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CHARTS FOR CRITICAL COMBINATIONS OF LONGITUDINAL AND TRANSVERSE DIRECT STRESS FOR FLAT RECTANGULAR PLATES

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

ADVANCE RESTRICTED REPORT

CHARTS FOR CRITICAL COMBINATIONS OF LONGITUDINAL AND TRANSVERSE DIRECT STRESS FOR FLAT RECTANGULAR PLATES

By Charles Libove and Manuel Stein

SUMMARY

Charts giving critical combinations of longitudinal and transverse direct stress are presented for isotropic flat rectangular plates for the following edge conditions: (a) all edges simply supported; (b) long edges simply supported, short edges clamped; (c) long edges clamped, short edges simply supported; and (d) all edges clamped. The charts are based upon energy solutions the accuracy of which is checked by comparisons with some particular exact results and lower-limit solutions. An example illustrating the use of the charts is given.

INTRODUCTION

Because the skin of an airplane in flight is subjected to combinations of stress, attention has been given to the problem of buckling of plates when more than one stress is present. Important cases of such buckling, involving the interaction of shear and one direct stress, have already been studied. In the present paper elastic buckling under the interaction of longitudinal and transverse direct stress is considered (see fig. 1).

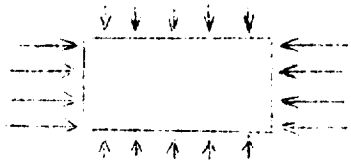

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Figure 1.- Interaction of longitudinal and transverse direct stress.

A combination of two direct stresses can arise in a plate if the Poisson effect produced by the application of one direct stress is restrained or partially restrained by members adjacent to the plate. If the individual stresses can be evaluated, the effect of the combination should be considered in investigating the stability of the plate.

The energy method is used to derive interaction equations that define the critical combinations of longitudinal and transverse direct stress for isotropic flat rectangular plates with the following four cases of edge conditions:

Case	Long edges	Short edges
(a)	Simply supported	Simply supported
(b)	Simply supported	Clamped
(c)	Clamped	Simply supported
(d)	Clamped	Clamped

Charts based upon the interaction equations are presented for these four cases.

The solution obtained for case (a) is exact. For the other three cases the solutions obtained are approximate and, as is characteristic of the energy method, unconservative. Comparisons with known exact solutions and lower-limit solutions indicate, however, that the errors in the present results are generally small.

SYMBOLS

a	length of plate
b	width of plate ($b<a$)
β	length-width ratio (a/b)
t	thickness of plate
w	deflection normal to plane of plate

A	arbitrary deflection coefficient
x	longitudinal coordinate
y	transverse coordinate
m	number of longitudinal buckles
n	number of transverse buckles
E	elastic modulus of plate material
μ	Poisson's ratio for plate material
D	flexural stiffness of plate $\left(\frac{Et^3}{12(1-\mu^2)}\right)$
σ_x	longitudinal direct stress, positive for compression
σ_y	transverse direct stress, positive for compression
k_x, k_y	dimensionless stress coefficients $\left(k_x = \sigma_x \frac{b^2t}{\pi^2D};\ k_y = \sigma_y \frac{b^2t}{\pi^2D}\right)$

INTERACTION CHARTS

General Description of Charts

The critical direct-stress combinations for the four cases of edge restraint considered in the present paper are defined by the solid curves in the interaction charts of figures 2 to 5. Each of the solid curves is a plot of the critical value of k_x against β for a given value of k_y. The parameters k_x and k_y are dimensionless measures of the longitudinal and transverse direct stress, respectively, and are defined by the formulas

$$k_x = \sigma_x \frac{b^2t}{\pi^2D} \tag{1}$$

$$k_y = \sigma_y \frac{b^2 t}{\pi^2 D} \tag{2}$$

The parameter β is the length-width ratio of the plate, that is

$$\beta = \frac{a}{b} \tag{3}$$

Any critical combination of k_x, k_y, and β, as represented by a point on one of the interaction charts, is associated with a definite buckle pattern. The different buckle patterns are indicated on the charts by sets of values of m and n assigned to different regions. These regions are separated by dashed curves. The integers m and n are defined as the number of longitudinal lobes (buckles) and the number of transverse lobes, respectively, into which the buckled shape of the plate is cut by the plane of the unbuckled plate.

Each of the interaction charts of figures 3 to 5 is based upon two energy solutions, which yield two different sets of interaction equations. One of these sets of equations was found to give more accurate results in one portion of the chart and less accurate results in another portion than the other set of equations. The dotted curve in each of figures 3 to 5 serves as a demarcation curve between these two portions of the chart. Although the dotted curves are continuous across the charts, parts are omitted in figures 4 and 5 where they would coincide with parts of the dashed curves.

The energy solutions upon which the interaction charts are based are given in the appendix. The data used in plotting the charts are given in tables 1 to 4.

Use of Charts

The interaction charts can be used to check the stability of a plate subjected to known longitudinal and transverse direct stresses or to determine the critical value of one direct stress corresponding to a given value of the other.

In order to check the stability of a plate with known values of σ_x and σ_y, it is necessary first to calculate

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the values of k_x, k_y, and β from equations (1), (2), and (3), respectively, and then to locate on the appropriate chart (depending on boundary conditions) that point which is defined by the calculated values of k_x and β. The value of k_y associated with this point is the critical value of k_y for the plate. If the actual value of k_y as calculated from equation (2) is higher than the critical value, the plate is unstable for the given loading. Conversely, if the actual value of k_y is lower than the critical value, the plate is stable.

If σ_x is known and it is required to find the corresponding critical value of σ_y, the critical value of k_y is first obtained as just described. The critical value of σ_y can then be calculated from the formula

$$\sigma_y = k_y \frac{\pi^2 D}{b^2 t} \tag{4}$$

If σ_y is known and it is required to find the critical value of σ_x for the plate, the values of k_y and β are first obtained from equations (2) and (3) and the point corresponding to these values is located on the appropriate chart. The ordinate of this point is the critical value of k_x. The corresponding critical value of σ_x can then be obtained from the formula

$$\sigma_x = k_x \frac{\pi^2 D}{b^2 t} \tag{5}$$

The use of the interaction charts will generally involve interpolation between k_y-curves. The form of the interaction equations (A3) to (A13), upon which the charts are based, indicates that for a given set of values of β, m, and n the critical value of k_y varies linearly with k_x. Accordingly, linear interpolation may be used along any vertical line segment between two adjacent dashed curves or between a dotted curve and an adjacent dashed curve. It can be shown, however, that linear interpolation between two points that lie on opposite sides of a dashed or a dotted curve introduces a small error which tends to offset the error inherent in the

energy solution. The use of linear interpolation between k_y-curves at all times is therefore recommended.

Accuracy of Charts*

The critical stress obtained by an energy method is characteristically unconservative; that is, it represents an upper limit to the true critical stress. If the assumed buckle pattern is the true one, however, the critical stress is exact. Only for case (a), all edges simply supported, are the present results exact. For the other cases it is important to have some idea of the degree of accuracy; for this reason the results in figures 3, 4, and 5 (cases (b), (c), and (d)) are compared with some results obtained from exact solutions in references 1, 2, 3, and 4. These comparisons are summarized in table 5 where it is seen that the errors in the present results are generally of the order of 1 or 2 percent.

The comparisons of table 5 are made for the special case of a plate subjected to only one direct stress. The table consequently furnishes comparisons principally along the line $k_x = 0$ and along the curve $k_y = 0$ in figures 3 to 5. Figure 6 is presented to furnish checks for cases in which both direct stresses are present. In figure 6 the interaction data for $\beta = 4$ in each of figures 3 to 5 are replotted in the form of a conventional interaction curve and compared with a curve that is known to represent a lower limit to the true interaction curve for the case. Since the interaction curve based on the energy method is characteristically unconservative and furnishes an upper limit to the true interaction curve, the true interaction curve must lie somewhere between the two curves compared. Consequently, if the two curves lie close together, the

*In the interval between the original publication of this report and its re-publication in the present form as a Wartime Report, extensive checks were made by James C. McCulloch of the Langley Laboratory staff on the results for case (d), all edges clamped. The checking was done by the method explained in NACA TN No. 1103 entitled "The Lagrangian Multiplier Method of Finding Upper and Lower Limits to Critical Stresses of Clamped Plates" by Bernard Budiansky and Pai C. Hu. The maximum error in any of the results shown in figure 5 or table 4 was found to be about 3 percent.

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maximum possible error in the upper-limit curve (present solution) is small.

The lower-limit interaction curve used in figure 6(a) is the exact interaction curve for a plate with all edges simply supported. This curve is known to represent a lower limit for the case under consideration because, all other things being equal, simple support along the short edges is a weaker boundary condition than clamping along the short edges. The lower-limit interaction curve used in figures 6(b) and 6(c), which is taken from figure 2 of reference 5, is the exact interaction curve for an infinitely long plate with clamped edges. This curve is known to represent a lower limit for the cases considered in figures 6(b) and 6(c) because an infinitely long plate must have a lower (or the same) buckling strength than a finite plate of the same width and thickness because of the absence of the restraining edges.

Comparisons similar to those just discussed would have little significance as an indication of accuracy for values of μ lower than 1/4 because the divergence between the true interaction curves and the lower-limit interaction curves generally increases as μ decreases.

ILLUSTRATIVE EXAMPLE

Problem

An isotropic flat rectangular plate is built into a structure in such a way that its long edges may be assumed to be clamped and its short edges simply supported. The plate is 24 inches long, 12 inches wide, and 1/8 inch thick. The modulus of elasticity E is 10,000,000 psi and Poisson's ratio μ is 1/3. The problem is to determine the longitudinal compressive stress σ_x that causes buckling on the assumption, first, that the Poisson effect (transverse expansion) takes place freely and, second, that the Poisson effect is completely prevented by members adjacent to the plate.

Solution

If the Poisson effect is allowed to take place freely, no transverse stress is induced in the plate. Consequently, $\sigma_y = 0$ and, from equation (2), $k_y = 0$. The

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length-width ratio β is equal to 24/12, or 2. The point defined by the values $k_y = 0$ and $\beta = 2$ is now located on the interaction chart of figure 4. The ordinate of this point, $k_x = 7.01$, is the critical value of k_x for the plate. The longitudinal stress which causes buckling is then found from equation (5) to be

$$\sigma_x = 7040 \text{ psi}$$

If the Poisson effect is prevented, a transverse stress equal to μ times the longitudinal stress is induced by the application of longitudinal stress. Accordingly, buckling will occur under a stress combination in which the transverse direct stress σ_y is one-third of the longitudinal direct stress σ_x. The ratio of stress coefficients k_y/k_x will also be 1:3. By trial and error, the point at which $\frac{k_y}{k_x} = \frac{1}{3}$ is found along the line $\beta = 2$ in figure 4. The value of k_x at this point is 5.93. The longitudinal direct stress corresponding to this value of k_x is given by equation (5) as

$$\sigma_x = 5950 \text{ psi}$$

It is noticed that prevention of the Poisson expansion reduces the buckling strength of the plate from 7040 psi to 5950 psi.

In this particular problem the transverse stress σ_y was assumed to be developed by prevention of the Poisson effect. The charts can also be used to determine the critical combinations of σ_x and σ_y when σ_y is established directly by a known loading.

CONCLUDING REMARKS

The principal results of the present investigation are embodied in the interaction charts. These charts can be used to obtain critical combinations of longitudinal

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and transverse direct stress for isotropic flat rectangular plates with several conditions of edge restraint.

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APPENDIX

CRITICAL COMBINATIONS OF DIRECT STRESS FOR FLAT RECTANGULAR PLATES DETERMINED BY ENERGY METHOD

The Energy Method

The critical stresses are determined from the principle that the work done by the applied loads during buckling is equal to the elastic-strain energy stored in the structure during buckling. If the structure under consideration is a flat rectangular plate with simple-support or clamped-edge conditions, subjected to two direct stresses in the plane of the plate, this equality can be written in the following form, which is adapted from equation (210) of reference 1:

$$\frac{\sigma_x t}{2}\iint\left(\frac{\partial w}{\partial x}\right)^2 dy\,dx + \frac{\sigma_y t}{2}\iint\left(\frac{\partial w}{\partial y}\right)^2 dy\,dx =$$

$$\frac{D}{2}\iint\left\{\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2 - 2\left(1-\mu\right)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]\right\}dy\,dx \tag{A1}$$

where the integrations are performed over the surface of the plate. It can be shown (reference 6) that for a rectangular plate with supported edges, the term $\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2$ on the right-hand side of equation (A1) vanishes upon integration. Consequently, equation (A1) can be reduced to

$$k_x\iint\left(\frac{\partial w}{\partial x}\right)^2 dy\,dx + k_y\iint\left(\frac{\partial w}{\partial y}\right)^2 dy\,dx =$$

$$\frac{b^2}{\pi^2}\iint\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2 dy\,dx \tag{A2}$$

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where

$$k_x = \sigma_x \frac{b^2 t}{\pi^2 D}$$

$$k_y = \sigma_y \frac{b^2 t}{\pi^2 D}$$

An approximate solution for the critical stresses can be obtained by assuming a deflection function $w = f(x,y)$ that satisfies the boundary conditions of the plate and substituting this function for w in equation (A2). An interaction equation defining the critical relationship between k_x and k_y results. This relationship is usually approximate and unconservative, as is generally the case in energy solutions. (See reference 7 or pp. 81-82 of reference 1.) When the assumed buckle pattern happens to be the true one (that is, satisfies the differential equation of equilibrium), the solution is exact. Conversely, the true buckle pattern is that one, of all possible buckle patterns satisfying the boundary conditions, which gives the lowest buckling stress. If there are several energy solutions to the same problem, therefore, the most accurate is the one that gives the lowest critical stress.

By using the energy method, the problem of buckling of flat rectangular plates under the action of two direct stresses was solved for four different cases of edge restraint. In each case, as many deflection functions were used as were necessary to include all the expected types of buckling configuration; and, for any given values of k_y and β, only the lowest of all the resulting values of k_x was used.

Boundary Conditions

The deflection functions were chosen to satisfy the following boundary conditions:

Along simply supported short edges

$$w = 0$$

$$\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} = 0$$

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Along clamped short edges

$$w = 0$$

$$\frac{\partial w}{\partial x} = 0$$

Along simply supported long edges

$$w = 0$$

$$\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2} = 0$$

Along clamped long edges

$$w = 0$$

$$\frac{\partial w}{\partial y} = 0$$

Deflection Functions and Interaction Equations

The deflection functions and interaction equations corresponding to the different buckle patterns can be obtained by assigning different combinations of positive integral values to m and n in the following general equations. In each of the cases (b), (c), and (d) two types of deflection function were used. Those functions which were found to give generally lower (and therefore more accurate) results when transverse compression predominates are called deflection functions 1. Those which were found to give lower results when longitudinal compression predominates are called deflection functions 2. In each of figures 3 to 5, a dotted demarcation curve separates that region (below the curve) governed by deflection functions 1 from that region (above the curve) governed by deflection functions 2. Portions of the dotted curves which would coincide with portions of the dashed curves are omitted in figures 4 and 5.

Case (a) all edges simply supported.-

Deflection functions:

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$$w = A \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

for $m = 1, 2, 3 \ldots$ and $n = 1, 2, 3 \ldots$.

Limits of plate:

$$x = 0, a; \quad y = 0, b$$

Interaction equations:

$$k_x = \frac{m^2}{\beta^2} + 2n^2 + \frac{\beta^2 n^2}{m^2}\left(n^2 - k_y\right) \tag{A3}$$

for $m = 1, 2, 3 \ldots$ and $n = 1, 2, 3 \ldots$. Equation (A3) yields exact results since the assumed deflection satisfies the differential equation of equilibrium (equation (209) of reference 1).

Case (b) long edges simply supported, short edges clamped.-

Deflection functions 1:

$$w = A\left[\frac{\pi}{2}\left(\frac{x^2}{a^2} - \frac{1}{4}\right) + \frac{1}{2}\cos\frac{\pi x}{a}\right]\sin\frac{n\pi y}{b}$$

for $m = 1$ and $n = 1, 2, 3 \ldots$.

Limits of plate:

$$x = -\frac{a}{2}, \frac{a}{2}; \quad y = 0, b$$

Interaction equations:

$$k_x = \frac{4.14}{n^2\beta^2} + 2 + 0.806\beta^2\left(n^2 - k_y\right) \tag{A4}$$

for $m = 1$ and $n = 1, 2, 3 \ldots$.

Deflection functions 2:

$$w = A\left[\cos\frac{(m - 1)\pi x}{a} - \cos\frac{(m + 1)\pi x}{a}\right]\sin\frac{n\pi y}{b}$$

for $m = 1, 2, 3 \ldots$ and $n = 1$.

Limits of plate:

$$x = 0, a; \quad y = 0, b$$

Interaction equations:

$$k_x = \frac{4}{\beta^2} + 2 + \frac{3\beta^2}{4}\left(1 - k_y\right) \tag{A5}$$

for $m = 1$ and $n = 1$.

$$k_x = \frac{m^4 + 6m^2 + 1}{\beta^2(m^2 + 1)} + 2 + \frac{\beta^2(1 - k_y)}{m^2 + 1} \tag{A6}$$

for $m = 2, 3, 4 \ldots$ and $n = 1$.

<u>Case (c) Long edges clamped, short edges simply supported.</u>-

Deflection functions 1:

$$w = A \sin\frac{m\pi x}{a}\left[\cos\frac{(n - 1)\pi y}{b} - \cos\frac{(n + 1)\pi y}{b}\right]$$

for $m = 1, 2, 3 \ldots$ and $n = 1, 2, 3 \ldots$

Limits of plate:

$$x = 0, a; \quad y = 0, b$$

Interaction equations:

$$k_x = \frac{1}{\beta^2} + 2\left(n^2 + 1\right) + \beta^2\left[\left(n^4 + 6n^2 + 1\right) - \left(n^2 + 1\right) k_y\right] \tag{A7}$$

for $m = 1$ and $n = 2, 3, 4 \ldots$

$$k_x = \frac{m^2}{\beta^2} + \frac{4}{3} + \frac{4\beta^2}{3m^2}\left(1 - k_y\right) \tag{A8}$$

for $m = 1, 2, 3 \ldots$ and $n = 1$.

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Deflection functions 2:

$$w = A \sin \frac{m\pi x}{a} \left[\frac{\pi}{2} \left(\frac{y^2}{b^2} - \frac{1}{4} \right) + \frac{1}{2} \cos \frac{\pi y}{b} \right]$$

for $m = 1, 2, 3 \ldots$ and $n = 1$.

Limits of plate:

$$x = 0, a; \quad y = -\frac{b}{2}, \frac{b}{2}$$

Interaction equations:

$$k_x = \frac{m^2}{p^2} + 2.12 + \frac{p^2}{m^2} \left(5.14 - 1.24 k_y \right) \tag{A9}$$

for $m = 1, 2, 3 \ldots$ and $n = 1$.

Case (d) all edges clamped.

Deflection functions 1:

$$w = A \left[\frac{\pi}{2} \left(\frac{x^2}{a^2} - \frac{1}{4} \right) + \frac{1}{2} \cos \frac{\pi x}{a} \right] \left[\cos \frac{(n-1)\pi y}{b} - \cos \frac{(n+1)\pi y}{b} \right]$$

for $m = 1$ and $n = 1, 2, 3 \ldots$.

Limits of plate:

$$x = -\frac{a}{2}, \frac{a}{2}; \quad y = 0, b$$

Interaction equations:

$$k_x = \frac{4.14}{p^2} + 2(n^2 + 1) + 0.806 p^2 \left[n^4 + 6n^2 + 1 - \left(n^2 + 1 \right) k_y \right] \tag{A10}$$

for $m = 1$ and $n = 2, 3, 4 \ldots$.

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$$k_x = \frac{4.14}{\beta^2} + 2.67 + 1.075\beta^2(4 - k_y) \qquad \text{(A11)}$$

for $m = 1$ and $n = 1$.

Deflection functions 2:

$$w = A\left[\cos\frac{(m-1)\pi x}{a} - \cos\frac{(m+1)\pi x}{a}\right]\left[\frac{\pi}{2}\left(\frac{y^2}{b^2} - \frac{1}{4}\right) + \frac{1}{2}\cos\frac{\pi y}{b}\right]$$

for $m = 1, 2, 3 \ldots$ and $n = 1$.

Limits of plate:

$$x = 0, a; \quad y = -\frac{b}{2}, \frac{b}{2}$$

Interaction equations:

$$k_x = \frac{4}{\beta^2} + 2.48 + \beta^2\,(3.85 - 0.93k_y) \qquad \text{(A12)}$$

for $m = 1$ and $n = 1$.

$$k_x = \frac{m^4 + 6m^2 + 1}{\beta^2(m^2 + 1)} + 2.48 + \frac{\beta^2(5.14 - 1.24k_y)}{m^2 + 1} \qquad \text{(A13)}$$

for $m = 2, 3, 4 \ldots$ and $n = 1$.

The interaction equations (A3) to (A13) were used in the preparation of the interaction charts of figures 2 to 5. The different regions of the charts, as marked by the dashed curves, are labeled with the values of m and n that gave minimum values of k_x for given values of k_y and β. These values of m and n also define the governing deflection function and indicate the buckle pattern.

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TABLE 1

CRITICAL COMBINATIONS OF LONGITUDINAL AND TRANSVERSE DIRECT-STRESS COEFFICIENTS FOR FLAT RECTANGULAR PLATES WITH ALL EDGES SIMPLY SUPPORTED

[Data for fig. 2]

k_y															
-3		-1		0		0.5		0.75		0.9		1.0		1.05	
β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x
1.000	[a]7.00	1.000	5.00	1.000	[b]4.00	1.000	3.50	1.000	3.25	1.000	3.10	1.000	3.00	1.000	2.95
1.200	6.22	1.100	5.24	1.200	4.13	1.189	[b]3.41	1.200	3.05	1.500	2.67	1.500	2.44	1.500	2.33
1.414	[b]6.00	1.189	[a]5.53	1.414	[a]4.50	1.500	3.57	1.414	[b]3.00	1.778	[b]2.63	2.000	2.25	2.000	2.05
1.600	6.12	1.682	[b]4.83	1.600	4.20	1.682	[a]3.77	1.800	3.12	1.900	2.64	3.000	2.11	3.000	1.66
1.732	[a]6.33	1.300	5.21	1.800	4.04	2.000	3.50	2.000	[a]3.25	2.515	[a]2.79	4.000	2.06	4.000	1.26
1.900	6.10	1.500	4.90	2.000	[b]4.00	2.378	[b]3.41	2.300	3.09	2.700	2.73	5.000	2.04	5.000	.79
2.122	[b]6.00	1.900	4.91	2.300	4.08	2.700	3.46	2.600	3.01	3.000	2.67				
2.300	6.05	2.060	[a]5.06	2.449	[a]4.17	2.912	[a]3.53	2.828	[b]3.00	3.300	2.64				
2.450	[a]6.17	2.300	4.87	2.700	4.04	3.300	3.43	3.200	3.03	3.557	[b]2.63				
2.600	6.05	2.523	[b]4.83	3.000	[b]4.00	3.568	[b]3.41	3.463	[a]3.08	3.800	2.64				
2.829	[b]6.00	2.700	4.85	3.464	[a]4.08	4.119	[a]3.47	4.243	[b]3.00	4.200	2.67				
3.163	[a]6.10	2.913	[a]4.95	4.000	[b]4.00	4.757	[b]3.41	4.899	[a]3.04	4.355	[a]2.68				
3.536	[b]6.00	3.100	4.86	4.472	[a]4.05					5.000	2.64				
3.873	[a]6.07	3.364	[b]4.83	5.000	[b]4.00										
4.243	[b]6.00	3.761	[a]4.90												
4.583	[a]6.05	4.205	[b]4.83												
		4.606	[a]4.87												
		5.046	[b]4.83												

k_y													
1.1		1.2		1.3		1.5		2		3		4	
β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x
1.000	2.90	1.000	2.80	1.000	2.70	1.000	2.50	1.000	2.00	1.000	1.00	1.000	0
1.200	2.55	1.500	1.99	1.500	1.77	1.500	1.32	1.500	.19	1.250	-.48	1.200	-1.62
1.500	2.22	2.000	1.45	2.000	1.05	2.000	.25	2.000	-1.75	1.500	-2.05	1.500	-4.30
2.000	1.85	2.500	.91	2.500	.28	3.000	-2.39	2.500	-4.09	1.750	-3.80		
2.500	1.53	3.000	.31	3.000	-.59	4.000	-5.94			2.000	-5.75		
3.000	1.21	3.500	-.36	3.500	-1.59								
4.000	.46	4.000	-1.14	4.000	-2.74								
5.000	-.46	4.500	-2.00	4.500	-4.02								

[a]Cusp, indicating change in the number of longitudinal buckles.

[b]Minimum point of a scallop.

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TABLE 2

CRITICAL COMBINATIONS OF LONGITUDINAL AND TRANSVERSE DIRECT-STRESS COEFFICIENTS FOR FLAT RECTANGULAR PLATES WITH LONG EDGES SIMPLY SUPPORTED AND SHORT EDGES CLAMPED

[Data for fig. 3]

k_y													
-10		-5		-2		-1		0		0.5		0.8	
β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x
1.390	[b]10.50	1.062	[a]10.66	1.000	8.25	1.000	7.50	1.000	6.75	1.000	6.37	1.000	6.15
1.489	[a]10.58	1.500	8.34	1.155	[b]8.00	1.100	7.12	1.200	5.86	1.200	5.32	1.200	4.99
1.800	9.76	1.617	[b]8.27	1.263	[a]8.10	1.278	[b]6.90	1.520	[b]5.46	1.500	4.62	1.500	4.11
1.875	[b]9.73	1.732	[a]8.33	1.500	6.99	1.398	[a]6.98	1.662	[a]5.52	1.807	[b]4.45	1.700	3.82
1.994	[a]9.79	2.000	7.80	1.700	6.57	1.600	6.23	2.000	4.85	1.977	[a]4.49	2.000	3.60
2.200	9.42	2.182	[b]7.71	1.923	[b]6.43	1.900	5.71	2.300	4.61	2.300	4.08	2.272	[b]3.55
2.380	[b]9.33	2.320	[a]7.75	2.060	[a]6.48	2.128	[b]5.62	2.530	[b]4.56	2.700	3.85	2.486	[a]3.57
2.523	[a]9.38	2.500	7.53	2.200	6.26	2.279	[a]5.65	2.711	[a]4.58	3.009	[b]3.81	3.000	3.27
2.700	9.18	2.769	[b]7.41	2.400	6.09	2.600	5.36	3.000	4.41	3.223	[a]3.83	3.500	3.16
2.898	[b]9.11	2.936	[a]7.45	2.595	[b]6.04	2.872	[b]5.30	3.415	[b]4.33	3.700	3.68	3.784	[b]3.14
3.061	[a]9.15	3.372	[b]7.25	2.760	[a]6.07	3.054	[a]5.32	3.632	[a]4.35	4.061	[b]3.65	4.053	[a]3.15
3.425	[b]8.97	3.561	[a]7.28	3.000	5.89	3.646	[b]5.12	4.335	[b]4.21	4.319	[a]3.66	4.500	3.08
3.603	[a]9.01	3.985	[b]7.15	3.290	[b]5.83	3.864	[a]5.14	4.596	[a]4.22	5.155	[b]3.56	5.107	[b]3.04
3.957	[b]8.89	4.193	[a]7.18	3.492	[a]5.85	4.438	[b]5.03	5.278	[b]4.14				
4.147	[a]8.92	4.604	[b]7.09	4.010	[b]5.71	4.688	[a]5.05						
4.492	[b]8.83	4.825	[a]7.11	4.236	[a]5.73	5.244	[b]4.97						
4.693	[a]8.86	5.228	[b]7.04	4.739	[b]5.64								
		5.460	[a]7.06	4.986	[a]5.66								
				5.475	[b]5.60								

k_y													
1		1.1		1.3		1.5		2		3		4	
β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x
1.000	6.00	1.000	5.92	1.000	5.77	1.000	5.62	1.000	5.25	1.000	4.5	1.000	3.72
1.100	5.30	1.100	5.21	1.100	5.03	1.100	4.85	1.100	4.40	1.500	.213	1.100	2.50
1.200	4.78	1.200	4.67	1.200	4.45	1.200	4.24	1.200	3.70	2.000	-3.41	1.200	1.39
1.300	4.36	1.400	3.89	1.300	3.98	1.300	3.73	1.400	2.53	2.300	-5.75	1.300	.36
1.500	3.78	1.600	3.37	1.400	3.60	1.400	3.30	1.500	2.02			1.400	-.63
1.750	3.30	1.800	2.99	1.500	3.27	1.500	2.93	1.600	1.55			1.600	-2.57
2.000	3.00	2.000	2.70	1.600	2.99	1.600	2.59	1.800	.66			1.700	-3.56
2.500	2.64	2.300	2.36	1.700	2.73	1.700	2.27	2.000	-.19				
3.000	2.44	2.500	2.16	1.800	2.47	1.800	1.97	2.200	-1.05				
4.000	2.25	3.000	1.73	2.000	2.03	2.000	1.42	2.400	-1.92				
5.000	2.16	3.500	1.35	2.200	1.65	2.200	.90	2.500	-2.38				
		4.000	.97	2.500	1.13	2.400	.39	2.600	-2.84				
		4.500	.57	3.000	.26	2.700	-.37	3.000	-4.80				
		5.000	.15	3.500	-.64	3.000	-1.17						
				4.000	-1.62	3.400	-2.30						
				4.500	-2.71	3.800	-3.53						

[a]Cusp, indicating change in number of longitudinal buckles.
[b]Minimum point of a scallop.

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TABLE 3

CRITICAL COMBINATIONS OF LONGITUDINAL AND TRANSVERSE DIRECT-STRESS COEFFICIENTS FOR FLAT RECTANGULAR PLATES WITH LONG EDGES CLAMPED AND SHORT EDGES SIMPLY SUPPORTED

[Data for fig. 4]

k_y													
-5		-2		0		1		2		3		3.5	
β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x
1.090	[b]9.21	1.000	8.38	1.000	7.76	1.000	7.38	1.000	6.14	1.000	4.90	1.058	[b]4.27
1.335	[a]9.78	1.100	8.08	1.100	7.34	1.006	[a]7.42	1.107	[a]6.56	1.100	5.02	1.300	4.38
1.635	[b]9.21	1.204	[b]8.00	1.200	7.11	1.100	6.92	1.200	6.21	1.200	5.22	1.400	4.48
1.888	[a]9.48	1.300	8.06	1.328	[b]7.01	1.200	6.66	1.300	5.97	1.295	[a]5.46	1.530	[a]4.65
2.180	[b]9.21	1.400	8.25	1.500	7.15	1.423	[b]6.43	1.400	5.82	1.400	5.21	1.700	4.44
2.437	[a]9.38	1.474	[a]8.46	1.627	[a]7.39	1.600	6.54	1.566	[b]5.74	1.500	5.06	1.900	4.31
2.600	9.24	1.600	8.16	1.800	7.11	1.743	[a]6.76	1.800	5.87	1.700	4.89	2.115	[b]4.27
2.725	[b]9.21	1.806	[b]8.00	1.992	[b]7.01	1.900	6.54	1.918	[a]6.01	1.832	[b]4.86	2.500	4.35
2.985	[a]9.33	2.085	[a]8.23	2.200	7.10	2.135	[b]6.43	2.100	5.82	2.100	4.95	2.630	[a]4.39
3.270	[b]9.21	2.200	8.09	2.301	[a]7.20	2.400	6.54	2.349	[b]5.74	2.244	[a]5.06	2.900	4.30
3.531	[a]9.29	2.408	[b]8.00	2.500	7.05	2.465	[a]6.59	2.600	5.81	2.500	4.90	3.173	[b]4.27
3.814	[b]9.21	2.692	[a]8.14	2.657	[b]7.01	2.700	6.45	2.712	[a]5.88	2.748	[b]4.86	3.600	4.32
4.078	[a]9.27	3.010	[b]8.00	2.970	[a]7.13	2.846	[b]6.43	3.132	[b]5.74	3.000	[a]4.90	3.685	[a]4.34
4.360	[b]9.21	3.297	[a]8.09	3.321	[b]7.01	3.182	[a]6.53	3.501	[a]5.82	3.173	[b]4.96	4.231	[b]4.27
4.624	[a]9.26	3.611	[b]8.00	3.637	[a]7.09	3.558	[b]6.43	3.915	[b]5.74	3.663	[a]4.86	5.288	[b]4.27
4.905	[b]9.21	3.901	[a]8.07	3.985	[b]7.01	3.897	[a]6.50	4.288	[a]5.80	4.096	[b]4.92		
5.170	[a]9.25	4.213	[b]8.00	4.305	[a]7.07	4.270	[b]6.43	4.698	[b]5.74	4.579	[a]4.86		
5.450	[b]9.21	4.504	[a]8.05	4.649	[b]7.01	4.612	[a]6.48	5.074	[a]5.78	5.017	[b]4.90		
		4.816	[b]8.00	4.970	[a]7.05	4.981	[b]6.43						
		5.108	[a]8.04	5.313	[b]7.01	5.324	[a]6.47						

k_y													
3.8		4.0		4.1		4.2		4.5		5		6	
β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x
1.000	3.91	1.000	3.66	1.000	3.53	1.000	3.40	1.000	3.00	1.000	2.33	1.000	1.00
1.200	3.74	1.250	3.31	1.250	3.10	1.250	2.89	1.100	2.68	1.100	1.88	1.100	.26
1.392	[b]3.70	1.500	3.11	1.500	2.81	1.500	2.51	1.250	2.27	1.250	1.23	1.250	-.85
1.400	3.70	2.000	2.91	2.000	2.38	2.000	1.85	1.500	1.61	1.500	.11	1.500	-2.89
1.600	3.74	3.000	2.78	2.500	1.99	2.500	1.16	1.800	.81	1.600	-.35		
1.800	3.84	4.000	2.73	3.000	1.58	3.000	.38	2.000	.25	2.000	-2.41		
1.945	[a]3.94	5.000	2.70	3.500	1.11	3.500	-.52	2.200	-.352				
2.000	3.91			4.000	.59	4.000	-1.54	2.500	-1.33				
2.200	3.81			4.500	.01	5.000	-3.96	3.000	-3.22				
2.300	3.77			5.000	-.62								
2.500	3.72												
2.700	3.70												
2.784	[b]3.70												
2.900	3.70												
3.100	3.72												
3.200	3.74												
3.408	[a]3.78												
4.176	[b]3.70												
4.820	[a]3.74												
5.568	[b]3.70												

[a]Cusp, indicating change in number of longitudinal buckles.
[b]Minimum point of a scallop.

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TABLE 4

CRITICAL COMBINATIONS OF LONGITUDINAL AND TRANSVERSE DIRECT-STRESS COEFFICIENTS

FOR FLAT RECTANGULAR PLATES WITH ALL EDGES CLAMPED

[Data for fig. 5]

k_y													
-10		-5		-1		0		1		2		3	
β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x
1.237	[b]13.21	1.200	11.44	1.045	[a]11.38	1.009	[b]10.33	1.000	9.40	1.000	8.47	1.000	7.54
1.325	[a]13.31	1.379	[b]11.10	1.200	10.01	1.103	[a]10.47	1.081	9.32	1.190	[b]8.13	1.200	6.79
1.669	[b]12.25	1.477	[a]11.19	1.400	9.16	1.200	9.65	1.183	[a]9.44	1.301	[a]8.23	1.392	[b]6.61
1.775	[a]12.32	1.600	10.70	1.592	[b]8.95	1.500	8.44	1.400	8.19	1.500	7.32	1.522	[a]6.68
2.000	11.80	1.861	[b]10.33	1.706	[a]9.01	1.680	[b]8.28	1.700	7.57	1.800	6.73	1.800	5.93
2.118	[b]11.74	1.979	[a]10.39	1.900	8.55	1.801	[a]8.34	1.801	[b]7.54	1.981	[b]6.66	2.100	5.59
2.246	[a]11.80	2.100	10.13	2.148	[b]8.37	2.000	7.93	1.929	[a]7.58	2.123	[a]6.70	2.318	[b]5.53
2.400	11.55	2.362	[b]9.92	2.285	[a]8.41	2.268	[b]7.76	2.200	7.18	2.400	6.37	2.483	[a]5.56
2.579	[b]11.46	2.504	[a]9.97	2.500	8.15	2.412	[a]7.81	2.430	[b]7.08	2.674	[b]6.28	2.800	5.33
2.724	[a]11.51	2.700	9.75	2.727	[b]8.06	2.600	7.59	2.584	[a]7.12	2.844	[a]6.31	3.128	[b]5.26
2.800	11.41	2.876	[b]9.69	2.892	[a]8.10	2.889	[b]7.49	2.800	6.93	3.000	6.19	3.327	[a]5.281
3.048	[b]11.29	3.038	[a]9.74	3.321	[b]7.89	3.052	[a]7.52	3.084	[b]6.84	3.390	[b]6.08	3.600	5.16
3.206	[a]11.33	3.399	[b]9.56	3.507	[a]7.92	3.506	[b]7.34	3.271	[a]6.87	3.599	[a]6.11	3.971	[b]5.11
3.521	[b]11.18	3.576	[a]9.59	3.924	[b]7.79	3.700	[a]7.37	3.756	[b]6.71	4.133	[b]5.97	4.210	[a]5.13
3.691	[a]11.22	3.927	[b]9.47	4.129	[a]7.82	4.143	[b]7.25	3.967	[a]6.74	4.366	[a]5.99	4.835	[b]5.03
3.979	[b]11.11	4.116	[a]9.50	4.534	[b]7.72	4.358	[a]7.27	4.438	[b]6.63	4.883	[b]5.91	5.107	[a]5.05
4.176	[a]11.14	4.459	[b]9.41	4.752	[a]7.75	4.785	[b]7.19	4.669	[a]6.65	5.138	[a]5.93	5.714	[b]4.98
		4.657	[a]9.44	5.148	[b]7.68	5.016	[a]7.21	5.127	[b]6.58	5.642	[b]5.87		
				5.377	[a]7.70	5.434	[b]7.15						

k_y													
3.5		3.8		4.0		4.2		4.5		5		6	
β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x	β	k_x
1.000	7.08	1.000	6.80	1.000	6.61	1.000	6.43	1.000	6.15	1.000	5.68	1.000	4.66
1.300	5.86	1.100	6.13	1.100	5.95	1.100	5.72	1.100	5.38	1.100	4.80	1.100	3.24
1.607	[b]5.58	1.200	5.72	1.200	5.45	1.200	5.18	1.200	4.78	1.200	4.00	1.200	2.28
1.757	[a]5.63	1.300	5.39	1.300	5.07	1.300	4.76	1.300	4.20	1.300	3.30	1.300	1.38
2.000	5.17	1.400	5.15	1.400	4.78	2.000	2.84	1.500	3.30	1.500	2.09	1.400	.53
2.300	4.87	1.500	4.98	1.500	4.51	3.000	1.19	1.700	2.56	1.700	.98	1.500	-.33
2.675	[b]4.77	2.000	4.56	1.700	4.10	4.000	-.51	2.000	1.55	2.000	-.60	1.600	-1.13
2.866	[a]4.79	2.250	[a]4.55	2.000	3.70	5.000	-2.54	2.200	.92	2.100	-1.13	2.000	-4.90
3.200	4.63	2.300	4.48	2.500	3.33			2.500	-.03				
3.611	[b]4.56	2.700	4.23	3.000	3.13			3.000	-1.71				
3.841	[a]4.58	3.128	[b]4.15	3.500	3.00								
4.200	4.49	3.351	[a]4.17	4.000	2.92								
4.583	[b]4.45	3.800	4.04	4.500	2.87								
4.859	[a]4.47	4.221	[b]4.00	5.000	2.83								
5.581	[b]4.39	4.490	[a]4.02										
		4.800	3.96										
		5.359	[b]3.92										

[a]Cusp, indicating change in the number of longitudinal buckles.
[b]Minimum point of a scallop.

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TABLE 5

COMPARISONS OF RESULTS OF PRESENT INVESTIGATION WITH EXACT SOLUTIONS

Case (b): long edges simply supported, short edges clamped

$k_x = 0$				$k_y = 0$			
	k_y				k_x		
β	Exact solution (1)	Present solution (2)	Percent difference	β	Exact solution (3)	Present solution (4)	Percent difference
1	7.69	7.76	0.9	1.0	6.74	6.75	0.1
$1\frac{1}{9}$	6.34	6.39	.8	1.2	5.84	5.86	.3
				1.4	5.45	5.51	1.1
				1.6	5.34	5.48	2.6
$1\frac{1}{4}$	4.66	4.69	.6	1.7	5.33	5.41	1.5
				1.73	5.33	5.34	.2
$1\frac{3}{7}$	3.43	3.45	.6	1.8	5.18	5.18	0
				2.0	4.85	4.85	0
$1\frac{2}{3}$	2.54	2.56	.8	2.5	4.52	4.56	.9
				2.83	4.50	4.50	0
2	1.92	1.94	1.0	3.0	4.41	4.41	0
$2\frac{1}{2}$	1.51	1.53	1.3				

Case (c): long edges clamped, short edges simply supported

$k_x = 0$				$k_y = 0$			
	k_y				k_x		
β	Exact solution (3)	Present solution (5)	Percent difference	β	Exact solution (6)	Present solution (7)	Percent difference
1	6.74	6.75	0.1	1.0	7.69	7.76	0.9
$1\frac{1}{4}$	5.59	5.59	0	1.2	7.05	7.11	.7
				1.4	7.00	7.04	.6
$1\frac{2}{3}$	4.82	4.82	0	1.6	7.31	7.33	.3
				1.8	7.06	7.11	.6
				2.0	6.98	7.01	.4
				2.1	7.00	7.04	.6
				3.0	7.05	7.11	.9

Case (d): all edges clamped

$k_x = 0$				$k_y = 0$			
	k_y				k_x		
β	Exact solution	Present solution (10)	Percent difference	β	Exact solution (8)	Present solution (11)	Percent difference
1	10.07[8]	10.33	2.6	1.0	10.07	10.33	2.6
$1\frac{1}{3}$	6.56[8]	6.62	.9	1.25	9.25	9.34	1.0
				1.50	8.33	8.44	1.3
2	4.84[9]	4.86	.4	1.75	8.11	8.31	2.5
				2.0	7.88	7.93	.6
				2.5	7.57	7.69	1.6
				3.0	7.37	7.51	1.9

[1]Reference 1, p. 345.
[2]Value for $\beta = 1$ based upon equation (A4) with $n = 2$; other values from equation (A4) with $n = 1$.
[3]Reference 1, p. 364.
[4]From equations (A5) and (A6).
[5]From equation (A8) with $m = 1$.
[6]Fig. 6 of reference 2.
[7]From equation (A9).
[8]Reference 3.
[9]Lower limit, reference 4.
[10]From equation (A11).
[11]From equations (A12) and (A13).

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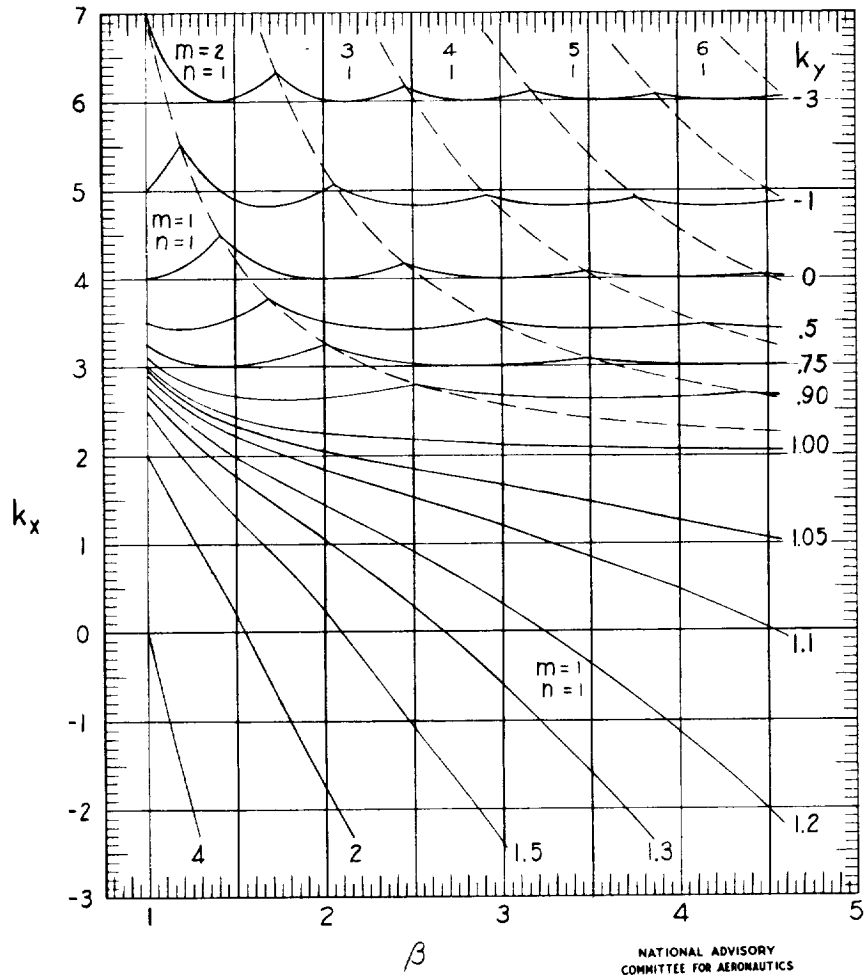
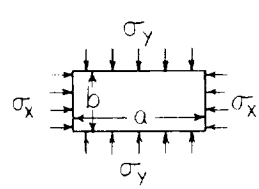




$$\sigma_x = k_x\left(\frac{\pi^2 D}{b^2 t}\right) \quad \sigma_y = k_y\left(\frac{\pi^2 D}{b^2 t}\right) \quad \beta = \frac{a}{b}$$

m = number of longitudinal buckles
n = number of transverse buckles

Figure 2.- Critical combinations of longitudinal and transverse direct-stress coefficients for flat rectangular plates with all edges simply supported.

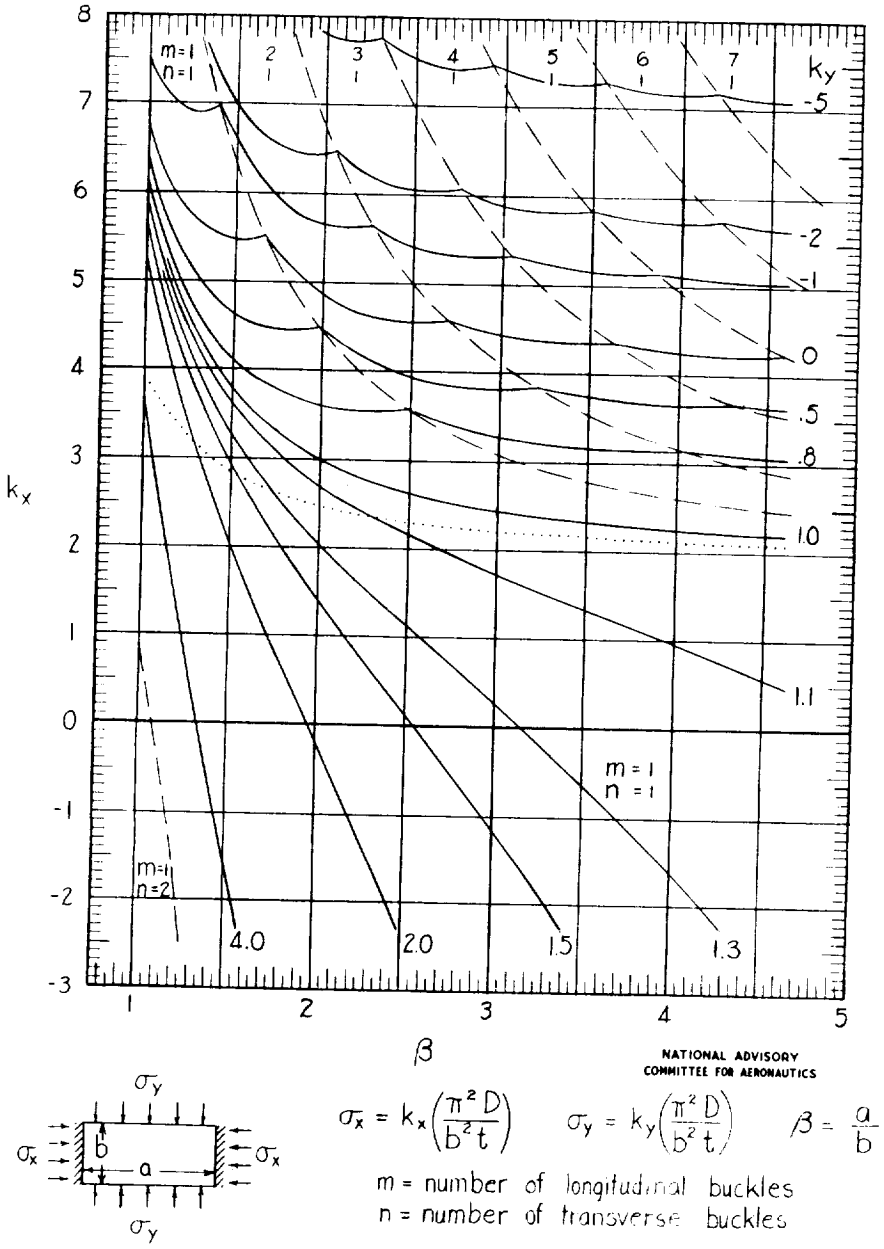


Figure 3.- Critical combinations of longitudinal and transverse direct-stress coefficients for flat rectangular plates with long edges simply supported and short edges clamped.

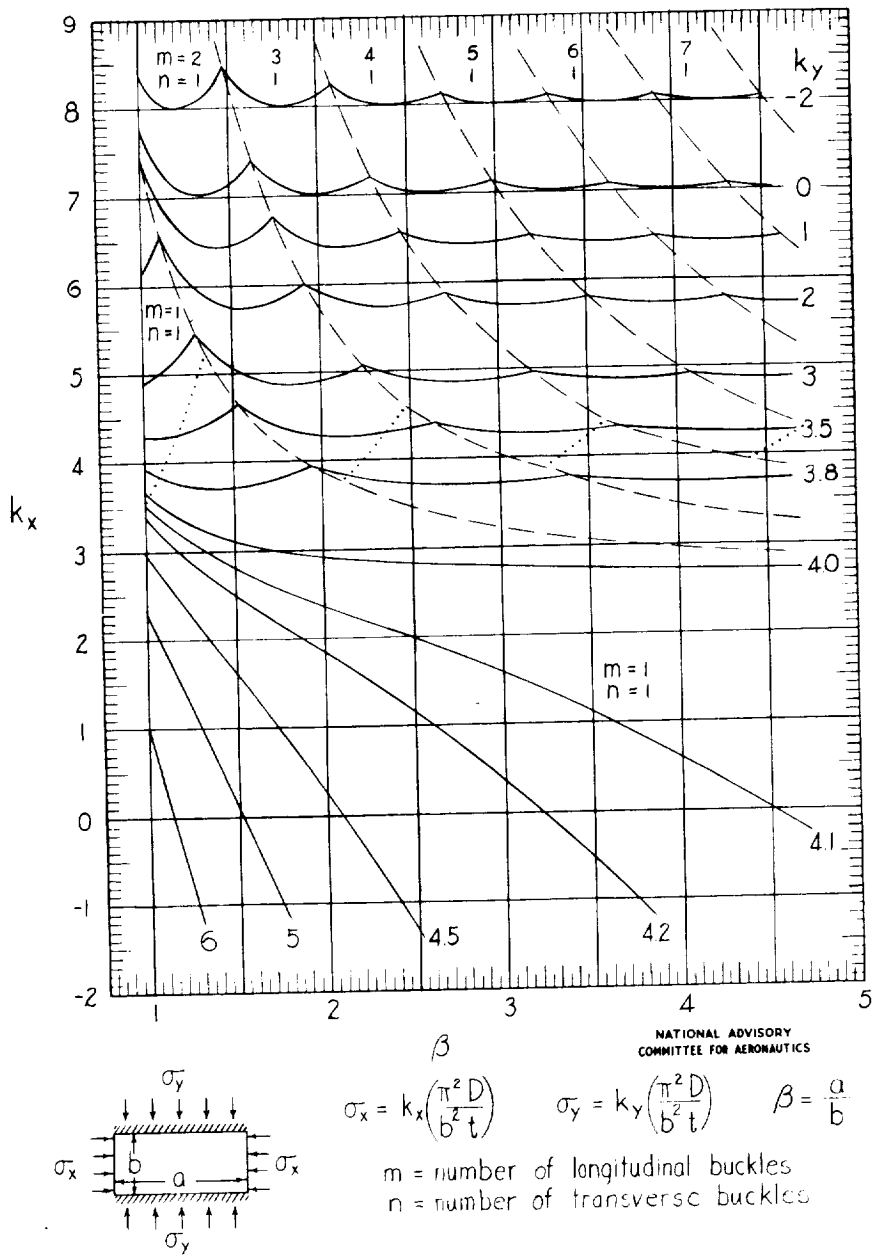
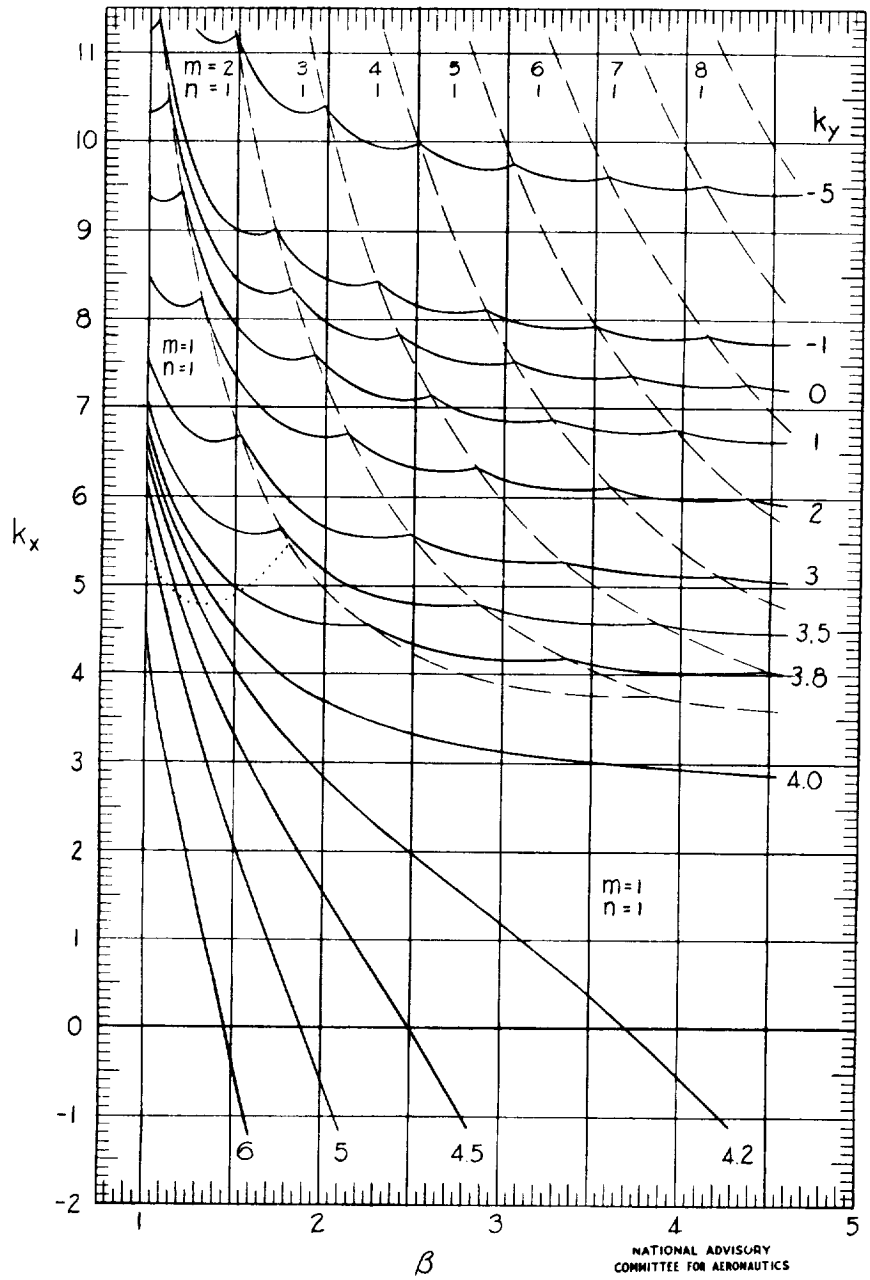
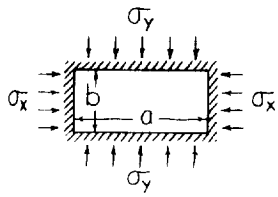


Figure 4.- Critical combinations of longitudinal and transverse direct-stress coefficients for flat rectangular plates with long edges clamped and short edges simply supported.

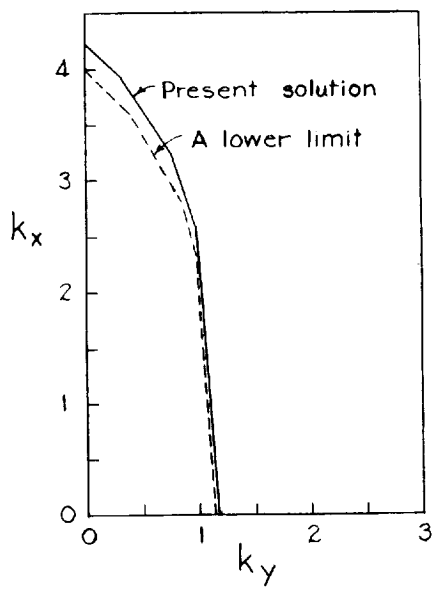




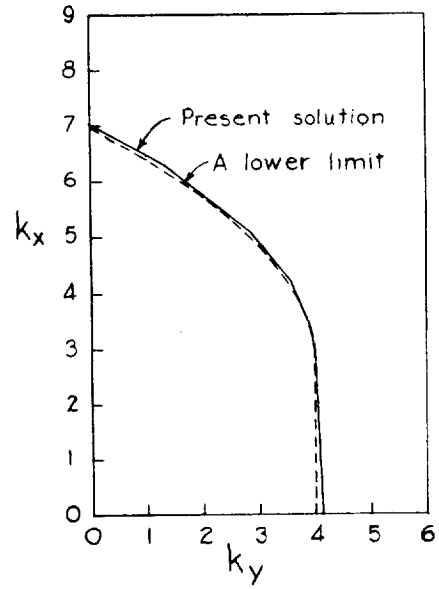
$$\sigma_x = k_x\left(\frac{\pi^2 D}{b^2 t}\right) \qquad \sigma_y = k_y\left(\frac{\pi^2 D}{b^2 t}\right) \qquad \beta = \frac{a}{b}$$

m = number of longitudinal buckles
n = number of transverse buckles

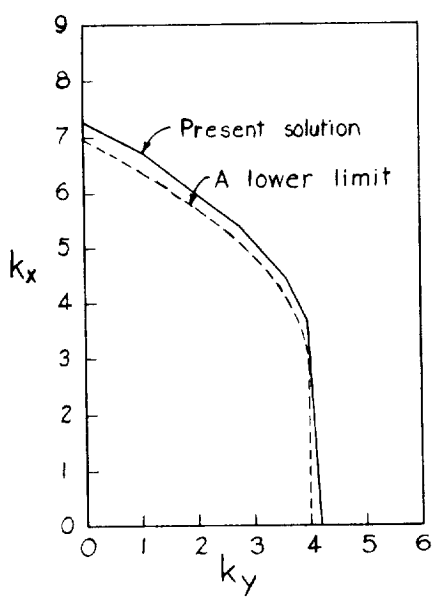
Figure 5.- Critical combinations of longitudinal and transverse direct-stress coefficients for flat rectangular plates with all edges clamped.



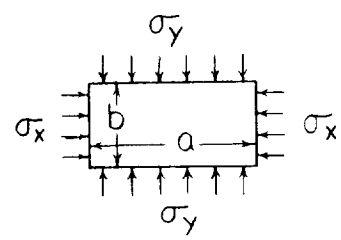
(a) Long edges simply supported, short edges clamped.



(b) Long edges clamped, short edges simply supported.



(c) All edges clamped.



$$\sigma_x = k_x\left(\frac{\pi^2 D}{b^2 t}\right)$$

$$\sigma_y = k_y\left(\frac{\pi^2 D}{b^2 t}\right)$$

$$\beta = \frac{a}{b}$$

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Figure 6.- Comparisons of present solutions with lower limits. $\beta = 4$.

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