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L-418

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SUMMARY

The problem of establishing a rational unsymmetrical gust-load criterion from available data on gust structure is briefly discussed and it is indicated that the problem cannot be solved at the present state of knowledge without extensive and questionable analysis. On interim semi-rational criterion, based on normal and angular acceleration data obtained in rough air on the XB-15 airplane, is suggested.

This interim criterion states (a) the intensity of the symmetrical component of gust, U_e', to be considered as acting simultaneously with the selected unsymmetrical component, and (b) the intensity of gust, U_t, to be considered as acting at the wing tips with an assumed linear unsymmetrical gust distribution. Values of b/k_X (viz, span divided by radius of gyration) to be used in the evaluation of angular accelerations with the proposed gust components are also suggested. The criterion, to be applied at the same speed as is used for the symmetrical gust and with the airplane in the lightly loaded condition, comprises the following values:

$$U_e' = 0.8\ U_e$$

$$U_t = 20\ \text{ft/sec}$$

Airplane type	b/k_X
Single-engine	8.25
Two-engine and three-engine	7.75
Four-engine	7.25

INTRODUCTION

The currently used 100/70 design criterion for unsymmetrical loads or, airplane wings has been admittedly unsatisfactory primarily because it is irrational aid also because it loads, in some cases, to obviously excessive strength requirements, The need and general requirements for a rational criterion have long been clear enough, but such a criterion has been impossible to establish because of the complexity and largely unknown character of gust distribution in the atmosphere, a thorough knowledge of which is essential to a truly rational solution of the problem. While solutions can be evolved in individual cases from extended calculations, such solutions are not generally applicable and, moreover, there is little or no basis for any gust distribution that may be assumed.

The purpose of this memorandum is to discuss briefly the nature of the general problem from the standpoint of available information on gust structure and to submit a suggestion for a generally applicable unsymmetrical gust criterion that is at least semi-rational in character and that yields design loads which appear to be reasonable and in accord with available acceleration data. The criterion suggested offers nothing of value with respect to the design of the primary wing structure, but it docs serve as a means of establishing design load factors for engine-mount supports and other supports for fixed masses located away from the plane of symmetry of the airplane.

The Nonuniform Gust Problem

A completely rational procedure for the determination of design wing loads in nonuniform gusts requires (a) a thorough knowledge of gust structure under various atmospheric conditions, (b) knowledge of unsteady lift phenomena under nonuniform conditions along the span, (c) knowledge of the airplane characteristics, including rolling moment of inertia, in advance of construction, and (d) the mathematical determination of the airplane motion in the important gust distributions. It probably is true that, in the case of small and moderate-size airplanes, the angular acceleration occurring in combination with some substantial linear acceleration is the only effect of importance arising in unsymmetrical or nonuniform gusts. However, in the case of possible future large airplanes, in which most of the load is carried in the mings and distributed approximately in conformity with the normal lift distribution, and to a lesser degree in the case of existing large airplanes, the existence of a localized gust of high intensity may result in critical wing stresses irrespective of the values of the angular or normal acceleration. The general problem, therefore, involves consideration of each of the more-or-less random gast distributions that may actually occur,

Let us consider briefly the gust structure or, more specifically, the relation between gust gradient, size or spatial extent of gust, and the gust intensity. Under certain hypothetical conditions, reference 1, it is possible to arrive at the theoretical result that

$$U_m^3 \propto H$$

where

U_m maximum gust velocity

and

H distance in which the gust velocity increases from zero to maximum

This relation docs not represent tho corsospondcncc of any gust with its gradient distance, but it represents rather the locus of the maximum gust intensities likely to be experienced. over a long period of time, provided. the conditions giving rise to tho gusts remain constant. Experimental data obtained in flight measurements seem to bear out the relationship approximately - even when taken with such a wide range of airplanes as from the little "Aeronca" to the large M-130 and XB-15.

On the other hand, there is no correlation found between gust intensity and gradient distance when dealing with tho data as a whole (reference 2). This result is to bo expected on the basis of certain physical considerations. For example, a thermal convection current may, in a simple case, be viewed as a free-air jet whose origin is rather indefinitely defined as compared nith the origin of a jet issuing from an orifice, But even in the latter case the jet has a boundary layer that grows thicker as the distance from the orifice increases, so that tho velocity gradient (gust gradient) nay have a wide range of values for a given velocity of the core, In the case of the actual. thermal, with its indefinite origin characteristics, there is not even the possibility of defining gradient distances in terms of height above tho origin, so that tho gradients may havc almost any valuo whatovor, being limited probably only as to their maximums.

Within the boundary zone of *a* thermal, masses of air of various sizes arc set in rolling or whirling motion as a result of the shearing of tho vertical current with respect to its environment. It is to bo oxpcctad. that; tho peripheral velocity of these rolling masses will bc approximately tho samo as tho velocity difference between the core and its environment - probably less owing to frictional losses, bur; possibly somewhat greater owing to conservation of momentum (if the whirling mass is somewhat compressed because of dynamic forces exerted by adjacent moving masses within the syston).

1t appears from the foregoing considerations that, in general, no relation between gust gradient and gust intensity exists or can exist and that the gradient may have a variety of valucs for any given value of the maximum velocity.

In regard to size or spatial extent of gusts, it may be said that acceleration data obtained on airplanes varying greatly in size indicate that gusts causing the highest accelerations havc gradient distances of' about 8 to 30 chord lengths (or about 1 span length). Without detailing tho reasons, it may further bo said that, in consequence of this result and of' the details of the records, gusts may bo pictured as varying in size in a random way fron dimensions that are small compared with the span to dimensions that are large compared with tho span, and also that the gradient distance in the direction of flight may, in general, be considered *as* of the same order *as* the gradient distance in the dirostion of the span, While admitting tho crudity of this information, we nag perhaps bo justified at this point in idealizing the coacopt of gust structure for the sake of attempting to provide a working basis.

Such *a* concept that docs not contradict the essential known facts is illustrated in figure 1. The largo gust shown is the one that would yield the maximum normal acceleration if the airplane penetrated it symmetrically.

This gust may be considered either as a complete or a truncated pyramid (or cone). In the truncated case the width of the flat top is about equal to the span and the gradient distance has a similar value - about 8 to 10 chord lengths. The airplane attains maximum acceleration just as it reaches maximum gust intensity, U, for the pitching alleviates the load beyond this position, while ahead of this position the pitching has little or no effect. Had the gust been much smaller, but geometrically similar and with the same maximum velocity, the acceleration would have been less primarily because of the reduced lateral dimension, and both the time and the distance from the start of the gust to peak load would have been less because of the reduced longitudinal dimension. Had the gust been much larger, but geometrically similar and with the same maximum velocity, the acceleration would likewise have been less because the alleviating effect of pitching would have come into play before the position of maximum gust intensity had been reached. These results are well in accord with the results of the acceleration measurements in flight previously mentioned. The assumption of completely pyramidal or conical, rather than truncated, gust form would have given similar qualitative agreement.

Now, if we discount the cubic parabola hypothesis and assume, on the basis of this discussion, that the gusts of various sizes have, under a given set of circumstances, the same maximum velocity, we may further crystallize our working concept. Such an assumption, so far as the nonuniform or unsymmetrical gust problem is concerned, is conservative relative to the cubic parabola hypothesis, which states that the smaller gusts have smaller intensities in accordance with the relation $U_m^3 \propto H$.

With the assumption of equal intensities, we may now picture gusts of various sizes, such as shown in figure 1, acting at any location along the span. These gusts may be of any size and distribution according to a perfectly random pattern. Small ones, such as shown, would lead to localized forces but slight disturbance of the airplane as a whole. Other sizes and combinations would lead to different results, and the motion of the airplane and stresses in the wing structure would depend not only on the sizes of tha gusts but also on their distribution along the span and in the direction of flight and on their vertical directions.

It can therefore be seen that no single, or few gust distributions can be selected from the flight data to use as criterions for unsymmetrical loads. The only possible manner in which this problem can be truly rationalized is to analyze the stresses in and motions of ai airplane undor a considcrablc number of the possible conbinations of gusts that can be visualized in accordance with the general idealized distribution pattern. Such an analysis would have to be based on "true" gust velocities as distinguished fron "effective" gust velocities, and the unsteady-lift effacts and airplane motions would 'nave Lo be takon into account. The unsteady-lift consideration alone poses quite a problem, for tho theory has been established only for uniform distribution along the span; rather drastic simplyfying assumptions would therefore have to be made to take into acccuot the unsteady-lift effect. Of course, the tremendously extensive analysis required for a solution for even one airplane could not possibly be undertaken as a design problem. It would, therefore, be necessary for someone to undertake a more general analysis in order to study the effects of the random gust distributions on tho wing beading moments and shears, and on the airplane motions for several airplanes of different sizes and typos. In this may it might be Pound possible ultimately to define simplified conditions and methods that would yield substantially correct results for the critical cases, Such analysis is reserved for the future, mien time and personnel will permit prosecution of the work.

For the present, a more simplified attack on tho problem nil1 have to suffice. The question of gust structure will be side-stepped as much as possible and the objective will be to set up a semi-rational criterion for combined normal and rolling accelerations that will yield reasonable values more or less in accord with limited flight data and with past design practice.

Derivation of Acceleration Criterions for Unsymmetrical Gusts

Table of Symbols

W weight, lb

S wing area, sq ft

q dynamic pressure, lb/sq ft

L-418

m slope of lift curve, radian measure

I_X moment of inertia about x axis, slug-ft^2

b wing span, ft

A aspect ratio b^2/S

U normal gust velocity, ft/sec

ρ mass density of air, $\frac{\text{lb sec}^2}{\text{ft}^4}$

α angular acceleration, rad/sec^2

C_{l_P} theoretical. rolling-moment coefficient

n load factor

f factor for correcting effective gust velocities; ratio of wing and contained weights to total airplane weight

C_L lift coefficient

k_X radius of gyration about x axis, ft

g acceleration of gravity, ft/sec^2

Y distance along span, ft

V airplane velocity, ft/sec

H gradient distance of gust in direction of flight, viz, distance from $U = 0$ to $U = U_{max}$

Subscripts

o sea level, initial

s symmetrical

s' reduced symmetrical

e effective

e' reduced effective

a

Subscripts (cont.)

- d design value
- t tip
- L high-speed level flight
- g maximum permissible gliding
- i indicated
- m maximum

General considerations.- Some unpublished tests made on the XB-15 airplane is rough air form the basis for the numerical values of the criterions to be established. In these tests, both resultant normal and angular accelerations were obtained from simultaneous acceleration measurements at the airplane center of gravity and at a point in the wing 29 feet 1 inch outboard from the plane of symmetry. From these data it is possible to deduce the magnitude of the maximum unsymmetrical effective gusts, based on the assumption of linear distribution along the span and corresponding to the maximum symmetrical effective gusts existing under the same weather conditions. The magnitude of the unsymmetrical gusts thus found can then be increased to correspond to the symmetrical effective gust intensity of 30 feet per second, which is approximately the value currently used as a 'basis for design.

The flight data are not sufficiently extensive to permit direct determination of the largest linear accelerations that may be superimposed on the maximum angular accelerations, owing to the low probability of such combinations. However, it is evident that the unsymmetrical gust cannot logically be superimposed on the design normal gust for the reason that such procedure implies the propriety of designing for a more severe normal gust then has been selected for design purposes. It is equally obvious that there is no point to superposing a normal gust of low intensity, suck as might be obtained directly from the test data, for then the combined accelerations at important stations out on the wings would be less than that resulting from the design normal gust alone.

We shall therefore select a normal gust of such value that, when it is combined with the unsymmetrical gust, the

L-418

resulting bending moments and shears at the wing root will be substantially equal to the moments and shears resulting from the design symmetrical gust alone. This process amounts to selecting the maximum symmetrical component possible without raising any question as to the necessity for designing the primary wing structure for the unsymmetrical condition. If the value selected appears reasonable in the light of past practice and in the light of the limited available test data, little concern need be felt over its arbitrary origin.

<u>Flight data</u>.- The applicable flight data were obtained during a period of flight of about 10 minutes, which was the roughest period in about 70 hours of cross-country flying. During this period the maximum normal acceleration increment at the center of gravity was 1.5g, which value corresponds to an effective gust velocity of 18.1 feet per second, obtained from the following expression:

$$U_e = \frac{\Delta n_s \frac{W}{S}}{\frac{\rho_o}{2} m V} \qquad (1)$$

in which

U_e	effective gust velocity, ft/sec
Δn_s	load factor increment, 1.5
W	weight at time of measurement, 52,000 lb
S	wing area, 2780 sq ft
m	slope of lift curve, 4.76
V_i	indicated speed, 274 ft/sec
ρ_o	standard mass density of air, 0.00238 slug-ft^3

The maximum differences between the simultaneous accelerations at one outboard nacelle and at the center of gravity were about 1.1g, which valuo corresponds to an angular acceleration of 1.2 radians per second^2 computed from

$$\alpha = \frac{(\Delta n_{nacelle} - \Delta n_{c.g.})\ g}{y} \qquad (2)$$

in which the distance, y, was 29 feet I inch.

Evaluation of effective linear unsymmetrical gust.- The effective lineally graded unsymmetrical gust corresponding to the measured angular acceleration of 1.2 radians per second^2 can be evaluated from

$$\frac{U_t}{V} = \frac{I_X \alpha}{C_{l_p}\ q\ S\ b} \qquad (3)$$

in which

U_t effective gust velocity at the tips; positive at one tip, negative at the other tip

I_X moment of inertia, 639,000 slug-ft^2, computed from equation (11), reference 3, as value existing under conditions of test and corresponding to a value of $b/k_X = 7.22$

q dynamic pressure, 89 lb/sq ft

C_{l_p} theoretical constant for 4:1 taper and aspect ratio, 8. The value is 0,455 from fig, 8 of reference 4.

b wing span, 149 ft

The value of U_t is found to be ±13.7 feet per second.

Since this value was obtained from measurements in rough air, in which the maximum normal effective gast velocity was 18.1 feet per second, it is probable that U_t would have been greater had the air been rough enough to have caused normal effective gusts of 30 feet per second to be measured. It seems reasonable, therefore, to increase the measured value of U_t by the ratio 30:18.1 in order to obtain a design value of U_t compatible with the design normal effective gust of 30 feet per second. The increased value of U_t is 22.6 feet por second,

Selection of normal gust component.- As previously

L-418

pointed out, the XB-15 test data are considored inadequate as an empirical basis for tho selection of a normal gust component to bo applied simultaneously with the maximum unsymmetrical component. It is perhaps worth rioting, however, that the normal component acting simultaneously with the maximum unsymmetrical condition measured in the tests was about 75 percent of the maximum symmetrical component measured. We should thesefore expect, on consideration of probabilities, that had the rough-air condition lasted over a longer period of time this figure would have boen somewhat greater.

Proceeding on the basis of equal bending moments and utilizing results given in references 3 and 4, the following expressions arc obtained for the banding moments at the wing root in the symmetrical and unsymmetrical casos, respectively:

$$M_{s_{root}} = \left(\frac{M_{a_z}}{nWb}\right) n_s Wb - \left(\frac{M_{w_z}}{fnWb}\right) f n_s Wb \tag{4}$$

and

$$M_{s'_{root}} = \left(\frac{M_{a_z}}{nWb}\right) n_s' Wb - \left(\frac{M_{w_z}}{fnWb}\right) f n_s' Wb + \left(\frac{M_{a_g}}{I_x \alpha}\right) I_x \alpha - \left(\frac{M_{w_\alpha}}{f \frac{W}{g} b^2 \alpha}\right) f \frac{W}{g} b^2 \alpha \tag{5}$$

in which

$\left(\frac{M_{a_z}}{nWb}\right)$ bending moment coefficient for the symmetrically distributed air load

$\left(\frac{M_{w_z}}{fnWb}\right)$ bcnding moment coefficient for symmetrically distribute?! wing inertia load

$\left(\frac{M_{a_g}}{I_x \alpha}\right)$ bending moment coefficent fos the lineally graded unsymmetrical gust

$\left(\frac{M_{w_\alpha}}{f \frac{W}{g} b^2 \alpha}\right)$ bending moment coefficient for the angular inertia load of the wing

n_s' reduced load factor

f ratio of wing and contained weights to total airplane weight

According to our premise, we wish to solve first for the ratio of n_s' to n_s when $M_s'_{root}$ equals $M_{s_{root}}$. Since, from equation (3),

$$\alpha = \frac{C_{l_p}\ q\ S\ b\ U_t}{I_X V}$$

and since

$$I_X = \frac{W}{g}\left(\frac{b}{b/k_X}\right)^2$$

and

$$\frac{2n_s W}{\rho_o V_g S} = C_{L_s}$$

where

V_g design gliding speed

and

C_{L_s} lift coefficient at this speed corresponding to n_s we arrive, through appropriate substitutions in equations (4) and (5) and by algebraic manipulation, at the ratio

$$\frac{n_s'}{n_s} = 1 - \frac{U_t}{C_{L_s} V_g}\left[C_{l_p}\ \frac{\left(\dfrac{M_{a_g}}{I_X \alpha}\right) - \left(\dfrac{M_{W_\alpha}}{f\frac{W}{g}b^2\alpha}\right) f \left(\dfrac{b}{k_X}\right)^2}{\left(\dfrac{M_{a_z}}{nWb}\right) - f\left(\dfrac{M_{W_z}}{fnWb}\right)}\right] \qquad (6)$$

This **equation** may be written

$$\frac{n_s'}{n_s} = 1 - k\ \frac{U_t}{C_{L_s} V_g} \qquad (7)$$

in which k represents the quantity appearing within the brackets of equation (6).

The terms within the Brackets nay be assignod numorical values from references 3 and 4 in harmong with the design characteristics 02 any airplane under consideration. Substitution or a number of different but possible values for these several terns indicates that k has a probable range from about 1.0 to 1.3. For example, the value for the XB-15 at the time of the tests previously mentioned has been found to bo 1.12.

Now, with U_t having a value tentatively assigned (say, 20 ft/sec) and V_g and C_{L_s} having values within a reasonable range, the value of n_s'/n_s is but little affected by substantial variations in k (viz, a change in k of 0.1 affects the load-factor ratio less than 2 percent). The quantity k may, for practical purposes, therefore, be taken as constant at its mean value. Since U_t and k now have fixed values, the load-factor ratio varies only with the prod-uct $C_{L_s} V_g$. Nor does this product change enough vith various $C_{L_s} V_g$'s to affect the ratio greatly, and it has been found by substituting values for a considerable range of conditions that the load-factor ratio will be very nearly 0.87 when U_t is taken as 20 feet per second.

The final step is to determine the reduced effective symmetrical gust velocity, U_e', corresponding to a load-factor ratio of 0.87. Since

$$n_s' = 1 + \Delta n_s' \tag{8}$$

we find by substituting in equation (7) that

$$1 + \Delta n_s' = n_s \left(1 - k \frac{U_t}{C_{L_s} V_g}\right) \tag{9}$$

But

$$\Delta n_s' = \rho_o \frac{m U_e' V_g}{2 W/S} \tag{10}$$

and

$$n_s = 1 + \Delta n_s = 1 + \frac{\rho_o m U_e V_g}{2 W/S} \tag{11}$$

so that

$$U_e' = U_e - \frac{kU_t}{C_{L_s} V_g}\left(U_e + \frac{2W}{\rho\ m\ V\ S}\right) \qquad (12)$$

Applying this expression to the evaluation of U_e' for several airplanes (taking $U_e = 30$, $U_t = 20$ and appropriate values for the remaining terms) reveals that, very nearly

$$U_e' = 0.8\ U_e$$

This value may, therefore, be selected as a practical design value that can be generally applied and that results in wing-root bending moments in the unsymmetrical condition very nearly equal to the bending moments in the design symmetrical gust condition.

In a similar way a value of U_e' can be found to give approximately equal shears. It has been found by the authors that the result is substantially the same and the derivation will not be presented here.

DISCUSSION

Alleviation resulting from angular velocity*.- In treating the unsymmetrical gust as an effective gust in a manner analogous to the treatment of the symmetrical gust before the introduction of the alleviation factor, f, (S.B. No. 67), the moment of inertia, I_X, plays a role analogous to that previously played by the wing loading in the symmetrical case; that is, other things being equal, the angular acceleration is found to be inversely proportional to the moment of inertia. The question arises as to whether the angular acceleration actually behaves in such a manner with changes in moment of inertia.

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*Subsequent to preparation of this section, conversations with Lt. Comdr. R. S. Hatcher, U.S.N., indicated that qualitative discussion might be insufficient to clarify the effect of changes in the moment of inertia on angular acceleration. Accordingly, an appendix has been included to show the effect quantitatively for an assumed gust distribution and with the lag in development of lift neglected.

In order to clarify this question somewhat, let us first consider briefly and qualitatively the influence of the wing loading in tho symmetrical case. According to the simple sharp-edge gust formula, tho load-factor incremoat is inversely proportional to tho wing loading, Two phenomena operate, however, to prevent the wing loading from having such a direct affect, namely, the lag in development of lift when the angle of attack suddenly changes, and the finite gradient of an actual gust. Both of these phenomena join in producing a finite period. of time of appreciable duration between tho initial onset of the gust and the maximum acceleration. As a result, tho airplane acquires a vortical. velocity during tho period of increasing acceleration and this velocity subtracts from tho gust velocity resulting in an alleviation of load, or in an offective gust velocity substantially less than the truo gust velocity. Now, it Is clear that a lightly loaded airplane acquires greater vertical velocity than a heavily loaded airplane, other things being equal, so that the relative alleviation. is greater in the former case. The effective gust velocity for a lightly loaded airplane is, therefore, less than the effective gust velocity for a heavily loaded airplane when the truo gust velocity remains constant, This phenomenon has been conservatively taken care of in the design requirements €or symmetrical gusts through tho use of an effective gust factor, f, which varies with the wing loading. Tho effect of this factor, when applied in the simple gust formula, is therefore the same as if tho wing loading itself were appropriately modified; that is, tho acceleration celcuiatcd with the factor f included shows less relative change with wing loading than nould bo the *case* if the factor were not used.

An analogous effect occurs in unsymmetrical conditions owing to the development of angular velocity in roll,, and since tho distance from the plane of symmetry to the lateral center of pressure of the unsymmetrical component of load is much greater than the radius of gyration (with a linear unsymmetrical distribution), the airplane all the more readily acquires an angular velocity in roll. The effect of tho unsymmetrical gust is therefore alleviated in about the same degree as tlic vertical gust is alleviated in the case of lightly loaded airplanes, It is quite possible, in fact, that the reason. for the experimental value of U_t, found fron the XB-15 measurements, being so much less than U_o is a result of the much greater alleviation fn roll than in translation - the ratio of U_t to

U_e being approximately the same as has been found from a rough quantitative estimate of the relative alleviation from the two conditions.

It is therefore evident that the use of a single value of effective gust velocity, U_t, for the unsymmetrical component will cause apparent changes in angular acceleration with moment of inertia that are not fully made good under actual flight conditions. There should, strictly speaking, be an alleviation factor applied to U_t in the same manner as the factor f is applied to the basic symmetrical effective gust of 30 feet per second. However, we are not in a position at the present time to derive such an alleviation factor except in a very rough and arbitrary way. An alternative and simpler procedure open to us is to allow for the alleviation effect by fixing the value of b/k_X for each significant airplane type, so that the angular acceleration will not vary at all with changes in moment of inertia when other quantities remain the same. Such a procedure yields a more nearly correct result than if a fixed value of U_t is used in conjunction with variable b/k_X. Certainly, at least, there is little justification for a meticulous determination and use of moment of inertia until such time as a reasonably correct alleviation factor for the unsymmetrical case can be worked out.

In view of the foregoing discussion, it seems sufficient for the present purpose to define the moment of inertia as

$$I_X = \frac{W}{g}\left[\frac{b}{\left(\frac{b}{k_X}\right)}\right]^2$$

in which the ratio b/k_X may be assigned values that take into account prono-meed differences in the nature of the design. After consideration of available data on moments of inertia and of the angular accelerations resulting from application of different values in conjunction with the selected value of U_t at speeds 25 percent greater than maximum speeds in level flight, the authors consider the following values of b/k_X to be reasonable:

L-418

Airplane type	b/k_X
Single-engine	8.25
Two-engine and three-engine	7.75
Four-engine	7.25

These values, of course, may not be applicable in some cases in which there are peculiarities in design or arrangement of mass.

Some calculated results.- For comparative purposes, engine-mount load factors have been computed for (a) the established symmetrical gust condition, (b) the proposed unsymmetrical gust condition, and (c) the 100/70 condition. In all cases the airplanes are taken in light condition at speeds 1.25 times the maximum speed in level flight. The design symmetrical gust velocity is taken as 30 feet per second and U_t is taken as 20 feet per second. Radii of gyration are taken in conformity with the ratios b/k_X suggested in the foregoing paragraph.

The data used and the results obtained are tabulated in table I. The angular accelerations are plotted in figure 2. Row 13 of table I gives the positive and negative load factors computed in the symmetrical gust condition, row 16, the engine-mount load factors for the outer engines in the proposed unsymmetrical condition, and row 20, the engine-mount load factors for the outer engines in the 100/70 unsymmetrical condition. It will be noted that in all cases but one the proposed unsymmetrical criterion yields a somewhat more conservative result than the 100/70 criterion. The exception is the single-engine fighter, in which case the high maneuver load factor results in an excessive angular component when the 100/70 criterion is used. That the proposed criterion yields the more conservative values when gust conditions govern does not signify that it is unduly severe. For example, figure 2 shows that, in the case of the XB-15, the angular acceleration computed from the 100/70 criterion is about the same as the value actually measured in air that was only moderately rough, whereas, as we have seen, the value computed from the proposed criterion is higher only by an amount sufficient to allow for air that is gusty enough to cause effective symmetrical gust velocities of 30 feet per second. It may also be mentioned here that V-G data taken on a B-247D airplane of Pennsylvania Central Airlines shoved, over a moderately long operating period, that tho accelerations at tho engine mount were about 0.5g higher than at the center of gravity. This result is in good agreement with the results computed for the 14-H and DC-3 (table I, rows 13 and 16).

The case of the airplane whose design is governed by maneuver requirements should, perhaps, be treated somewhat differently than the case of the airplane designed by gust conditions alone. In the opinion of the authors, it mould be logical to determine the angular acceleration resulting from the unsymmetrical gust and to superimpose it upon the normal maneuver load factor reduced by an amount equivalent to 20 percent of the symmetrical gust load factor that would apply at the design speed and weight. Such procedure tacitly assumes that the maneuver load factor is made up of two components, one a true maneuver load factor and the other a gust component; since the combination is constant for a given design, there is a further tacit assumption that the components are variable, the maneuver component becoming smaller as the gust component increases, and vice versa, so that the load factor is realized by a maneuver alone only when the air is smooth. These implications are present in the existing normal load requirements, if not expressly stated, and they are not unreasonable when considered in the light of actual flying practice.

Before concluding, further reference is made to figure 2, which shows a number of values of angular acceleration plotted against a scale of airplane span. The hyperbolic curves shown have a special significance explained in the appendix. The features of greatest interest on figure 2 are the values of angular acceleration for tho XB-15 and for the XF13C-3. In the former case, as previously mentioned, the proposed unsymmetrical criterion yields a properly higher acceleration than the 100/70 criterion when considered in the light of the measured value. In the latter case, the proposed criterion yields an angular acceleration that is also higher than an experimental value obtained in an abnormally abrupt aileron maneuver; at the same time, the 100/70 criterion yields a value that is obviously excessive both with respect to probable aileron and to probable gust conditions. It appears that the proposed criterion gives results that are not only reasonable for the gust conditions but that it also probably yields a result that is adequate to take care of any normal aileron maneuver so far as the mere question of angular accelerations is concerned.

L-418

CONCLUSIONS AND RECOMMENDATIONS

The present status of knowledge of gust structure in the atmosphere and the nature of the general nonuniform gust problem do not permit selection at this time of a single, or a few simple gust distributions to apply to a rational solution of the problem. However, a criterion with a rational framework can be established for the determination of outboard load factors resulting from symmetrical and unsymmetrical gust components.

Such a criterion, the numerical values of which are based on angular acceleration measurements made on the XB-15 airplane in rough air, is suggested. It states:

$$U_e' = 0.8\ U_e$$
$$U_t = 20\ \text{ft/sec}$$

Airplane type	b/k_X
Single-engine	8.25
Two-engine and three-engine	7.75
Four-engine	7.25

where

U_e' symmetrical effective gust component to be used concurrently with the unsymmetrical component

U_e design symmetrical effective gust

U_t effective gust velocity at the tips; positive on one tip and negative at the other with linear distribution between

b/k_X ratio of span to radius of gyration

In applying the criterion, U_e' is used in the usual way to obtain the normal component of acceleration; the angular acceleration, a, is computed from

$$a = \frac{C_{l_p}\, q\, S\, b}{\frac{W}{g}\left(\frac{b}{\frac{b}{k_X}}\right)^2}\, \frac{U_t}{V_g}$$

where C_{l_p}, the rolling nonent coefficient for linoar unsymmetrical load distribution, is obtained fron reference 4.

Tho use of constant values of b/k_X for each type is recommended in place of the actual calculated values because such use yields a result more nearly in accord nith angular accelerations to be expected in flight than the use of actual values, and because it is simpler. It has, in Pact, been shown that, other things being equal, tho angular acceleration varies but little nith moment of inertia, whereas the use of the simple formula $\alpha = M/I$ would yield an angular acceleration varying inversely as the moment of inertia.

The values of U_t, etc., suggested, should be applied with the airplane in light condition at maximum permissible gliding speed, In the case of airplanes designed by a maneuver load factor, the symmetrical gust load-factor increment corresponding to U_e at tho design speed and weight should be determined and the maneuver load factor should be reduced by 20 percent of tho gust-load component before applying the unsymmetrical gust distribution.

It nay bc found preferable, if the proposed criterion is accepted in substance as a design requirement, to specify that

$$U_t = 0.67\, U_e$$

where the ratio 0.67 is the ratio of 20 to 30 which nay be considered, in conjunction with the use of fixed values of b/k_X, as a temporary alleviation factor for the unsymmetrical case corresponding to the variable factor f used for the symmetrical case. In this manner tho basic unsymmetrical gust is tied in nith the basic symmetrical gust and the way is left open for improvements or modifications to the unsymmetrical alleviation term when further information becomes available.

L-418

APPENDIX

Quantitative Illustration of Rate of Change of Angular Acceleration with Moment of Inertia in Gusts Having Finite Gradients

In order better to demonstrate the manner in which the moment of inertia affects the angular acceleration in unsymmetrical gust conditions, an expression is herein derived for the angular acceleration in lineally graded unsymmetrical gusts having linear gradients in the direction of flight as well as in the diraction of the span. The unsteady-lift effect is neglected, so that the results are only approximately correct for the "true" gust conditions assumed. However, illustrative calculations based on the expression derived are adequate to show the influence of the time element in causing alleviation of the angular acceleration and in suppressing the effect of the moment of inertia.

Figure 3 illustrates the type of gust assumed. At any instant the rolling moment

$$L = I\alpha$$

consists of two components: that resulting from the distribution of angle of attack associated directly with the assumed gust, and a component resulting from a similar distribution of angle of attack associated with the rolling velocity about the X axis. We therefore write

$$I\alpha = C_{l_p} \frac{VtU_{max}}{H} V \frac{\rho}{2} Sb - C_{l_p} \left[\int_0^t \frac{b}{2} \alpha \, dt\right] V \frac{\rho}{2} Sb \qquad (1)$$

from which

$$\alpha = \left[\frac{C_{l_p} \, q \, U_{max} \, g\left(\frac{b}{k_X}\right)^2}{H \frac{W}{S} b}\right] t - \left[\frac{C_{l_p} \, V \frac{\rho}{2} \, g\left(\frac{b}{k_X}\right)^2}{2 \frac{W}{S}}\right] \int_0^t \alpha \, dt \qquad (2)$$

Differentiating,

$$\frac{d\alpha}{dt} = \left[\frac{C_{l_p}\ q\ U_{max}\ g\left(\frac{b}{k_X}\right)^2}{H\ \frac{W}{S}\ b}\right] - \left[\frac{C_{l_p}\ V\ \frac{\rho}{2}\ g\left(\frac{b}{k_X}\right)^2}{2\ \frac{W}{S}}\right]\alpha \qquad (3)$$

The solution of this differential oquation is

$$\alpha = \frac{2U_{max}\ V}{H\ b}\left[1 - e^{-\left(\frac{C_{l_p} V\ \frac{\rho}{2}\ g\left(\frac{b}{k_X}\right)^2}{2\ \frac{W}{S}}\right)t}\right] \qquad (4)$$

Angular accelerations **have** been computed at time $t = \frac{H}{V}$ according to this expression for the **following** conditions:

b	120 ,ft
$\frac{W}{S}$	25 lb/sq ft
V	200 ft/sec
Umax	18 ft/sec
C_{l_P}	0 ,455
H	variable
$\frac{b}{k_X}$	variable

The results **arc** plotted in figuro 4, It is apparent **at** a glance that whon H has approciable valuo tho rate of changò of angular acceleration with momont of inortia $\left[\text{viz, with } \left(\frac{b}{k_X}\right)^2\right]$ is groatly reduced fron the rate of change with H = 0. For the size of airplanc solocted, a value of H of **about** 100 to 150 feet (certainly aot **less** than 60 ft) **would** be about right for a gust extending completely across the span in the manner **assumed.** Betwoen normal limits of $(b/k_X)^2$, namely, between about 50 and 80, and with $H = 100$ feet, the change in angular acceleration is only about ±10 percent of the mean value as against a change of ±33 percent with the condition $H = 0$.

The effects pointed out in the foregoing paragraph should not be construed to mean that the moment of inertia has no great effect when it changes as a result of a change in the airplane size. In order to show this influence, curve segments have been calculated and are shown plotted on figure 2.

The dashed segments are based on the assumption of constant wing loading and gradient distance, H, using the suggested values of b/k_X for the three-size categories. The ordinates of these curves have been adjusted so that the right-hand segment passes through the "criterion" point for the XB-15 airplane. The curve segments therefore show the effect of reduced size on angular acceleration when all other quantities remain constant. The points plotted for the DC-3, Lockheed 14-H, and XF13C-3 airplanes on the basis of the suggested criterion do not fall exactly on these curve segments because of differences in speed and wing loading between these airplanes and the values for the XB-15.

The dotted segments are based on the assumption that H is proportional to the span, the wing loading remaining constant. The effect of this assumption which, as we have seen, is probably more nearly correct than the assumption of constant H, is to cause the angular acceleration to increase at a substantially more rapid rate with decreasing span than occurs with the assumption of constant H implicitly underlying the suggested criterion. This behavior suggests that the criterion yields unconservative angular accelerations when applied to the smaller airplanes. However, it should be borne in mind that the criterion also conservatively assumes that U_t remains constant with airplane size. If H were assumed proportional to the span, it probably would be more logical also to assume that U_t was reduced according to some relationship such as $U^3 \propto H$. This combination of assumptions would substantially reduce the ordinates of the dotted-curve segments. It is also worth reiterating at this point that the criterion, as it stands, yields a value of angular acceleration for the XF13C-3 airplane that is substantially greater than that measured at high speed in an abnormally abrupt aileron maneuver. This result leads us to feel that the criterion is adequate for all practical purposes.

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L-418

TABLE I

	XF13C-3	Lockheed 14-H	Douglas DC-3	Douglas DC-4	Boeing XB-15	Boeing 314
Number of engines	1	2	2	4	4	4
Gross weight, lb		17,500	24,400	47,000	69,000	82,500
Light weight, lb	4662	12,700	19,400	42,500	52,000	55,000
Wing area., sq ft	205	545	987	2,150	2,780	2,867
W/S (light), lb/sq ft	22.7	23.3	19.7	13.8	18.7	19.2
Span, b, ft	35.0	65.5	95	138	149	152
Span/radius gyration	8.25	7.75	7.75	7.25	7.25	7.25
Indicated max. level speed, V_L, mph	237	216	197	190	190	190
Indicated max. glide speed, V_G, mph	296	270	247	237	238	238
Slope lift curve, m	4.55	4.76	4.76	4.76	4.76	4.75
Gust factor, f	1.07	1.07	1.04	1.04	1.02	1.03
Distance to outer engine	0	7.6	9.3	27.5	27.5	29.0
Gust load factor, n_s	4.32 −2.32	4.08 −2.06	4.25 −2.25	4.10 −2.13	4.23 −2.23	4.18 −2.18
Reduced gust load factor, n_s'	3.65 −1.65	3.47 −1.47	3.60 −1.60	3.48 −1.48	3.59 −1.59	3.54 −1.54
Load factor due to ang. accel., n_α		±1.28	±1.17	4 1.99	f1.98	±1.98
$n_s' + n_\alpha$	3.65 −1.65	4.73 −2.73	4.77 −2.77	5.47 −3.47	5.57 −3.57	5.52 −3.52
Angular acceleration, a	12.5	5.42	4.06	2.33	2.29	2.20
n_s' (100-70)	3.67 −1.55	3.47 −1.77	3.62 −1.92	3.48 −1.78	3.60 −1.90	3.55 −1.85
n_α (100-70)		± .75	± .65	± .96	± .91	± .93
$(n_s'+n_\alpha)$(100-70)	3.67 −1.55	4.22 −2.52	4.27 −2.57	4.44 −2.74	4.51 −2.81	4.48 −2.78
Ang. accel., α(100-70)	17.00*	3.16	2.27	1.22	1.07	1.04

*Based on an applied maneuver load factor of 8.00.

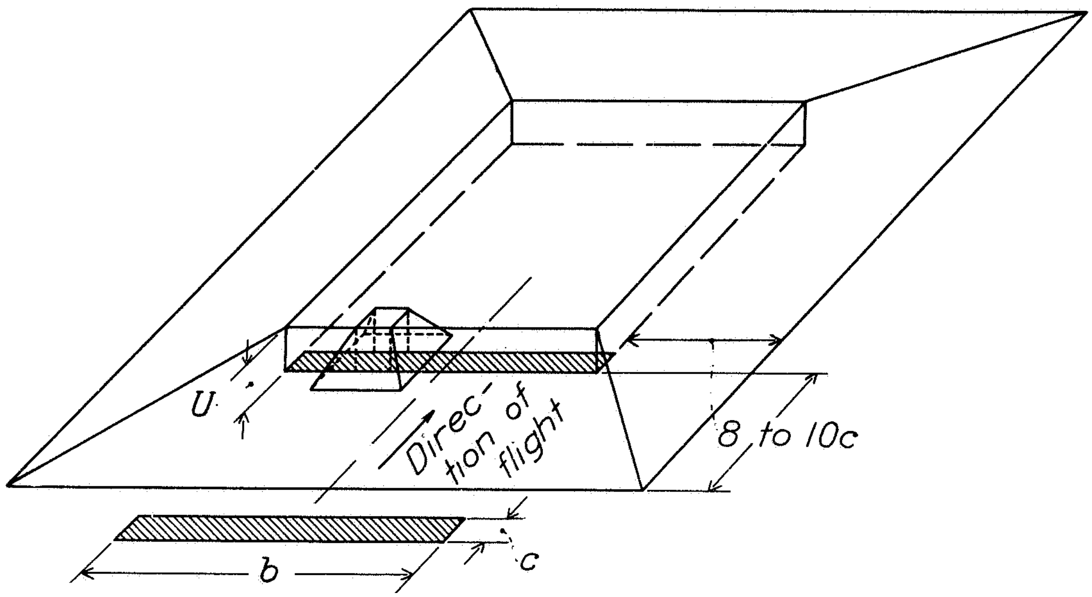

Figure 1.- **Possible gusts acting** along span.

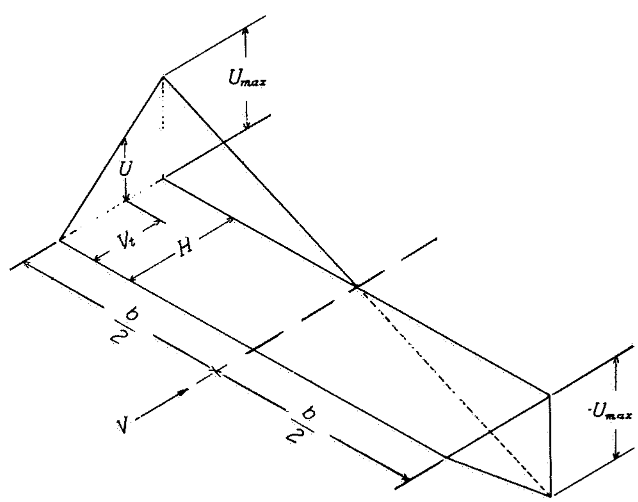

Figure 3.- Assumed gust distribution.

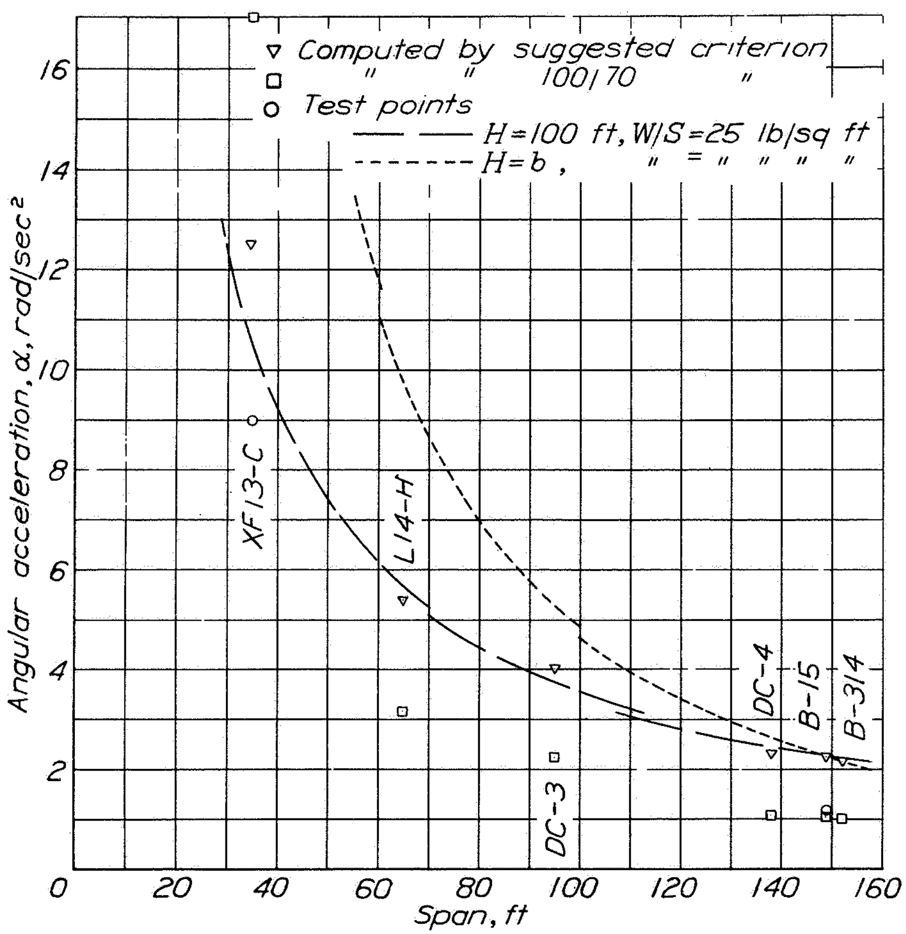

Figure 2.- Computed angular accelerations.

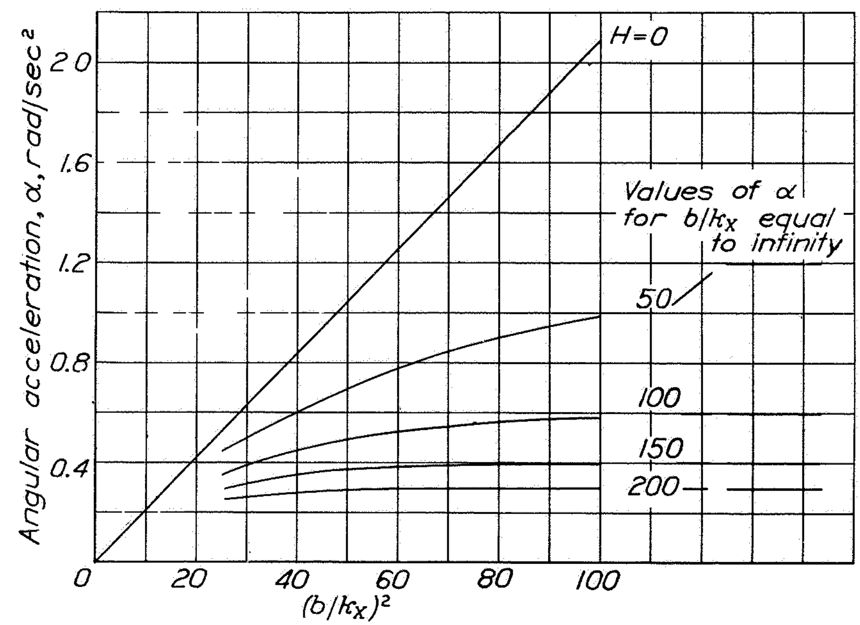

Figure - Computed angular accelerations for various gust gradients.