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SECONDARY STRESSES IN OPEN BOX BEAMS

SUBJECTED TO TORSION

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SUMMARY

In open box beams subjected to torsion, secondary
stresses arise owing to lateral bending of the spar caps.
The present paper outlines a simple method for estimating
the magnitude of these stresses and gives the results of
tests of an open box beam in the neighborhood of a dis-
continuity where the cover changed from the top to the
bottom of the box.

INTRODUCTION

The main strength element of a wing is frequently
a closed box of approximately rectangular cross section.
In some designs, large portions of the box cover are
removable and therefore nonstructural; the closed box
is thus converted into an open box, or channel, for some
distance along the span. The stress analysis of such an
open box under torsional loads is commonly based on the
assumption that the side walls of the box act as
independent spars. Such an analysis is essentially cor-
rect for computing the average stresses in the spar caps
but fails to disclose the existence of secondary stresses
caused by lateral bending of these caps. The present
paper discusses the nature of these stresses and shows
how their magnitude can be estimated.

THEORETICAL ANALYSIS

Open Box without Bulkheads

If an open box without bulkheads is subjected to
torque loads, the deformation will be essentially as

shown in figure 1. The previously rectangular cross section distorts into a parallelogram. The side walls or spars develop the resisting torque by bending - one spar bending up; the other spar, down. The cover, being connected to the lower spar caps, is stretched along one edge and compressed along the other edge. The cover stresses may be assumed to vary linearly from the maximum tensile stress along one edge to the maximum compressive stress along the other edge. On this assumption, the cover may be replaced for purposes of computation by two equivalent concentrated flanges attached to the lower spar caps as indicated in figure 1(b) - each equivalent flange having a cross-sectional area equal to one-sixth the area of the cover. The open box is thereby converted into a structure consisting of two independent spars that can be analyzed by the elementary bending theory. In a practical structure, the cross-sectional area of the cover should be understood to mean the area effective in resisting longitudinal stresses (stringers and effective width of skin). The results obtained by this simple procedure are identical with those obtained by Cambilargiu (reference 1) in a rather elaborate analysis employing the calculus of variations.

At a station where the cover is interrupted by a chordwise cut, the normal stresses in the cover must be zero and, consequently, the equivalent cross-sectional area of the cover is zero. With increasing distance from the cut, the equivalent area increases. The rate of this increase may be calculated by the methods of the shear-lag theory (reference 2); in many practical cases it will be sufficiently accurate, however, to assume that the rate of increase is uniform and that the theoretical value of one-sixth the effective area is applicable at some estimated distance from the cut. If the cover is not buckled and has no stringers attached to it, this dis-tance may be taken as one-half the width of the box. If the cover is buckled, or if stringers are attached to the cover sheet, greater distances must be assumed.

Open Box with One Bulkhead

If an open box with a bulkhead is subjected to torque loads (fig. 2), the major part of the resisting torque will be developed by bending of the two spars as in a box without bulkheads. The bulkhead, however, now keeps the cross section rectangular and, as a result,

the vertical deflections of the spars must be accompanied
by lateral deflections of the upper and lower faces of
the box. The internal forces associated with these
lateral deflections furnish the minor part of the
resisting torque.

The forces acting on the parts of the structure are
shown in figure 3. The statically indeterminate unknown
is the force X acting between each vertical edge of the
bulkhead and the adjacent spar. The internal forces and
moments acting on the structure can be expressed without
difficulty in terms of the applied torque and of the
unknown force X. The force X can then be calculated
by any standard method such as internal work, dummy unit
load, consistent deformations, or least work. When the
expressions for deformation or internal work are being
written, it is generally sufficient to confine attention
to bending of the spars caused by the applied torque and
lateral bending of the free spar caps caused by the
horizontal force Y exerted by the upper edge of the
bulkhead. In some cases, of course, it may be advisable
to include terms for shear deformation of the walls and
for lateral bending of the horizontal beam, which con-
sists of the cover and the spar caps attached to it.

Open Box with Many Bulkheads

The open box with many bulkheads can be treated by
the same principles as the box with one bulkhead.
Because an analysis with many statically indeterminate
unknowns is laborious, it is advisable to simplify the
calculation by assuming that the lateral bending of the
spar caps is confined to the root bay; that is, only
the bulkhead next to the root is assumed to be active.
Comparative calculations indicate that the maximum
values of the lateral bending stresses at the root are
not changed appreciably by changing from one active
bulkhead to two, provided the bulkheads are equally and
not too closely spaced.

Open Box with Wandering Cover

In some wing structures of the open-box type the
cover wanders from top to bottom. In some wings on fighter
airplanes, for instance, the root bay is covered only at
the top and the bottom is left open to permit retraction

of the landing gear. In the adjacent. bay, the bottom is
covered and the top has a large opening to give access
to the wing guns. In such a structure, the bulkhead at
the discontinuity, where the cover changes from top to
bottom, is analogous to the root bulkhead of the simple
open box discussed in the preceding paragraphs. The
analysis of the stresses in the vicinity of the discon-
tinuity may therefore be made by the same methods as
outlined for the simple open box; the only difference is
that all four spar caps appear in the calculation instead
of only two. The sample analysis given in the appendix
is of this type.

EXPERIMENTAL INVESTIGATION

Test Specimen

The test specimen built to verify the theory pre-
sented in the present paper was an open box of 24S-T
aluminum alloy with wandering cover. (See fig. 4.)
The specimen was symmetrical about the plane of the root
bulkhead, at which it was supported. The load was applied
in the form of equal couples at the ends of the box.
Because the structure and the loading were symmetrical
about the root bulkhead, the spars could be considered
to be perfectly fixed at the root bulkhead.

The box was tested first with bulkheads spaced
28 inches, as shown in figure 4. In these tests, the
bulkhead at the discontinuity,where the cover changed
from top to bottom, was of 0.040-inch-thick 24S-T aluminum
alloy like the cover and the angles A (fig. 4) were not
attached. The second series of tests was made after
intermediate bulkheads had been added and the bulkhead
spacing thus reduced to 14 inches. The maximum lateral
bending stresses at the discontinuity measured in this
second series of tests were lower than the calculated
values. It was believed that the discrepancy might be
caused by elastic deformation of the bulkhead at the
discontinuity; this factor becomes more important as the
bulkhead spacing decreases. The aluminum-alloy bulkhead
was therefore replaced by the $\frac{1}{8}$-inch-thick steel bulkhead
shown in figure 4, and the steel angles A were added
to give the spar caps firm support against lateral bending.
The third and fourth series of tests were run with the

bulkhead at the discontinuity modified as described and
with bulkhead spacings of 14 and 28 inches, respectively.

Test Procedure

Strain measurements were made on the spar caps in
the vicinity of the discontinuity with 2-inch Tuckerman
optical strain gages. The load was applied in three
equal increments. The strain readings were plotted
against load and a straight line was drawn through the
points. If the line did not pass through the origin of
the plot, a parallel line was drawn to fulfill this con-
dition and a new value of strain obtained from the new
line at maximum load was used for the computation of
the corresponding stress. Check runs were made if the
straight line drawn through the experimental points in
the load-strain plot missed the origin by more than
0.2 ksi.

Because it would be difficult to specify the
location of the strain gages on the individual final
plots, figure 5 was prepared to show schematically the
location of the gages for all tests and the numbering
system adopted for the flanges or spar caps. For con-
venience, most measurements were made on the outermost
fibers of the outer angles. For these fibers, the   .
stresses caused by lateral bending of the flanges increase
the stresses caused by bending of the spars (the principal
action) on flanges 1 and 2 and decrease the stresses on
flanges 3 and 4. Some measurements were taken on the
innermost fibers of the inner angles of flanges 3 and 4
as a check.

Results and Discussion

The main test results are shown in figures 6 to 9.
The results are not shown in the test sequence because
this sequence was dictated simply by convenience of
altering the specimen. Two sets of calculated curves
are shown: one based on the assumption that only the
first bulkhead was active on either side of the discon-
tinuity, the other one based on the assumption that the
first two bulkheads on either side of the discontinuity
were active. For design purposes, only the peak stresses
that occur at the discontinuity need be known. The
discussion is therefore confined mainly to these peak
stresses.

The experimental stresses are lower than the calculated peak stresses in some cases and higher in other cases. A study of the results led to the conclusion that there are at least two counteracting factors influencing the stresses caused by lateral bending. One factor was mentioned in the section "Test specimen": namely, the relief of the lateral bending by elastic deformation of the bulkhead - in particular of the parts of the bulkhead and adjacent cover that support the spar caps directly against lateral bending. This relief becomes more pronounced as the bulkhead spacing decreases because the angle of twist of the first bulkhead with respect to the discontinuity bulkhead decreases and, consequently, a given amount of deformation will result in relatively larger relief.

The factor that tends to increase the lateral bending stresses is lack of integral action of the riveted spar cap. In an effort to estimate the magnitude of this factor, a few measurements were made to establish the variation of lateral bending stress over the width of the spar caps. The results of these measurements are shown in figure 10. It will be seen that the stresses in flanges 1 and 2 agree quite well with the calculated stresses. In flanges 3 and 4, the experimental points are displaced from the calculated straight line in such a way that they lie approximately on two lines parallel to the calculated line, one above it and one below it. The nature of the displacement tends to indicate that there is lack of integral action more in the primary two-spar action than in the lateral bending action. (This lack of integral action in lateral bending would result in steepening the slopes of the two lines to produce a saw-tooth diagram.) The data are insufficient to draw definite conclusions, but in any case it may be noted that the largest discrepancies shown in figures 6 and 8 occurred on flanges 3 and 4.

The simplified theory assumes that the stresses caused by lateral bending do not extend beyond the plane of the discontinuity into the region where the spar cap is not free. The stresses actually do extend beyond this point, of course, but they rapidly approach zero with increasing distance from the discontinuity. A study of figures 6 to 9 indicates that the lateral bending stresses

become negligible at a distance of about 8 inches from
the discontinuity; that is, at a distance equal to about
four times the width of the spar cap.

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APPENDIX

ANALYSIS OF OPEN BOX WITH WANDERING COVER

As an example of the method of·analysis, the stresses are analyzed near the discontinuity of an open box with wandering cover like the test specimen discussed in the main part of the present paper. Only the first bulkhead on either side of the discontinuity is assumed to be active in relieving the spars by converting some of the torque into lateral bending of the spar caps; it is also assumed that the two active bulkheads are equidistant from the discontinuity.

The vertical forces and bending moments acting on the spar caps are shown in figure 11. If W denotes the internal work, the unknown X can be obtained by the theorem of least work from the expression

$$\frac{\partial W}{\partial X} = 0 \qquad\qquad (1)$$

If only bending terms are considered, as is sufficiently accurate in most practical cases, this expression may be written

$$\frac{\partial W}{\partial X} = \int \frac{M}{EI} \frac{\partial M}{\partial X} \, dx = 0 \qquad\qquad (2)$$

where

M   bending moment

E   Young's modulus

I   moment of inertia associated with bending moment M

The coordinate x increases from the tip toward the root. The origin of x is chosen for each bay in such a way that a convenient expression for the bending moment is obtained.

Inspection of the expressions used shows the origins chosen for each bay.

The necessary expressions for M and $\dfrac{\partial M}{\partial X}$ are:

From B to D (fig. 11) (vertical moment in spar)

$$M = \frac{T}{b}L_1 + \frac{T}{b}x - Xx \tag{3}$$

$$\frac{\partial M}{\partial X} = -x \tag{4}$$

From D to E (vertical moment in spar)

$$M = \frac{T}{b}x - 2aX \tag{5}$$

$$\frac{\partial M}{\partial X} = -2a \tag{6}$$

From B to C or from C to D (lateral moment in spar caps)

$$M = X\frac{bx}{2c} \tag{7}$$

$$\frac{\partial M}{\partial X} = \frac{bx}{2c} \tag{8}$$

where

T   applied torque (fig. 3)

c   depth of box (fig. 3)

b   width of box (fig. 3)

The meanings of a, L_0, L_1, and L_3 are explained by figure 11. The expressions (3) to (8) are inserted in equation (2). Care must be taken to associate the

moment of inertia of the spar I_S with the bending moments in the spars, and the lateral moment of inertia of the spar cap I_F with the lateral bending moments in the spar caps; it must also be remembered that there are two spars and four spar caps. The integrations indicated in equation (2) lead to the formula

$$X = \frac{T}{ba} \frac{\frac{8}{3}a^2 + 2aL_1 + L_0^2 - L_3^2}{\frac{8}{3}a + 4(L_0 - L_3)\frac{b^2a}{6c^2}\frac{I_S}{I_F}} \qquad (9)$$

This formula was used in the analysis of the test specimen under the assumption that only one bulkhead was active at either side of the discontinuity. When two bulkheads are assumed to be active, it is not practicable to write final formulas corresponding to formula (9); it is more convenient to write and solve the equations replacing expression (1) numerically for each individual problem.

In the calculations for the test specimen, the equivalent area of the cover sheet was taken as zero at the tip and at the discontinuity. At stations 28 inches from these locations and at the root of the beam, the equivalent area was taken as one-sixth the actual area. Between the values defined in this manner, linear variation was assumed.

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1. Cambilargiu, E.: The Torsion of Box Beams with One Side Lacking. NACA TM No. 939, 1940.

2. Kuhn, Paul, and Chiarito, Patrick T.: Shear Lag in Box Beams - Methods of Analysis and Experimental Investigations. NACA Rep. No. 739, 1942.
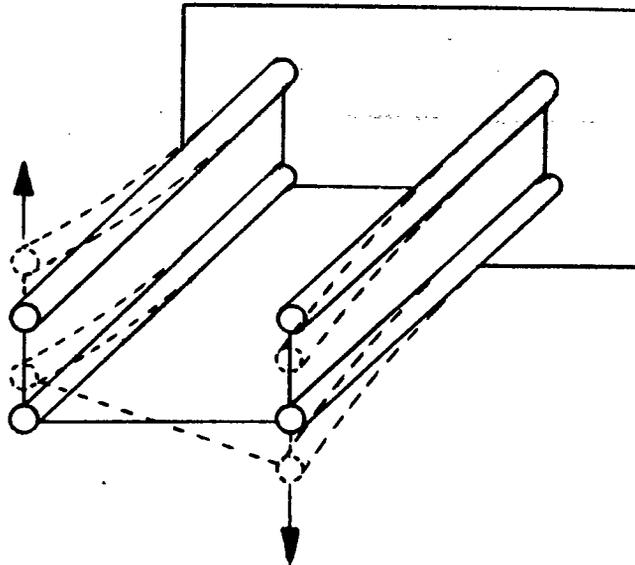
(a) Idealized structure.
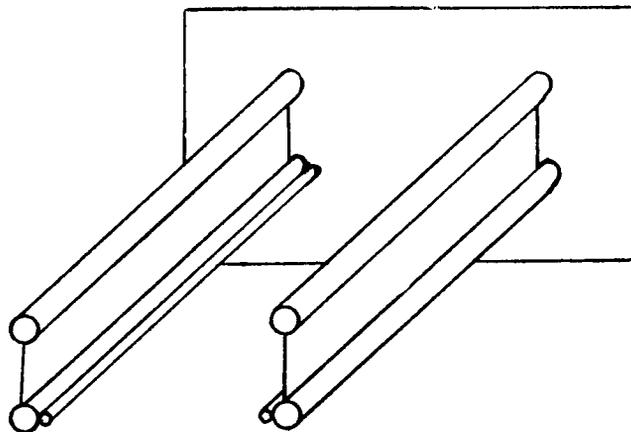
(b) Substitute structure.   NATIONAL ADVISORY
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Figure 1.- Open box without bulkheads subjected
to torsion.
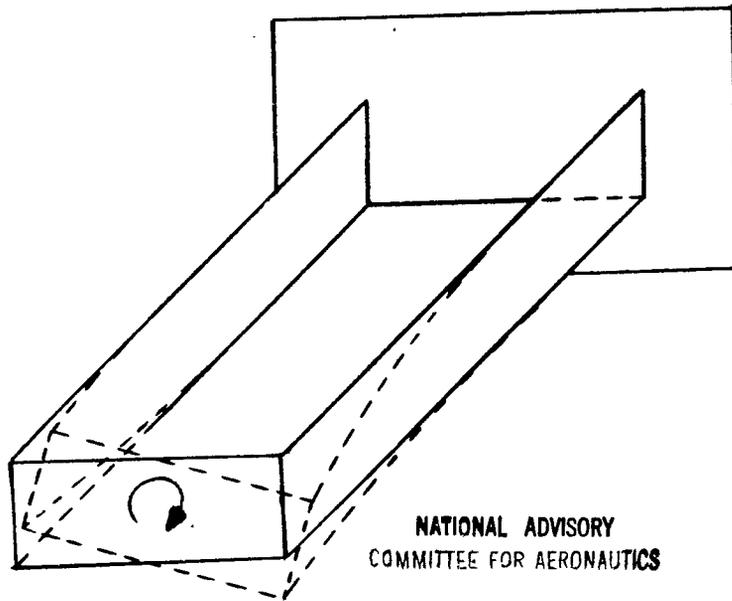
Figure 2.- Open box with a bulkhead subjected to torsion.
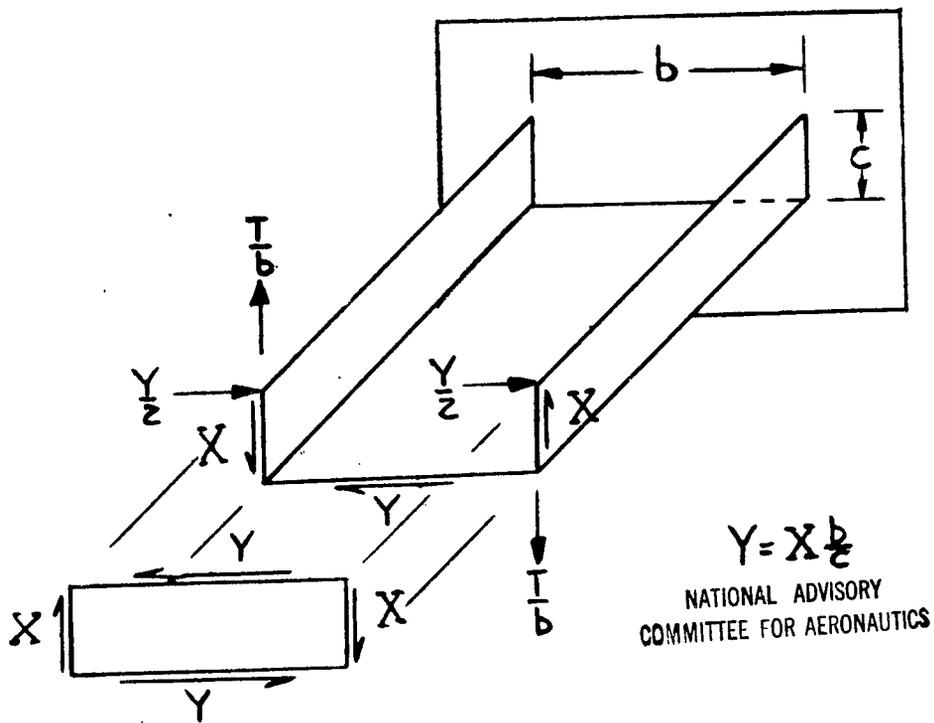
Figure 3.- Free-body diagrams of open box with bulkhead.

$$Y = X\frac{b}{c}$$

Fig. 4

Box and loading are symmetrical about plane of root bulkhead.

Root bulkhead

1×1×⅛ angle (steel)

.040 cover

⅛ bulkhead (steel)

1×1×⅛ angle

Angle A

Angle A

Angle A

.040 cover

.081 web

Root fixture

10

112

45

56

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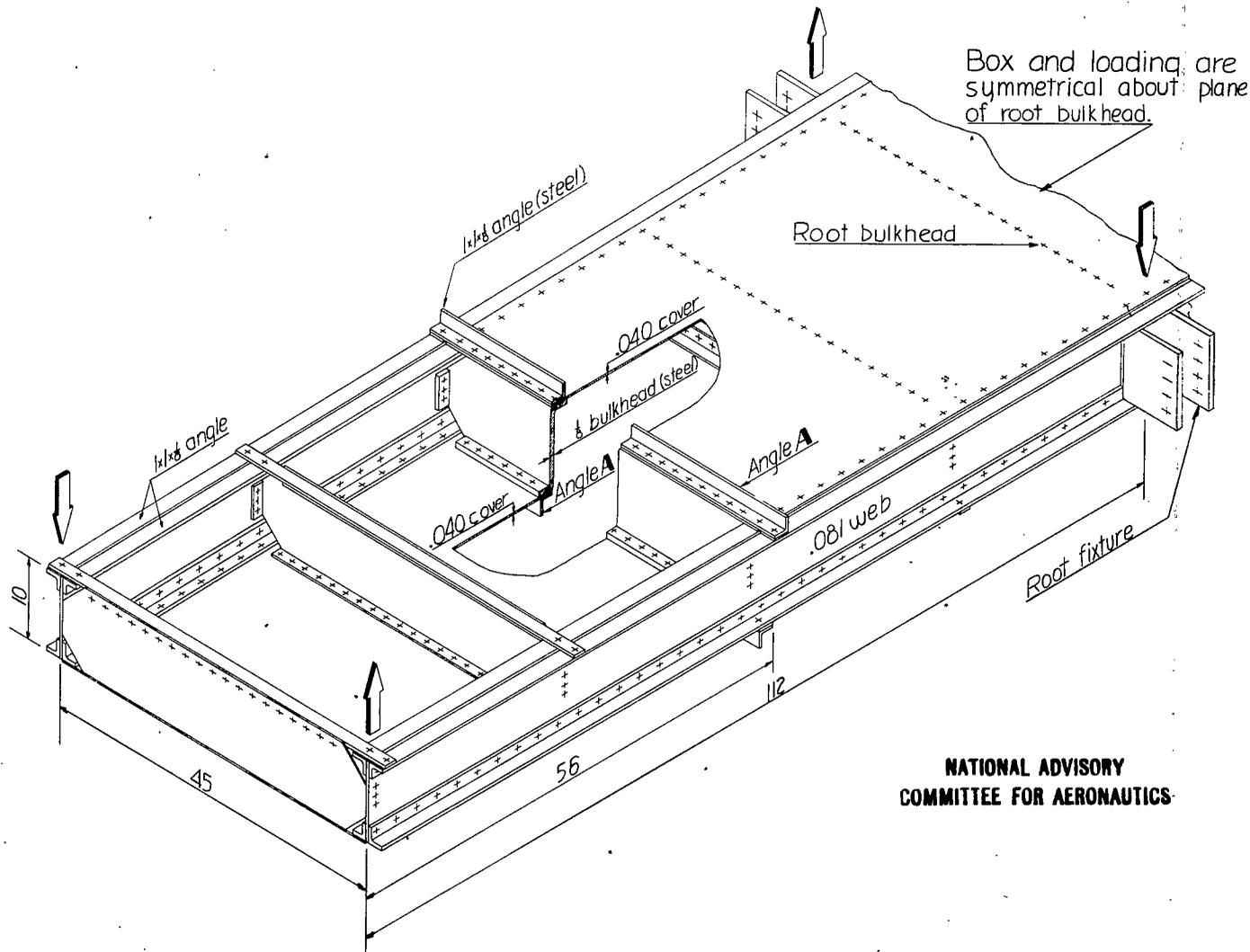
Figure 4.- Open box with wandering cover used as test specimen. All material 24S-T aluminum alloy unless otherwise specified.

Fig. 5
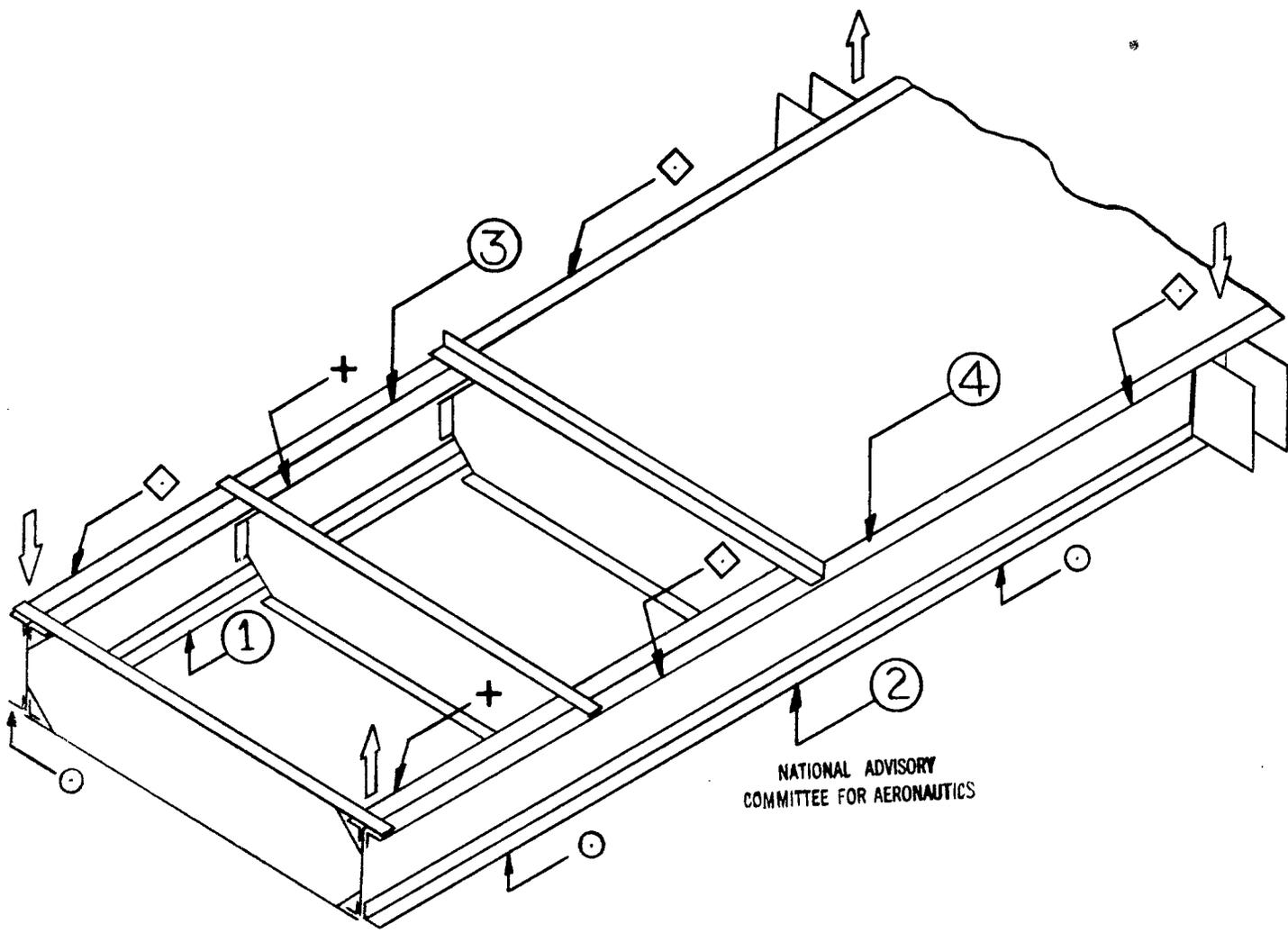
Figure 5.- Location of strain gages and designations of spar caps.
Symbols indicate strain gages; numbers, spar caps.

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Fig. 6
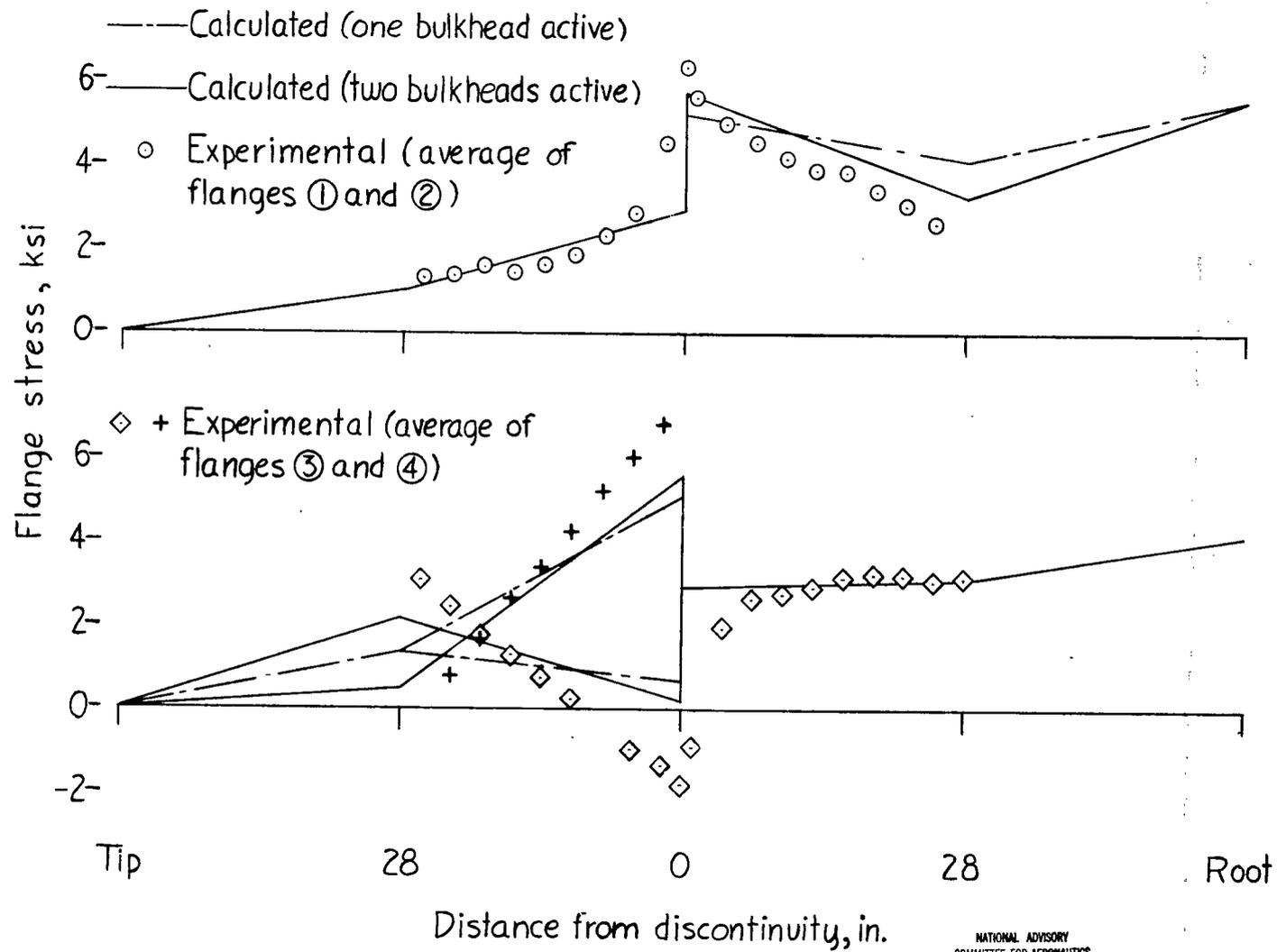
Figure 6.–Stresses in spar caps of open box with steel bulkheads spaced 28 inches.

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Fig. 7
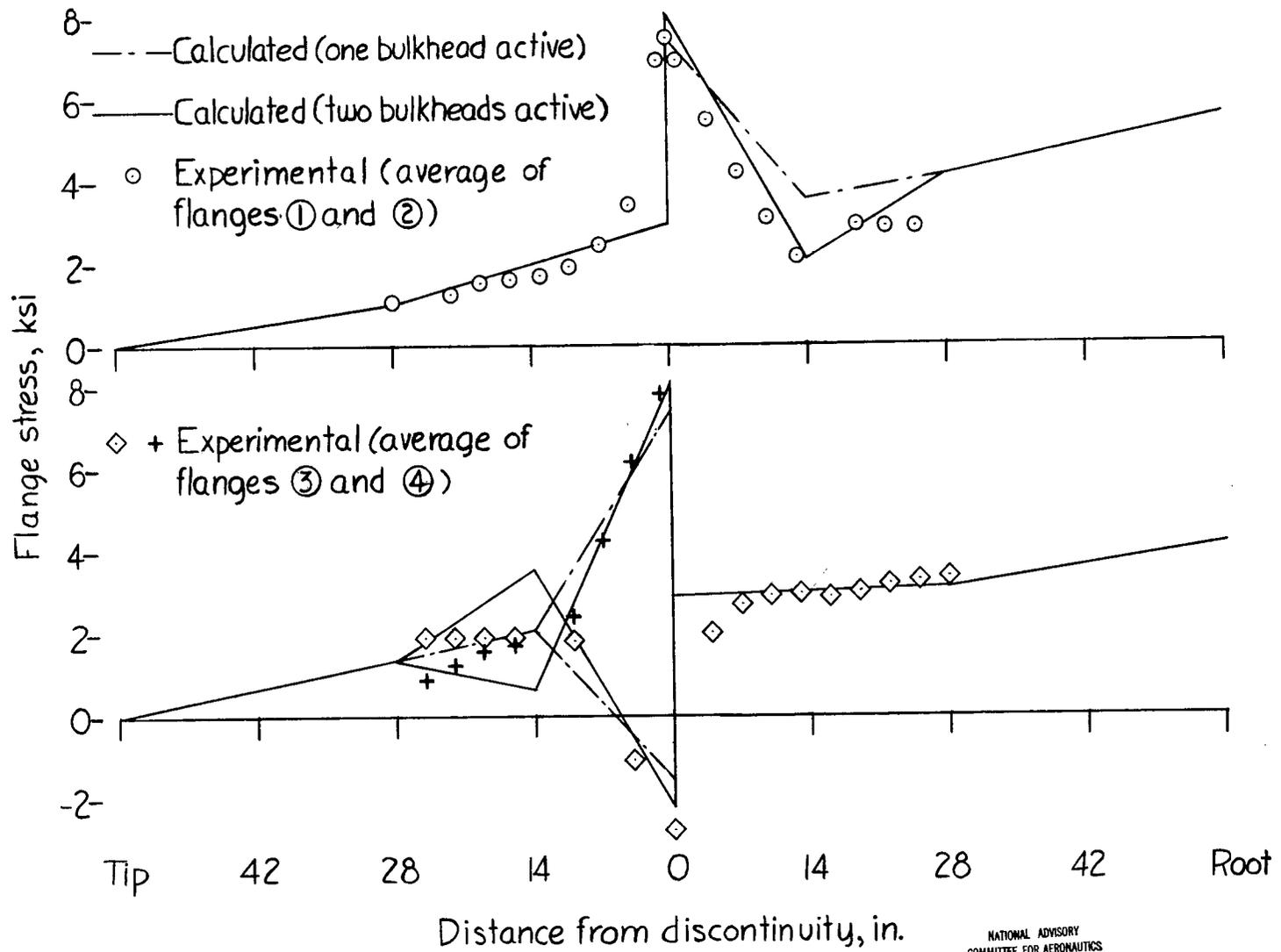
Figure 7.–Stresses in spar caps of open box with steel bulkheads spaced 14 inches.

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Fig. 8
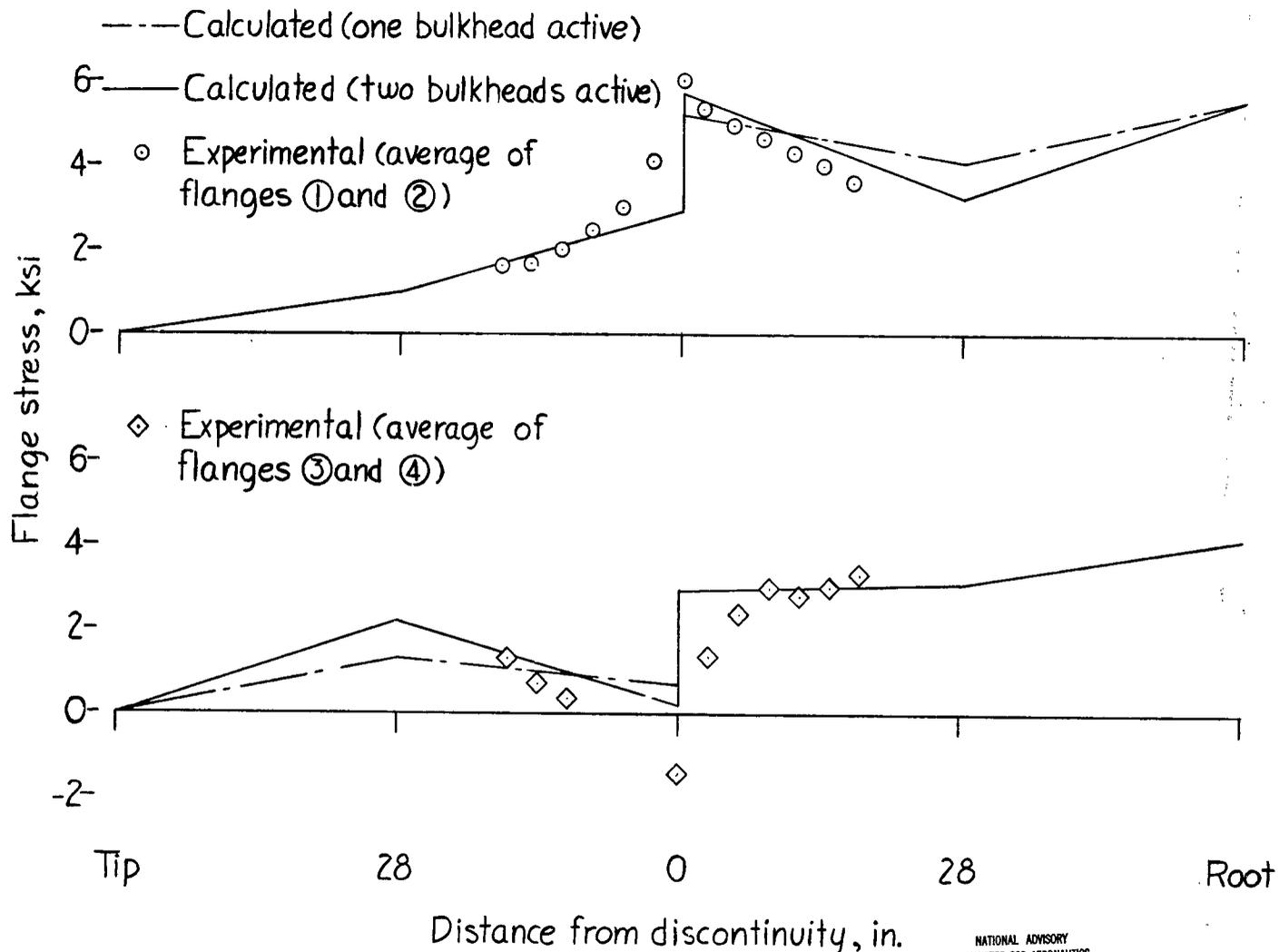
Figure 8.—Stresses in spar caps of open box with dural bulkheads spaced 28 inches.

Fig. 9

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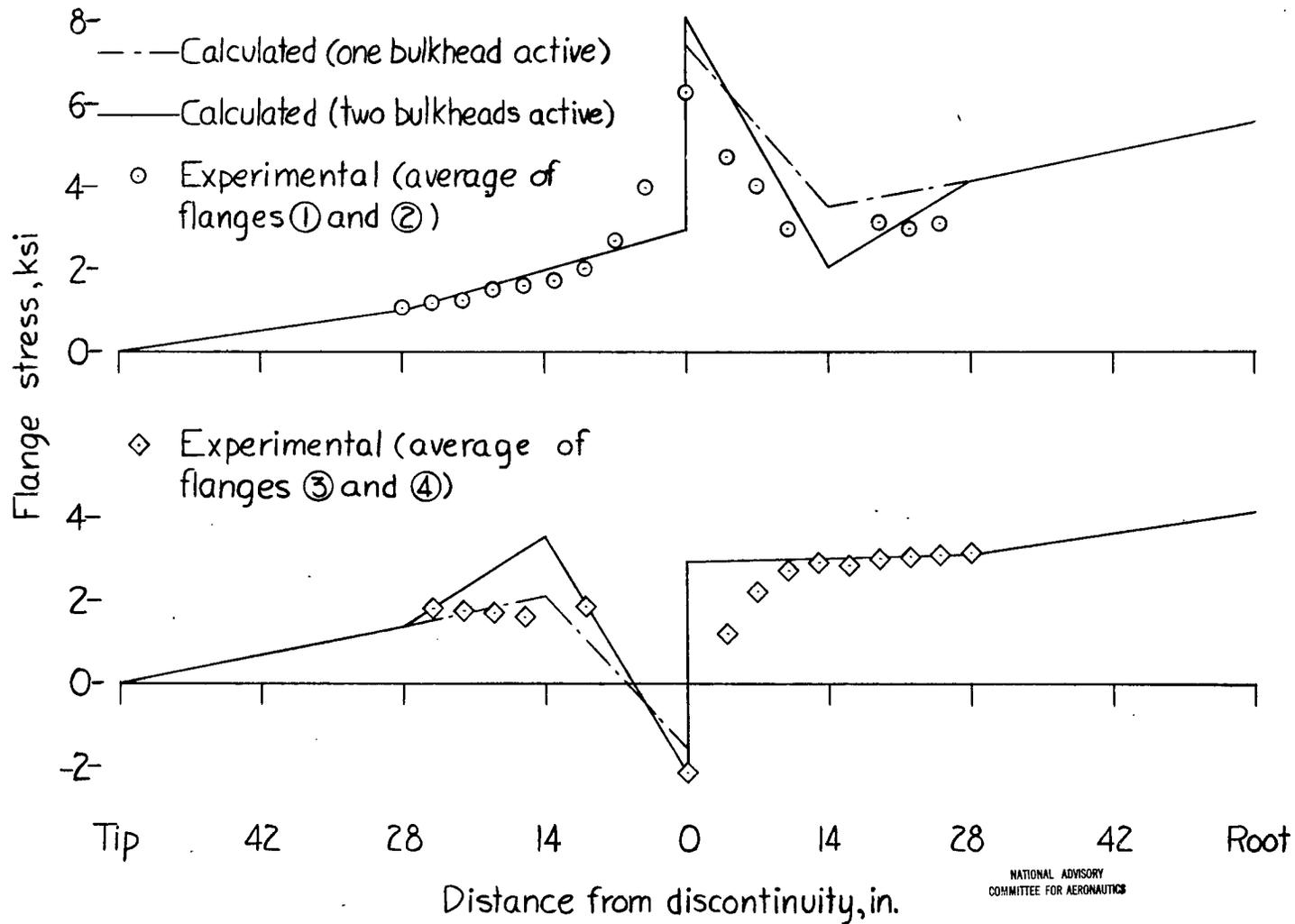
Figure 9.-Stresses in sparcaps of open box with dural bulkheads spaced 14 inches.
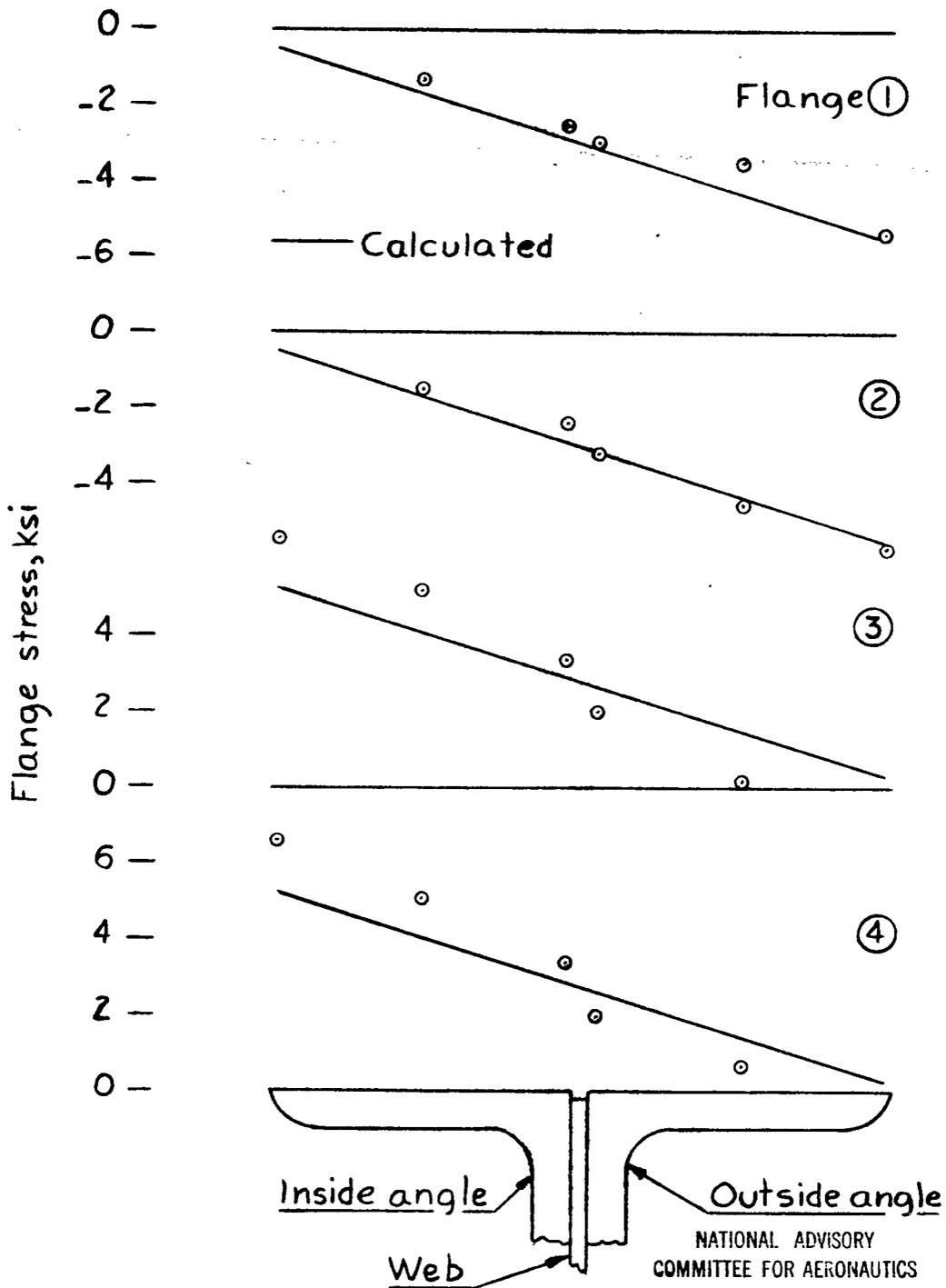
Figure 10.— Stress distribution across free spar caps
2 inches from the discontinuity. Dural
bulkheads spaced 28 inches.

Fig. 11   NACA ARR No. L4I23
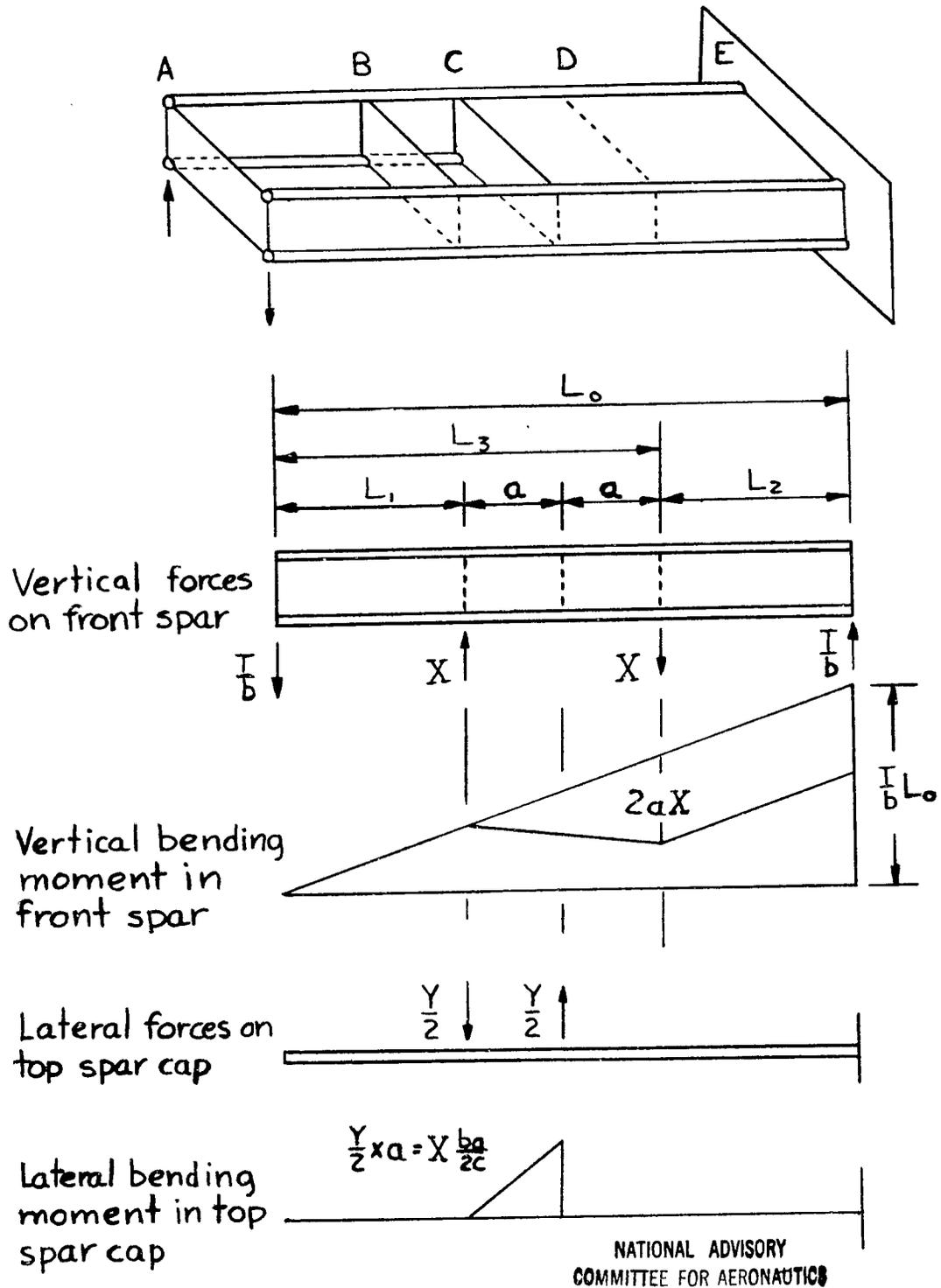
Figure 11.–Force and bending-moment diagrams.