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TESTS AND APPROXIMATE ANALYSIS OF BENDING STRESSES DUE TO TORSION IN A D-SECTION BOX

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ADVANCE RESTRICTED REPORT

TESTS AND APPROXIMATE ANALYSIS OF BENDING STRESSES DUE TO TORSION IN A D-SECTION BOX

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SUMMARY

Strain surveys were made on the stringers reinforcing a D-section box loaded in torsion and restrained from warping at the root. On the basis of the tests, a procedure is suggested for estimating the bending stresses due to torsion that occur near the flange of a D-section box. A similar test of a triangular-section box showed that for such a section there were no significant bending stresses due to torsion, a fact which was also shown theoretically.

INTRODUCTION

The problem of the analysis of the bending stresses due to torsion in box beams of rectangular cross section, in which the root cross section is restrained from warping, has been thoroughly investigated by several authors. (See references 1 and 2.) The solutions proposed have been made on the assumption that, when the box is twisted, the deformations of the box can be separated into two parts. One part is the deformation caused by the twisting of the box when each cross section is allowed to warp freely; the other is the deformation caused by the bending of each wall according to the simple engineering theory. Relatively simple solutions for the stress distribution can then be obtained by applying the conditions of restraint at the root and of consistent deformations along the common edge of adjacent walls. When the section is unsymmetrical and reinforced with stringers, however, a theory based on even these simplifying assumptions becomes too involved for routine stress analysis. Consequently, the stress analyst, when confronted with the analysis of an unsymmetrical section, must resort to some approximate procedure by which the simple solutions for symmetrical boxes can be adapted to the problem at hand. Such a procedure for a D-section box is given in the present paper. The results of tests on a D-section box and on a triangular-section box, which indicate the applicability and limitations of the approximation are also discussed.

SYMBOLS

A_F equivalent area of flange of rectangular box, square inches

E Young's modulus of elasticity, ksi

G shear modulus of elasticity, ksi

K $\sqrt{\dfrac{8G}{A_F E\left(\dfrac{b}{t_b} + \dfrac{c}{t_c}\right)}}$

T torque, kip-inches

b width of rectangular box, inches

c depth of rectangular box, inches

t_b thickness of cover sheet, inches

t_c thickness of web, inches

x spanwise distance from root, inches

σ_F stress in flange, ksi

METHOD OF ANALYSIS

The Doubly Symmetrical Rectangular Box

The method of analysis of the bending stresses due to torsion of the doubly symmetrical box restrained from warping at the root is based directly on the theory presented in reference 1. The structure analyzed is a rectangular box that is stiffened by bulkheads and has a distinct flange in each corner. The theory assumes that the webs and cover sheets of the box carry only shear stresses. The ability of the actual web and cover sheets to resist bending stresses is provided for by adding to the area of each corner flange one-sixth of the area of the adjacent web and cover sheet.

If the spacing of the bulkheads is large compared with the dimensions of the cross section, or the torque is not constant along the length of the box, the more exact analyses of reference 1 should be applied. When the bulkhead spacing is small and the box is subjected to a constant torque, however, the solution for the stresses in the flanges due to torsion reduces to the simple formula

$$\sigma_F = \pm \frac{TKE}{8bcG}\left(\frac{b}{t_b} - \frac{c}{t_c}\right)e^{-Kx}$$

where the coordinate x is measured spanwise from the root. Figure 1 shows the notation for the details of the cross section of the doubly symmetrical rectangular box.

The bending stresses due to torsion in the rectangular box are linearly distributed chordwise from the maximum positive value in one flange to the maximum negative value in the adjacent flange. The sign of the bending stresses in any flange can readily be determined by assuming that the walls with the smaller ratio of width to thickness carry the applied torque by bending.

The D-Section Box

Because no simple procedure is available for the precise analysis of the bending stresses due to torsion in unsymmetrical sections, some method of adapting the solution for the doubly symmetrical rectangular box to these sections is desirable. Fortunately, the bending stresses due to torsion are small compared with the primary bending stresses; consequently, a rough estimation of the bending stress due to torsion is sufficient to obtain an accurate value for the total bending stresses. Furthermore, in a D-section box beam, the maximum primary bending stresses occur in the flange at the root and it is necessary, therefore, to determine only the flange stresses due to torsion.

The results of a theoretical study indicate that the bending stresses due to torsion which occur at the flange of a D-section box may be assumed approximately equal to the stresses occurring at the flange of an equivalent rectangular box. The depth and width of this equivalent rectangular box, as well as the web- and cover-sheet thickness, may be assumed equal to the corresponding dimensions of the D-section box. The corner-flange area of the rectangular box may be assumed equal to the sum of the flange area of the D-section box and the equivalent flange area of the web, cover sheet, and stringers of the D-section box.

As a first approximation, the equivalent flange area of the webs and cover sheets may be taken as one-sixth of the area of the webs and cover sheets of the equivalent rectangular box. The equivalent flange area of the stringers may be obtained by first spreading the cross-sectional area of the stringers across the curved cover of the D-section box to form a fictitious stringer sheet. The equivalent flange area of the stringers may then be taken as one-sixth the area of a sheet as wide as the equivalent rectangular box and as thick as the stringer sheet.

In order to determine the stresses in the stringers adjacent to the flange, the stresses may be assumed to decrease linearly in the chordwise direction from a maximum stress at the flange to zero at a distance from the flange equal to one-half the width of the equivalent rectangular box.

TEST SPECIMENS AND TEST PROCEDURE

The details of the D-section and the triangular-section boxes that were tested are given in figures 2 and 3. The material for the boxes was 24S-T aluminum alloy. The bulkheads were made of $\frac{3}{64}$-inch steel sheet the edges of which were stiffened by a steel flange 1 inch by 3/32 inch. For the tests, each box was supported at its center and equal torques were applied at the tips of the box. The condition of restrained warping of the center cross section of the box was obtained from the symmetry of the box and the load.

Strain surveys were made on both boxes with 2-inch Tuckerman optical strain gages except at the stations 1 inch from the root where 1-inch gages were used. Because the boxes were closed, strains could be measured only on the outside. For each test run, strain-gage readings were taken at zero torque and after each of three equal increments of torque. The torque was then released and a second set of zero readings taken. If the two sets of zero readings did not agree within 200 psi, a new test run was made. The strain readings from each gage were plotted against torque and a straight line was drawn through the plotted points. If this line as drawn did not intersect the origin of coordinates of the plot, a new line parallel to the original line was drawn through the origin. A corrected value of strain at maximum torque was then obtained with the new line.

TEST RESULTS

The results of the strain surveys were converted to stresses with an assumed modulus of 10,500 ksi and the spanwise distribution of the measured stringer stresses for the D-section box are given in figure 4. The average observed stresses were obtained by combining at each station the average of the stresses in the pair of symmetrical stringers with the average of the stresses of the corresponding pair of stringers in the other half of the box. The largest and the smallest stresses measured at these points are also indicated in figure 4. Because no significant stringer strains could be measured in the triangular-section box, no stress distributions for that box are given.

DISCUSSION

A comparison of the observed and calculated stresses (see fig. 4) indicates that the procedure suggested for the analysis of the D-section box gives a conservative value for the maximum bending stresses due to torsion near the flange of this section. In the region close to the root station, the observed stringer stresses do not continue to rise in the spanwise direction as the theory predicts. This deviation may be partly explained by the local disturbances in the stress distribution caused by the concentration of the center reaction at this station. Some local bending of the stringers may be present, which could also account for the wide range of stresses measured at equivalent points.

No calculated stress distribution is given for the bending stresses due to torsion near the leading edge of the D-section box; in general, these stresses will be higher than those at the flange of the D-section box. This conclusion follows from the conditions of statics, which require that bending stresses due to torsion produce no resultant force or moment on any cross section. Because a large percent of the total cross-sectional area is usually concentrated near the flanges, higher stresses must exist in the less concentrated area near the leading edge to balance the forces and moments produced by the stresses at the flange.

A comparison of the experimental results indicates that the highest stresses were measured in the stringer nearest the leading edge. (See fig. 4.) Fortunately, these stresses occur where the primary bending stresses are smaller than at the flange, so that the total stresses near the leading edge will be smaller than the total stresses at the flange. The stresses near the leading edge of the D-section box, however, will usually not be critical for design purposes.

In order to determine the effect on the stresses of decreasing the depth of one wall of a box beam that had originally been rectangular in cross section, a theoretical study based on the theory of reference 2 was made. Calculations were made of maximum bending stresses due to torsion at the root of a box beam that had walls of constant thickness and was braced by closely spaced, rigid bulkheads. Its over-all dimensions were approximately those of the D-section box. The spanwise distribution of torque on the box varied as a sine curve from zero at the root to a maximum at the tip.

Figure 5 presents the dimensions of the box and the results of the calculations showing that the decrease in depth of one wall of the box beam does not seriously affect the maximum stresses in the opposite wall until the cross section is almost triangular. As the cross section approaches the triangular shape, the bending stresses due to torsion rapidly approach zero. The maximum stresses in the shallow wall increase rapidly as the wall depth decreases and reach a maximum value about 20 percent greater than that for the rectangular box; however, these stresses also become zero when the section is triangular.

CONCLUDING REMARKS

An approximation to the maximum bending stresses due to torsion that occur near the flange of a D-section box may be obtained by computing the stresses in an equivalent rectangular box that circumscribes the D-section box. The approximation may not be conservative for the stresses near the leading edge of the D-section box, but these stresses will usually not be critical for design purposes.

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2. Goodey, W. J.: Thin Sections in Torsion. The Direct Stresses Developed in Open Sections with One End Subject to Constraint. Aircraft Engineering, vol. VIII, no. 94, Dec. 1936, pp. 331-334.

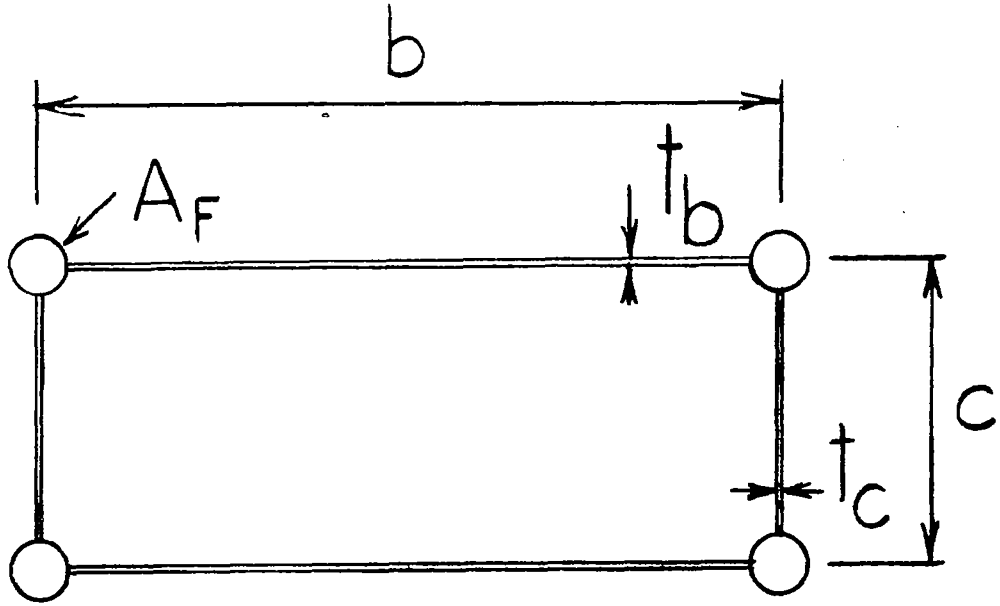

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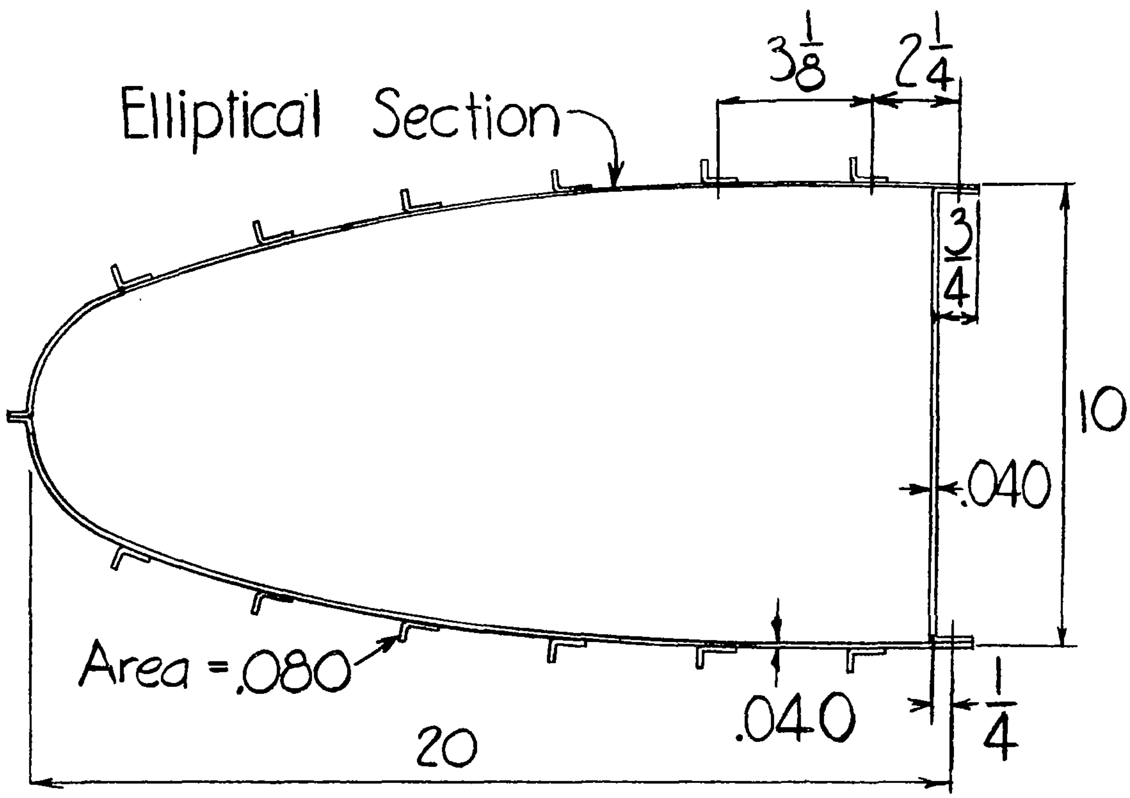
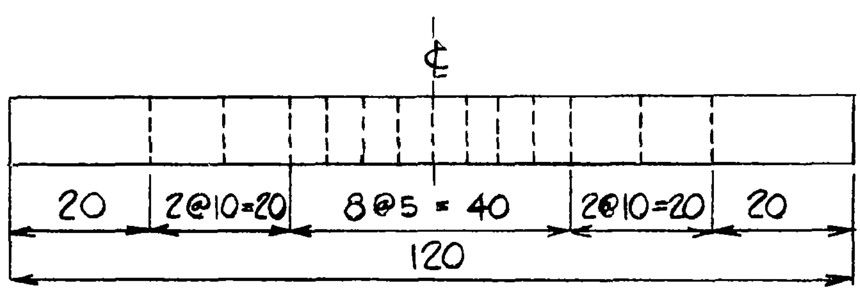
Figure 1.- Cross section of doubly symmetrical rectangular box.



(a) Cross section.



(b) Bulkhead spacing.

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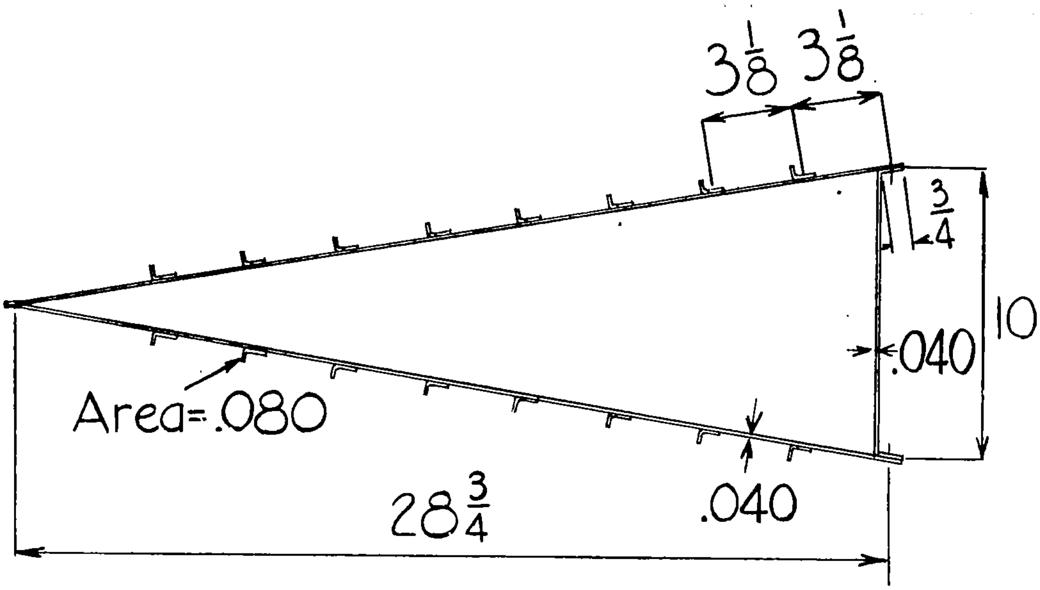
Figure 2.- Details of D-section box.



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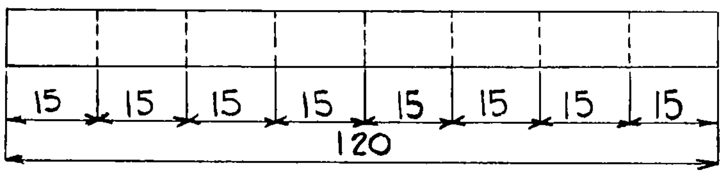
(a) Cross section.



(b) Bulkhead spacing.

Figure 3. -Details of triangular-section box.

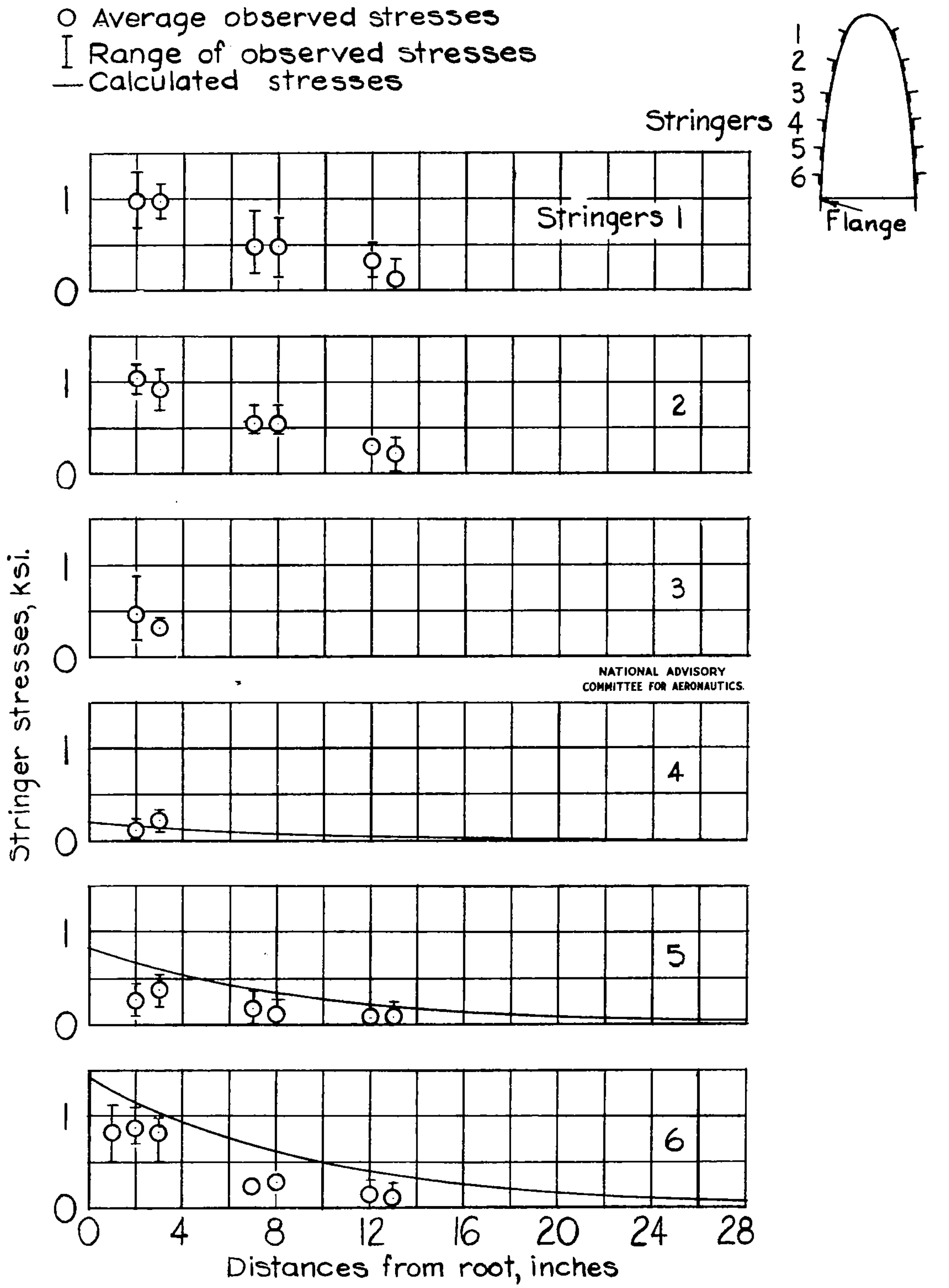


Figure 4 - Spanwise distribution of stringer stresses in D-section box, T = 42.8 kip-inches.

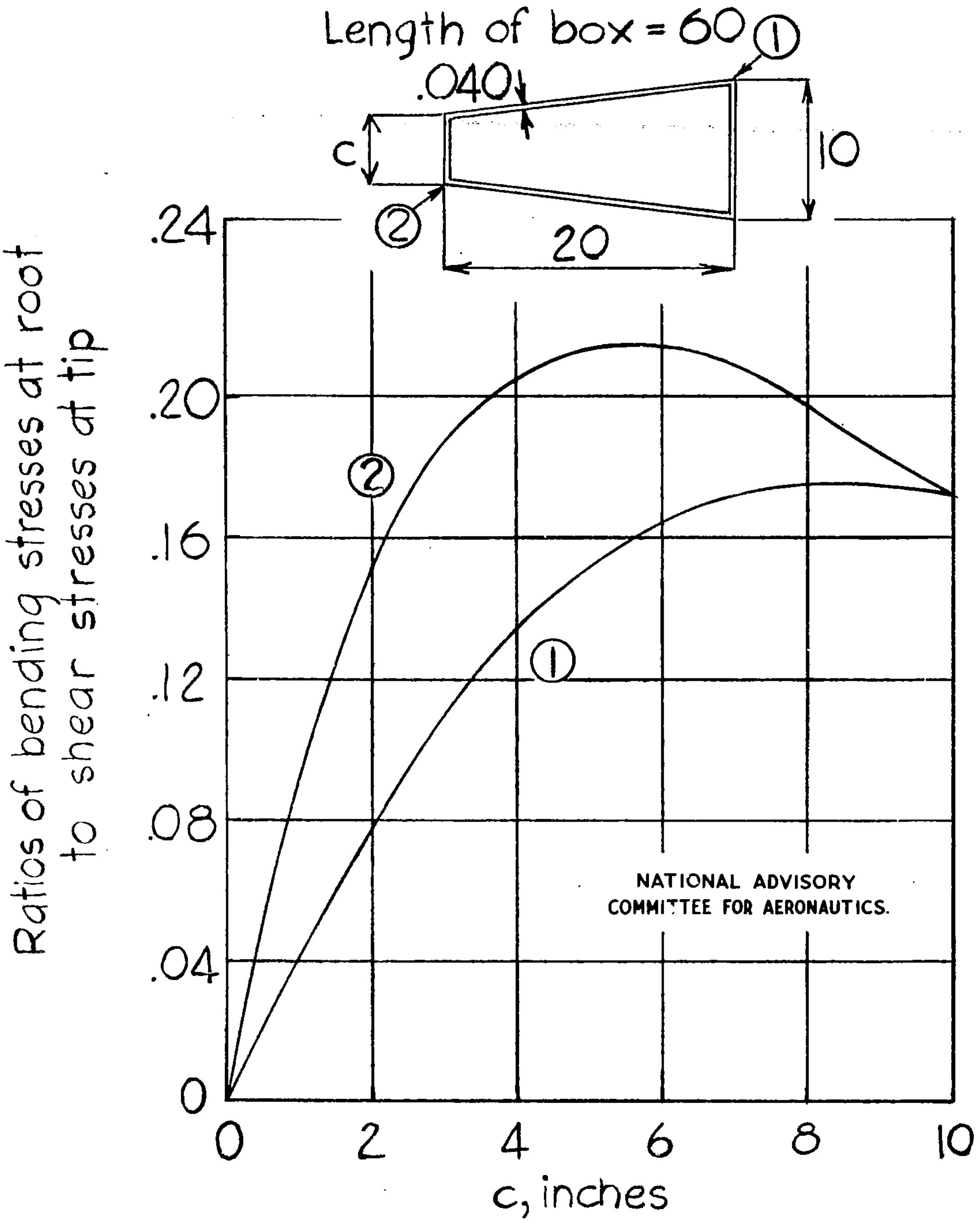


Figure 5.- Effect of changing depth of web at leading edge on maximum bending stresses due to torsion.

Sinusoidal Spanwise variation of Torque, from zero at root to max. at tip