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AERODYNAMICS FACTORS AFFECTING THE ABILITY OF A PILOT TO RETURN AN AIRPLANE TO LEVEL FROM A BANKED ATTITUDE BY USE OF THE RUDDER ALONE AND WITHOUT CHANGE OF HEADING

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INTRODUCTION

Considerable importance is attached by the Navy to the ability of a pilot to return the wing of a banked airplane to level by use of the rudder alone and without a change of heading of the airplane, particularly at low speeds, where the airplane may be in close proximity to the ground or to the deck of an aircraft carrier, and a banked attitude might be produced by turbulent-air conditions. Because of the resulting interest, a study has been made of the aerodynamic factors upon which the ability to perform the maneuver depends in an attempt to obtain a better understanding of the conditions involved and, if possible, to evolve criterions that may be used during design.

NOTATION

m mass of airplane

v sideslip velocity

g acceleration due to gravity

Y_v rate of change of lateral force with sideslip velocity $(\partial Y/\partial v)$

L_v rate of change of rolling moment with sideslip velocity $(\partial L/\partial v)$

L_p rate of change of rolling moment with rolling angular velocity

p rolling angular velocity

mk_x^2 moment of inertia about the X axis, or the plane of symmetry

ϕ angle of bank, subscript 0 (initial condition)

The forces and the moments are referred to the stability axes, a system in which the X axis is in the plane of symmetry and along the relative wind for steady conditions of unyawed flight, the Y axis perpendicular to the plane of symmetry, and the Z axis in the plane of symmetry and perpendicular to the X axis. (See reference 1.)

β angle of sideslip in radians

C_L lift coefficient

μ $2m/\frac{\rho}{2} Sb$

$C_{l_\beta} = \partial C_l/\partial \beta$

$C_{l_p} = \partial C_l / \partial \frac{pb}{2V}$

$C_{Y_\beta} = \partial C_Y/\partial \beta$, C_Y (lateral force coefficient)

$e = 2.71828$

$s = \frac{2Vt}{b}$, t (time in seconds)

V forward velocity

$\tau = m/\frac{\rho}{2} SV$

S wing area

b wing span

ANALYSIS

The maneuver consists essentially in a sideslip in the direction of the low wing. The action of the dihedral rolls the wing back to the level position. The rudder is employed to counteract the turning tendency resulting from the inherent weathercock stability of the airplane.

In the analytical treatment of the problem, it is first assumed that the maneuver can be performed. On the

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basis of this assumption, the yawing motion will be zero and can be eliminated from further consideration. Simplified equations of motions are then set up and solved for the angles of sideslip that will occur during the maneuver. Whether or not the wings will tend to return to the level position depends on the effective dihedral of the wings. The ability to perform the maneuver otherwise depends on whether the rudder is sufficiently powerful to produce the angles of sideslip that must occur. The magnitudes of the angles of sideslip are dependent on the relation between the effective dihedral and the cross-wind and drag forces.

For a given set of conditions, the angles of sideslip vary somewhat with the flight-path angle of the airplane. Experimental data, however, have indicated that the application of power generally tends to reduce the dihedral effect, so that the critical condition occurs with power on. The quantitative solutions, therefore, have only been obtained for the low-speed level-flight condition, where the conditions are probably most critical.

When the yawing is zero, the equations of lateral motion for the condition of initial level flight are reduced to the form:

$$\left.\begin{aligned} m\frac{dv}{dt} - vY_v - mg(\phi - \phi_o) &= mg\,\phi_o \\ mk_x^2\frac{d^2\phi}{dt^2} - vL_v - pL_p &= 0 \end{aligned}\right\} \qquad (1)$$

This equation implies that at the start of the maneuver the airplane is in a banked attitude but is neither rolling nor sideslipping. The effect of initial roll or sideslip is discussed later. The complete solution of this equation indicates the existence of two superimposed modes of motion, one a heavily damped subsidence of any initial rolling and the other, the oscillation involving skidding and some residual rolling, similar to the usual lateral oscillation but involving no yawing motion. Because of the heavy damping of the first mode, only the oscillation appears to be important for the maneuver being considered.

In reference 1 it has been suggested that the solution for this oscillation can be approximated by assuming that the airplane is swinging laterally as a pendulum about a center of oscillation with a radius of curvature equal to $-L_p/L_v$. When this assumption is made, the sideslipping and rolling motions are related by the equation:

$$v = (-L_p/L_v)p$$

When this relationship is applied to equation (1), it is seen that the angle of sideslip, $\beta = \tan^{-1} v/V$, or approximately v/V, varies linearly with the initial angle of bank, ϕ_o. Consequently, it has been found more convenient to deal with the relation β/ϕ_o than with β. The final solution in the nondimensional form then becomes:

$$\beta/\phi_o = \frac{2C_{L_o}}{\sqrt{\dfrac{4\mu\, C_{l_\beta}\, C_{L_o}}{C_{l_p}} - C_{Y_\beta}{}^2}}\, e^{\frac{C_{Y_\beta}}{2\mu} s} \tag{2}$$

$$\times \sin\left(\frac{1}{2\mu}\sqrt{\frac{4\mu\, C_{l_\beta}\, C_{L_o}}{C_{l_p}} - C_{Y_\beta}{}^2}\right) s$$

The differentiation of equation (2) with respect to s when set equal to zero determines s when the angle of sideslip, β, is a maximum, and the result is:

$$s_{\beta_{max}} = -\frac{2\mu}{\sqrt{\dfrac{4\mu\, C_{l_\beta}\, C_{L_o}}{C_{l_p}} - C_{Y_\beta}{}^2}} \tan^{-1} \frac{\sqrt{\dfrac{4\mu\, C_{l_\beta}\, C_{L_o}}{C_{l_p}} - C_{Y_\beta}{}^2}}{C_{Y_\beta}} \tag{3}$$

The significance of equation (3) is that $s_{\beta_{max}}$ gives the time or distance in semispan lengths for the banked wing

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to become level. The maximum angle of sideslip that will occur during the maneuver:

$$\beta_{max}/\phi_o = -\sqrt{\frac{C_{L_o}\,C_{l_p}}{\mu C_{l_\beta}}}\; e^{-\frac{C_{Y_\beta}}{\sqrt{\frac{4\mu C_{l_\beta}\,C_{L_o}}{C_{l_p}} - C_{Y_\beta}^{2}}}\tan^{-1}\frac{\sqrt{\frac{4\mu C_{l_\beta}\,C_{L_o}}{C_{l_p}} - C_{Y_\beta}^{2}}}{C_{Y_\beta}}} \tag{4}$$

In the limiting case where the effective dihedral is zero the equation reduces to

$$\beta_{max}/\phi_o = -\frac{C_{L_o}}{C_{Y_\beta}} \tag{5}$$

This equation (5) also applies to the case where the ailerons are employed to balance the rolling moment due to sideslip; that is, for the steady sideslip condition.

DESIGN CHART

The data obtained by solving equation (4) have been plotted on figure 1. The values for the parameters have been chosen to exceed any values likely to be encountered in practice. The figure has been constructed for a lift coefficient of 2. This value was chosen as representative of the better installations of high-wing devices on modern airplanes. Various sources have dealt with means for computing all of the factors needed for using the chart. The chart, however, is intended primarily for use with wind-tunnel data for specific designs. As wind-tunnel tests do not usually include evaluation of C_{l_p}, this value will still have to be obtained by computation. Figure 8 of reference 2 is recommended as a convenient source for values for this factor. The effective dihedral, $C_{l_\beta} = -\frac{\partial C_l}{\partial \psi} \times 57.3$, where $\frac{\partial C_l}{\partial \psi} = \frac{\partial C_l'}{\partial \psi} + \frac{C_m'}{A}$, if the data are referred to the wind-tunnel system of axes (A = aspect ratio). The lateral-force coefficient $C_{Y_\beta} = -\frac{\partial C_c}{\partial \psi} \times 57.3 - C_D$. The values of C_c used in the preceding equation should be taken when the yawing moment is zero as a result of an appropriate rudder deflection for the particular angles of ψ chosen.

DISCUSSION

The chart indicates that conditions are critical only for low values of the effective dihedral. (1° dihedral in the average case will give a value of the abscissa of the order of 2.2.) The effect of lateral force in this range is also critical. For values of the abscissa above 6, the lateral force has only a small effect on the absolute angles of sideslip. For very low values of dihedral such as are encountered with many modern airplanes at low speeds with power on, the effect of the lateral force is large, β_{max}/ϕ_o varying from 2 to infinity for the values of the parameter employed in the chart.

As previously mentioned, the chart strictly applies only when the sideslip velocity at the start of the maneuver is zero. In the actual case some rolling or sideslipping will probably be present at the start of the maneuver. The rolling motion, according to the complete solution of equation (1), will be quickly damped and will probably have a negligible effect on the ensuing motion. The sideslipping velocity at the start of the maneuver may vary from the value obtainable in the steady sideslip to zero, depending on the prior history of the motion, which in turn depends on the violence of the gust or the manipulation of the controls producing the banked attitude. In gusty air, where most interest attaches to the maneuver, it may be presumed that the disturbance will be violent and the time to obtain the banked attitude small. In this case conditions will approach those assumed for the chart. For the demonstration of the capabilities of an airplane to perform the maneuver, however, there may be relatively large amounts of initial sideslip, depending on how the banked attitude is obtained. If a turn is entered and the angular velocity in yaw is stopped when the angle of bank becomes the desired value, the sideslip velocity will approach zero and the conditions of the chart will be approximated. The values of sideslip accompanying the attainment of the banked attitude by using the ailerons will depend on the speed with which the banked attitude is obtained and the time allowed for "conditions to become steady." The value of the ordinate, for zero value of the abscissa, will apply as previously noted for the steady sideslip condition. If, for example,

$$C_{Y_\beta} = -0.5$$

and

$$\mu C_{l_\beta}/C_{l_p} = 4$$

β_{max}/ϕ_o will lie between 4 and 0.6, a considerable spread depending on the manner in which the initial banked attitude was obtained. The probable value will, of course, more nearly approach the lower limit. The effect of any initial sideslip can be taken into account by assuming a higher angle for the initial bank than is specified. These assumed higher angles of initial bank, ϕ_o', for values of β_o (initial sideslip angle) greater than zero may be estimated from figures 1, 2, and 3. For zero lateral force, the value of ϕ_o'/ϕ_o is given by the formula

$$\frac{\phi_o'}{\phi_o} = \sqrt{1 + \left(\frac{\beta_o}{\beta_{max}}\right)^2}$$

where β_{max} is obtained from figure 1. When $C_{Y_\beta} = -0.50$ and -1.00, figures 2 and 3 are used to obtain ϕ_o'/ϕ_o by first connecting the origin to the point having the ordinate β_o/ϕ_o and the abscissa 1. This line will intersect the curve having the desired value of $\mu C_{l_\beta}/C_{l_p}$, and the reciprocal of the abscissa at this point of intersection gives ϕ_o'/ϕ_o. Computations of motion during aileron maneuvers indicate that where the banked attitude is obtained by use of the ailerons with the rudder being employed to hold a constant heading β_o/ϕ_o will vary from about 0.2 to 0.35, depending on whether the specified attitude is attained in 1 or 3 seconds.

Figure 4 shows, for the range of the parameter $\mu C_{l_\beta}/C_{l_p}$ considered herein, the time to level the wing from the initially banked position for values of $C_{Y_\beta} = 0$ and -1.00. The time is taken as equivalent to that required to attain the maximum sideslip angle and is given in the nondimensional form, t/τ, where τ, the characteristic time unit for the airplane, is equal to $m/\frac{\rho}{2}SV$. The chart indicates that conditions are only critical for low values of the dihedral angle where the time to level the wing increases rapidly with decreasing values of the dihedral angle. Beyond a value of the abscissa of the order of 4.5 (equivalent in the average case to an effective dihedral angle of about 2° to 3°), the curve for t/τ flattens and the dihedral effect becomes small.

ILLUSTRATIVE COMPUTATION

As an illustration of the use that can be made of figure 1, the following illustrative example is included. From unpublished wind-tunnel data on a modern low-wing monoplane with landing gear extended, flaps fully deflected, and power on, the following data were taken:

$W/S = 39.5$

$b = 40.8$ ft

$A = 5.55$

$\lambda = 0.50$

$C_{l_\beta} = -0.05$

$C_{Y_\beta} = -0.456$

$C_L = 2$

Maximum angle of sideslip that can be held with rudder = 17°

C_{l_p} from reference 2, figure 8, = -0.43

μ by computation for sea level = 50.7

$\mu C_{l_\beta}/C_{l_p} = 5.9$

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From figure 1, for $\mu C_{l_\beta}/C_{l_p} = 5.9$ and $C_{Y_\beta} = -0.456$ $\beta_{max}/\phi_o = 0.52$. For the steady sideslip condition, $\beta_{max}/\phi_o = 4.39$ from equation (5) or by interpolation from figure 1. If $\phi_o = 10^\circ$, then the maximum angle of sideslip will fall between 5.2° and 43.9°. Even with allowance for the possible initial angle of sideslip, the particular airplane should be able to perform the maneuver satisfactorily. If β_o/ϕ_o is 0.35 as previously noted as probable for a normal entry by use of the ailerons and rudder, according to figure 2, ϕ_o'/ϕ_o equals 1/0.81 or 1.25. (Note dash lines on figure.) The value β_{max} under these conditions will equal 6.5°. A steady sideslip with the wing down 10°, however, could not be made.

The time to return the wing to level is computed from figure 4. This figure for the specified values of the parameters gives t/τ as 0.44. As τ equals 8.1, 3.6 seconds will be required for the wings to level out.

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REFERENCES

1. Jones, Robert T.: A Study of the Two-Control Operation of an Airplane. Rep. No. 579, NACA, 1936.

2. Pearson, Henry A., and Jones, Robert T.: Theoretical Stability and Control Characteristics of Wings with Various Amounts of Taper and Twist. Rep. No. 635, NACA, 1938.

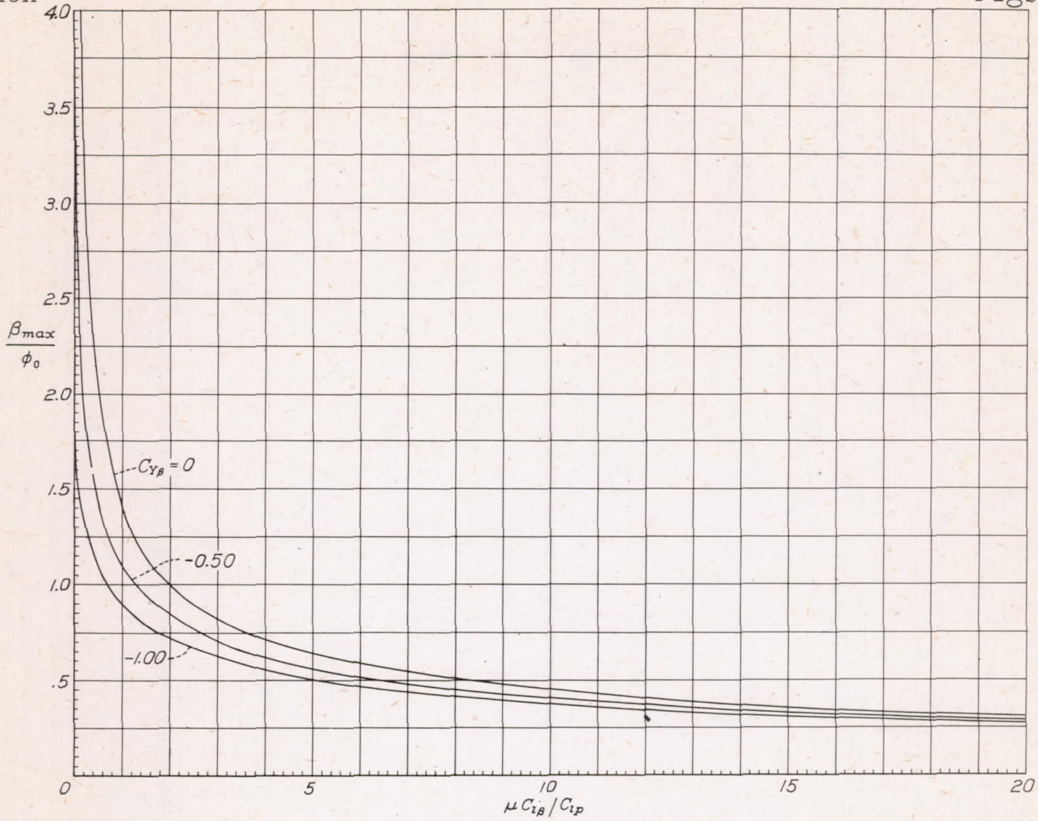

Figure 1.- Variation of maximum sideslip angle for a given initial bank with parameter $\mu C_{l_\beta}/C_{l_p}$; $C_L = 2.00$.

$$\mu = \frac{2m}{\frac{\rho}{2}Sb}\ ;\ C_{l_\beta} = \frac{\partial C_l}{\partial \beta}\ ;\ C_{l_p} = \frac{\partial C_l}{\partial \frac{pb}{2V}}\ ;\ C_{Y_\beta} = \frac{\partial C_Y}{\partial \beta}\ .$$

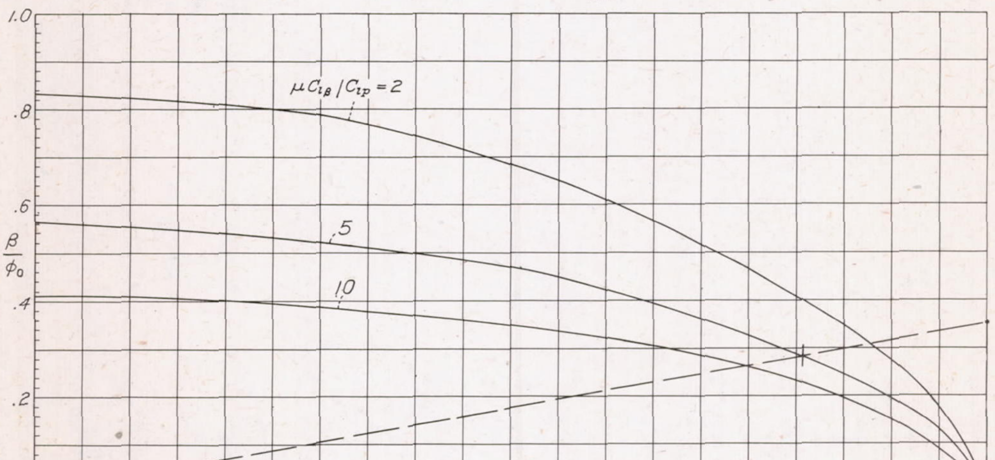

Figure 2.- Variation of sideslip angle with bank for various values of the parameter $\mu C_{l_\beta}/C_{l_p}$; $C_L = 2.00$; $C_{Y_\beta} = -0.50$.

$$\mu = \frac{2m}{\frac{\rho}{2}Sb}\ ;\ C_{l_\beta} = \frac{\partial C_l}{\partial \beta}\ ;\ C_{l_p} = \frac{\partial C_l}{\partial \frac{pb}{2V}}$$

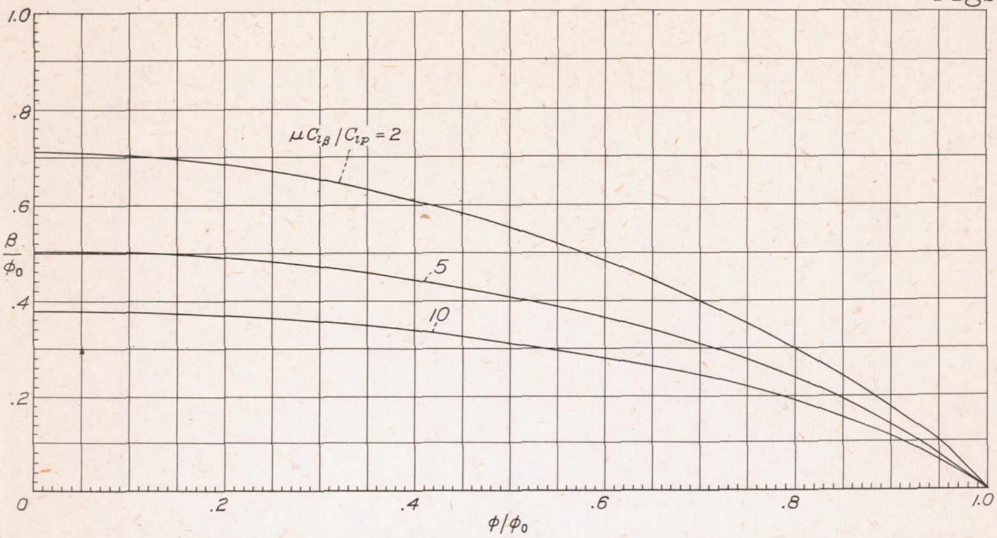

Figure 3.- Variation of sideslip angle with bank for various values of the parameter $\mu C_{l_\beta}/C_{l_p}$; $C_L = 2.00$; $C_{Y_\beta} = -1.00$.

$$\mu = \frac{2m}{\frac{\rho}{2}Sb}\ ;\ C_{l_\beta} = \frac{\partial C_l}{\partial \beta}\ ;\ C_{l_p} = \frac{\partial C_l}{\partial \frac{pb}{2V}}\ .$$

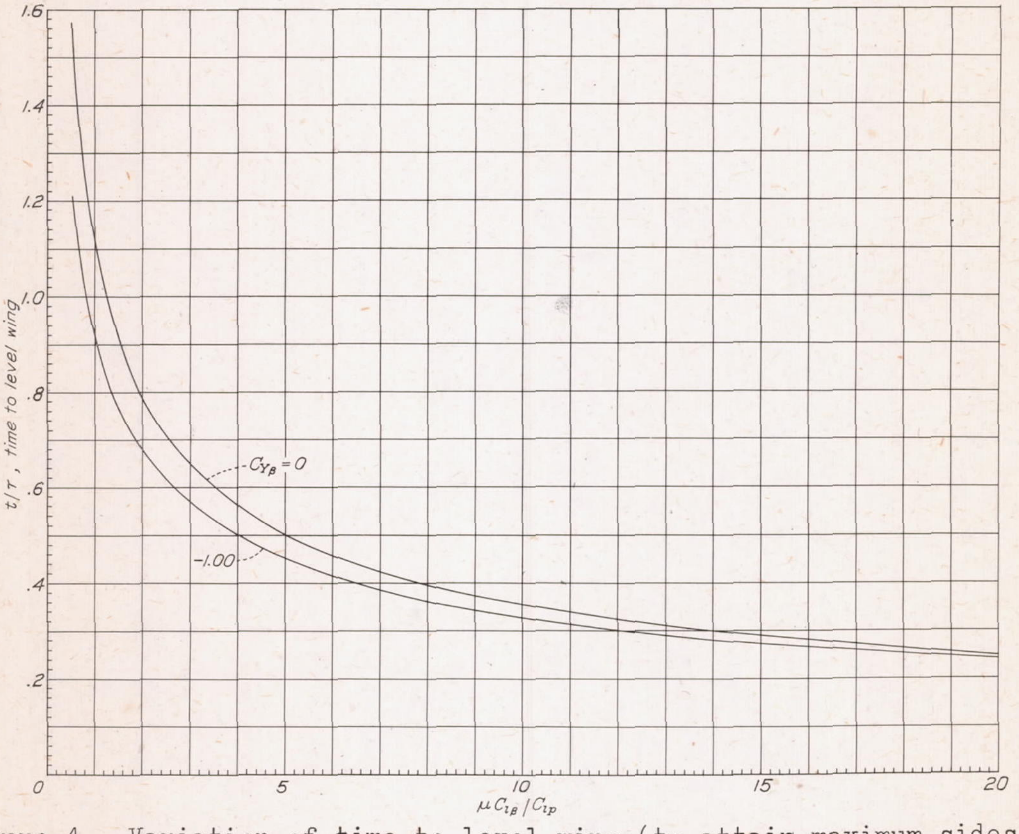

Figure 4.- Variation of time to level wing (to attain maximum sideslip angle) with parameter $\mu C_{l_\beta}/C_{l_p}$; $C_L = 2.00$.

$$\mu = \frac{2m}{\frac{\rho}{2}Sb}\ ;\ C_{l_\beta} = \frac{\partial C_l}{\partial \beta}\ ;\ C_{l_p} = \frac{\partial C_l}{\partial \frac{pb}{2V}}\ ;\ t/\tau = \frac{t}{m/\frac{\rho}{2}SV_o}\ .$$