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THE PORPOISING CHARACTERISTICS OF A PLANING SURFACE

REPRESENTING THE FOREBODY OF A FLYING-BOAT HULL

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THE PORPOISING CHARACTERISTICS OF A PLANING SURFACE REPRESENTING THE FOREBODY OF A FLYING-BOAT HULL

By James M. Benson

SUMMARY

A V-bottom planing surface representing the forebody of a flying-boat hull was used in an investigation of the low-angle type of porpoising. Controllable tail surfaces were fitted on an outrigger that supported them in a position roughly the same as they would have been on a complete model. The planing surface was considered as though it were part of a complete dynamic model and for each test it was balanced to bring the center of gravity of the assembly to the desired position, and the pivot about which it was free to turn was located there. The model was towed in the same manner as a complete dynamic model.

The porpoising characteristics of the planing surface were observed for different combinations of load, speed, moment of inertia, location of pivot, elevator setting, and tail area. The model was found always to be stable above and unstable below a rather well-defined critical trim and showed no tendency to porpoise in the high-angle condition that is commonly observed with flying boats. The critical trim was found to be determined mainly by the speed and load and, to a smaller extent, by the location of the pivot and the radius of gyration. Moving the pivot either forward or down or increasing the radius of gyration lowered the critical trim. When porpoising did occur it was observed that a decrease in the radius of gyration caused the amplitudes of the oscillations in trim to increase markedly. An increase in the mass and moment of inertia without changing the radius of gyration or other variables resulted in an increased amplitude of the oscillations. Increasing the tail area to about twice normal size did not appear to affect the critical trim.

By a comparison of the data from these tests, in which the effect of a wing was completely absent, with data from a complete model and from theoretical computations, it was
concluded that the effect of the wing of a complete model upon the lower limit of stability at the lower planing speeds was relatively small.

INTRODUCTION

A theoretical approach to the problems of porpoising has been made by Perring and Clauert (reference 1), who modified the conventional analysis of aerodynamic stability and applied it to the study of the stability on the water of the idealized case of a single flat plate and also to the case of two flat plates in tandem. They treated the two cases without aerodynamic surfaces and with wing and tail plane. They demonstrated by comparison of their theoretical values with data from one full-scale test that the conventional stability analysis could be applied to the study of porpoising, provided that experimental data were available for evaluating the stability derivatives for the hull forms actually used.

Considerable experimental work has been done in towing basins with dynamic models of seaplanes to determine their stability characteristics. Most of the experimental work on porpoising has been concerned with specific designs and, in many cases, attention has been given principally to the high-angle type of porpoising. A probable reason for doing the larger part of the experimental work on the one type of porpoising has been the fact that slight alterations in design, such as changes in the depth and the form of the step, frequently have been effective in markedly changing the porpoising characteristics at the higher angles but have had little if any effect upon the trim at which low-angle porpoising occurs. Another probable reason for giving more attention in experimental work to high-angle instability is that it has usually proved to be the more violent and dangerous of the two types. Inasmuch as low-angle porpoising may in itself become unsafe during a take-off or may lead to violent instability at higher angles, it appears that insufficient attention has been given to the basic problem of the stability of a simple planing surface. The present investigation was therefore planned to study experimentally the stability characteristics of a V-bottom planing surface representing the forebody of a seaplane.

A planing surface fitted with a controllable elevator and having an angle of dead rise of 22½° was towed in the NACA tank at constant speeds from a towing gear that permitted the model to trim and rise freely. The range of
trim that would cause porpoising was observed for various combinations of speed, load, moment of inertia, location of pivot, and mass. In addition to the experimental investigation, the effect of a wing and of increased tail area upon low-angle porpoising at a speed above the hump was analytically computed by the method of reference 1.

**PLANNING MODEL**

A sketch of the model and the arrangement for testing is shown in figure 1. The model has a V-bottom planning surface with an angle of dead rise of 22° and a beam of 16 inches. The keel is straight for a distance of 36 inches forward of the stern and is faired into a bluff bow having a developable bottom. The model was fitted with a "normal" tail plane of NACA 0015 airfoil section of rectangular planform and with a span of 41 inches. The chord of the stabilizer was 5.7 inches, and that of the elevator was 5.5 inches. The moment arm of the tail plane varied with the location of the pivot, averaging about 4 feet. For tests with increased tail area, a second tail plane having about 1.5 times the area of the normal tail plane was attached to the model and was located about one chord length above it so as to form a biplane tail having 2.5 times the normal area.

The moment of inertia, the load on the water, and the mass moving vertically could be independently adjusted.

**TEST PROCEDURE**

The model was towed at the low water level in the NACA tank using a procedure similar to that followed in references 2 and 3 for the towing of dynamic models to determine trim limits. Runs were made at constant speed and with fixed loads on the water while the trim of the model was adjusted by means of the elevator to obtain the critical trim.

**Critical Trim**

For the purpose of this report "critical trim" may be defined as the trim separating the stable range from the unstable range. At trims above the critical value the planning surface ran stably and, if it was momentarily disturbed, the resulting oscillations decayed to zero after
a few cycles. At trim below the critical value porpoising began spontaneously and continued indefinitely at a fairly constant amplitude of the oscillation in trim. This concept of a definite critical trim may not be strictly true. Instead, a narrow range of trim wherein the model is neutrally stable may separate the stable from the unstable range. Coombes (reference 3) has described a range of neutral stability for a dynamic model of a twin-float seaplane.

In the determination of the critical value of the planing surface, the trim of the model was gradually lowered from a stable attitude until oscillations began spontaneously and continued regularly through an amplitude of about 2°. The trim was then increased while the accompanying decrease in amplitude was noted. At the point where the oscillation seemed to disappear, the trim was noted and compared with the corresponding attitude at which the model began to porpoise during the decrease of trim. In general, the two readings did not differ by more than about ±0°. The average of the two readings thus obtained was recorded as the critical trim.

During a constant-speed run of the carriage, the critical trim was obtained for several combinations of load and mass moving vertically. In this manner the variations of critical trim with load, speed, and mass were determined. Changes in moment of inertia and location of pivot were made and the tests repeated. For certain locations of the pivot the aerodynamic control was inadequate to balance the hydrodynamic moment at the critical trim and a gravity moment was used to obtain the desired trim. This use of a gravity moment caused the model to pivot about a point different from the center of mass of the rotating system. No allowance was made for the variation in loading introduced by the aerodynamic forces on the tail surfaces.

Porpoising Oscillations

For a limited number of tests, observations were made of the amplitude of the oscillations in trim that followed after the model was trimmed below the critical value. Tests were made at constant load, speed, mass, and moment of inertia; and the amplitudes were obtained for various elevator settings. For a particular elevator setting the model was restrained in pitch by pulling lightly on lines attached to the bow and the stern. This damping in pitch was barely sufficient to prevent porpoising but still per-
mitted the model to assume the attitude required for equilibrium of the hydrodynamic and aerodynamic forces. The trim at equilibrium was observed and the model was then released to permit porpoising. After a few cycles the oscillations would reach a maximum in amplitude and would usually continue indefinitely between the two limiting values of trim. The upper and lower limits of oscillation were recorded.

In the tests to determine the effect of radius of gyration on amplitude of porpoising the critical trim was determined for a particular configuration of the model, and the elevator was lowered enough to cause the model to trim at about $\frac{3}{4}$ below that value. The pitching oscillations were prevented by damping as previously described. The model was then released and the amplitude of oscillation observed. The radius of gyration was changed during the run by varying the mass moving vertically without making any change in moment of inertia, load, speed, or elevator setting. The amplitude of oscillation was again noted after the change in mass. This method did not completely isolate the effect of the radius of gyration upon the amplitude because the changes in the radius of gyration were, in general, accompanied by a small change in the critical trim.

**TEST RESULTS**

**Porpoising Characteristics of a Single Planing Surface**

The combination of planing surface with tail plane exhibited the presence of one critical trim value and had no tendency to porpoise at higher trims. The porpoising that occurred when the trim was less than the critical value closely resembled the motions in the porpoising of a complete dynamic model when planing on the forebody alone. When the trim of the model was increased from the unstable to the stable range, the critical trim was found to be practically the same as the value determined by lowering the trim from the stable to the unstable range. Recovery from porpoising always followed the application of sufficient positive moment to trim the model above the critical value. The absence of the high-angle condition of instability with the single planing surface is in agreement with the commonly accepted concept that the high-angle type of porpoising of a flying boat is a phenomenon that always involves the afterbody.
When porpoising of the planing surface did occur, the motion was usually constant in amplitude but, in some cases, it would wax and wane in a manner similar to that resulting from the addition of two simple harmonic motion's of slightly different periods.

**Critical Trim**

The variation of the critical trim with speed of the planing surface is plotted in Figure 2, where all variables are expressed as follows:

Load coefficient, \( C_A = \Delta/wb^3 \)

Gross load coefficient, \( C_{\Delta_0} = \Delta_0/wb^3 \)

Speed coefficient, \( C_V = V/\sqrt{gb} \)

where

\( \Delta \) load on water, pounds
\( \Delta_0 \) initial load on water, gross load, pounds
\( b \) maximum beam, feet
\( w \) specific weight of water, pounds per cubic foot
\( g \) acceleration of gravity, feet per second per second

and

\( k \) radius of gyration, fraction of beam
\( l_1 \) location of pivot forward of trailing edge (T.E.), fraction of beam
\( l_2 \) location of pivot above keel, fraction of beam.

The load on the water was selected as the parameter and each plot represents a particular combination of location of pivot and radius of gyration. The mass moving vertically is given in the same nondimensional units as the load coefficient. The moment of inertia is not listed in each case but may be obtained from the values given for the mass and the radius of gyration.

**Effect of tail area.**—The variations in critical trim with speed for the model having the normal tail and for the
model having the tail area increased 125 percent are plotted in figure 3. Increasing the tail area to more than twice normal size appeared in the present tests to change the critical trim very little. A more precise exploration of the region of neutral stability might reveal a definite effect but it does not appear that any practicable increase in the tail area of a flying boat beyond the area that would be required for suitable aerodynamic characteristics would have any marked effect upon the lower limit.

**Effect of radius of gyration.**—The variation of critical trim is plotted as a function of the radius of gyration in figure 4, which shows the results of tests at two different loads at a speed coefficient of 6.0 with the pivot located 0.38 beam forward of the trailing edge and 1.26 beams above the keel. The radius of gyration as here used is $\sqrt{I/M}$, in which the mass $M$ includes the model, counterweights, and fittings. (See fig. 1.) The curves show that increases in the radius of gyration lower the critical trim but that the effect is small when compared with the effect of speed and load.

**Effect of location of pivot.**—Figure 5 shows the variation of critical trim caused by changes in the location of the pivot for two combinations of load and radius of gyration. There is a definite tendency for the critical trim to be lowered when the pivot is moved either forward or down. This effect is also small when compared with the effect of load and speed.

**Amplitude of Porpoising Oscillations**

**Effect of elevator setting.**—Figure 6 shows how the amplitude of the porpoising is affected by the elevator setting. The axis of ordinates represents the trim assumed by the model for a particular elevator setting when porpoising was restrained by the application of external damping. The amplitude of the oscillation that followed the removal of the damping is represented by a horizontal line between the values of the upper and the lower limits of the trim occurring in the oscillation.

**Effect of tail area.**—The variation of amplitude with radius of gyration for the model having a biplane tail with 125 percent increased area is shown in figure 7(c). A comparison of this plot with figure 7(b) shows that the larger tail area resulted in somewhat greater amplitudes of oscillation.
Effect of radius of gyration.— The variation in amplitude with changes in the radius of gyration \( k \), when the elevator setting was constant, is shown in figure 7. The marked increase in amplitude with a decrease in radius of gyration may be due in part to the fact that the decrease in \( k \) causes an increase in the critical trim. An interpretation of this figure should include the effect of elevator setting upon critical trim.

The difference in handling characteristics caused by the changes in the radius of gyration was more marked than might appear from the plots. During the tests at constant speed, the elevator was set to trim the model at about \( \pm 10^\circ \) below the critical trim. The amplitude of the resulting oscillation was observed and the mass moving vertically was changed without changing the speed, the load, or the elevator setting. Throughout the tests it was noted that, with either an increase in mass or a decrease in moment of inertia, the porpoising motions of the model when trimmed below the critical values were definitely more violent and more likely to lead to dangerous amplitudes.

In figures 7(b) and 7(c) are shown the results of two tests to determine if, for a particular value of the radius of gyration, the amplitude would be affected by the moment of inertia and the mass. In figure 7(b) at \( k = 1.09b \), a 50-percent increase in the moment of inertia is shown to have caused a small increase in the amplitude of the motion. In figure 7(c) at \( k = 0.76b \), a 100-percent increase in the moment of inertia is shown to have caused a large increase in the amplitude.

Comparison of Results with Data for Complete Models Having Afterbody and Wing

Experimental.— In figure 8 is plotted a curve showing the lower limit of stability for a particular flying boat, which has a dead rise of \( 22^\circ \) measured at the keel. The curve is taken from data obtained at the NACA tank during tests of a \( \frac{1}{8} \)-scale model dynamically similar to the full-size craft, which has a gross load coefficient of 0.82 and a get-away speed coefficient of 7.0. The model was towed at constant speeds and the trim limits were determined in the same way as the critical trims were determined for the planing surface. The load carried by the model was computed for different speeds by deducting the aerodynamic lift from the gross weight. The aerodynamic lift was
computed from data obtained by towing the complete model a short distance above the water in the tank. The critical trim of the planing surface at speeds and loads corresponding to the values for the dynamic model is shown for comparison. The critical trim of the planing surface agrees well with the lower limit of stability at the lower speeds.

Theoretical.—The data given in reference 4 were used for computing the effect of a wing, the area of the tail, and the moment of inertia upon the critical trim for a limited number of conditions comparable with some of the conditions included in the tests of the planing surface. Reference 4 describes the methods used in deriving the data and in computing the values of the stability derivatives and the values of the terms in the discriminant equation. The data from reference 4 and the computed results are given in table I.

The notation given in table I is the same as is customarily used in porpoising analysis, in which axes are assumed to be fixed relative to the undisturbed water surface with Z positive downward, X positive forward, and M positive when tending to raise the bow. The force Z is then defined as a force along the OZ axis and Z is written for \( \frac{\partial Z}{\partial t} \). The other derivatives are similarly defined. A, B, C, D, and E are the coefficients of the stability equation.

\[ A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0 \]

where \( \lambda \) is used for the differential operator, \( \frac{\partial}{\partial t} \). R is Routh's discriminant and is defined by

\[ R = BCD - AD^2 - E^2 \]

The criterion for stability is that A, B, C, D, E, and R shall all be positive. These terms have been calculated for a speed coefficient of 5.65 and a load coefficient of 0.61 on the water. (See table I.)

At a trim of 5° the airplane is unstable and an increase of 300 percent in the moment of inertia has not resulted in stability, although the change in the value of R indicates that the margin of instability may have been reduced. At a trim of 7° the normal condition is
stable but a decrease of 50 percent in the moment of inertia results in instability, as shown by the negative value of R.

The trend shown by these calculations is in agreement with the experimental result obtained with the planing surface; that an increase in the radius of gyration causes a definite but relatively small decrease in the critical trim.

The validity of the stability tests on a planing surface with tail plane but without wing has been examined by calculating the derivatives and the terms of the discriminant equation, assuming that the aerodynamic effect of the wing is confined to $Z_w$ and $Z_b$. Making the aerodynamic values of all the $Z$ derivatives equal to zero and keeping the $W$ derivatives unchanged caused no change in sign of any of the terms at either $5^\circ$ or $7^\circ$. A change in the value of $R$, however, indicates that the wing may have a slight stabilizing effect. These results are in agreement with the comparison made in figure 8 and with the results reported in reference 2. At higher speeds the aerodynamic terms would be larger and more important in comparison with the hydrodynamic and the effect of the wing might become more pronounced.

The effect of a 100-percent increase in tail area has been calculated for the airplane with wing by doubling the aerodynamic values of $N_q$, $N_w$, and $N_b$. The increase in the value of $R$ indicates that the larger tail area has resulted in stability at $5^\circ$ and an increased margin of stability at $7^\circ$.

**DISCUSSION**

Comparison with reference 1.—From the tests reported in reference 1, Perring and Glauert reached the following conclusions regarding the porpoising characteristics of a seaplane with only the forebody in contact with the water (single-step case):

A seaplane, traveling on one step, tends to porpoise as the speed over the water is increased and this is generally due to the angle of incidence decreasing as the speed increases. The following factors may lead to instability:
(a) Center of gravity too far in front of the main step.
(b) Center of gravity too high.
(c) Excessive aerodynamic weathercock stability.
(d) Moment of inertia too small.
(e) Aerodynamic moment forcing the nose down.

In order to compare the results of the present tests of a planing surface with the conclusions reached in reference 1 from tests of a seaplane, a differentiation must be made between the effects that the variables have upon the critical trim and the effects that they have upon the trim assumed by the seaplane at a particular speed.

The aerodynamic weathercock stability referred to in the foregoing and hereinafter is the weathercock stability in pitch.

The moment of inertia and the amount of weathercock stability do not affect the trim that the seaplane will assume, but variations in these quantities may cause porpoising by lowering the critical trim. The results of the present tests of a planing surface with a tail plane indicate that a decrease in moment of inertia for a particular value of the mass tends to decrease the critical trim and thereby may lead to instability. The results of the tests also indicate the effect of excessive weathercock stability as can be seen from a comparison of figures 7(b) and 7(c). With the normal tail the porpoising motions were not so violent as when the tail area was increased 125 percent. The tests showed that the critical trim was not measurably affected in the one particular case in which the tail area was increased although the analysis of reference 1 indicated that, in general, an increase in tail area should increase the critical trim. The results of computing the effect of tail area as recorded in table I do not agree with the conclusion of reference 1 regarding the effect of weathercock stability.

The results of the tests are in agreement with the conclusions that instability might be caused if the center of gravity were too far in front of the main step or if the nose were forced down by an aerodynamic moment because either condition would tend to cause the seaplane to trim below the critical value.

Figure 5 shows that the critical trim is increased
by raising the center of gravity and this result is in agreement with the conclusion of reference 1.

Dynamic Models Having Excess Mass and Moment of Inertia

The tests of the planing surface indicated that an increase in both mass and moment of inertia, without any change in the radius of gyration, tends to increase the amplitude of the pitching oscillations during porpoising. This effect is of particular interest because tests of dynamic models are occasionally made with the model having excess weight and moment of inertia. From the analysis of porpoising in reference 1, the authors concluded that a test of that type, in which the ratio of mass to moment of inertia was correct, would accurately reproduce the porpoising characteristics of the full-size craft except that the frequency of oscillation would be reduced by the excess mass and moment of inertia. The lowered frequency may account for the increased amplitudes observed in the present tests. In a porpoising oscillation of a given amplitude the amount of energy dissipated by the damping forces decreases rapidly with decrease in frequency. With an excess in mass and moment of inertia of a dynamic model, it is to be expected that the damping would be less effective and therefore the amplitudes of the fully developed oscillations would be larger than if the frequencies were correctly reproduced.

Effect of Radius of Gyration

The results of the tests indicate that variations in the radius of gyration have a twofold effect upon the porpoising characteristics of a flying boat. Large increases in the radius of gyration reduce the critical trim and also reduce the amplitude of porpoising that occurs when the flying boat is trimmed below the critical value. The plots of amplitude in figure 7 show a definite tendency to converge as the radius of gyration is increased. Whether the convergence would continue to zero at some value beyond the range of the present tests is not known.

A comparison of the radius of gyration and the beam loading for several flying boats and float seaplanes of recent design has been tabulated as follows:
The plot in figure 10 of the foregoing values suggests that there may be a relationship between radius of gyration and load coefficient which would be useful in predicting porpoising characteristics. The dashed line in figure 10 was drawn through arbitrarily selected points corresponding to radii of gyration somewhat above the average of the values in the plot. Two of the points that lie below the dashed line are values for flying boats which have shown unusually severe instability on the water. The equation for the straight line is

\[ k = 0.9 C_{\Delta_0} + 0.5 \]

where \( k \) is the radius of gyration, fraction of beam.

The fact that the float seaplanes have larger values of the radius of gyration and larger beam loadings than the flying boats is of particular interest. In recent years the porpoising of flying boats has been much more of a problem than the porpoising of float seaplanes. In view of this fact the results of the present tests indicate a very significant effect, which may have been given insufficient attention heretofore. For some of the heavily loaded flying boats that have exhibited severe porpoising it would appear well worth while to carry out a full-scale investigation of the effect of increasing the radius of gyration by as much as 50 percent.
Effect of Location of Pivot

The results indicate that, in some cases, a definite but relatively small decrease of the lower limit of stability of a seaplane may be obtained by moving the center of gravity either forward or downward or by increasing the radius of gyration.

Trend toward Increased Beam Loadings

The continued trend toward increased beam loadings of flying boats is illustrated by figure 9, which is a plot of the loading curves for several notable designs that have appeared during the past 10 years. Design A appeared about 1932, designs E and F appeared in 1939, and the others appeared during the interval between 1938 and 1939. The values showing load on the water as a function of speed were computed in each case by assuming for all speeds a lift coefficient equal to that at the stalling speed. The speed of maximum trim (approximate the hump speed) as determined in tank tests of dynamic models is indicated by an arrow on each curve except for design A for which data were not available. It is noteworthy that the hump speed does not vary greatly with load over the wide range of loadings. In 1932 a gross load coefficient of 0.45 was considered sufficient but by 1939 a load coefficient of 1.0 was being used. The increase in beam loading is even more striking at higher speeds. For example, at $C_Y = 4.0$ the increase is from about 0.2 to 0.75, or nearly fourfold. An increase in the hazards resulting from porpoising has accompanied the large increase in loading at the planing speeds.

With the conventional form of forebody having a 22° V-bottom and transverse step it appears that, in order to avoid low-angle porpoising as the gross load is increased, the speed at which the seaplane gets on the step must be increased. In general, this change would increase the time required for the take-off and would probably present additional difficulties in the control of spray.

Recent designs of flying boats have approached rather closely the limit of beam loading that is permissible with a conventional form of hull. Further increases in beam loading appear to require a form of hull that will considerably reduce the probability of the low-angle porpoising inherent in the form of forebody represented by the planing surface used in these tests. Some improvement in the form of the hull may be obtained by an investigation of the
effect of dead rise and the effect of the plan form of the step. A better solution may be found in some arrangement designed to plane efficiently on two steps at speeds considerably beyond the hump or in some arrangement of hydrofoils that reduces the load carried by the planing bottom.

CONCLUSIONS

The conclusions listed below apply over the range of variables included in the tests of a V-bottom planing surface having a 22° angle of dead rise and fitted with tail surfaces to provide aerodynamic damping. The conclusions are also believed to apply to the case of a seaplane having a forebody of the form represented by the planing surface. The conditions of particular interest and application are those arising when the seaplane gets on the step during take-off or planes on the forebody during a landing.

1. For a given set of variables that include speed, load, moment of inertia, and position of center of gravity, there is a rather sharply defined critical trim below which the system is unstable and above which it is stable.

2. The critical trim is determined mainly by the speed and the load on the water.

3. Increasing the radius of gyration decreases the critical trim.

4. Moving the center of gravity either forward or down decreases the critical trim.

5. Increasing the amount of aerodynamic damping to about twice the amount normally occurring on flying boats does not appreciably alter the critical trim.

6. Decreasing the radius of gyration may have two effects. It may increase the critical trim and it may also cause a marked increase in the amplitude of porpoising that follows when the system is trimmed below the critical value.

7. An increase in the mass and moment of inertia, without any change in the radius of gyration, tends to increase the amplitude of porpoising oscillations.

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REFERENCES


TABLE I.- VALUES OF THE STABILITY DERIVATIVES AND THE TERMS IN THE DISCRIMINANT EQUATION, $C_v=5.65; \quad C_d=0.61$

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<td>Normal</td>
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<td>-114</td>
<td>-194</td>
<td>-3.03</td>
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<td>0</td>
<td>-40</td>
<td>-6.6</td>
<td>0</td>
<td>0</td>
<td>-1.92</td>
<td>-0.028</td>
<td>-0.784</td>
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<td>-do--</td>
<td>-----do-----</td>
<td>Total$^a$</td>
<td>-114</td>
<td>-234</td>
<td>-3.63</td>
<td>3.2</td>
<td>3.92</td>
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<td>-0.05</td>
<td>-0.66</td>
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<td>-do--</td>
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<td>Total$^b$</td>
<td>-114</td>
<td>-234</td>
<td>-3.63</td>
<td>3.2</td>
<td>7.84</td>
<td>-6.64</td>
<td>-1.01</td>
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<td>-do--</td>
<td>Normal</td>
<td>Total$^b$</td>
<td>-114</td>
<td>-194</td>
<td>-3.0</td>
<td>3.2</td>
<td>3.92</td>
<td>-3.32</td>
<td>-0.05</td>
<td>-0.66</td>
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<td>Increased 100 percent</td>
<td>-----do-----</td>
<td>Total$^b$</td>
<td>-114</td>
<td>-234</td>
<td>-3.63</td>
<td>3.2</td>
<td>3.92</td>
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<td>-114</td>
<td>-194</td>
<td>-3.0</td>
<td>3.2</td>
<td>3.92</td>
<td>-5.2</td>
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$^a$Computed in reference 4.

$^b$Computed from data in reference 4.
Figure 1.- Arrangement of planing surface for tests.

Figure 3.- Critical trim as a function of speed for a load coefficient of 0.40 with a normal tail and with increased tail area. $k$, 0.76b; $l_1$, 0.12b; $l_2$, 1.00b.
Figure 2.—Critical trim as a function of speed with load as a parameter for different combinations of location of pivot, radius of gyration, and mass moving vertically. Location of pivot is specified as fraction of beam forward of the trailing edge and above the keel.
(f) \( C_{\Delta o} = 0.53; k = 0.76b; l_1 = 0.38b; \)
\[ l_2 = 1.26 b. \]

(g) \( C_{\Delta o} = 0.53; k = 1.07b; l_1 = 0.38b; \)
\[ l_2 = 1.00b. \]

(h) \( C_{\Delta o} = 0.67; k = 0.97b; l_1 = 0.38b; \)
\[ l_2 = 1.26 b. \]

Figure 2.- Continued.
Figure 2 \textit{- Continued.}
Figure 2.- Continued.

- (m) $C_{Ao}$, 0.53; $k$, 0.76$b$; $l_1$, 0.62$b$; $l_2$, 0.83$b$.
- (n) $C_{Ao}$, 1.07; $k$, 0.54$b$; $l_1$, 0.62$b$; $l_2$, 1.00$b$.
- (o) $C_{Ao}$, 1.20; $k$, 0.73$b$; $l_1$, 0.62$b$; $l_2$, 1.00$b$.
- (p) $C_{Ao}$, 0.53; $k$, 0.76$b$; $l_1$, 0.62$b$; $l_2$, 1.26$b$.

Critical trim, deg

Speed coefficient, $C_v$
Figure 2.- Concluded.

Radius of gyration, $k$, fraction of beam

Figure 4.- Variation of critical trim with change in radius of gyration. $C_v$, 6.0; $l_1$, 0.38b; $l_2$, 1.26b.

Distance forward of trailing edge, $l_1$, fraction of beam

Figure 5.- Variation of critical trim with change in the location of the pivot. $C_A$, 0.40; $C_v$, 5.0.
Figure 6. - Effect of elevator setting upon the amplitude of porpoising for three values of the radius of gyration. $G_1$, 0.27; $G_2$, 0.12; $G_3$, 1.26. Oscillations 1 and 2 are minimum and maximum amplitudes occurring in a waxing and waning oscillation.

Figure 7. - Effect of radius of gyration and of increased tail area upon the amplitude of porpoising. $G_4$, 4.5.
Radius of gyration, $k$, fraction of beam

![Graph of Radius of Gyration](image)

Amplitude of oscillation, deg

Critical trim, deg

![Graph of Critical Trim](image)

Speed coefficient, $C_v$

![Graph of Speed Coefficient](image)
Figure 9.- Comparison of the load coefficient on the water of several flying boats during take-off, assuming the value of $C_L$ to be constant and equal to $C_L$ at get-away.

Figure 10.- Variation of radius of gyration and gross load coefficient for several recent designs of flying boats and seaplanes.