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SUMMARY

Equations are presented for the stick force per g in maneuvers obtained with a geared spring tab. A geared spring tab, as defined herein, differs from an ordinary spring tab in that, when the elevator is moved with the stick free at zero airspeed, the tab moves with respect to the elevator in the same manner as a conventional geared, or balancing, tab.

The geared spring tab is shown to present the theoretical possibility of obtaining a value of force per g independent of speed regardless of the spring stiffness. If the geared spring tab is used in conjunction with an elevator that has zero variation of hinge moment with angle of attack, the force per g may be made independent of speed at any center-of-gravity location. A suitably designed geared spring tab will provide adequate ground control, small sensitivity of the control forces to slight changes in the elevator hinge-moment parameters, and substantially no variation of stick force per g with speed. The geared spring tab is shown to be most suitable for application to large airplanes.

INTRODUCTION

An analysis of the elevator control forces obtained with spring tabs was presented in reference 1. Two types of spring tab were discussed: the ordinary, or ungeared, spring tab (fig. 1) and the geared spring tab (fig. 2). The geared spring tab differs from the ordinary spring tab in that when the elevator is moved with the stick free at zero airspeed, the tab moves with respect to the elevator in the same manner as a conventional geared, or balancing, tab. Although the calculations and discussion of reference 1 were concerned mainly with ordinary spring tabs, the advantages of geared spring tabs were pointed out. The geared spring tab presents the theoretical possibility of obtaining a value of force per g in maneuvers that does not vary with speed even though a stiff spring is used to provide adequate ground control. The present report briefly outlines the theory of the geared spring tab, gives formulas for use in design, and indicates more completely the practical possibilities and limitations of the device.

DISCUSSION

Difficulties have been experienced with conventional types of elevator balance on large airplanes, because the elevator must be very closely balanced and because small changes in the hinge-moment parameters cause large changes in the control forces. The possibility of using a servotab to avoid these difficulties was explained in reference 1. In tests of a control surface equipped with a servotab, which is defined as the system shown in figure 1 with the spring omitted, the pilots considered this arrangement undesirable because the elevator did not follow smoothly movements of the stick when the airplane was on the ground, taxying, or making landings and take-offs. A banging action of the control was experienced because the elevator did not move until the tab had hit its stops. A spring tab provides a mechanical connection between the stick and the elevator that relieves this difficulty. When a spring tab is used, the force per g varies with speed. This variation may be reduced to an acceptable amount by using a tab spring sufficiently flexible to make the control behave essentially as a servotab at normal flight speeds. The ground control provided by this flexible spring might be considered acceptable but a stiffer spring would be very desirable, especially on large airplanes that have elevators with high moments of inertia.

The equations for the stick forces with a spring tab were presented in reference 1. In the appendix of the present paper, these equations are extended to allow calculation of the stick forces with a geared spring tab. The force per g obtained with an ordinary spring tab has been shown to vary with speed. As the speed approaches zero the force per g approaches that obtained with the tab fixed and, at very high speeds, approaches the value for a servotab. With a geared spring tab, as the speed approaches zero the force per g is shown to approach that of an equivalent balancing tab and, at very high speeds, is shown to approach the value for a servotab. The geared spring tab therefore provides a means of reducing the force per g at low speeds while leaving the force per g at high speeds unchanged. The force per g may theoretically be made to remain constant throughout the speed range, no matter what spring stiffness is used. This arrangement therefore embodies the advantage provided by either the conventional balance or the servotab, namely, that the stick-force gradient does not vary with speed. The undesirable sensitivity of the conventional balance to small changes in hinge-moment characteristics and the poor ground control of the servotab are avoided by the geared spring tab.

In order to compare the merits of conventional types of balance, ungeared spring tabs, and geared spring tabs, the stick-force characteristics have been computed for an airplane of the medium-bomber class (weight, 50,000 lb) with the various types of elevator control. The results of these calculations are shown in figure 3. The characteristics of the airplane and of the tab systems that were used in the calculations are given in tables I and II, respectively. (All symbols are defined in appendix A.) The stick forces of a closely balanced elevator with conventional balance (as, for example, a balancing tab) are shown in figure 3(a). The critical nature of the balance is also shown by the large changes in stick-force gradients caused by changes in $\partial C_{h_e}/\partial \delta_e$ and $\partial C_{h_e}/\partial \alpha_T$ of -0.001 per degree. Variations of this order of magnitude may result from slight differences in contours of the elevator, within production tolerances, on different airplanes of the same type. The characteristics of an ungeared spring tab are illustrated in figure 3(b). The spring constant has been chosen to provide a fair degree of ground control without excessive variation of force per g with speed at normal flight speeds. The criterion for the choice of this spring stiffness was presented in reference 1. For the airplane under consideration, the spring stiffness is such as to require a stick force of 100 pounds to deflect the tab 1 radian at zero airspeed when the elevator is held fixed.

The characteristics of a geared spring tab that was designed to provide the same control-force characteristics as the conventional balance are shown in figure 3(c). The method of calculating the values of the hinge-moment parameters and gear ratio that were used to obtain stick-force gradients independent of speed is given in appendix B. The same characteristics will be obtained with any spring stiffness.

The exact values of hinge-moment parameters required to give the characteristics shown in figure 3(c) will not be attained in practice. It is therefore desirable to investigate the effects of changing the hinge-moment parameters slightly. If the spring in the geared spring tab had infinite stiffness, the system would be identical with the balancing tab (fig. 3(a)) and the stick forces would be equally sensitive to small changes in hinge-moment parameters. The spring stiffness must therefore be limited to a point at which normal changes in $\partial C_{h_e}/\partial \delta_e$ and $\partial C_{h_e}/\partial \alpha_T$ do not cause large changes in the stick-force characteristics.

In order to determine the effects of errors in the values of $\partial C_{h_e}/\partial \delta_e$ and $\partial C_{h_e}/\partial \alpha_T$ when a finite value of spring stiffness is used, the stick forces have been computed for a geared spring tab that has the same spring stiffness as the ungeared spring tab of figure 3(b). The effects of changing $\partial C_{h_e}/\partial \delta_e$ and $\partial C_{h_e}/\partial \alpha_T$ by -0.001 for the geared spring tab are shown in figures 4(a) and 4(b), respectively. Some variation of force per g with speed is introduced but the variation is considerably smaller than that normally encountered with the ungeared spring tab (fig. 3(b)). Inasmuch as a greater variation of force per g with speed probably can be tolerated, an increase in spring stiffness to improve the ground control appears desirable.

The changes in $\partial C_{h_e}/\partial \delta_e$ and $\partial C_{h_e}/\partial \alpha_T$ cause changes in the order of magnitude of the stick forces as well as some variation in force per g with speed. These changes are, however, much smaller than those that occur with the conventional balance (fig. 3(a)). At high speeds, in fact, they approach the changes that would occur if a servotab were used.

The effect of changing the gear ratio of the geared spring tab from its ideal value is shown in figure 4(c).

The effect of changing the gear ratio is nearly equivalent to changing the value of $\partial C_{h_e}/\partial \delta_e$. An error in providing the ideal value of $\partial C_{h_e}/\partial \delta_e$ on an actual airplane may therefore be corrected by suitable adjustment of the gear ratio.

The geared spring tab used to obtain the characteristics shown in figure 3(c) had values of the hinge-moment parameters $\partial C_{h_e}/\partial \alpha_T$ and $\partial C_{h_t}/\partial \alpha_T$ equal to zero. The equations given in appendix B show that this condition must be satisfied if the stick-force gradient is to be independent of speed at any center-of-gravity location. The value of $\partial C_{h_e}/\partial \alpha_T$, in practice, may be made equal to zero by use of elevators with a beveled trailing edge or with horn balances. The value of $\partial C_{h_t}/\partial \alpha_T$ is normally very small and may likewise be adjusted by varying the trailing-edge angle. If the values of $\partial C_{h_e}/\partial \alpha_T$ and $\partial C_{h_t}/\partial \alpha_T$ are not equal to zero, the force per g may still be made independent of speed by use of a geared spring tab for one particular center-of-gravity location, but the force per g will vary somewhat with speed at other center-of-gravity locations.

The effect of an increase in altitude on the stick-force gradients obtained with a geared spring tab is to shift forward the center-of-gravity location for zero force per g (the maneuver point) and to leave the slopes of the curves of force per g against center-of-gravity location unchanged. In this respect, the geared spring tab may be shown to follow the same rules as a conventional elevator. The stick-force variation with speed in straight flight is related to the force per g in maneuvers in the same way for a spring-tab elevator as for a conventional elevator.

The application of spring tabs to airplanes of various sizes was considered in reference 1. The results of this analysis, in general, may be applied to the geared spring tab. In order to avoid excessive stick-force variation with speed with an ordinary spring tab, the spring must be sufficiently flexible to make the control behave essentially as a servotab in the normal-flight speed range. The stick-force gradient obtained with a geared spring tab must also equal that of a servotab if force variation with speed is to be avoided. Because the stick forces obtained with a servotab result from the aerodynamic hinge moments on the tab, some difficulty may be encountered in providing sufficiently heavy stick-force gradients with normal tab designs on airplanes much smaller than the 50,000-pound airplane considered in the present report. The calculations of reference 1 indicated that sufficiently heavy stick forces may be provided on an airplane weighing about 16,000 pounds, but a large tab-to-stick gear ratio and a tab having a rather wide chord are required. These features increase the difficulty of preventing flutter.

Because the stick-force gradients obtained with a spring tab on small airplanes are undesirably low, the use of a bobweight in conjunction with the spring tab has been proposed to obtain desirable stick-force gradients in steady maneuvers. Flight tests showed this arrangement to be unsatisfactory because of undue lightness of the stick forces for sudden or rapid movements of the control stick. The reason for this undesirable control "feel" is that the elevator may be suddenly moved to large deflections because the aerodynamic hinge moments on the tab are small. After a certain time lag, the acceleration builds up and causes the bobweight moment to be felt by the pilot. These effects are discussed more fully in reference 2. The preceding considerations indicate that the geared spring tab may prove unsatisfactory on small airplanes. On large airplanes, for which sufficiently large stick forces result from the aerodynamic hinge moments on the tab, the geared spring tab should be satisfactory.

CONCLUSIONS

An analysis of the characteristics of geared spring tabs for elevator control has led to the following conclusions:

1. By means of a geared spring tab, it is theoretically possible to provide a value of stick-force gradient in maneuvers that does not vary with speed, no matter what spring stiffness is used. If the geared spring tab is used in conjunction with an elevator that has zero variation of hinge moment with angle of attack, the force

per g may be made independent of speed at any center-of-gravity location.

2. A geared spring tab may be designed to provide adequate ground control and small sensitivity of the control forces to slight changes in the hinge-moment parameters. The poor ground control associated with a servotab and the sensitivity of a conventional balance to small changes in hinge-moment parameters may therefore be avoided.

3. The geared spring tab appears to be most suitable for application to large airplanes.

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APPENDIX A

SYMBOLS

W	weight
b	span
S	wing area
c	chord
l	tail length
S_T	tail area
$\left(\frac{dC_L}{d\alpha}\right)_w$	slope of lift curve of wing
ϵ	downwash angle
q	dynamic pressure
C_L	lift coefficient
I	elevator moment of inertia
$\frac{\partial C_{L_T}}{\partial \delta_e}$	variation of lift coefficient of tail with elevator angle
τ	elevator effectiveness factor $\left(\frac{\partial C_{L_T}/\partial \delta_e}{\partial C_{L_T}/\partial \alpha_T}\right)$
K_1	ratio of stick movement to elevator deflection, tab fixed; normally positive
K_2	ratio of stick movement to tab deflection, elevator fixed; normally negative
K_3	ratio of stick force to tab angle at zero airspeed, elevator fixed; normally positive

K_4 ratio of stick force to elevator angle at zero airspeed; elevator held in deflected position by external means, tab deflection held at zero by application of required force at control stick; positive for balancing tab

H hinge moment

C_h hinge-moment coefficient $\left(\frac{H}{qbc^2}\right)$

δ_e elevator deflection (positive down)

δ_t tab deflection from elevator (positive down)

x_s stick deflection (positive forward)

F stick force (pull force positive)

α angle of attack of wing

α_T angle of attack of tail

ρ mass density of air

n normal acceleration in g units

g acceleration of gravity $\left(32.2 \text{ ft/sec}^2\right)$

x distance between center of gravity and stick-fixed neutral point in straight flight (positive when center of gravity is rearward)

$$A = \frac{W\left(1 - \frac{d\epsilon}{d\alpha}\right)}{\left(\frac{dC_L}{d\alpha}\right)_w S} + g\frac{\rho}{2}l$$

$$B = \frac{Wx}{\frac{\partial C_{L_T}}{\partial \delta_e}\frac{q_T}{q}S_T l} - \frac{1}{T}g\frac{\rho}{2}l$$

$\left(\frac{dC_{h_e}}{d\alpha_T}\right)_{tf}$ variation of elevator hinge-moment coefficient with angle of attack of tail, measured with tab free

$\left(\frac{dC_{h_e}}{d\delta_e}\right)_{tf}$ variation of elevator hinge-moment coefficient with elevator angle, measured with tab free

Subscripts:

t tab

e elevator

T tail

b value for equivalent balancing tab

APPENDIX B

EQUATIONS FOR ELEVATOR FORCES WITH GEARED SPRING TAB

The tab system considered is shown in figure 2. The mechanical characteristics of the linkage are completely determined when four constants are specified. These constants are defined by the following equations:

$$x_s = K_1\delta_e + K_2\delta_t \tag{1}$$

$$F = K_3\delta_t + K_4\delta_e \tag{2}$$

Equation (2) applies when the airspeed is zero. The ratio between the tab deflection and the elevator deflection, stick fixed, equals $\frac{-K_1}{K_2}$ and the ratio between the tab deflection and the elevator deflection at zero airspeed, stick free, equals $\frac{-K_4}{K_3}$. The ratio $\frac{K_4}{K_3}$ is defined as the linkage ratio of an equivalent balancing tab. When the system is in equilibrium, the relations between stick force, elevator hinge moments, and tab hinge moments are given in terms of these constants by the expressions

$$\left.\begin{aligned} \Delta F &= \frac{\Delta H_e - \Delta H_t \frac{K_4}{K_3}}{K_1 - \frac{K_4}{K_3}K_2} \\ \Delta F &= \frac{\Delta H_t}{K_2} + K_3\left(\Delta\delta_t + \frac{K_4}{K_3}\,\Delta\delta_e\right) \end{aligned}\right\} \tag{3}$$

The changes in elevator and tab hinge moments for any type of maneuver are given by the equations

$$\Delta H_e = \left(\Delta\alpha_T\frac{\partial C_{h_e}}{\partial \alpha_T} + \Delta\delta_e\frac{\partial C_{h_e}}{\partial \delta_e} + \Delta\delta_t\frac{\partial C_{h_e}}{\partial \delta_t}\right)q_T b_e c_e^{\,2} \tag{4}$$

$$\Delta H_t = \left(\Delta\alpha_T\frac{\partial C_{h_t}}{\partial\alpha_T} + \Delta\delta_e\frac{\partial C_{h_t}}{\partial\delta_e} + \Delta\delta_t\frac{\partial C_{h_t}}{\partial\delta_t}\right)q_T b_t c_t^2 \qquad (5)$$

In the present report, as in reference 1, the stick force required in a gradual pull-up is used as a criterion of the elevator-control characteristics. The stick force required in a pull-up, or any other maneuver, may be determined by substituting the appropriate values for $\Delta\alpha_T$ and $\Delta\delta_e$ and solving equations (3), (4), and (5) simultaneously. If the tab is assumed to have a negligible effect on the lift of the tail, the values of $\Delta\alpha_T$ and $\Delta\delta_e$ in a gradual pull-up are

$$\Delta\alpha_T = \left[\frac{W\left(1 - \frac{d\epsilon}{d\alpha}\right)}{\left(\frac{dC_L}{d\alpha}\right)_W qS} + \frac{g\frac{\rho}{2}l}{q}\right](n - 1) \qquad (6)$$

$$\Delta\delta_e = \left(\frac{Wx}{\frac{\partial C_{L_t}}{\partial\delta_e} q_T S_t l} - \frac{1}{\tau}\,\frac{g\frac{\rho}{2}l}{q}\right)(n - 1) \qquad (7)$$

For convenience, the solution of these equations for an ungeared spring tab ($K_4 = 0$), which was derived in reference 1, is presented first. The force per g is given by

$$\frac{\partial F}{\partial n} = \frac{\frac{1}{K_1}\left\{A\left[\left(\frac{\partial C_{h_e}}{\partial\alpha_T}\right)_{tf} + \frac{K_2K_3\frac{\partial C_{h_e}}{\partial\alpha_T}}{\frac{\partial C_{h_t}}{\partial\delta_t}q_T b_t c_t^2}\right] + B\left[\left(\frac{\partial C_{h_e}}{\partial\delta_e}\right)_{tf} + \frac{K_2K_3\frac{\partial C_{h_e}}{\partial\delta_e}}{\frac{\partial C_{h_t}}{\partial\delta_t}q_T b_t c_t^2}\right]\right\}\frac{q_T}{q}b_e c_e^2}{1 - \frac{K_2\frac{\partial C_{h_e}}{\partial\delta_t}b_e c_e^2}{K_1\frac{\partial C_{h_t}}{\partial\delta_t}b_t c_t^2} + \frac{K_2K_3}{\frac{\partial C_{h_t}}{\partial\delta_t}q_T b_t c_t^2}} \qquad (8)$$

where

$$\left.\begin{aligned} A &= \frac{W\left(1 - \frac{d\epsilon}{d\alpha}\right)}{\left(\frac{dC_L}{d\alpha}\right)_w S} + g\frac{\rho}{2}l \\ B &= \frac{Wx}{\frac{\partial C_{L_T}}{\partial \delta_e}\frac{q_T}{q}S_T l} - \frac{1}{\tau}g\frac{\rho}{2}l \end{aligned}\right\} \tag{9}$$

The terms $\left(\frac{dC_{h_e}}{d\alpha_T}\right)_{tf}$ and $\left(\frac{dC_{h_e}}{d\delta_e}\right)_{tf}$ are the values of $\frac{\partial C_{h_e}}{\partial \alpha_T}$ and $\frac{\partial C_{h_e}}{\partial \delta_e}$ that would be measured on the elevator with the tab free and are given by the expressions

$$\left.\begin{aligned} \left(\frac{dC_{h_e}}{d\alpha_T}\right)_{tf} &= \frac{\partial C_{h_e}}{\partial \alpha_T} - \frac{\frac{\partial C_{h_t}}{\partial \alpha_T}\frac{\partial C_{h_e}}{\partial \delta_t}}{\frac{\partial C_{h_t}}{\partial \delta_t}} \\ \left(\frac{dC_{h_e}}{d\delta_e}\right)_{tf} &= \frac{\partial C_{h_e}}{\partial \delta_e} - \frac{\frac{\partial C_{h_t}}{\partial \delta_e}\frac{\partial C_{h_e}}{\partial \delta_t}}{\frac{\partial C_{h_t}}{\partial \delta_t}} \end{aligned}\right\} \tag{10}$$

The force per g for a geared spring tab, obtained by simultaneous solution of equations (3), (4), and (5), may be expressed by the same equation as was derived for an ordinary spring tab (equation (8)), provided that certain substitutions are made for some of the parameters. These substituted values may be interpreted physically as the characteristics of the equivalent balancing tab previously defined. The complete equation is

$$\frac{\partial F}{\partial n} = \frac{\dfrac{1}{(K_1)_b}\left\{A\left[\left(\dfrac{dC_{h_e}}{d\alpha_T}\right)_{tf} + \dfrac{K_2K_3\left(\dfrac{\partial C_{h_e}}{\partial \alpha_T}\right)_b}{\dfrac{\partial C_{h_t}}{\partial \delta_t} q_T b_t c_t^2}\right] + B\left[\left(\dfrac{dC_{h_e}}{d\delta_e}\right)_{tf} + \dfrac{K_2K_3\left(\dfrac{\partial C_{h_e}}{\partial \delta_e}\right)_b}{\dfrac{\partial C_{h_t}}{\partial \delta_t} q_T b_t c_t^2}\right]\right\}\dfrac{q_T}{q} b_e c_e^2}{1 - \dfrac{K_2\left(\dfrac{\partial C_{h_e}}{\partial \delta_t}\right)_b b_e c_e^2}{(K_1)_b \dfrac{\partial C_{h_t}}{\partial \delta_t} b_t c_t^2} + \dfrac{K_2K_3}{\dfrac{\partial C_{h_t}}{\partial \delta_t} q_T b_t c_t^2}} \tag{11}$$

where the quantities with the subscript b are defined in the following table:

Quantity	Definition	Physical significance
$(K_1)_b$	$K_1\left(1 - \frac{K_4K_2}{K_3K_1}\right)$	Ratio between stick travel and elevator deflection for equivalent balancing tab
$\left(\frac{\partial C_{h_e}}{\partial \delta_e}\right)_b$	$\frac{\partial C_{h_e}}{\partial \delta_e} - \frac{K_4}{K_3}\frac{\partial C_{h_e}}{\partial \delta_t} - \frac{K_4}{K_3}\frac{\partial C_{h_t}}{\partial \delta_e}\frac{b_t c_t^2}{b_e c_e^2} + \left(\frac{K_4}{K_3}\right)^2\frac{\partial C_{h_t}}{\partial \delta_t}\frac{b_t c_t^2}{b_e c_e^2}$	Value of $\partial C_{h_e}/\partial \delta_e$ for equivalent balancing tab
$\left(\frac{\partial C_{h_e}}{\partial \alpha_T}\right)_b$	$\frac{\partial C_{h_e}}{\partial \alpha_T} - \frac{K_4}{K_3}\frac{\partial C_{h_t}}{\partial \alpha_T}\frac{b_t c_t^2}{b_e c_e^2}$	Value of $\partial C_{h_e}/\partial \alpha_T$ for equivalent balancing tab
$\left(\frac{\partial C_{h_e}}{\partial \delta_t}\right)_b$	$\frac{\partial C_{h_e}}{\partial \delta_t} - \frac{K_4}{K_3}\frac{\partial C_{h_t}}{\partial \delta_t}\frac{b_t c_t^2}{b_e c_e^2}$	Value of $\partial C_{h_e}/\partial \delta_t$ for equivalent balancing tab, measured with tab link connected. Physical significance may be visualized as effect of deflecting tab as a trim tab by changing length of tab link

The stick-force characteristics of an ordinary spring tab were discussed in reference 1. At very high speeds, the stick force per g normal acceleration was shown to approach the value obtained with a servotab and, at low speeds, the force per g was shown to approach the value obtained with the tab fixed. By similar reasoning, the stick-force gradient with a geared spring tab may be shown to approach that of a servotab at high speeds and to approach that obtained with the equivalent balancing tab at low speeds. By varying the gear ratio, the force per g at low speeds may be adjusted to any desired value without affecting the force per g at high speeds. In particular, the force per g at low speeds may be adjusted to the value obtained at high speeds. The stick-force gradient, in this case, is found to be independent of the speed.

The conditions that must be satisfied in order to provide a force gradient independent of speed may be found from equation (11). The assumption is made that the ratio q_T/q is independent of speed - a condition approximately true at maneuvering speeds. The force per g will be independent of speed if the ratio of the terms in the numerator that contain q_T to the terms in the denominator that contain q_T is the same as the ratio of the remaining terms in the numerator to the remaining terms in the denominator. For one particular center-of-gravity location, this condition may always be satisfied by suitable choice of the gear ratio. If it is desired to provide a force gradient independent of speed at any center-of-gravity location, the following relations must be satisfied:

$$\frac{\left(\frac{\partial C_{h_e}}{\partial \alpha_T}\right)_{tf}}{1 - \frac{K_2\left(\frac{\partial C_{h_e}}{\partial \delta_t}\right)_b b_e c_e^2}{(K_1)_b \frac{\partial C_{h_t}}{\partial \delta_t} b_t c_t^2}} = \left(\frac{\partial C_{h_e}}{\partial \alpha_T}\right)_b \qquad (12)$$

$$\frac{\left(\frac{dC_{h_e}}{d\delta_e}\right)_{tf}}{1 - \frac{K_2\left(\frac{\partial C_{h_e}}{\partial \delta_t}\right)_b b_e c_e^2}{(K_1)_b \frac{\partial C_{h_t}}{\partial \delta_t} b_t c_t^2}} = \left(\frac{\partial C_{h_e}}{\partial \delta_e}\right)_b \tag{13}$$

In practice, equation (12) can be satisfied only by making $\partial C_{h_e}/\partial \alpha_T$ and $\partial C_{h_t}/\partial \alpha_T$ very close to zero. Equation (13) may then be used to determine the gear ratio K_4/K_3 that must be employed to provide a value of force per g which does not vary with speed.

An example of the application of equation (13) to the airplane with the characteristics given in table I is presented. If the values for $(K_1)_b$, $\left(\frac{\partial C_{h_e}}{\partial \delta_t}\right)_b$, and $\left(\frac{\partial C_{h_e}}{\partial \delta_e}\right)_b$ given in the preceding table are substituted in formula (13), the following relation is obtained:

$$\frac{\left(\frac{dC_{h_e}}{d\delta_e}\right)_{tf}}{1 - \frac{K_2\left(\frac{\partial C_{h_e}}{\partial \delta_t} - \frac{K_4}{K_3}\frac{\partial C_{h_t}}{\partial \delta_t}\frac{b_t c_t^2}{b_e c_e^2}\right) b_e c_e^2}{K_1\left(1 - \frac{K_4 K_2}{K_3 K_1}\right)\frac{\partial C_{h_t}}{\partial \delta_t} b_t c_t^2}} = \frac{\partial C_{h_e}}{\partial \delta_e} - \frac{K_4}{K_3}\frac{\partial C_{h_e}}{\partial \delta_t} - \frac{K_4}{K_3}\frac{\partial C_{h_t}}{\partial \delta_e}\frac{b_t c_t^2}{b_e c_e^2}$$

$$+ \left(\frac{K_4}{K_3}\right)^2 \frac{\partial C_{h_t}}{\partial \delta_t}\frac{b_t c_t^2}{b_e c_e^2} \tag{14}$$

All of the quantities are assumed to be known except the gear ratio K_4/K_3. It will be noted that, because the quantities K_4 and K_3 always occur as a ratio and K_4

and K_3 are both increased in the same ratio when the spring stiffness is increased, no limitation is placed on the spring stiffness.

If an attempt is made to solve equation (14) explicitly for the gear ratio K_4/K_3, a fifth-degree equation is obtained. This complication may be avoided, however, by solving equation (14) by a method of successive approximations. This process is applicable because a change in the value of K_4/K_3 has a marked effect on the value of the right-hand side of equation (14) and a relatively small effect on the value of the left-hand side. As a first approximation, the value of K_4/K_3 is assumed to be zero where it occurs in the left-hand side of equation (14) and the equation (now a quadratic) is solved for K_4/K_3. This approximate value is substituted in the left-hand side of the equation, and the equation is again solved for K_4/K_3. The process converges very rapidly and this second approximation generally will be sufficiently accurate.

If the values for the airplane and elevator characteristics given in table I are substituted in equation (14) and the values of K_4/K_3 on the left-hand side are assumed to equal zero, the following equation is obtained:

$$\frac{-0.003}{1 - \dfrac{(-0.45)(-0.003)(34.0)(2.2)^2}{(1.80)(-0.005)(7.35)(0.8)^2}} = -0.003 - \frac{K_4}{K_3}(-0.003) - \frac{K_4}{K_3}(0)\frac{(7.35)(0.8)^2}{(34.0)(2.2)^2} + \left(\frac{K_4}{K_3}\right)^2(-0.005)\frac{(7.35)(0.8)^2}{(34.0)(2.2)^2}$$

This quadratic equation has the solutions:

$$\frac{K_4}{K_3} = 0.868$$

$$\frac{K_4}{K_3} = 20.2$$

Of these two solutions, only the smaller value is of practical interest. The larger value would result in excessive tab deflections that would cause the lift increment due to the tab, which has been neglected in the present analysis, to reverse the direction of lift on the surface. If the value $\frac{K_4}{K_3} = 0.868$ is substituted in the left-hand side of equation (14) and the equation is again solved for K_4/K_3, the second approximation for the gear ratio is obtained as $\frac{K_4}{K_3} = 0.85$. Further approximations do not change this value appreciably.

The following criterion for determining approximately the minimum value of the spring stiffness required for satisfactory ground control was given in reference 1:

$$\frac{1}{I}\frac{\partial H_e}{\partial x_s} = 200 \text{ foot-pounds per foot per slug-foot}^2$$

For a geared spring tab, the variation of elevator hinge moment with stick deflection when the elevator is held fixed is given by the formula

$$\frac{\partial H_e}{\partial x_s} = \frac{-(K_1)_b K_3}{K_2} + \frac{\left(\frac{\partial C_{h_e}}{\partial \delta_t}\right)_b q_T b_e c_e^2}{K_2} - \frac{(K_1)_b \frac{\partial C_{h_t}}{\partial \delta_t} q_T b_t c_t^2}{K_2^2} \tag{15}$$

If it is desired to satisfy the criterion at zero airspeed, the terms containing q_T may be neglected and the following relation is obtained:

$$\frac{1}{I}\frac{\partial H_e}{\partial x_s} = 200$$

$$= \frac{-(K_1)_b K_3}{K_2 I}$$

This expression may be used to solve for K_3, which determines the spring stiffness. For the example under consideration,

$$K_3 = \frac{K_2(200)I}{-(K_1)_b}$$

$$= \frac{(-0.45)(200)(1.5)}{-1.80\left[1 - \frac{(0.85)(-0.45)}{1.80}\right]}$$

$$= 95.0 \text{ pounds per radian}$$

A value of K_3 of 100 pounds per radian has been used in the examples of this paper. From the value of K_4/K_3 determined previously, the value of K_4 may be readily obtained.

REFERENCES

1. Phillips, William H.: Application of Spring Tabs to Elevator Controls. NACA ARR No. L4H28, 1944.

2. Jones, Robert T., and Greenberg, Harry: Effect of Hinge-Moment Parameters on Elevator Stick Forces in Rapid Maneuvers. NACA ARR No. L4J12, 1944.

TABLE I

AIRPLANE CHARACTERISTICS

W, lb	50,000
S, sq ft	1,000
c, ft	11.18
l, ft	35
S_T, sq ft	200
b_e, ft	34
c_e, ft	2.2
b_t, ft	7.35
c_t, ft	0.8
$\left(\frac{dC_L}{d\alpha}\right)_W$, per radian	4.5
$1 - \frac{d\epsilon}{d\alpha}$	0.55
τ	0.5
$\frac{\partial C_{L_T}}{\partial \delta_e}$, per radian	1.7
$\frac{q_T}{q}$	1.0
I, slug-ft^2	1.5

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TABLE II

CONTROL-SYSTEM CHARACTERISTICS

	Conventional balance (fig. 3(a))	Ungeared spring tab (fig. 3(b))	Geared spring tab (fig. 3(c))
K_1, ft per radian	2.18	1.80	1.80
K_2, ft per radian	------------	-0.45	-0.45
K_3, lb per radian	------------	100	100
K_4, lb per radian	------------	------------	85
$\frac{\partial C_{h_e}}{\partial \alpha_T}$, per deg	0	0	0
$\frac{\partial C_{h_e}}{\partial \delta_e}$, per deg	-0.00058	-0.003	-0.003
$\frac{\partial C_{h_e}}{\partial \delta_t}$, per deg	------------	-0.003	-0.003
$\frac{\partial C_{h_t}}{\partial \alpha_T}$, per deg	------------	0	0
$\frac{\partial C_{h_t}}{\partial \delta_e}$, per deg	------------	0	0
$\frac{\partial C_{h_t}}{\partial \delta_t}$, per deg	------------	-0.005	-0.005

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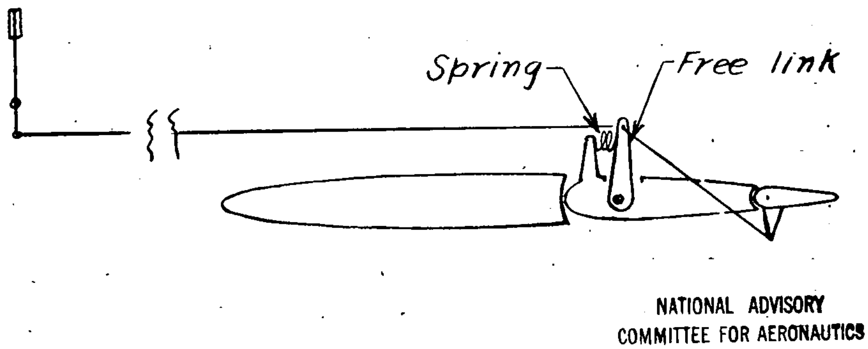


Figure 1.- Mechanism for ordinary spring tab.

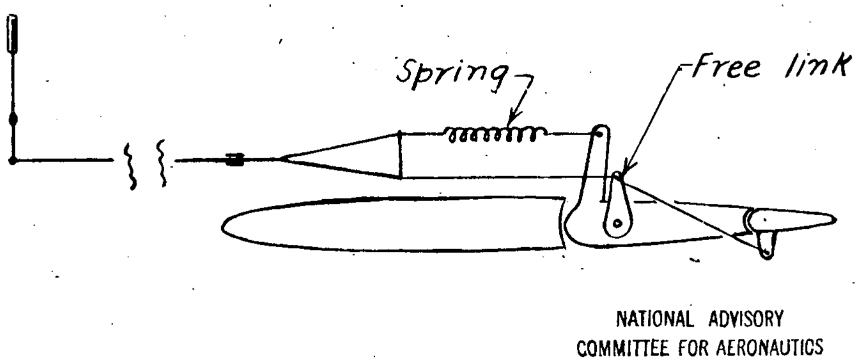
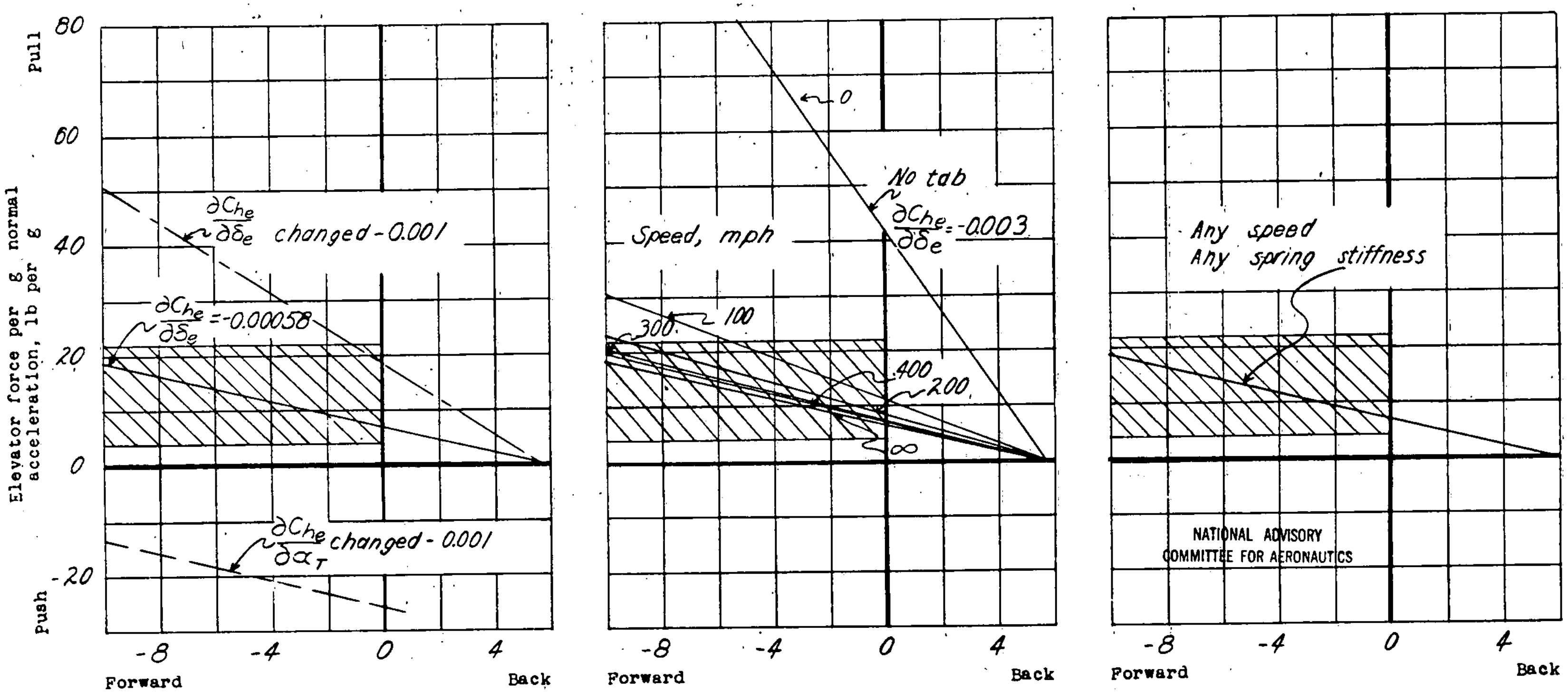


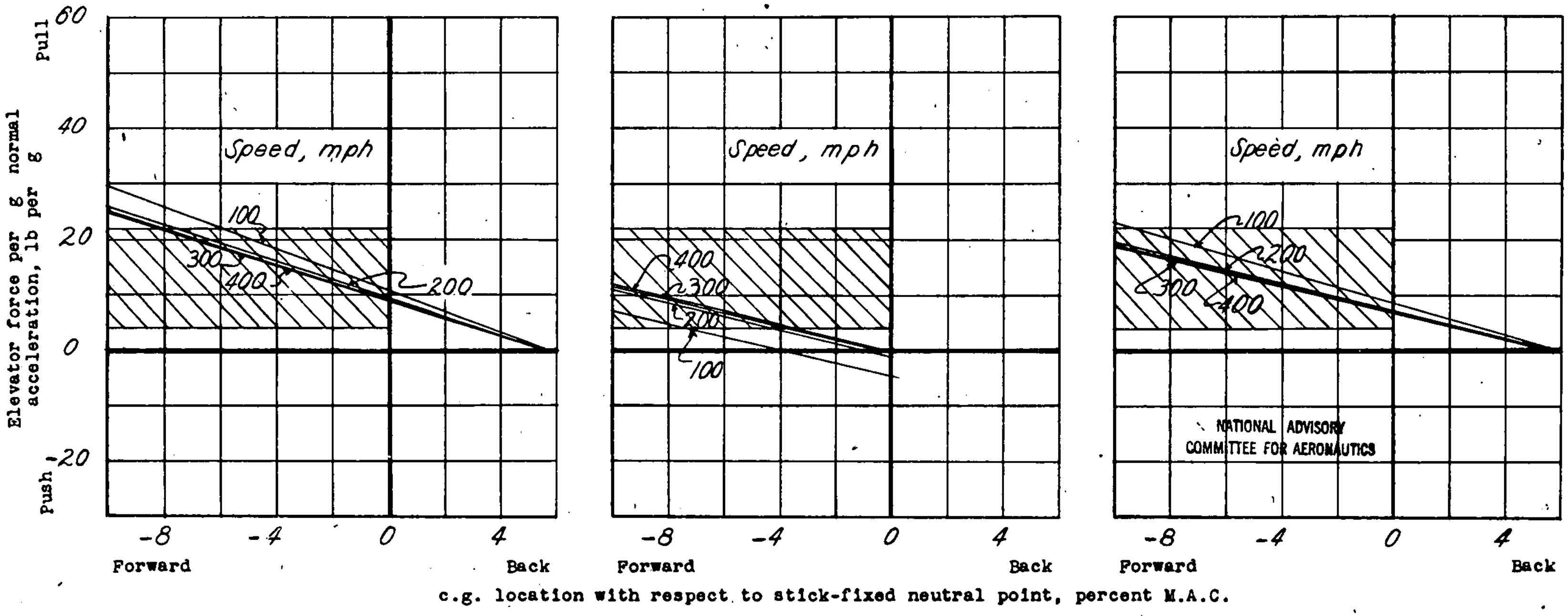
Figure 2.- Mechanism for geared spring tab.



c.g. location with respect to stick-fixed neutral point, percent M.A.C.

(a) Conventional balance. (b) Ungeared spring tab. (c) Geared spring tab.

Figure 3.- Stick-force characteristics of various types of elevator control. Desirable range of stick forces indicated by shaded area.



(a) $\frac{\partial C_{h_e}}{\partial \delta_e}$ changed -0.001.

(b) $\frac{\partial C_{h_e}}{\partial \alpha_T}$ changed from 0 to -0.001.

(c) Gear ratio changed 0.2.

Figure 4.- Effects of design variations on stick-force characteristics of geared spring tab of figure 3(c).