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CALCULATION OF STICK FORCES FOR

AN ELEVATOR WITH A SPRING TAB

By Harry Greenberg

Langley Memorial Aeronautical Laboratory

Langley Field, Va.

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SUMMARY

Formulas for the calculation of hinge-moment characteristics of an elevator with a spring tab have been developed in terms of basic aerodynamic parameters, spring stiffness, and airspeed. The formulas have been used in a study of the stick-force gradients on a pursuit airplane equipped with an elevator with a spring tab. Charts are presented showing the variation of stick-force gradient in accelerated flight over a large range of speed and the complete range of spring stiffness for various center-of-gravity locations, altitudes, and airplane sizes.

It is shown that the stick-force gradient for the elevator with spring tab tends to decrease as the speed increases and for weak springs tends to approach the value corresponding to a pure servotab (no spring). This tendency is independent of altitude, size, or center-of-gravity location although the magnitudes vary with these parameters. The variation of stick-force gradient with center-of-gravity location is less for the spring-tab than for a linked-tab type of balance.

INTRODUCTION

On most types of control surface, balanced or unbalanced, the control force per unit deflection of the surface increases approximately as the square of the speed. On a spring-tab type of balanced control (reference 1), the amount of aerodynamic balance increases with speed; this condition results in a control force that increases less rapidly than the square of the speed. This type of control can be used to advantage on ailerons since it reduces the difference between the control force per unit helix angle $pb/2V$ at the high and low ends of the speed range.
The question has arisen as to whether the known advantages of the spring tab on the aileron could be realized for the elevator. The purpose of this report is to analyze the characteristics of the spring-tab control used as an elevator. General expressions, by which either the static or maneuvering stick forces for an elevator with a spring tab may be calculated, are developed and applied to the calculation of maneuvering forces for a typical pursuit airplane. The maneuvering stick forces for the same elevator arrangement with a servotab and with no tab are also presented for comparison. The effects of variations in spring stiffness, airspeed, altitude, center-of-gravity location, airplane size, and tab size are considered.

DESCRIPTION OF ELEVATOR-TAB SYSTEM

In the spring-loaded elevator-tab arrangement referred to herein, the control is connected directly to the tab, as in a servotab, and to the elevator through a spring. (See fig. 1.) This arrangement gives the control system characteristics that are between those of a servocontrolled elevator and an ordinary unbalanced elevator. A weak spring approaches the case of no spring, or pure servocontrol. A stiff spring approaches the case of a rigid connection, or an ordinary unbalanced elevator. As the speed is increased, the aerodynamic forces increase while the spring effect remains constant; effectively, the spring becomes weaker in comparison with the aerodynamic forces and the condition of pure servocontrol is approached.

In figure 1, BC is an idler that is free to pivot at the hinge of the elevator B. The control rod AC operates the tab through the linkage BCDE and operates the elevator through the spring and crank HC. The lengths of BC and BC are assumed equal in the analysis.

SYMBOLS

\[ A_w \] wing aspect ratio

\[ A_T \] tail aspect ratio
b wing span

\[ C_{He} \text{ hinge-moment coefficient about elevator hinge } \left( \frac{1}{2} \rho v^2 S_e c_e \right) \]

\[ C_{ht} \text{ hinge-moment coefficient about tab hinge } \left( \frac{H_t}{2} \right) \]

\[ C_L \text{ lift coefficient of wing } \left( \frac{L}{\frac{1}{2} \rho v^2 S_w} \right) \]

\[ C_{LT} \text{ lift coefficient of tail } \left( \frac{L_T}{\frac{1}{2} \rho v^2 S_T} \right) \]

\[ C_m \text{ pitching-moment coefficient about airplane center of gravity } \left( \frac{\text{Pitching moment}}{\frac{1}{2} \rho v^2 S_w} \right) \]

\[ \overline{c} \text{ mean chord of wing } \]

\[ \overline{c_e} \text{ mean chord of elevator } \]

\[ \overline{c_t} \text{ mean chord of tab } \]

\[ F_n \text{ stick-force gradient in maneuvers } \left( \frac{dF_s}{dn} \right) \]

\[ F_s \text{ stick force } \]

\[ F_1 \text{ force in spring; positive when in compression } \]

\[ F_2 \text{ force in control rod AC at C; positive in same sense as } F_3 \]

\[ F_3 \text{ force in control rod AC at A; positive as in figure 1 } \]

\[ g \text{ acceleration of gravity } \]

\[ H_e \text{ hinge moment about elevator hinge } \]
\( H_t \)  hinge moment about tab hinge
\( K \)  linkage ratio \( (l_1/l_2) \)
\( k_1 \)  spring constant, pounds per foot
\[ k_2 = \frac{\sqrt{1 - k^2} k_1 l_1^2}{\rho S_\epsilon \sigma_s} \]
\( L \)  lift of wing
\( l_s \)  length of control stick
\( L_n \)  distance between wing and tail
\( L_T \)  lift of tail
\( l_1 \)  length of arm BC
\( l_2 \)  length of arm DE
\( M \)  Mach number
\( m \)  mass of airplane
\( n \)  normal acceleration per g of airplane due to curvature of flight path; accelerometer reading minus component of gravity force
\( q \)  dynamic pressure
\( r \)  gearing ratio between control stick and rod
\( S_w \)  area of wing
\( S_e \)  area of elevator
\( S_T \)  area of tail surface
\( S_t \)  area of tab
\( V \)  airspeed
\( W \)  weight of airplane
\( x/c \)  distance between center of gravity of airplane and neutral point in fraction of mean wing chord
\( \alpha \) angle of attack at wing
\( \alpha_T \) angle of attack at tail
\( \delta_e \) deflection of elevator
\( \delta_s \) deflection of elevator control arm BC
\( \delta_t \) deflection of tab with respect to elevator
\( \theta \) angle of pitch of airplane
\( \varepsilon \) pitching velocity
\( D\theta \) nondimensional pitching velocity \((\bar{c}\dot{\theta}/2V)\)
\( \mu \) airplane-density parameter \((m/\rho S_w b)\)
\( \rho \) mass density of air

Whenever \( \delta_e, \delta_t, \alpha_T, \delta_s, \alpha, \) and \( D\theta \) are used as subscripts, a derivative is indicated; for example, 
\( C_{m\alpha} = \frac{\delta C_m}{\delta \alpha} \) and \( C_{mD\theta} = \frac{\delta C_m}{\delta D\theta} \). When a derivative or coefficient is written with a bar above it - for example, \( \bar{C}_{m\alpha} \) - the total derivative or coefficient is indicated, that is, the resultant or effective value which takes into account the floating tendency and spring action of the elevator with stick fixed.

All angles are measured in radians.

METHODS OF ANALYSIS

The basic assumptions involved in the analysis are as follows:

(1) Linkage ratio is constant

(2) Aerodynamic derivatives are constant over the range of deflections involved

(3) Effect of speed on the aerodynamic derivatives is given by the factor \( \frac{1}{\sqrt{1 - M^2}} \)
(4) Effect of power is neglected

(5) Effect of changes in forward speed during a pull-up is neglected

(6) Effect of horizontal tail flexibility is ignored

Assumption (1) is valid because the linkage ratio does not change appreciably within the small range of deflections that occur in flight. Assumption (2) is valid, according to low-speed wind-tunnel tests of the particular arrangement considered, for elevator and tab deflections up to 5°. A 10g pull-up is not likely to involve deflections greater than 5° on the airplane considered herein. The method of accounting for the effect of speed (assumption (3)), although approximately correct for factors involving lift of the wing and tail, is of doubtful validity for the factors involving hinge moments. As pointed out later, this correction for speed does not affect the comparison of plain elevator, spring-loaded tab, and servotab. The effect of power is to increase slightly the stick-force gradient. This increase would tend to counteract the effect of the spring, which decreases the stick-force gradient with increase in speed. Figure 2 is therefore strictly applicable only to gliding flight. The error involved in approximation (5) is believed to be small. The flexibility of the tail will increase the stick force gradient slightly at high speeds, according to reference 5. On the basis of these assumptions, the relations developed herein hold for small deflections.

From the geometry of the elevator-tab arrangement, it is obvious that

\[ \delta_t = K(\delta_s + \delta_e) \]

and that

\[ F_3 = F_1 + F_2 \]  

(1)

From the condition for equilibrium of the elevator system,

\[ F_3 L_1 = \left( \delta_e C_{he} \delta_e + \delta_t C_{ht} \delta_t + a_T C_{he\alpha_T} \right) q \frac{S}{e} \]  

(2)
From the condition for equilibrium of the tab, 

\[
F_{2T} = -\left( \delta_t C_{ht} \delta_t + \delta_e C_{ht} \delta_e + a_T C_{ht} a_T \right) q s_{T c} \tag{3}
\]

Combining expressions (1) to (3) gives

\[
\frac{F_{lll}}{q s_{e c} e} = \delta_e C_{he} \delta_e + \delta_t C_{he} \delta_t + a_T C_{he} a_T
\]

\[+ \frac{S_{t c}}{s_{e c}} \left( \delta_t C_{ht} \delta_t + \delta_e C_{ht} \delta_e + a_T C_{ht} a_T \right) \tag{4}
\]

The compression of the spring is \( l_1 (\delta_s + \delta_e) \); hence, \( F_1 = k_1 l_1 (\delta_s + \delta_e) \). Substituting in equation (4) gives

\[
\frac{k_1 l_1^2 (\delta_s + \delta_e)}{q s_{e c} e} = \delta_e C_{he} \delta_e + \delta_t C_{he} \delta_t + a_T C_{he} a_T
\]

\[+ \frac{S_{t c}}{s_{e c}} \left( \delta_t C_{ht} \delta_t + \delta_e C_{ht} \delta_e + a_T C_{ht} a_T \right) \tag{5}
\]

If the values of the aerodynamic coefficients on the right-hand side of equation (5) are obtained from low-speed data, they should be multiplied by \( \frac{1}{\sqrt{1 - M^2}} \) to apply to high speeds, according to the Glauert approximation or, if

\[
k_2 = \frac{\sqrt{1 - M^2} k_1 l_1^2}{q s_{e c} e}
\]
equation (5) may be written as

\[ k_2(\delta_s + \delta_e) = \delta_e C_{\theta e} \delta_e + \delta_t C_{\theta t} \delta_t + \alpha T C_{\alpha T} \]

\[ + \frac{S_t c_t}{S_e c_e} (\delta_t C_{\theta t} \delta_t + \delta_e C_{\theta t} \delta_e + \alpha T C_{\alpha T}) \]  

(6)

By substituting for \( \delta_t \) in terms of \( \delta_s \) and \( \delta_e \) in equation (6) and solving for \( \delta_e \), there is obtained

\[ \delta_e = \frac{\left( k_2 - KC_{\theta e} \delta_e - \frac{k_2 S_t c_t}{S_e c_e} C_{\theta t} \delta_t \right) \delta_s - \left( C_{\theta e T} + \frac{S_t c_t}{S_e c_e} C_{\theta t} \delta_e \right) \alpha T}{C_{\theta e} \delta_e - k_2 + KC_{\theta e} \delta_e + \frac{k_2 S_t c_t}{S_e c_e} C_{\theta t} \delta_t + \frac{S_t c_t}{S_e c_e} C_{\theta t} \delta_e} \]  

(7)

which determines the angle \( \delta_e \) at which the elevator floats in response to a control deflection \( \delta_s \) and angle of attack \( \alpha_T \). The tab angle is then determined by the linkage ratio. Equation (7) can be written in abbreviated form as

\[ \delta_e = A \delta_s + 3 \alpha_T \]  

(8)

Then,

\[ \delta_t = x(\delta_s + \delta_e) \]

\[ = k(1 + A) \delta_s + kE \alpha_T \]  

(9)
The substitution of equations (8) and (9) in the expression for the control force $F_3$ gives

$$
\bar{C}_{h_e} = \frac{F_3 l}{qS_e C_e}
$$

$$
= \left[ A_{h_e} + K(1 + A)C_{h_e} \delta_t \right] \delta_s + \left( B_{h_e} + KBC_{h_e} \delta_t + C_{h_e} \alpha_T \right) \alpha_T \tag{10}
$$

which gives the fundamental control hinge-moment derivatives $\bar{C}_{h_e} \delta_s$ and $\bar{C}_{h_e} \alpha_T$ as

$$
\bar{C}_{h_e} \delta_s = A_{h_e} + K(1 + A)C_{h_e} \delta_t \tag{11}
$$

and

$$
\bar{C}_{h_e} \alpha_T = B_{h_e} + KBC_{h_e} \delta_t + C_{h_e} \alpha_T \tag{12}
$$

When the stick is fixed, the elevator moves with changes in angle of attack in accordance with equation (7). As a result, the static-stability derivative $\bar{C}_{m_\alpha}$ and the damping in pitching $\bar{C}_{m_D\theta}$ are affected. They may be calculated by

$$
\bar{C}_{m_\alpha} = C_{m_\alpha} + C_{m_\delta_e} \frac{da_T}{d\alpha} + C_{m_\delta_t} \frac{KB}{d\alpha} \tag{13}
$$

and

$$
\bar{C}_{m_D\theta} = C_{m_D\theta} + C_{m_\delta_e} \frac{da_T}{d\theta} + C_{m_\delta_t} \frac{KB}{d\theta} \tag{14}
$$
The control effectiveness \( C_{m\delta_s} \) similarly depends on the motion of the elevator with respect to the control arm. The relation is

\[
\frac{C_{m\delta_s}}{C_{m\delta_e}} = C_{m\delta_e} A + C_{m\delta_t} K(1 + A) \tag{15}
\]

After the five fundamental derivatives are obtained by equations (11) to (15), the stick force per unit normal acceleration in a pull-up may be calculated. The formula for this stick-force gradient, which is taken from equations on page 14 of reference 2, is

\[
F_n = \frac{\rho S_e \overline{\sigma} \overline{\theta}}{\mu g} \left( \frac{\mu a_w \overline{C_{\theta e}}}{c_{L\alpha}} + \frac{\mu a_w \overline{C_{\theta e}}}{c_{L\alpha}} \frac{C_{ma}}{C_{m\delta_s}} \frac{C_{\theta e\delta_s}}{C_{m\delta_s}} \right) (16)
\]

for a mass-balanced elevator. In formula (16), total derivatives are used. Values of \( \overline{C_{\theta e\alpha}} \) and \( \overline{C_{\theta e\delta\theta}} \) are obtained from

\[
\overline{C_{\theta e\alpha}} = \frac{\partial C_{\theta e\alpha}}{\partial \alpha}, \quad \overline{C_{\theta e\delta\theta}} = \frac{\partial C_{\theta e\delta\theta}}{\partial \theta} \tag{17}
\]

The effect of compressibility must again be taken into account in using formula (16). All the derivatives in that expression should be multiplied by \( \frac{1}{\sqrt{1 - M^2}} \), if the data used in computing these derivatives are based on low-speed measurements. The factor cancels out except in the second and fourth terms. The corrected formula is

\[
F_n = \frac{\rho S_e \overline{\sigma} \overline{\theta}}{\mu g} \left( \frac{\mu a_w \overline{C_{\theta e}}}{c_{L\alpha}} + \frac{1}{\sqrt{1 - M^2}} \frac{\mu a_w \overline{C_{\theta e\delta\theta}}}{c_{L\alpha}} \right) (18)
\]
Spring, elevator, and tab deflections corresponding to any acceleration are calculated by the following formulas, derived by using equations (2) of reference 2:

\[
\frac{\delta_s}{n} = -\frac{\bar{\alpha}}{2v^2} \left( \frac{4A_w\mu c_m\alpha}{c_{Lq}c_m\delta_s} \sqrt{1 - \frac{\bar{\alpha}^2}{v^2}} + \frac{c_m\bar{\theta}}{c_m\delta_s} \right)
\]

\[
\frac{\alpha_T}{n} = \frac{\bar{\alpha}}{2v^2} \left( \frac{2A_w\mu + \frac{d\alpha_T}{d\theta}}{c_{Lq}} \right)
\]

A number of computations, based on typical airplane characteristics, have been made to illustrate the effect of spring stiffness on the characteristics of an elevator with a spring tab. The following airplane dimensions and derivatives are used:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W/S_w$, pounds per square foot</td>
<td>40</td>
</tr>
<tr>
<td>$\sigma$, feet</td>
<td>7</td>
</tr>
<tr>
<td>$l_a$, feet</td>
<td>2</td>
</tr>
<tr>
<td>$S_T/S_w$</td>
<td>0.18</td>
</tr>
<tr>
<td>$A_w$</td>
<td>6</td>
</tr>
<tr>
<td>$A_T$</td>
<td>4.5</td>
</tr>
<tr>
<td>$l_h/\sigma$</td>
<td>3.3</td>
</tr>
<tr>
<td>$c_{Lq}$</td>
<td>4.3</td>
</tr>
<tr>
<td>$d\alpha_T/d\alpha$</td>
<td>1/2</td>
</tr>
<tr>
<td>$d\alpha_T/d\theta$</td>
<td>6.6</td>
</tr>
<tr>
<td>$C_{ma}$ (for c.g. location 0.058 ahead of neutral point)</td>
<td>-0.232</td>
</tr>
<tr>
<td>$C_{mp\theta}$</td>
<td>-15.3</td>
</tr>
<tr>
<td>Altitude, feet</td>
<td>20,000</td>
</tr>
<tr>
<td>$\mu$</td>
<td>23.3</td>
</tr>
</tbody>
</table>

The following elevator and tab dimensions and derivatives are used:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_e$, square feet</td>
<td>17.44</td>
</tr>
<tr>
<td>$\sigma_e$, feet</td>
<td>1.48</td>
</tr>
<tr>
<td>$c_{T}/c_e$</td>
<td>0.386</td>
</tr>
<tr>
<td>$S_e/S_T$</td>
<td>0.323</td>
</tr>
<tr>
<td>$S_T/S_e$</td>
<td>0.114</td>
</tr>
</tbody>
</table>
\[ \frac{S_t^{\alpha_t}}{S_e^{\alpha_e}} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots

\begin{align*} 
S_t^{\alpha_t} &amp; = 0.044 \\
\kappa_1, \ \text{foot} &amp; = 0.5 \\
K &amp; = 1 \\
\alpha_{e_l} &amp; = -0.487 \\
\alpha_{e_r} &amp; = 1.84 \\
\alpha_{e_d} &amp; = -0.115 \\
\alpha_{e_d} &amp; = 0.115 \\
\alpha_{e_d} &amp; = 3.09 \\
\alpha_{e_d} &amp; = -1.044 \\
\alpha_{e_d} &amp; = -0.0615 \\
\alpha_{e_d} &amp; = 0 \\
\end{align*}

From equation (7),

\[
A = \frac{k_2 + 0.130}{-k_2 - 0.622} \quad B = \frac{0.115}{-k_2 - 0.622}
\]

From equation (10),

\[
\alpha_{\delta e} = \frac{-0.487k_2 - 0.0067}{-k_2 - 0.622} \quad \alpha_{\delta t} = \frac{0.115k_2 + 0.0023}{-k_2 - 0.622}
\]

From equations (13) to (15),

\[
\begin{align*}
\bar{C}_{\alpha} &amp; = -0.232 - \frac{0.0635}{-k_2 - 0.622} \\
\bar{C}_{\alpha} &amp; = -15.3 - \frac{0.838}{-k_2 - 0.622} \\
\bar{C}_{\alpha} &amp; = -1.106 \frac{k_2 + 0.130}{-k_2 - 0.622} - 0.0615
\end{align*}
\]

These values can be substituted in formulas (17) and (18) to obtain \( F_n \).
RESULTS AND DISCUSSION

The computed values of the stick-force gradient in maneuvers $F_n$ for the assumed airplane and elevator are plotted as a function of speed in figure 2, for various values of the spring constant $k_1$. The top curve, for infinite spring stiffness, applies to an ordinary unbalanced elevator, for which the spring is replaced by a rigid rod. The bottom curve applies to a pure servocontrol, for which the spring is removed. The intermediate curves are for the cases in which springs of various stiffnesses are connected between the control rod and elevator. The increase in stick-force gradient with speed for very high and very low values of spring stiffness is based on the assumption of the effect of compressibility mentioned previously and is not important for the purposes of this report. The important fact is that the addition of a spring reduces the stick-force gradient in the manner shown. A very weak spring reduces the stick-force gradient at the high end of the speed range to a value only slightly higher than that of the pure servocontrol. In the case corresponding to complete servo-operation (no spring) in figure 2, the stick force is less than the minimum value considered desirable. This value could be increased by using a tab of increased chord. Increasing the span of the tab would have no appreciable effect on the stick-force gradients because the increased forces on the tab are compensated by the reduced deflections needed. Other methods of reducing the stick forces, such as the linked balancing tab, would result in a slight increase of stick-force gradient with speed as is the case of the top and bottom curves of figure 2.

It is sometimes considered desirable to have direct control until a certain stick force is reached after which the tab control begins to function. This direct control is accomplished by preloading the spring by an amount that depends on the stick force at which it is desired to have the tab come into action. The stick force varies with acceleration in the manner shown in figure 3. The curve has the slope for infinite spring stiffness up to a certain point, as indicated by the solid line, and then has the slope corresponding to the spring stiffness used, as indicated by the dashed line.
The point where the slope changes of course depends on the preload in the spring. Such an arrangement might be useful in maintaining reasonable control forces for pull-outs at very high speeds.

The effect of increased airplane size is shown in figure 14. The wing loading and control gearing $L_s$ are assumed the same as in figure 2, but all lengths are assumed doubled. The stick-force gradient for pure servo-operation ($K_1 = 0$) is somewhat higher than the value considered desirable and any appreciable amount of spring stiffness would make the stick-force gradient too large. For this case, a tab with smaller chord could be used to give lower stick forces.

The effect of altitude on the variation of stick-force gradient with speed is shown in figure 5. An increase in altitude reduces the stick-force gradient by an amount that does not vary appreciably with speed. The loss in stick force in pounds decreases as the spring stiffness is decreased.

The effect of center-of-gravity location on the stick-force gradient for several types of balanced elevator is shown in figure 6. The elevator with spring tab shows the smallest change of stick-force gradients. The linked-tab balance chosen for comparison was assumed to be so linked as to give the same stick-force gradient as the elevator with spring tab for one particular center-of-gravity location ($\frac{X}{C} = 0.05$). The variation of stick-force gradient with center-of-gravity location is less with the spring tab than with the linked tab because $C_{he_{\alpha}}$ is reduced as well as $C_{he_{\delta}}$; this condition permits a smaller $C_{he_{\delta}}$ for a given stick-force gradient. As shown in equation (18), the variation in stick-force gradient with $C_{m_{\alpha}}$, which depends on center-of-gravity location, is proportional to $C_{he_{\delta}}$. 
CONCLUSIONS

Formulas have been developed for the calculation of hinge-moment characteristics of an elevator with a spring tab. The analysis included basic aerodynamic parameters, spring stiffness, and airspeed and indicated the following conclusions:

1. The stick-force gradients for an elevator with a spring tab tend to decrease as the speed increases. For a weak spring at high speeds, the stick force approaches that of a pure servocontrol.

2. The variation of stick-force gradient with center-of-gravity location is less for an elevator with a spring tab than with a linked tab.

3. Increase in altitude reduces the stick-force gradients by a nearly constant amount over the speed range, for a given spring stiffness. The amount of reduction in the stick-force gradient decreases as the spring stiffness decreases.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va.

REFERENCES


Figure 1. - Spring-loaded elevator-tab combination.
Figure 2.- Variation of stick-force gradient with speed for elevator with spring tab. $k_1$, spring stiffness; wing span, 42 feet.
Figure 3.— Effect of spring preload on stick-force gradient.
Figure 4.—Variation of stick-force gradient with speed. $k_1$, spring stiffness; wing span, 84 feet.
Figure 5.— Variation of stick-force gradient with speed at sea level and 20,000 feet. $k_1$, spring stiffness; wing span, 42 feet.
Figure 6. - Variation of stick-force gradient with center-of-gravity location for various types of balance.