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A THEORETICAL ANALYSIS OF THE EFFECT OF AILERON INERTIA AND HINGE MOMENT ON THE MAXIMUM ROLLING ACCELERATION OF AIRPLANES IN ABRUPT AILERON ROLLS

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A THEORETICAL ANALYSIS OF THE EFFECT OF AILERON INERTIA AND HINGE MOMENT ON THE MAXIMUM ROLLING ACCELERATION OF AIRPLANES IN ABRUPT AILERON ROLLS

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SUMMARY

Data on the rolling accelerations of airplanes of different types and sizes in abrupt aileron rolls have been obtained and reported by the NACA in connection with a number of maneuverability and flying qualities investigations. Preliminary to an attempt to correlate these data, an effort was made to analyze theoretically the relation between the torque applied to the aileron system by the pilot, the resulting motion of the system, and the subsequent rolling motion of the airplane in an abrupt aileron roll. The analysis, which is presented herein, covers only the simplest possible case, in which a constant increment of torque is instantly applied by the pilot to a rigid control system having a moment of inertia that does not vary, and a hinge-moment coefficient that varies directly, with aileron deflection. The results are presented in the form of a chart showing the fraction of the maximum rolling acceleration theoretically attainable with instantaneous aileron deflection that can actually be realized with different combinations of pilot's effort, control inertia, and hinge-moment coefficient.

Quantitative comparison of the theoretical results with existing experimental data, to determine how closely the simple case assumed approximates actual flight conditions, is not possible. Essential information as to the magnitude of the effective moment of inertia and the hinge-moment coefficient of the aileron systems of the airplanes tested is not available. Qualitatively, the theoretical indications that the increase of rolling acceleration is reduced and ultimately limited by the increase of hinge moment with airspeed are in agreement with flight observations on the only airplane with which abrupt rolls were made at high speed.

INTRODUCTION

It is well known that the rolling velocity acquired by an airplane during deflection of the ailerons reduces the maximum rolling acceleration attainable in an abrupt aileron roll to a value below that corresponding to the rolling moment of the ailerons at zero rate of roll. Aside from the physiological and psychological characteristics of the pilot, the factors governing the time required to deflect the ailerons are the friction and inertia of the aileron control system, and the aerodynamic moment that acts on the ailerons whenever they are in the deflected position. The extent to which these factors, by altering the minimum time required to deflect the controls, contribute to the reduction of the maximum rolling acceleration is difficult to determine experimentally. Very often it is obscured by other factors such as control flexibility, wing flexibility, and so forth, which also reduce the maximum rolling acceleration in the case of an actual airplane. For a hypothetical case, however, in which the pilot applies a constant increment of torque, in excess of control friction, to a rigid conventional aileron system, it is possible to express theoretically, in general form, the relation between the pilot's torque, the control inertia, the aileron hinge moment, and the rolling acceleration of the airplane. The equations and assumptions involved are presented in the following pages.

SYMBOLS

I_c moment of inertia of the ailerons and their control system

δ control deflection

t time

Q_P torque, in excess of friction, applied to the control system by the pilot

ρ mass density of the air

V true airspeed

S_a aileron area

c aileron chord

C_H aileron hinge-moment coefficient at δ_{max}

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x	δ/δ_{max}
G	$\dfrac{Q_P}{\frac{1}{2}\rho V^2 S_a c C_H}$
N	$\dfrac{Q_P}{I_c \delta_{max}}$
L_a	rolling moment due to ailerons
L_p	damping moment due to rolling velocity
$L_{\frac{dp}{dt}}$	inertia moment due to rolling acceleration
S	wing area
b	wing span
C_l	rolling-moment coefficient of ailerons
p	rolling velocity
C_L	lift coefficient of airplane
α	angle of attack of airplane
K	$\dfrac{1}{32}\rho S V b^2 \dfrac{dC_L}{d\alpha}$
I_x	moment of inertia of airplane about the longitudinal axis
D	$1 - \dfrac{x}{G}$
E	$\dfrac{K}{I_x\sqrt{\frac{N}{G}}}$

ANALYSIS

In an aileron roll the moments acting upon the aileron control are

$$0 = -I_c \frac{d^2\delta}{dt^2} + Q_P - \frac{1}{2}\rho V^2 S_a c C_H \frac{\delta}{\delta_{max}} \tag{1}$$

This equation implies the following assumptions: angular and linear acceleration of the airplane does not tend to deflect the ailerons; the hinge-moment coefficient is directly proportional to δ and is unaffected by rate of roll; and I_c is the same at all values of δ.

Let

$$\frac{\delta}{\delta_{max}} = x \tag{2}$$

then

$$\frac{d^2\delta}{dt^2} = \delta_{max}\frac{d^2x}{dt^2}$$

Also, let

$$\frac{Q_P}{\frac{1}{2}\,\rho\,V^2\,S_a\,c\,C_H} = G \tag{3}$$

$$\frac{Q_p}{I_c\,\delta_{max}} = N \tag{4}$$

where G and N are constants. It is assumed that Q_P is instantly applied and remains constant throughout the deflection of the control.

After substitution from formulas (2), (3), and (4) into formula (1), formula (1) becomes

$$\frac{d^2x}{dt^2} = N - \frac{N}{G}\,x \tag{5}$$

The solution of this equation is

$$x = G\left[1 - \cos\left(\sqrt{\frac{N}{G}}\,t\right)\right] \tag{6}$$

Formula (6) expresses the relationship between control deflection, time, and the ratios expressed by formulas (3) and (4). By means of this relationship the element of time can replace aileron deflection in the equation of moments acting about the longitudinal axis of the airplane during roll.

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Up to the time of maximum rolling acceleration, the equation of moments acting about the longitudinal axis during roll may be written with sufficient accuracy as:

$$0 = L_a + L_p + L_{\frac{dp}{dt}} \tag{7}$$

where

$$L_a = \frac{1}{2} \rho \, S \, V^2 \, b \, C_l \tag{8}$$

and

$$L_p = - \frac{1}{32} \rho \, S \, V \, b^2 \, p \, \frac{dC_L}{d\alpha} \tag{9}$$

or

$$L_p = - K p \tag{10}$$

where

$$K = \frac{1}{32} \rho \, S \, V \, b^2 \, \frac{dC_L}{d\alpha}$$

By definition,

$$L_{\frac{dp}{dt}} = - I_x \frac{dp}{dt} \tag{11}$$

After substitution from equations (8), (10), and (11) into equation (7)

$$0 = \frac{1}{2} \rho \, S \, V^2 \, b \, C_l - Kp - I_x \frac{dp}{dt} \tag{12}$$

If it is assumed that C_l varies directly with control deflection, then

$$C_l = C_{l_{\delta_{max}}} x$$

where $C_{l_{\delta_{max}}}$ is the aileron rolling-moment coefficient at full aileron-control deflection, or where $x = 1$. After substitution for x from formula (6),

$$C_l = C_{l_{\delta_{max}}} G \left[1 - \cos\left(\sqrt{\frac{H}{G}} t \right) \right] \tag{13}$$

After substitution from formula (13) into formula (12) and division by I_x;

$$0 = \frac{\frac{1}{2} \rho S V^2 b C_{l_{\delta_{max}}}}{I_x} G \left[1 - \cos\left(\sqrt{\frac{N}{G}}\, t\right)\right] - \frac{Kp}{I_x} - \frac{dp}{dt} \qquad (14)$$

If the ailerons were instantaneously deflected, then the equation of moments about the longitudinal axis of the airplane would be

$$0 = L_a + L_{\left(\frac{dp}{dt}\right)_o}$$

where $L_{\left(\frac{dp}{dt}\right)_o}$ is the inertia moment at zero rolling velocity with $x = 1$. When the moments are replaced by their formulas and the resulting equation is solved for the acceleration

$$\left(\frac{dp}{dt}\right)_o = \frac{\frac{1}{2} \rho S V^2 b C_{l_{\delta_{max}}}}{I_x} \qquad (15)$$

where $\left(\frac{dp}{dt}\right)_o$ is the rolling acceleration at zero rolling velocity with $x = 1$.

When formula (15) is substituted into formula (14) and the resulting expression is solved for the rolling acceleration

$$\frac{dp}{dt} = \left(\frac{dp}{dt}\right)_o G \left[1 - \cos\left(\sqrt{\frac{N}{G}}\, t\right)\right] - \frac{Kp}{I_x} \qquad (16)$$

The solution of this differential equation gives the formula for rolling velocity:

$$p = \left(\frac{dp}{dt}\right)_0 \frac{GI_x}{K} - \frac{\left(\frac{dp}{dt}\right)_0 G\sqrt{\frac{N}{G}}}{\frac{K^2}{I_x^2} + \frac{N}{G}} \sin\left(\sqrt{\frac{N}{G}}\, t\right)$$

$$- \frac{\left(\frac{dp}{dt}\right)_0 GK}{I_x\left(\frac{K^2}{I_x^2} + \frac{N}{G}\right)} \cos\left(\sqrt{\frac{N}{G}}\, t\right) - \frac{\left(\frac{dp}{dt}\right)_0 I_x N}{K\left(\frac{K^2}{I_x^2} + \frac{N}{G}\right)} e^{-\frac{K}{I_x} t} \qquad (17)$$

Differentiation of equation (17) gives

$$\frac{dp}{dt} = \frac{\left(\frac{dp}{dt}\right)_0 I_x^2 N}{K^2 + I_x^2 \frac{N}{G}} \left[- \cos\left(\sqrt{\frac{N}{G}}\, t\right) + \frac{K}{I_x\sqrt{\frac{N}{G}}} \sin\left(\sqrt{\frac{N}{G}}\, t\right) + e^{-\frac{K}{I_x} t}\right] \qquad (18)$$

This equation is the expression of the rolling acceleration in terms of the physical factors governing it.

Differentiation of equation (18) gives

$$\frac{d^2p}{dt^2} = \frac{\left(\frac{dp}{dt}\right)_0 N}{\frac{K^2}{I_x^2} + \frac{N}{G}} \left[\sqrt{\frac{N}{G}} \sin\left(\sqrt{\frac{N}{G}}\, t\right) + \frac{K}{I_x} \cos\left(\sqrt{\frac{N}{G}}\, t\right) - \frac{K}{I_x} e^{-\frac{K}{I_x} t}\right] \qquad (19)$$

At maximum rolling acceleration $\frac{d^2p}{dt^2} = 0$. The time of maximum rolling acceleration is then given by

$$0 = \sqrt{\frac{N}{G}} \sin\left(\sqrt{\frac{N}{G}}\, t\right) + \frac{K}{I_x} \cos\left(\sqrt{\frac{N}{G}}\, t\right) - \frac{K}{I_x} e^{-\frac{K}{I_x} t} \qquad (20)$$

Unfortunately, formula (20) cannot be solved explicitly for t. By substitution of the value of t satisfying formula (20) into formula (19), the value of the maximum rolling acceleration is found.

Formulas (18) and (20) may also be expressed in terms of the aileron control position x. Expressed in this manner, formula (18) becomes

$$\frac{dp}{dt} = \frac{\left(\frac{dp}{dt}\right)_0 N}{\frac{K^2}{I_x^2} + \frac{N}{G}} \left[-\left(1 - \frac{x}{G}\right) + \frac{K}{I_x\sqrt{\frac{N}{G}}} \sqrt{1 - \left(1 - \frac{x}{G}\right)^2} + e^{-\frac{K}{I_x\sqrt{\frac{N}{G}}} \cos^{-1}\left(1 - \frac{x}{G}\right)} \right] \tag{21}$$

Formula (20) expressed in terms of x is

$$0 = \sqrt{\frac{N}{G}} \sqrt{1 - \left(1 - \frac{x}{G}\right)^2} + \frac{K}{I_x}\left(1 - \frac{x}{G}\right) - \frac{K}{I_x} e^{-\frac{K}{I_x\sqrt{\frac{N}{G}}} \cos^{-1}\left(1 - \frac{x}{G}\right)} \tag{22}$$

By formulas (18) and (20) the maximum rolling acceleration is expressed in terms of time, t, and by formulas (21) and (22) in terms of aileron-control deflection x. The influence of the several factors entering into these formulas can be shown by expressing the maximum rolling acceleration as a fraction of $\left(\frac{dp}{dt}\right)_0$, the maximum theoretically attainable. For this purpose the formulas for rolling acceleration in terms of control deflection have been chosen.

From formula (21) the rolling acceleration expressed as a fraction of the maximum theoretically attainable is

$$\frac{\frac{dp}{dt}}{\left(\frac{dp}{dt}\right)_o} = \frac{N}{\frac{K^2}{I_x^2} + \frac{N}{G}}\left[-\left(1 - \frac{x}{G}\right) + \frac{K}{I_x\sqrt{\frac{N}{G}}}\sqrt{1 - \left(1 - \frac{x}{G}\right)} + e^{-\frac{K}{I_x\sqrt{\frac{N}{G}}}\cos^{-1}\left(1 - \frac{x}{G}\right)}\right] \tag{23}$$

Let

$$D = 1 - \frac{x}{G} \tag{24}$$

$$E = \frac{K}{I_x\sqrt{\frac{N}{G}}} \tag{25}$$

When those substitutions are made in formula (23)

$$\frac{\frac{dp}{dt}}{\left(\frac{dp}{dt}\right)_o} = \frac{G}{E^2 + 1}\left[- D + E\sqrt{1 - D^2} + e^{-E \cos^{-1} D}\right] \tag{26}$$

After substitution from formulas (24) and (25) into formula (22), the condition for maximum rolling acceleration becomes

$$0 = \frac{1}{E}\sqrt{1 - D^2} + D - e^{-E \cos^{-1} D} \tag{27}$$

Formula (27) cannot be expressed as an explicit function of D or E. Values satisfying this equation have been determined and are given as a graph in figure 1.

The physical significance of E is found to be

$$E = \frac{\frac{1}{32}\rho S b^2 \frac{dC_L}{d\alpha}}{I_x\sqrt{\frac{\frac{1}{2}\rho S_a c C_H}{I_c \delta_{max}}}} = \rho^{\frac{1}{2}}\left(\frac{\frac{1}{32} S b^2 \frac{dC_L}{d\alpha}}{I_x\sqrt{\frac{\frac{1}{2} S_a c C_H}{I_c \delta_{max}}}}\right) \tag{28}$$

For a given airplane flying at a given altitude, E is a constant, and may therefore be considered as a characteristic of the airplane itself. For geometrically similar airplanes, on the assumption that I_x and I_c vary with the fifth power of the linear dimensions, the factor in parenthesis, and hence the value of E, is independent of airplane size.

The physical significance of D is

$$D = 1 - \frac{x\ (\frac{1}{2}\ \rho\ V^2\ S_a\ c\ C_H)}{Q_P}$$

From this expression it is seen that D is the torque applied by the pilot in excess of that required to overcome the opposing aileron hinge moment.

The physical significance of G is

$$G = \frac{Q_P}{\frac{1}{2}\ \rho\ V^2\ S_a\ c\ C_H}$$

From this expression it is seen that G is the ratio of the torque applied by the pilot to the torque required to maintain full aileron deflection at zero rate of roll, or the ratio of the pilot's effort to the airplane stiffness.

The preceding equations do not lend themselves to an easy visualization of the factors influencing the maximum rolling acceleration of an airplane. By a graphic representation of the maximum rolling acceleration as a function of the governing factors, the influence of the several factors is more easily seen and certain important results are brought out.

As shown above, E may be considered as a characteristic of the airplane itself which varies only with altitude. A value of E may be chosen to represent an airplane. By formula (27) or by figure 1, the value of D for maximum rolling acceleration may be found.

It is to be noticed that in the derivation of the preceding formulas no account has been made of the aileron control being limited in its deflection. In an actual airplane the aileron control cannot move beyond the position of full deflection, or where $x = 1$. Examination of formula (26) shows that the maximum rolling acceleration is directly proportional to G. Formula (24) gives

$$G = \frac{x}{1 - D}$$

This expression shows x to be directly proportional to G and, therefore, the maximum rolling acceleration is directly proportional to x. From this fact it is concluded that maximum rolling acceleration occurs when the control deflection is a maximum.

Because x is limited to a maximum value of 1, the maximum rolling acceleration attained by an airplane will be divided into two types: that for which the torque applied by the pilot is insufficient to attain full aileron-control deflection, and that for which the torque applied by the pilot is more than sufficient to attain full aileron-control deflection. The boundary between these two conditions is found by letting $x = 1$, and with the value of D for maximum rolling acceleration determined from formula (27) or by figure 1, the value of G is found by formula (24). After substitution of these values of E, D, and G in formula (26), the maximum rolling acceleration is found for the condition when the torque applied by the pilot is just sufficient to attain full control deflection. The maximum rolling accelerations found in this way for airplanes of various values of E are plotted as the broken line in figure 2. This curve shows that full control deflection may be attained without the pilot applying sufficient torque to maintain full control deflection. This property has been found to be due to the inertia of the ailerons and their control system by deriving formulas similar to formulas (26) and (27) based on the assumption that the ailerons and their controls have no inertia.

When the value of G, or the torque applied by the pilot, is less than that shown by the broken line in figure 2, the maximum aileron-control deflection attained is less than 1, and therefore the limiting condition that x cannot exceed 1 does not enter the problem. For an airplane represented by a value of E, the value of D for maximum rolling acceleration is found by formula (27) or figure 1. Substitution of the values of D and E into formula (26) then shows that the maximum rolling acceleration is directly proportional to G, or the torque applied by the pilot. In figure 2, between the value of $G = 0$ and the value of G required for full control deflection to be attained, as given by the broken line, the curve of maximum rolling acceleration against G for any given value of E is a straight line. That region to the left of the broken line in figure 2 therefore shows the variation of the maximum rolling acceleration attained by an airplane in an abrupt aileron roll with variation of the magnitude of the constant torque applied by the pilot to the control when the torque applied by the pilot is insufficient to attain full control deflection.

When the torque applied by the pilot is in excess of that required to reach full control deflection, the limiting condition that x cannot exceed 1 must be considered. As shown above, when the value of D for maximum rolling acceleration of an airplane having a given value of E is substituted into formula (26), the maximum rolling acceleration is directly proportional to the control deflection attained. Therefore, when the torque applied by the pilot is greater than that required to reach full control deflection and the control is stopped at $x = 1$, the maximum rolling acceleration occurs at the instant the control deflection reaches the value $x = 1$. For this condition, the maximum rolling acceleration of an airplane of a given value of E is found by substituting the value of D found from formula (24) when $x = 1$ into formula (26). The maximum rolling acceleration may then be found for any value of G greater than that required to attain a control deflection of $x = 1$. In figure 2, to the right of the broken line is plotted the variation of the maximum rolling acceleration with G, the torque applied by the pilot, for several airplanes represented by various values of E.

Based on the curves of figure 2 and formulas (26), (27), and (28), the following observations may be made concerning the maximum rolling accelerations of airplanes.

(1) The maximum rolling acceleration of an airplane is dependent upon the characteristics of the airplane itself expressed by E. As discussed above, immediately following formula (28) the value of E for any given airplane varies only with altitude or as $\rho^{\frac{1}{2}}$. Therefore, for a given airplane the value of E will become smaller the greater the altitude and, consequently, as shown by figure 2, the maximum rolling acceleration expressed as $\dfrac{\left(\dfrac{dp}{dt}\right)_{max}}{\left(\dfrac{dp}{dt}\right)_0}$ will become greater for any given value of G. Whether or not $\left(\dfrac{dp}{dt}\right)_{max}$ becomes greater at altitude depends upon how

$\left(\frac{dp}{dt}\right)_o$ varies with altitude for the particular airplane. Since $\left(\frac{dp}{dt}\right)_o$ is given by

$$\left(\frac{dp}{dt}\right)_o = \frac{-\frac{1}{2}\rho S_w V^2 b C_l}{I_x}$$

it is seen that $\left(\frac{dp}{dt}\right)_o$ remains constant for all altitudes if the indicated airspeed at all altitudes is the same. For an airplane flying at a given indicated airspeed, $\left(\frac{dp}{dt}\right)_{max}$ will increase with increase in altitude.

(2) Geometrically similar airplanes differing only in absolute size should have approximately the same value of E.

(3) Further examination of figure 2 shows that for an airplane flying at a given altitude and a given velocity, there is a practical limit to the maximum rolling acceleration which the pilot may attain by increasing the torque he applies to the aileron control. For example, consider the airplane whose value of E is 1. If the pilot increases the torque applied in deflecting the aileron control, the maximum angular acceleration which the airplane attains increases up to a value of G of about 3. Beyond this value, further increase in G does not produce much increase in the maximum rolling acceleration.

The effect of airspeed on maximum rolling acceleration is easily deduced from figure 2. As an illustration, assume an airplane in which the rolling-moment coefficient of the ailerons is capable of producing a rolling acceleration of 10 radians per second^2 at 200 miles per hour for instantaneous control deflection. Assume that 300 miles per hour is the maximum speed at which the pilot would be able to hold the control fully deflected at zero rate of roll. For a given airplane and a given altitude E is constant and for a given pilot's effort G varies inversely and $\left(\frac{dp}{dt}\right)_o$ directly with V^2. The resulting variation of $\frac{dp}{dt}$ with V obtained from figure 2 is shown graphically in figure 3 for two values of the parameter E.

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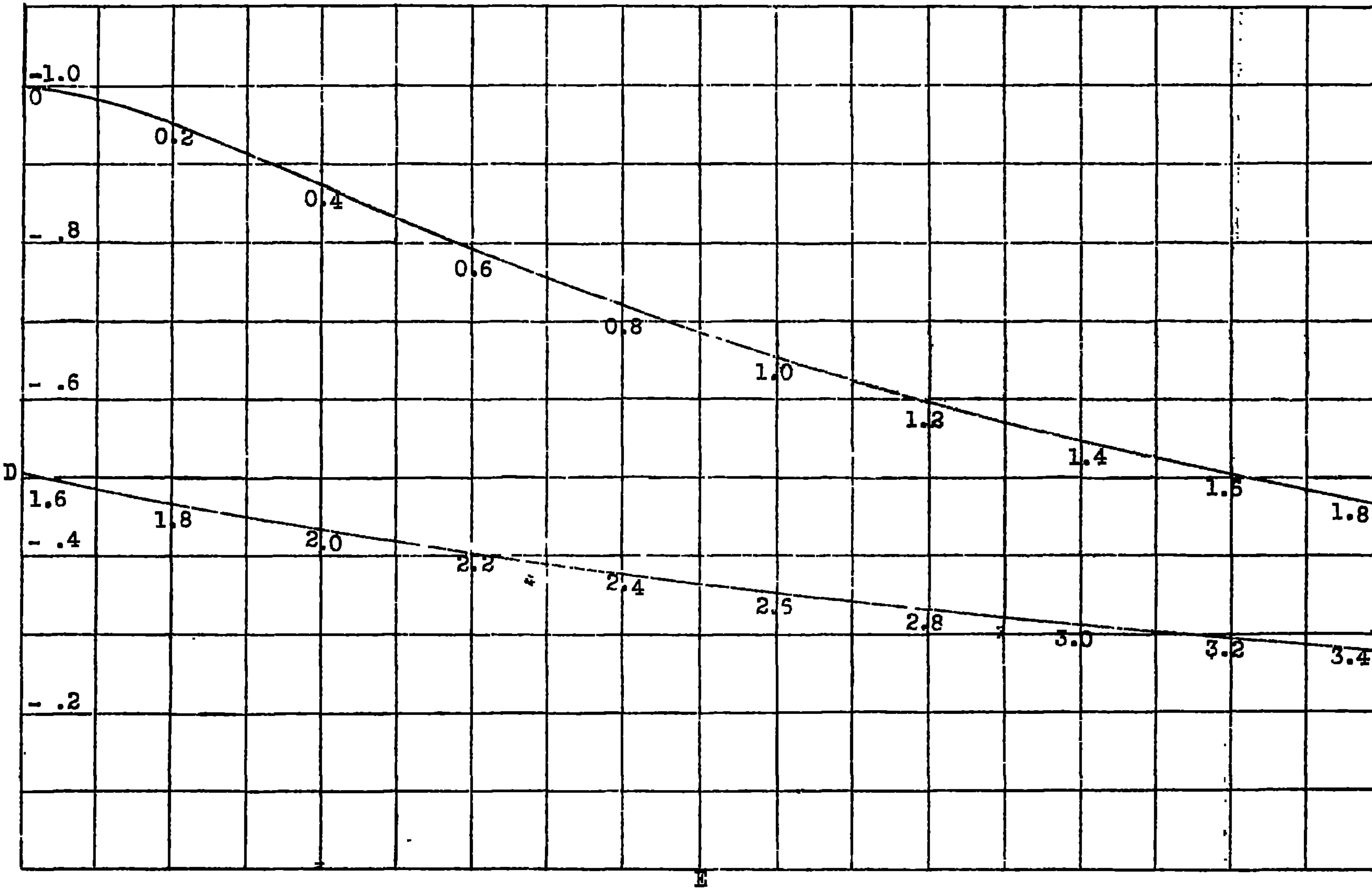

Figure 1._ Values of D and E satisfying equation 27 and valid for equation 26.

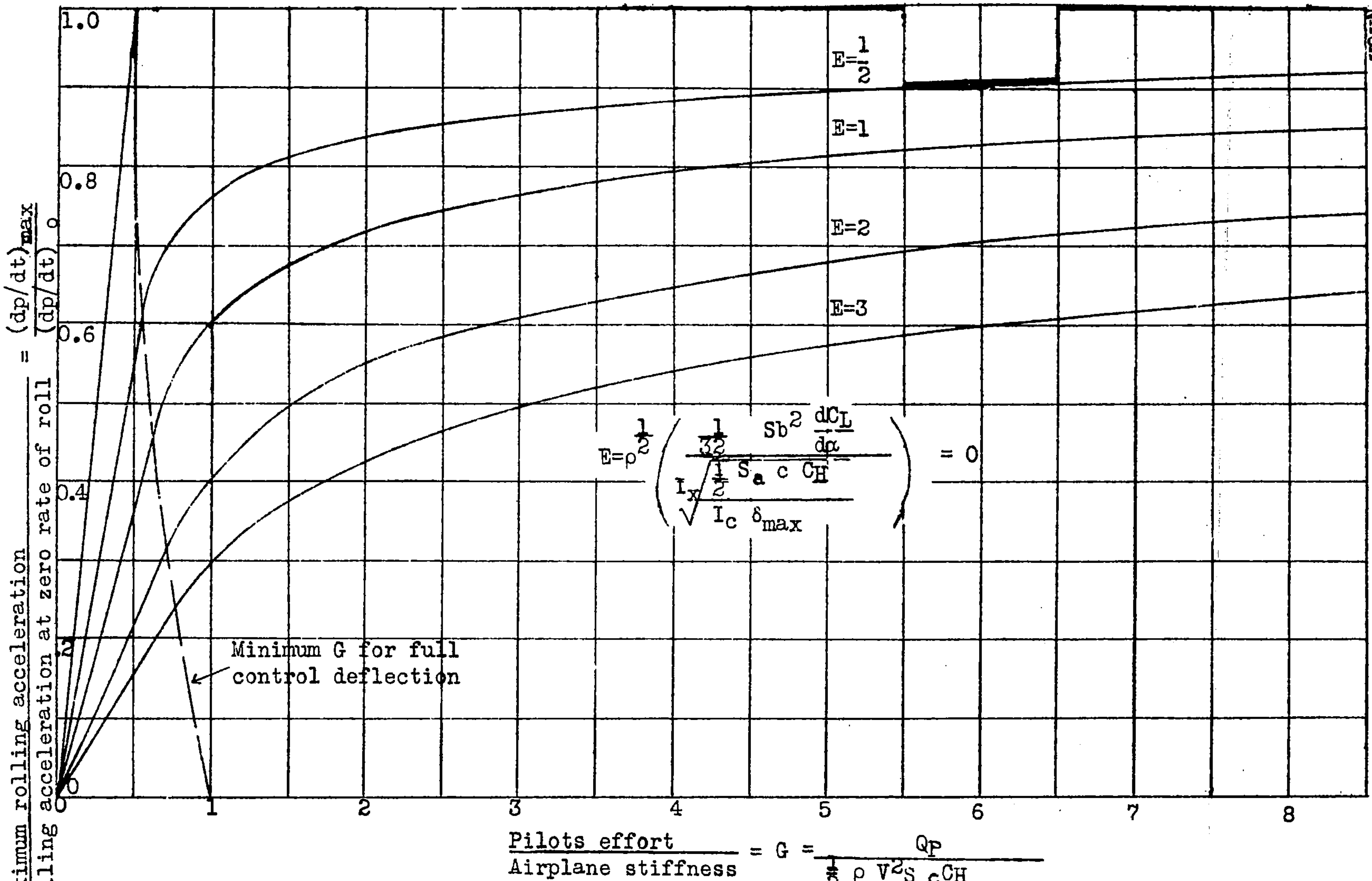

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Figure 2.- The maximum rolling acceleration as a function of the pilots effort and the characteristic of the airplane,E.

Fig. 2

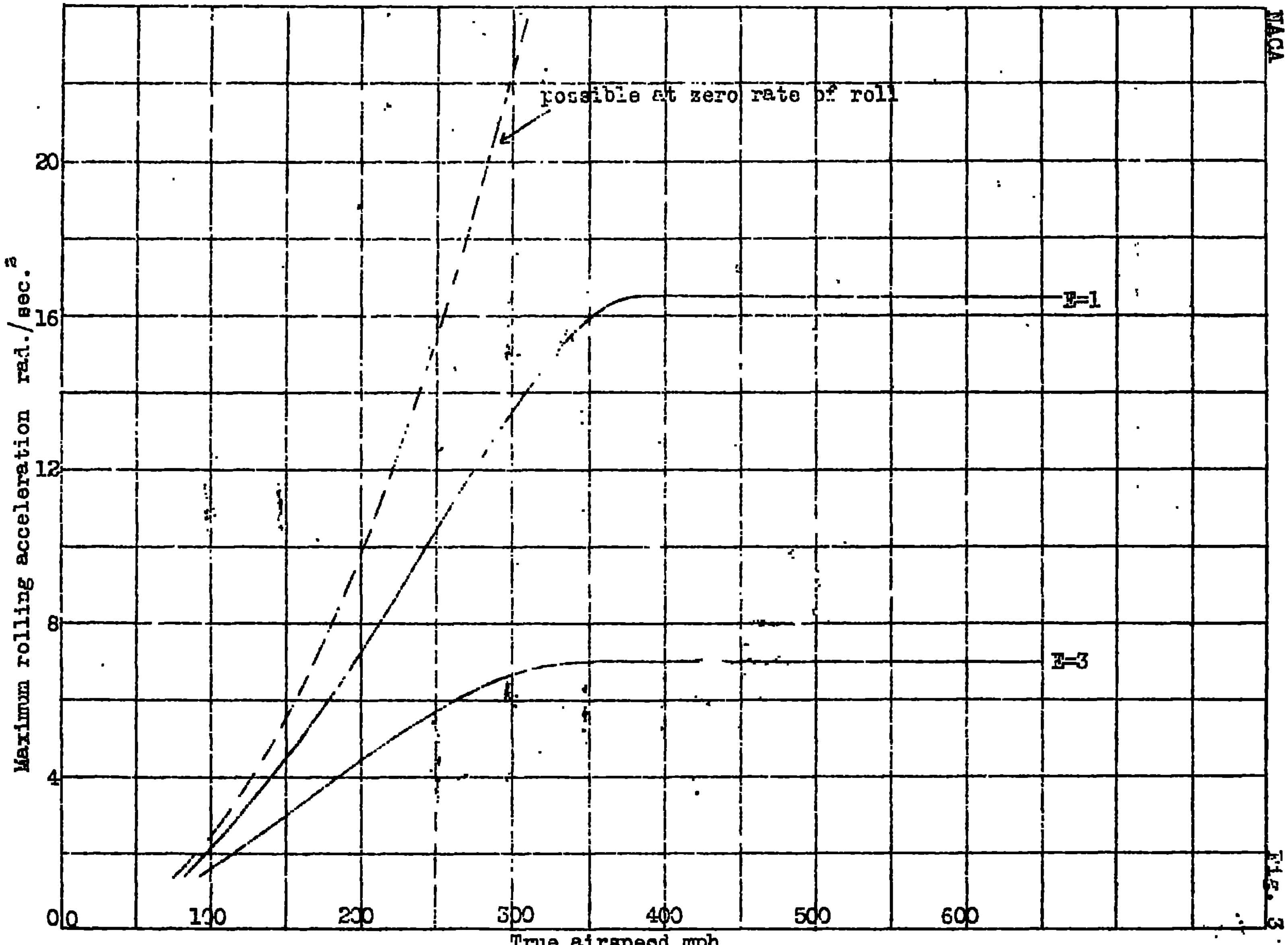

Figure.-3.-Calculated effect of airspeed on maximum rolling acceleration for a typical case

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