$  is designated  $C_Y' \psi$ .) Unpublished flight data

obtained at the Langley Laboratory on a number of airplanes indicates that with propeller windmilling and at the airspeeds for which the present methods are applicable the propeller thrust coefficient  $T_c$  is on the average approximately -0.03. The data also show that when the propeller pitch setting is set for high rotational speeds the propeller blades are, at these airspeeds, still against the low-pitch stops, which correspond on the average to a propeller-blade angle at 0.75 propeller radius of about  $20^\circ$ . It has been found sufficiently accurate for the airplanes considered in the present analysis, to use directly the

values of  $\left( \frac{dC_N}{d\alpha} \right)_p$  for a propeller blade angle

of  $20^\circ$  given in figure 2 of reference 6. This figure presents data for a Hamilton Standard 3155-6 propeller at a value of  $T_c$  equal to zero and a side-force factor (defined in reference 6) of 80.7. For convenience the values given in this figure for a propeller blade angle of  $20^\circ$  are tabulated as follows:

| Propeller                   | $\left(\frac{\partial C_N}{\partial \alpha}\right)_p$ |
|-----------------------------|---|
| Two-blade                   | 0.095   |
| Three-blade                 | .135  |
| Four-blade                  | .170  |
| Six-blade (single rotating) | .240  |
| Six-blade (dual rotating)   | .275  |

It should be emphasized, however, that when the propeller characteristics are known the proper values of side-force factor and blade angle should be used in reference 6 to deter-

mine the value of  $\left(\frac{\partial C_N}{\partial \alpha}\right)_p$ .

$\left(\frac{\partial \beta}{\partial \alpha}\right)_p$  Values of  $\left(\frac{\partial \beta}{\partial \alpha}\right)_p$  are obtained from figure 2(b).

A value of  $x_1$  that corresponds to the plane of the propeller is used.

For propeller locations ahead of the center of gravity the value of  $\partial C_m / \partial C_L$  due to the propeller normal force is positive.

The added downwash over the horizontal tail resulting from the normal force on the propeller contributes a further change in pitching-moment slope that is assumed to be given by

$$\left(\frac{\partial C_m}{\partial C_L}\right)_{p_2} = -\left(\frac{\partial C_m}{\partial C_L}\right)_t \frac{\left(\frac{\partial C_N}{\partial \alpha}\right)_p \left(\frac{\partial \beta}{\partial \alpha}\right)_p}{L_t \left(1 - \frac{d\epsilon}{d\alpha}\right)_t} \quad (6)$$

Equation (6) is derived by calculating the downwash in the slipstream from momentum considerations and reducing the value by an empirical coefficient to take into account the effects of the wing, fuselage, and other factors. In all cases in which a value of  $\partial C_m / \partial C_L$  exists because of this factor - that is, when the horizontal tail may be considered as being within the downwash field due to the propeller, as on conventional tractor-propeller arrangements with tail at rear of fuselage - the value will be positive.

The increased rate of change of downwash over the tail accounted for by equation (6) should also be considered in estimations of the absolute angles of attack of the horizontal tail or of absolute elevator trim angles. In order to include this effect, the rate of change of horizontal tail angle of attack with wing angle of attack

$$\frac{d\alpha_t}{d\alpha} = \left(1 - \frac{d\epsilon}{d\alpha}\right)_t$$

should be reduced by the quantity

$$\frac{\left(\frac{dC_N}{d\alpha}\right)_p \left(\frac{d\beta}{d\alpha}\right)_p}{4}$$

The change in absolute downwash angle due to this effect will of course be zero when the resultant angle of attack of the propeller axis, including the effect of wing upwash, is zero.

The effect on the bodies within the influence of the propeller wake of the added downwash due to the propeller was ignored.

#### COMPARISON WITH FLIGHT DATA

The methods described in the preceding section and illustrated in figure 1 have been applied to the estimation of the stick-fixed neutral points of the airplanes shown in figure 5; dimensions of these airplanes are given in table I. The estimated values of  $\partial C_m / \partial C_L$  due to each component and the estimated resultant neutral point are given for each airplane in table II. The stick-fixed neutral point for each airplane as determined from

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flight tests for a value of  $C_L = 0.3$  using the method described in reference 2 is also shown in table II.

Flight values and estimated values of neutral-point location are compared in figure 6. The results generally agree within  $\pm 1.5$  percent  $c$ , which represents the accuracy with which the flight neutral points shown are believed to have been evaluated. The lack of agreement between estimated and flight neutral points for airplane 9 may be due to the fact, shown by wind-tunnel tests (reference 7), that at normal flight attitudes the horizontal tail of this airplane is within an unusually low-energy wing wake.

#### CONCLUDING REMARKS

The present report gives a method of estimating the stick-fixed neutral point of an airplane with propeller windmilling, flaps neutral, and landing gear retracted. The method may be applied best at low lift coefficients. A study of available flight data indicates that at higher lift coefficients the neutral point generally remains the same or moves back; thus, the neutral point predicted by the present method will be conservative throughout the flight range for this flight condition.

No reliable methods for predicting the effects of power are at the present time available. For preliminary estimates of the shift in stick-fixed neutral point due to power, however, the data given in NACA CB No. L4H01 "Effect of Power on the Stick-Fixed Neutral Points of Several Single-Engine Monoplanes as Determined in Flight" may be applied.

The shift in neutral point due to freeing the elevator may be computed if the elevator hinge-moment characteristics are known.

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TABLE I.- DIMENSIONS OF AIRPLANE TESTED

| Air-plane | Test-point symbol | Wing area, $S_w$ (sq ft) | Wing span (ft) | Aspect ratio of wing, $A_w$ | Mean aerodynamic chord of wing, $\bar{c}$ (ft) | Horizontal tail area, $S_t$ (sq ft) | Horizontal tail span (ft) | Aspect ratio of tail, $A_t$ (ft) | Longitudinal distance from wing a.c. to $1/4$ mean chord of horizontal tail (ft) | Propeller diameter, $D$ (ft) | Longitudinal distance from wing a.c. to propeller plane (ft) | Fuselage length (ft) | Maximum fuselage width (ft) | Nacelle length (ft) | Maximum nacelle width (ft) |
|-----------|-------------------|--------------------------|----------------|-----------------------------|--|-------------------------------------|---------------------------|----------------------------------|--|------------------------------|--|----------------------|-----------------------------|---------------------|----------------------------|
| 1         | o                 | 155                      | 34.0           | 7.5                         | 4.75   | 28.0                                | 9.3                       | 3.1                              | 11.97  | 6.00                         | 6.04   | 22.2                 | 3.5                         | ----                | ---                        |
| 2         | +                 | 236                      | 37.3           | 5.9                         | 6.80   | 48.0                                | 12.8                      | 3.4                              | 16.47  | 10.00                        | 6.71   | 28.9                 | 4.1                         | ----                | ---                        |
| 3         | x                 | 236                      | 37.3           | 5.9                         | 6.80   | 48.0                                | 12.8                      | 3.4                              | 16.48  | 11.00                        | 8.93   | 31.7                 | 3.2                         | ----                | ---                        |
| 4         | □                 | 213                      | 34.0           | 5.4                         | 6.72   | 40.0                                | 13.0                      | 4.2                              | 15.52  | 10.38                        | 9.65   | 30.6                 | 2.8                         | ----                | ---                        |
| 5         | ◇                 | 300                      | 40.8           | 5.5                         | 7.29   | 58.3                                | 16.0                      | 4.4                              | 19.83  | 12.17                        | 8.71   | 35.3                 | 4.5                         | ----                | ---                        |
| 6         | △                 | 236                      | 37.0           | 5.8                         | 6.64   | 41.4                                | 13.2                      | 4.2                              | 15.34  | 10.76                        | 9.10   | 32.2                 | 2.8                         | ----                | ---                        |
| 7         | ▽                 | 248                      | 38.3           | 5.9                         | 6.88   | 45.2                                | 13.3                      | 3.9                              | 15.44  | 11.58                        | 10.84  | 32.9                 | 2.9                         | ----                | ---                        |
| 8         | ∇                 | 328                      | 52.0           | 8.3                         | 7.02   | 54.0                                | 21.8                      | 8.8                              | 21.46  | 11.50                        | 7.72   | 19.5                 | 3.1                         | 33.8                | 3.5                        |
| 9         | ∇                 | 258                      | 39.3           | 6.0                         | 6.94   | 61.2                                | 14.8                      | 3.6                              | 15.79  | 9.00                         | 7.56   | 26.9                 | 4.9                         | ----                | ---                        |
| 10        | ▷                 | 334                      | 42.8           | 5.5                         | 8.11   | 77.8                                | 18.5                      | 4.4                              | 19.43  | 13.08                        | 8.50   | 33.8                 | 5.1                         | ----                | ---                        |
| 11        | △                 | 422                      | 49.7           | 5.9                         | 9.11   | 107.4                               | 19.0                      | 3.4                              | 17.90  | 12.00                        | 11.04  | 36.7                 | 4.7                         | ----                | ---                        |
| 12        | ▷                 | 490                      | 54.2           | 6.0                         | 9.76   | 110.8                               | 20.8                      | 3.9                              | 22.69  | 13.00                        | 11.56  | 41.0                 | 5.0                         | ----                | ---                        |
| 13        | ◁                 | 959                      | 89.5           | 8.4                         | 11.74  | 203.0                               | 26.7                      | 3.5                              | 32.40  | 11.50                        | 11.93  | 56.0                 | 6.3                         | 15.4                | 4.3                        |
| 14        | ◊                 | 465                      | 61.3           | 8.1                         | 8.36   | 100.0                               | 21.2                      | 4.5                              | 24.02  | 11.25                        | 5.81   | 47.9                 | 4.3                         | 19.2                | 4.0                        |
| 15        | ◊                 | 538                      | 61.3           | 7.0                         | 9.68   | 122.3                               | 20.2                      | 3.3                              | 25.58  | 12.08                        | 7.48   | 48.5                 | 3.5                         | 16.9                | 4.1                        |
| 16        | ◊                 | 540                      | 70.0           | 9.1                         | 8.13   | 116.1                               | 23.1                      | 4.6                              | 27.76  | 12.50                        | 9.64   | 50.5                 | 5.2                         | 23.8                | 4.8                        |

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TABLE II.- ESTIMATED VALUES OF  $\partial C_m / \partial C_L$  AND  
RESULTANT NEUTRAL POINT

| Airplane | Wing aerodynamic center (fraction $\bar{c}$ ) | Estimated values of increment in pitching-moment slope, $\partial C_m / \partial C_L$ |  |  |  |  |  | Airplane neutral point (fraction $\bar{c}$ ) |              |
|----------|---|---|--|--|--|--|--|--|--------------|
|          |   | $\left(\frac{\partial C_m}{\partial C_L}\right)_w$                                    | $\left(\frac{\partial C_m}{\partial C_L}\right)_f$ | $\left(\frac{\partial C_m}{\partial C_L}\right)_{nac}$ | $\left(\frac{\partial C_m}{\partial C_L}\right)_{p_1}$ | $\left(\frac{\partial C_m}{\partial C_L}\right)_{p_2}$ | $\left(\frac{\partial C_m}{\partial C_L}\right)_t$ | Estimated $x_o$                              | Flight $x_o$ |
|          |   | 1   | 0.245  | 0.110  | 0.054  | -----  | 0.006  | 0.007  | -0.177       |
| 2        | .246  | .084  | .047   | -----  | .013   | .013   | -.157  | .33  | .32          |
| 3        | .246  | .094  | .031   | -----  | .019   | .012   | -.156  | .34  | .32          |
| 4        | .238  | .113  | .029   | -----  | .022   | .011   | -.175  | .35  | .37          |
| 5        | .238  | .103  | .051   | -----  | .023   | .019   | -.196  | .34  | .33          |
| 6        | .240  | .077  | .027   | -----  | .019   | .011   | -.134  | .32  | .33          |
| 7        | .240  | .087  | .021   | -----  | .030   | .012   | -.150  | .33  | .32          |
| 8        | .241  | .170  | .048   | 0.040  | .030   | .025   | -.313  | .41  | .41          |
| 9        | .243  | .111  | .039   | -----  | .011   | .012   | -.173  | .35  | .33          |
| 10       | .238  | .128  | .048   | -----  | .018   | .016   | -.210  | .37  | .36          |
| 11       | .238  | .090  | .039   | -----  | .017   | .012   | -.158  | .33  | .34          |
| 12       | .238  | .108  | .037   | -----  | .013   | .014   | -.172  | .35  | .34          |
| 13       | .246  | .113  | .020   | .025   | .009   | .015   | -.182  | .36  | .37          |
| 14       | .238  | .195  | .041   | .016   | .015   | .024   | -.291  | .43  | .45          |
| 15       | .238  | .085  | .015   | .038   | .016   | .019   | -.173  | .32  | .33          |
| 16       | .240  | .144  | .050   | .048   | .022   | .020   | -.284  | .38  | .39          |

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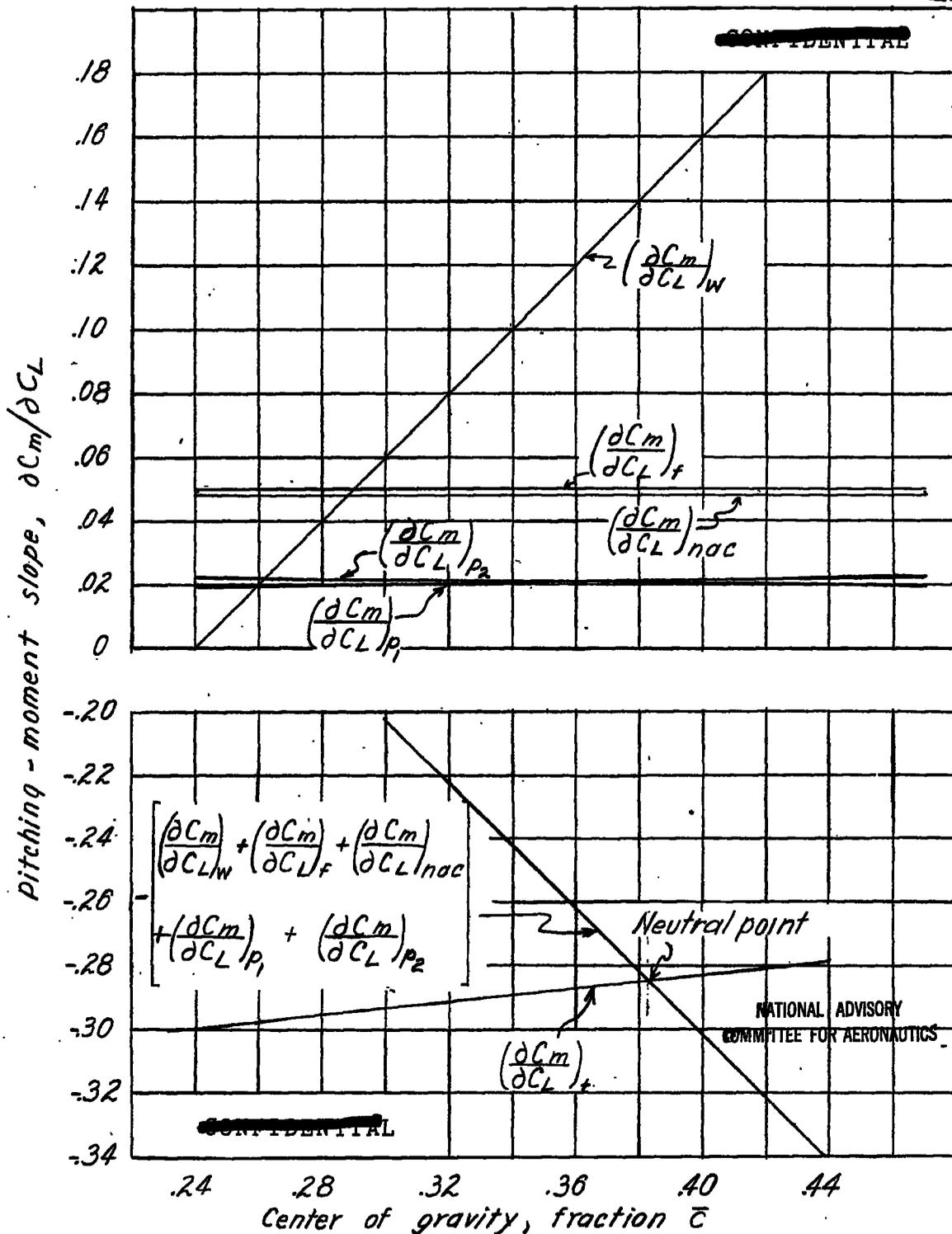


Figure 1.-Illustration of procedure for determining neutral point for typical airplane.

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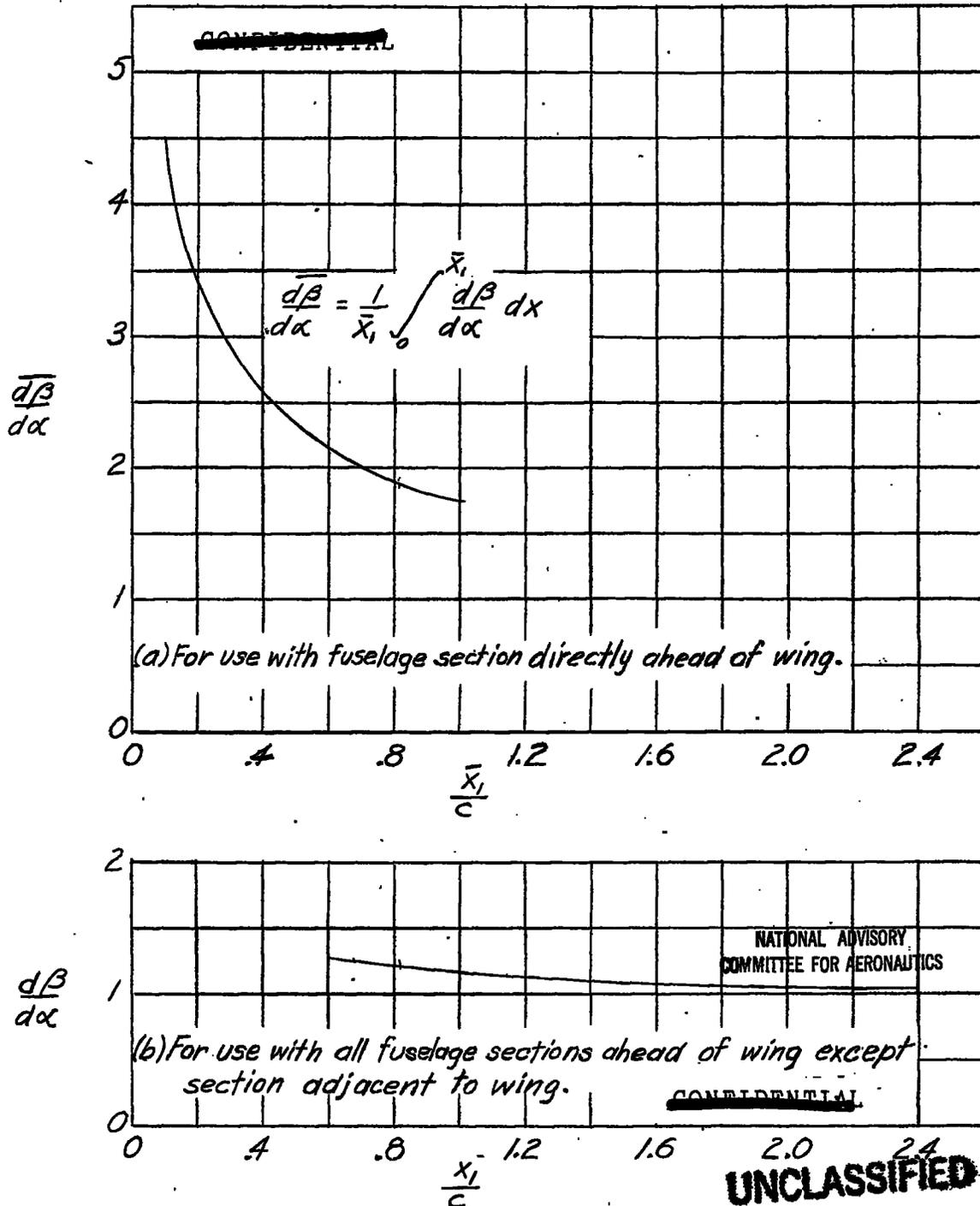


Figure 2.- Variation of  $d\beta/d\alpha$  with distance from wing leading edge.  $\frac{dCL}{d\alpha} = 4.5$  radians. (Data from reference 3.)

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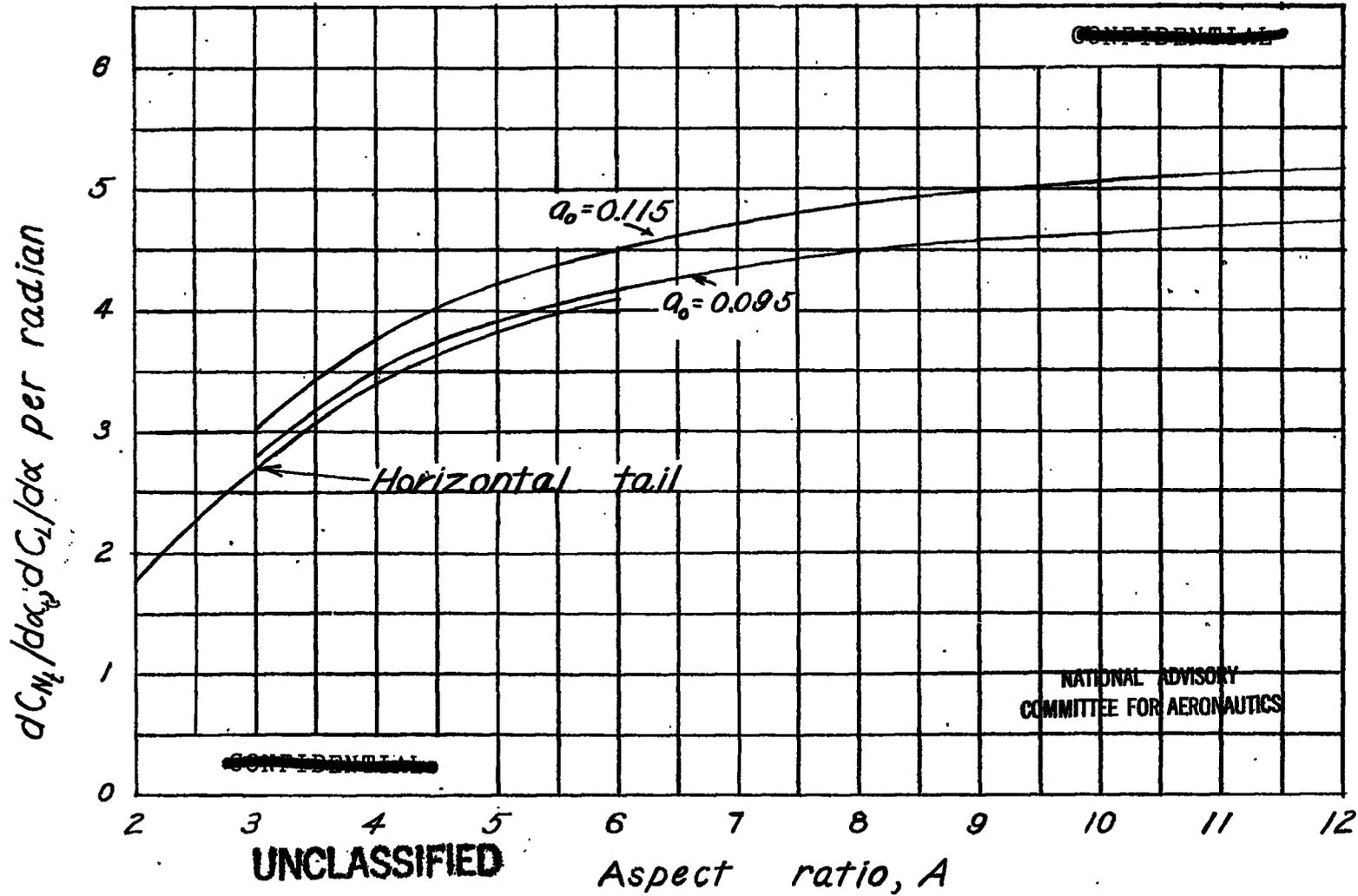
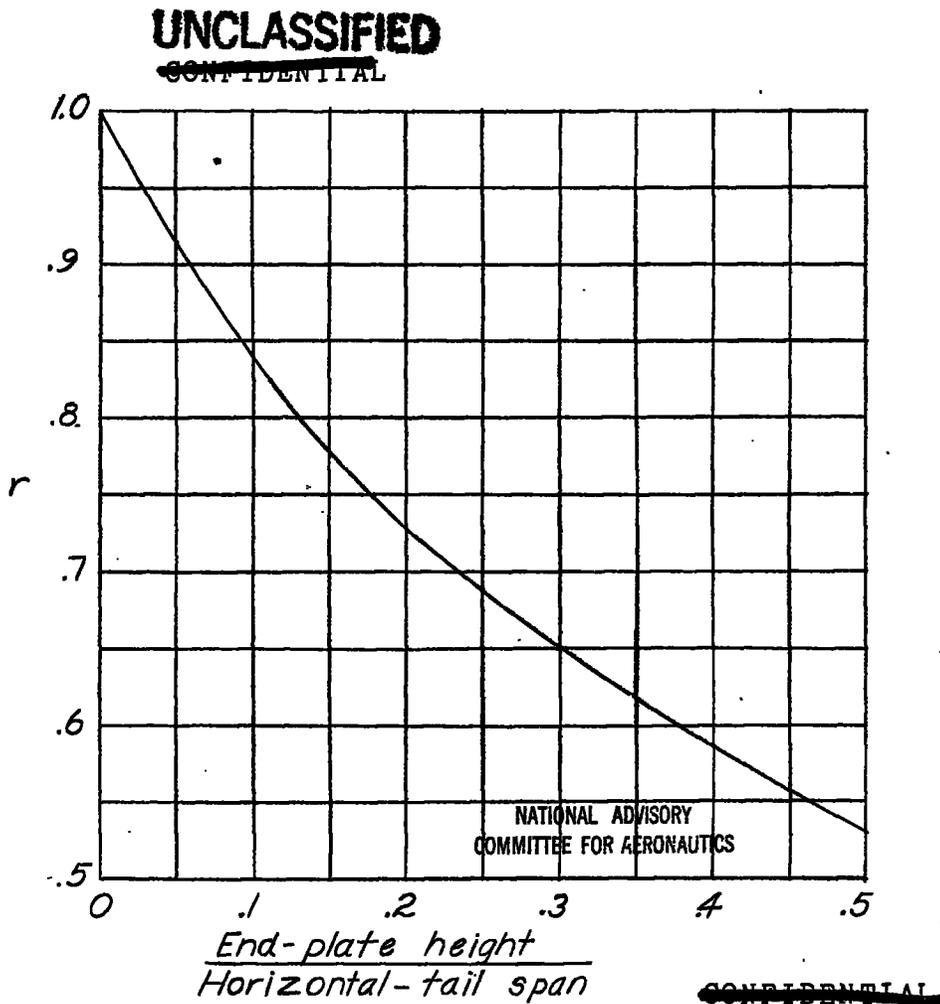


Figure 3. - Variation of  $dC_M/d\alpha$  and  $dC_L/d\alpha$  with aspect ratio. (Data from reference 4.)

FIG. 3



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Figure 4.— Correction for  $dC_{N_t}/d\alpha_t$  of horizontal tail with end plates. (Data from reference 4.)

$$\frac{dC_{N_t}}{d\alpha_t} = \frac{a_0 \times 57.3}{1 + \frac{r a_0 \times 57.3}{\pi A}} \text{ per radian}$$

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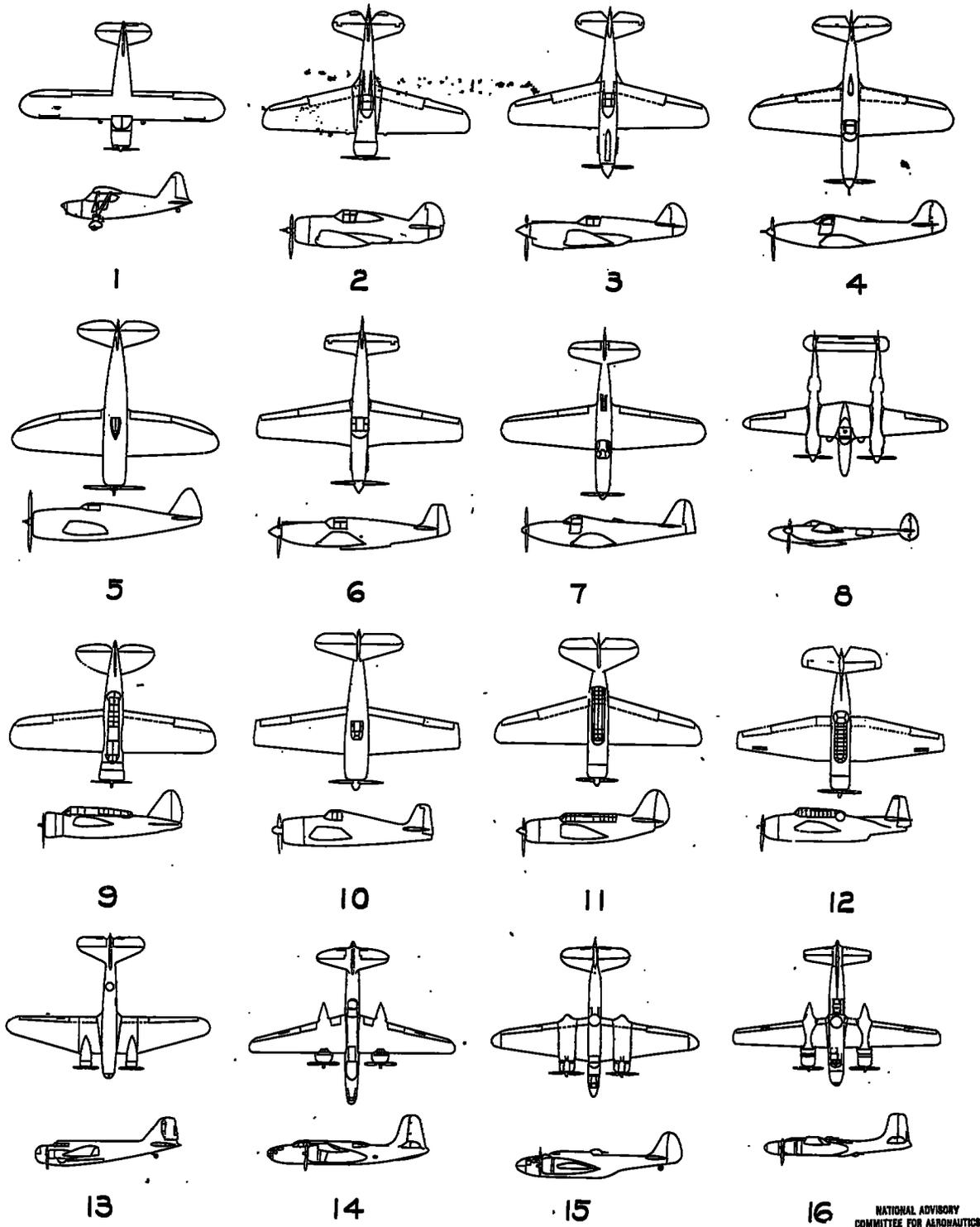


Figure 5. - Views of airplanes tested.

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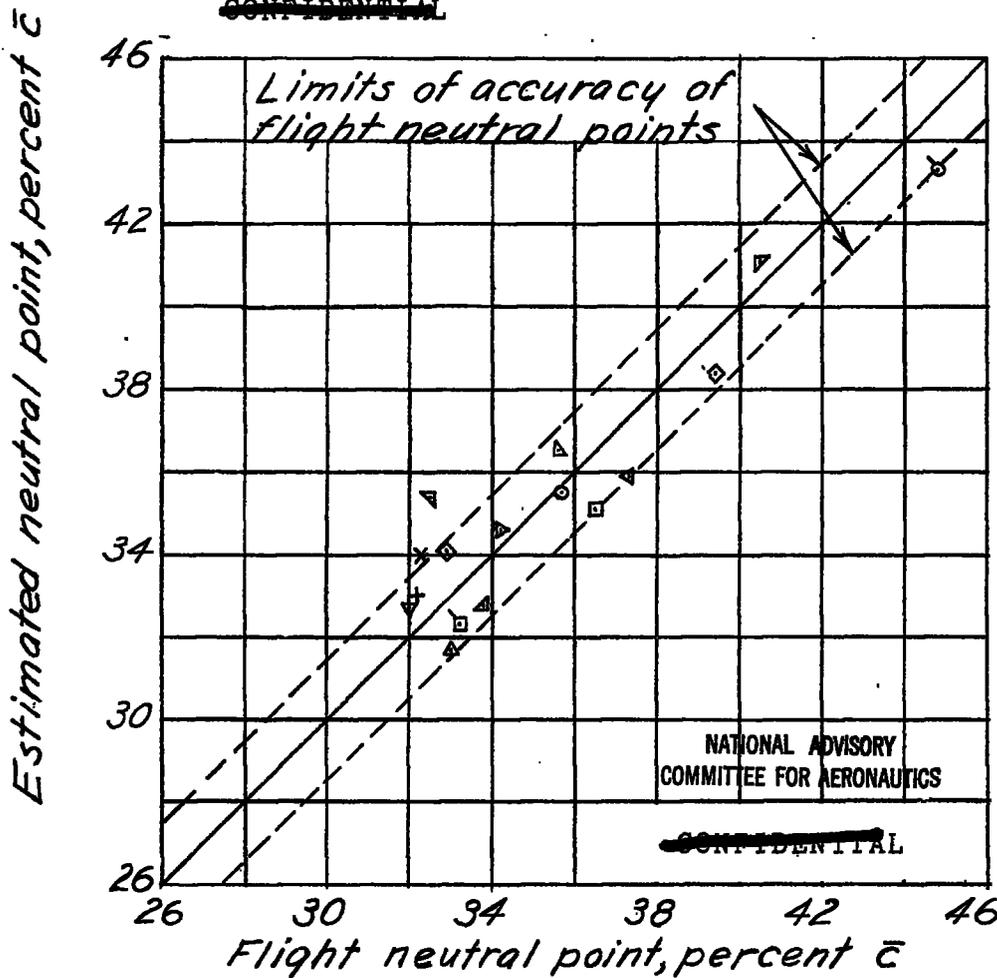


Figure 6.- Comparison of estimated stick-fixed neutral points with neutral points from flight tests.  $C_L = 0.3$ . (Test-point symbols are identified in table I.)

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