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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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ORIGINALLY ISSUED

February 1945 as
Advance Confidential Report E5A29

ADDITION OF HEAT TO A COMPRESSIBLE FLUID IN MOTION

By L ce L. Hicks

Aircraft Engine Research Laboratory
Cleveland, Ohio

NACA

WASHINGTON

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ADDITION OF HEAT TO A COMPRESSIBLE FLUID IN MOTION

By Bruce L. Hicks

SUMMARY

The nature of nonadiabatic, frictionless, steady flow of a compressible fluid in a pipe of constant cross section is summarized. The flow conditions can most conveniently be described in terms of the local Mach number M. It was found that steady flow cannot occur when heat is being added to the fluid at a point where its velocity equals the local velocity of sound $(M = 1)$, and that the maximum temperature of the fluid occurs at $M = 1/\sqrt{\gamma}$ where γ is the ratio of specific heats. These results are illustrated by application to high-speed flow through combustion chambers.

INTRODUCTION

The study of combustion phenomena in a flowing gas must include consideration of the thermal and the dynamical properties of the fluid as well as of its chemical nature. The flow pattern of a burning gas, for example, is dependent upon the quantity of heat liberated at each internal point by the combustion of the moving fluid.

The purpose of this report is to summarize, without extended proofs, the results of a study of a simplified model of nonadiabatic, compressible fluid flow, both subsonic and supersonic, and to state these results in a form that will make them immediately useful in providing a theoretical background for current technical problems of high-speed combustion. The model chosen is the flow, in a tube of constant cross section, of a compressible fluid to which heat is being added. This model is a natural generalization of fluid-flow models commonly used and offers a logical point of departure for more advanced calculations.

In the present treatment, the local Mach number M of the flow is extensively used as a variable and the effects of heat addition upon the limiting conditions of flow of a compressible fluid are emphasized. Apparently these effects have not previously been explicitly described in the literature in terms of M, although related subjects in one-dimensional compressible flow have been treated using M by Bailey (reference 1) and Nielsen (reference 2).

THEORY

Statement of the Basic Equations

The characteristics of the model of nonadiabatic compressible flow, which were chosen in order to restrict and simplify the treatment, may be stated explicitly as follows:

1. The flow is steady (independent of time).

2. Pressure, temperature, and velocity do not vary within any section normal to the flow. The rate of heat addition to the fluid is therefore constant within a section.

3. The coefficients of viscosity and of thermal conductivity are equal to zero; hence, the fluid flow is not affected by friction or by conduction of heat in the direction of flow.

4. The fluid obeys the thermal equation of state for a perfect gas.

5. The values of the specific heats and of the gas constant are invariant throughout the flow.

With these restrictions, the fluid must move in accordance with the following equations, which describe its nonadiabatic frctionless flow in a pipe (fig. 1):

$$c_p T_1 + \frac{1}{2}V_1^2 + Q = c_p T_2 + \frac{1}{2}V_2^2 \text{ (conservation of energy)} \qquad (1)$$

$$p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2 \text{ (equation of motion)} \qquad (2)$$

$$\rho_1 V_1 = \rho_2 V_2 \text{ (conservation of mass)} \qquad (3)$$

$$\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2} \text{ (thermal equation of state)} \qquad (4)$$

The symbols p, ρ, T, V, and c_p in these equations represent pressure, density, temperature, velocity, and specific heat at constant pressure with consistent units used throughout. The subscripts 1 and 2 indicate that the symbols refer to sections 1 and 2 of the flow pictured in figure 1. The symbol Q represents the heat added per unit mass of fluid between sections 1 and 2.

A more convenient form of the equations is obtained by introducing the local Mach number $M = V/\sqrt{\gamma RT}$ in place of the velocity where R is the gas constant. Equations (1), (2), and (3) then become

$$T_1\left(1 + q + \frac{\gamma - 1}{2} M_1^2\right) = T_2\left(1 + \frac{\gamma - 1}{2} M_2^2\right) \tag{5}$$

$$p_1\left(1 + \gamma M_1^2\right) = p_2\left(1 + \gamma M_2^2\right) \tag{6}$$

$$\rho_1 M_1\sqrt{T_1} = \rho_2 M_2\sqrt{T_2} \tag{7}$$

The quantity q is equal to $Q/(c_pT_1)$ and γ is equal to c_p/c_v.

Through multiplication and division of equations (5), (6), and (7) it is possible to derive the fundamental equation that describes the addition of heat to a compressible fluid in one-dimensional motion.

$$\frac{\left(1 + q + \frac{\gamma - 1}{2} M_1^2\right)M_1^2}{\left(1 + \gamma M_1^2\right)^2} = \frac{\left(1 + \frac{\gamma - 1}{2} M_2^2\right)M_2^2}{\left(1 + \gamma M_2^2\right)^2} \tag{8}$$

Application of the Second Law of Thermodynamics

An important characteristic of the model described by the basic equations can be derived directly from the Second Law of Thermodynamics. The change in entropy per unit mass ΔS of a perfect gas between the two thermodynamic states in which it exists at sections 1 and 2 (fig. 1) is given by the expression

$$\Delta S = c_p \log_e \frac{T_2}{T_1}\left(\frac{p_2}{p_1}\right)^{\frac{1-\gamma}{\gamma}} \tag{9}$$

By use of equations (4), (6), and (7) the specific-entropy change of the fluid in passing from section 1 to section 2 may be rewritten in terms of the Mach numbers M_1 and M_2 as

$$\Delta S = c_p \log_e \frac{M_2{}^2 \left(1 + \gamma M_1{}^2\right)^{\frac{\gamma+1}{\gamma}}}{M_1{}^2 \left(1 + \gamma M_2{}^2\right)^{\frac{\gamma+1}{\gamma}}} \tag{10}$$

It is seen that the entropy change of the moving fluid depends only upon the Mach number. Detailed examination of equation (10) discloses the fact that the entropy when considered as a function of M_2 possesses only one maximum, which occurs at $M_2 = 1$. When $M_2 = 1$, therefore, the entropy cannot increase. Because the entropy change of a system that behaves in accordance with the Second Law of Thermodynamics must be positive when the quantity of heat added to it is positive, it is concluded that addition of a positive amount of heat to the fluid is impossible when $M_2 = 1$.

RESULTS AND DISCUSSION

From equation (8) and previous equations, a number of detailed characteristics of the steady nonadiabatic flow of a gas in a pipe of constant section (fig. 1) may be derived.

Mathematical Description of the Flow

The heat added $q = Q/(c_pT_1)$ and the changes in pressure, temperature, density, and velocity between sections 1 and 2 may each be expressed as a simple rational function of M_1 and M_2, as was done for q in equation (8). One physically significant value of M_2 exists for each M_1 and $q > 0$ if $M_1 < 1$ and two such values if $M_1 > 1$. If $M_1 < \sqrt{\dfrac{\gamma - 1}{2\gamma\ (\gamma q + 1)}}$, there is only one real value of M_2; if $1 > M_1 > \sqrt{\dfrac{\gamma - 1}{2\gamma\ (\gamma q + 1)}}$, there are two real values of M_2, one of which corresponds to an expansion shock and may be shown to be impossible on thermodynamic grounds. For $M_1 > 1$, transition from the larger of the two possible values of M_2 to the smaller one corresponds to the formation of a compression shock.

The direction of change of the quantities p, ρ, T, and Q between sections 1 and 2 can be specified by the signs of the derivatives of these quantities with respect to M. For the entire range of M from zero to infinity

$$\left.\begin{array}{lll} \dfrac{dp}{dM} \leqq 0 & \dfrac{dV}{dM} \geqq 0 & \dfrac{d\rho}{dM} \leqq 0 \\[2ex] (1 - M^2)^{-1} \dfrac{dQ}{dM} \geqq 0 & (1 - \gamma M^2)^{-1} \dfrac{dT}{dM} \geqq 0 & \end{array}\right\} \qquad (11)$$

As heat is added to the compressible fluid, the Mach number will increase for all values of $M < 1$; on the other hand, the temperature will rise only for $M < 1/\sqrt{\gamma}$ or $M > 1$ but will fall in the range of Mach number $1 > M > 1/\sqrt{\gamma}$. This behavior can be derived directly from equations (11).

Limiting Flow Conditions

If the values of $M_2^{\,2}$ given by equation (8) are to be real, q must be limited and therefore the heat that can be added to a fluid moving with Mach number M_1 is limited; the limit is expressed by the inequality

$$q \leqq \frac{\left(M_1 - \dfrac{1}{M_1}\right)^2}{2(\gamma + 1)} \qquad (12)$$

Thus, in agreement with the conclusion based on the Second Law of Thermodynamics, the heat added $q = \dfrac{Q}{c_p\,T_1}$ cannot be positive for $M_1 = 1$, that is, for flow velocity equal to the local velocity of sound. The quantity of heat indicated by the equality sign in equation (12) is the quantity that is necessary to increase or decrease the Mach number to unity from its initial value M_1. If heat is added to but not removed from the fluid, this value of unity for the Mach number will occur at the downstream end of the tube, provided that the heat is supplied entirely by heat transfer from the wall of the tube.

The limit to heat addition for a given entrance Mach number may also be thought of as a limit to the entrance Mach number for a given heat addition. The effect of heat addition upon the limit to entrance Mach number is represented in figure 2 by the curve for $M_2 = 1$.

Because the mass flow is proportional to $(M_1p_1)/\sqrt{T_1}$, the limit to heat addition may be alternatively described as a limit to the mass flow for a given amount of heat added. If $M_1 < 1$ and M does not exceed unity between sections 1 and 2, there will be an upper limit to the mass flow, as discussed in reference 3. It may be shown that the same upper limit exists for $M_1 < 1$ if M does exceed unity between sections 1 and 2. If $M_1 > 1$, a lower limit to the mass rate of flow without shock exists, which is the same whether or not M is everywhere greater than unity between sections 1 and 2. The possibility of continuous transitions from $M < 1$ to $M > 1$ and vice versa are discussed in the next section. In all instances the limiting flow conditions can be most easily expressed analytically in terms of M_1 as the independent varible. There is greater physical reality, however, in the expression of the limiting flow conditions in terms of a limit on the mass flow. Consequently M_1, which always bears a simple relation to the mass flow, has been taken to be the dependent variable in figure 2 and in other similar figures.

The temperature ratio $\frac{T_2}{T_1}$ cannot exceed $\frac{\left(1 + \gamma M_1^2\right)^2}{4\gamma M_1^2}$ and attains this maximum value at $M_2 = 1/\sqrt{\gamma}$ for all values of M_1 and q. This limitation can be expressed alternatively as a limit to the entrance Mach number, which is then a function of T_2/T_1 alone and which occurs when $M_2 = 1/\sqrt{\gamma}$. A graphical representation of the limiting M_1 is given in figures 2 and 3 for both $M_2 = 1/\sqrt{\gamma}$ and $M_2 = 1$. If the temperature ratio T_2/T_1 is specified, the maximum entrance Mach number $(M_1 < 1)$ is larger for $M_2 = 1/\sqrt{\gamma}$ than for $M_2 = 1$. If q rather than T_2/T_1 is specified, the limiting entrance Mach number $(M_1 < 1)$ is less for $M_2 = 1/\sqrt{\gamma}$ than for $M_2 = 1$, as is evident from figure 2 and the previous discussion.

The minimum value of the pressure ratio p_2/p_1 corresponding to the maximum allowable heat addition and to $M_2 = 1$ is equal to $\left(1 + \gamma M_1^2\right)/(1 + \gamma)$ for subsonic flow. If the pressure ratio is decreased below this limit by decreasing p_2, the flow will not change because the effects of the pressure reduction cannot be propagated upstream through the region where $M_2 = 1$ at the downstream section 2. For supersonic flow, the same expression gives the maximum pressure ratio that can occur with steady flow.

Transition between Subsonic and Supersonic Flow

The theory indicates that a continuous transition from subsonic to supersonic flow or vice versa can be effected by addition of heat until $M = 1$ followed by removal of heat. If heat is added to but not removed from the moving fluid, such a continuous transition is not possible. A discontinuous transition from subsonic to supersonic flow (adiabatic or nonadiabatic expansion shock) may be shown to be impossible on the basis of the Second Law of Thermodynamics. A discontinuous transition from supersonic to subsonic flow is, however, always possible through the action of a compression shock, whether the heat added is positive, zero, or negative. The action of viscosity or of thermal conductivity, which are omitted from consideration in the present treatment, would provide the mechanism whereby such a discontinuous transition would be effected (reference 4).

Thermal Choking

It has been shown that steady flow cannot occur unless the flow conforms to limitations on the pressure and temperature ratios. When conditions that exceed these limitations are initially imposed on the flow, the flow will adjust itself in such a way as to again reach conformity with the limitations. This self-adjustment of the flow is somewhat similar to the adjustment that occurs when a perfect compressible fluid flows (1) isentropically through a tube of varying section, (2) adiabatically with frictional pressure drop through a tube of constant section (reference 5), or (3) isothermally with frictional pressure drop through a tube of constant section (reference 6). By analogy with these types of flow, the term "thermal choking" might be appropriately applied to the phenomenon of limited mass flow discussed in the present paper. The similarity between the various types of choking phenomenon is being investigated in more detail.

APPLICATIONS

The theoretical results indicate that the mass flow of a compressible fluid cannot exceed a limit that is fixed by the values of p_1, T_1, and q. For each value of M_1 or for given values of p_1 and T_1, at the first section, there is only one value of q or of Q, which corresponds to the limiting flow condition. These results can be applied to the flow of a burning mixture through the combustion chamber of a jet-propulsion power plant.

It is assumed that the cross sections of the ducts which will be considered are constant and that frictional effects, change of gas constant and of γ can be neglected. The maximum mass flow G for subsonic velocities throughout the chamber has been computed for the inlet conditions $p_1 = 15$ pounds per square inch and $T_1 = 500^\circ$ F absolute and is plotted in figures 4, 5, and 6 for $M_2 = 1/\sqrt{\gamma}$ and $M_2 = 1$. The curves in figures 4 and 5 were obtained from figures 2 and 3, respectively, by changing the scales in accordance with the chosen entrance conditions. The data in figure 6 were derived from the expression $\left(\frac{p_2}{p_1}\right) \geq \frac{1 + \gamma M_1^2}{1 + \gamma}$, which is discussed in a previous section. Because the fuel-air ratio determines the value of Q, the heat released per pound of mixture for complete combustion of a given fuel, points can be specified on the curves of maximum mass flow that correspond to a range of values of fuel-air ratio. Although the very lean mixtures corresponding to fuel-air ratios less than about 0.02 would not burn under ordinary circumstances, the values of fuel-air ratio can still be used as a convenient measure of heat release. In figures 4, 5, and 6 the fuel-air ratios are indicated for a fuel whose heat of combustion is 18,000 Btu per pound.

The determination of the conditions for maximum flow for complete combustion $(M_2 = 1)$ will serve to illustrate the use of the figures. If the limit on the total gas outflow (including secondary air) from the combustion chamber is desired, an over-all fuel-air ratio of about 0.02 is appropriate. The limit on the gas flow through a burner duct containing a richer mixture with fuel-air ratio equal to 0.05 is calculated for comparison. From the curve for $M_2 = 1$ (fig. 4), the maximum flow rates for these two fuel-air ratios are found to be 22.2 and 14.8 pounds per square foot per second, respectively. The final temperatures determined from figure 5 are 1630° and 3440° F absolute, and the final pressures from figure 6 are 6.80 and 6.48 pounds per square inch. The entrance Mach numbers $M_1 = \frac{G}{89.0}$ corresponding to these flow rates are 0.250 and 0.166 or, because the velocity of sound at the entrance temperature of 500° F absolute is 1096 feet per second, the entrance velocities are 274 and 182 feet per second. The values of M_1 can be checked by calculating the values of $q = \frac{Q}{c_p T} = \frac{Q}{120}$, which are 2.94 and 7.14, and by reading the values of M_1 (fig. 2). It is evident that the mass flow of burning gases can be considerably restricted by the effects of thermal choking.

If the upstream pressure is increased above 15 pounds per square inch, the downstream pressure and the flow should increase proportionately with the pressure because M_1 and M_2 remain constant. If the downstream pressure is decreased while p_1 remains equal to 15 pounds per square inch, then M_1, M_2, and the flow should each remain constant. The theoretical limits placed by the occurrence of thermal choking on the flow are therefore similar to those found for isentropic flow from converging nozzles.

Aircraft Engine Research Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio.

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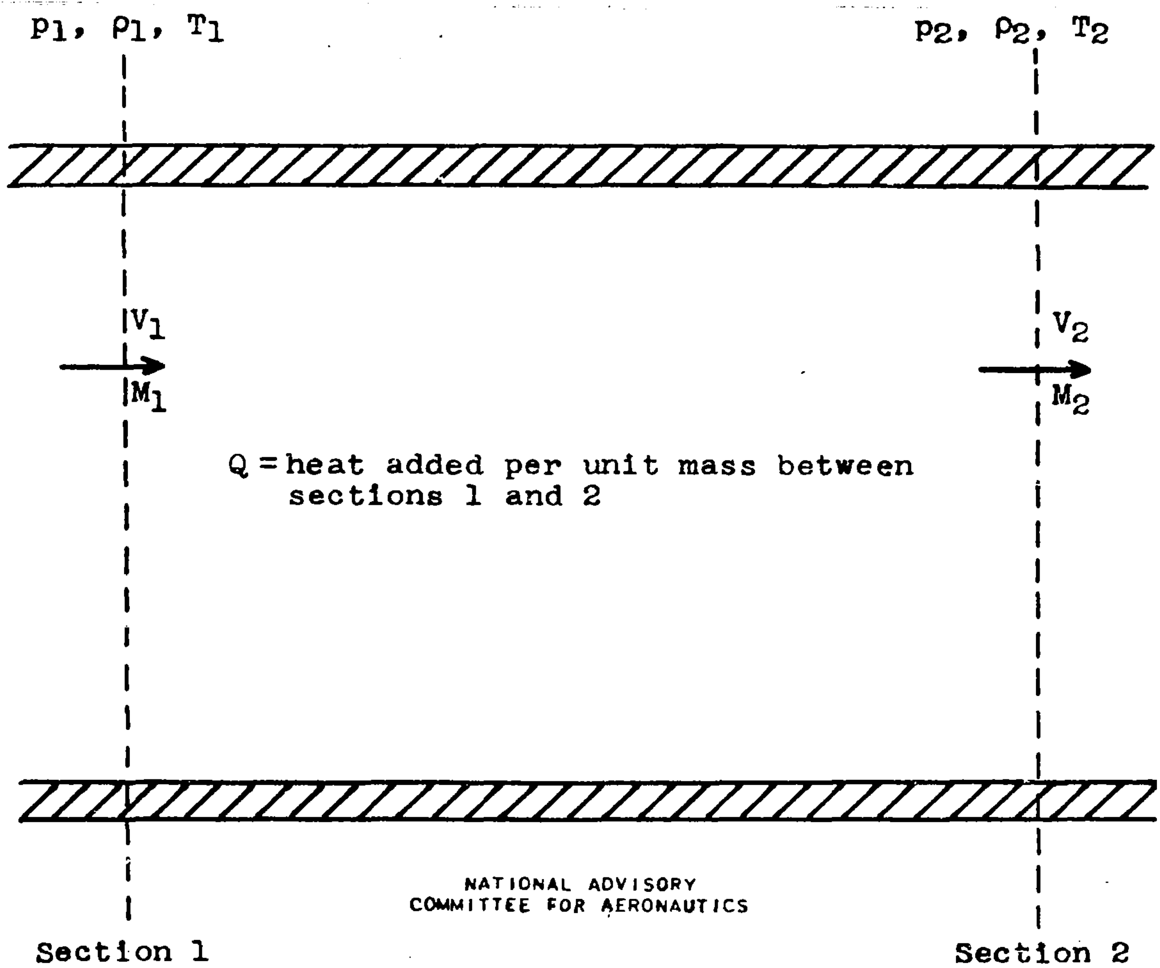

Figure 1. - One-dimensional fluid flow with heat addition.

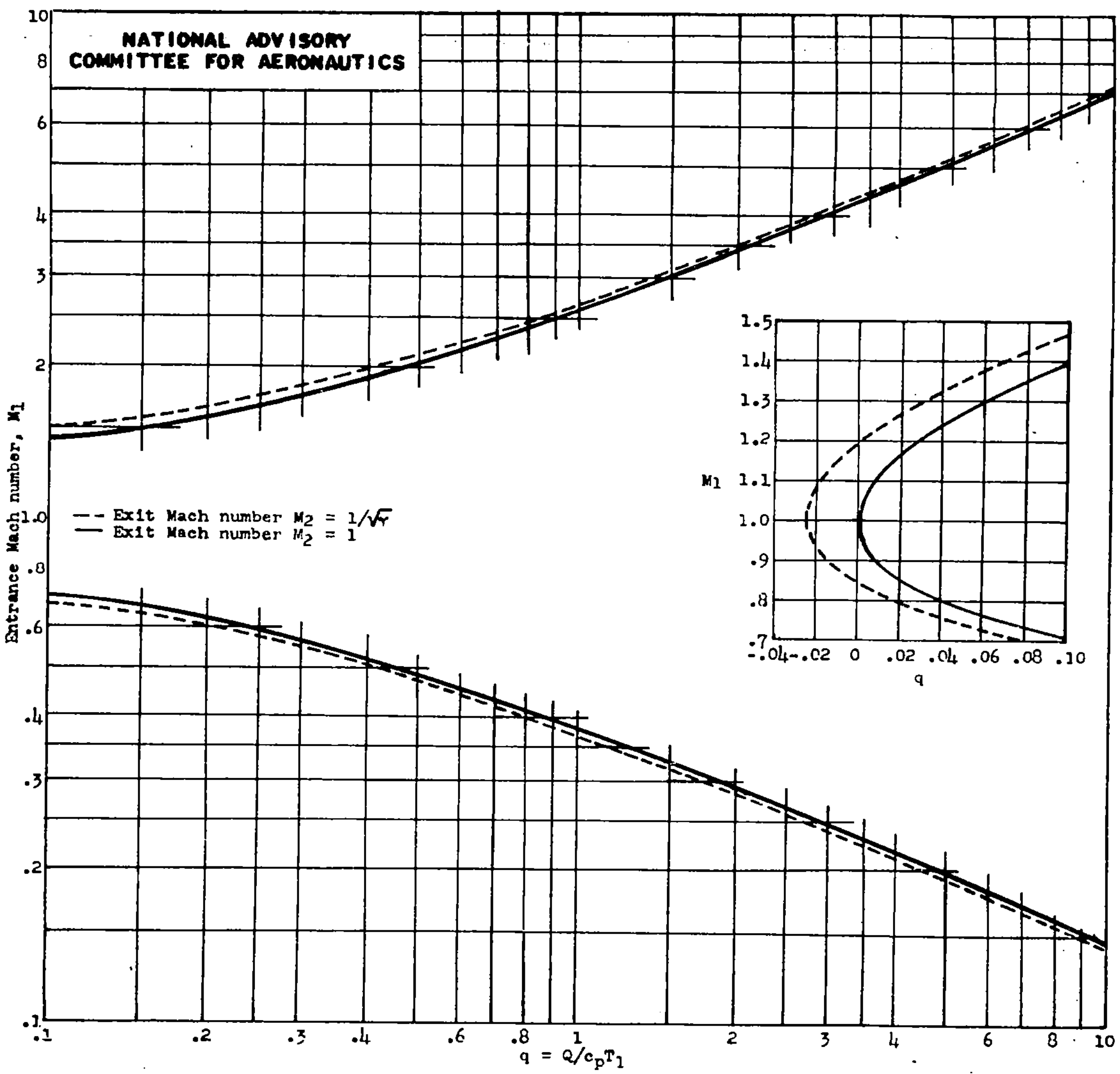

Figure 2. - Effect of heat addition on limit to entrance Mach number M_1 of a gas in steady flow. $\gamma = 1.4$. Q is the heat added per pound of mixture.

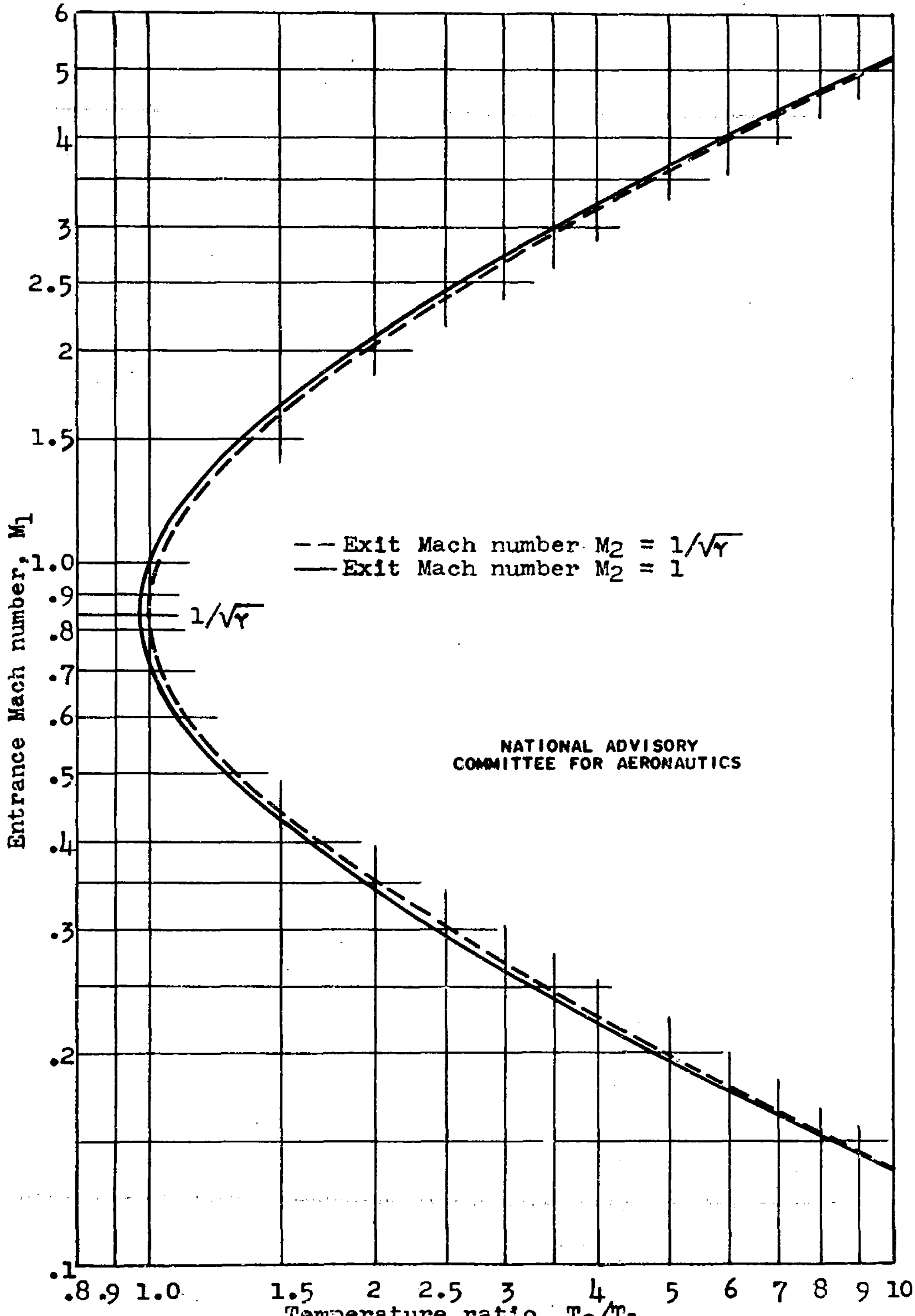

Figure 3. - Relation between temperature ratio and limit to entrance Mach number M_1 of a gas in steady flow. $\gamma = 1.4$.

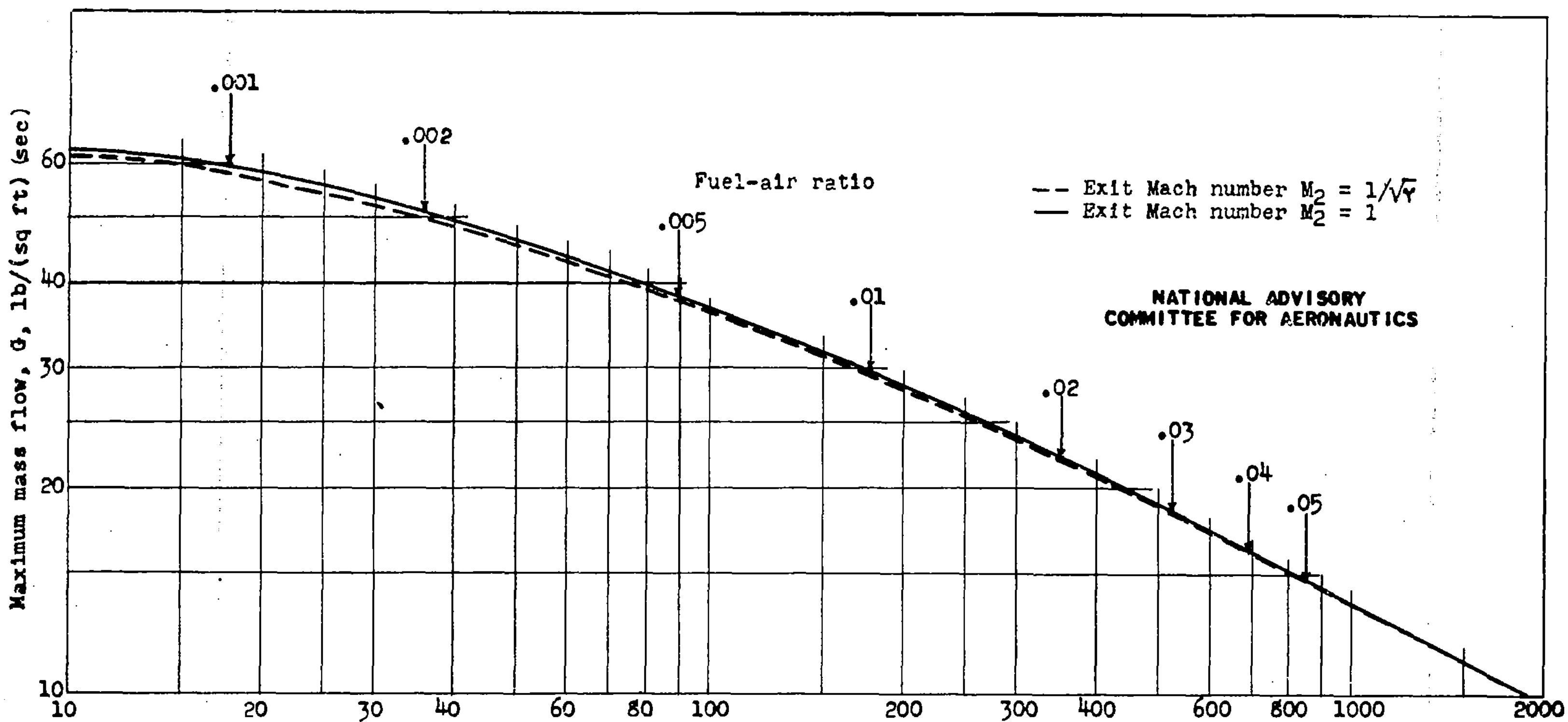

Figure 4. - Effect of heat addition on maximum mass flow for entrance pressure of 15 pounds per square inch and entrance temperature of 500° F absolute.

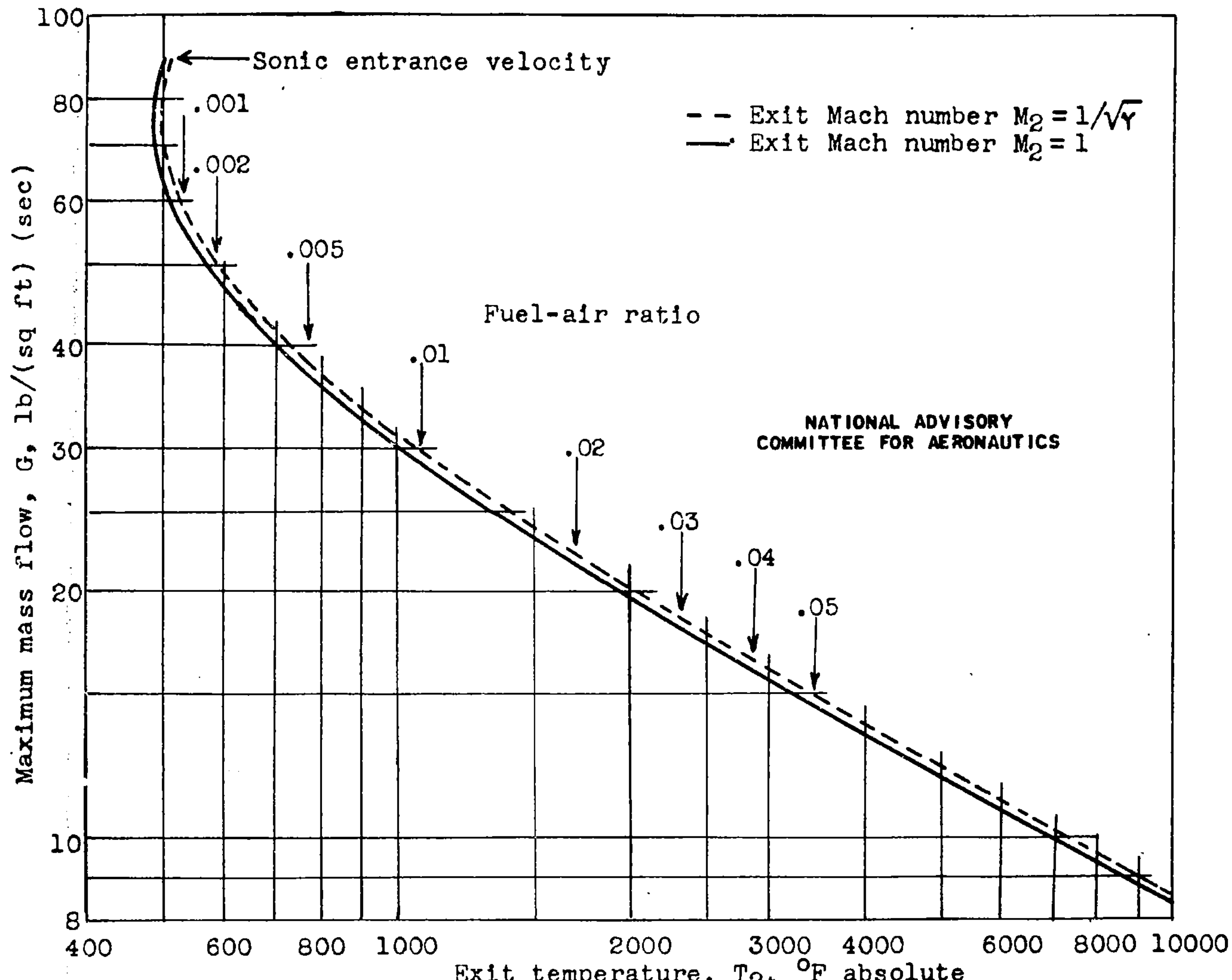

Figure 5. - Relation between maximum mass flow and exit temperature for entrance pressure of 15 pounds per square inch and entrance temperature of 500° F absolute.

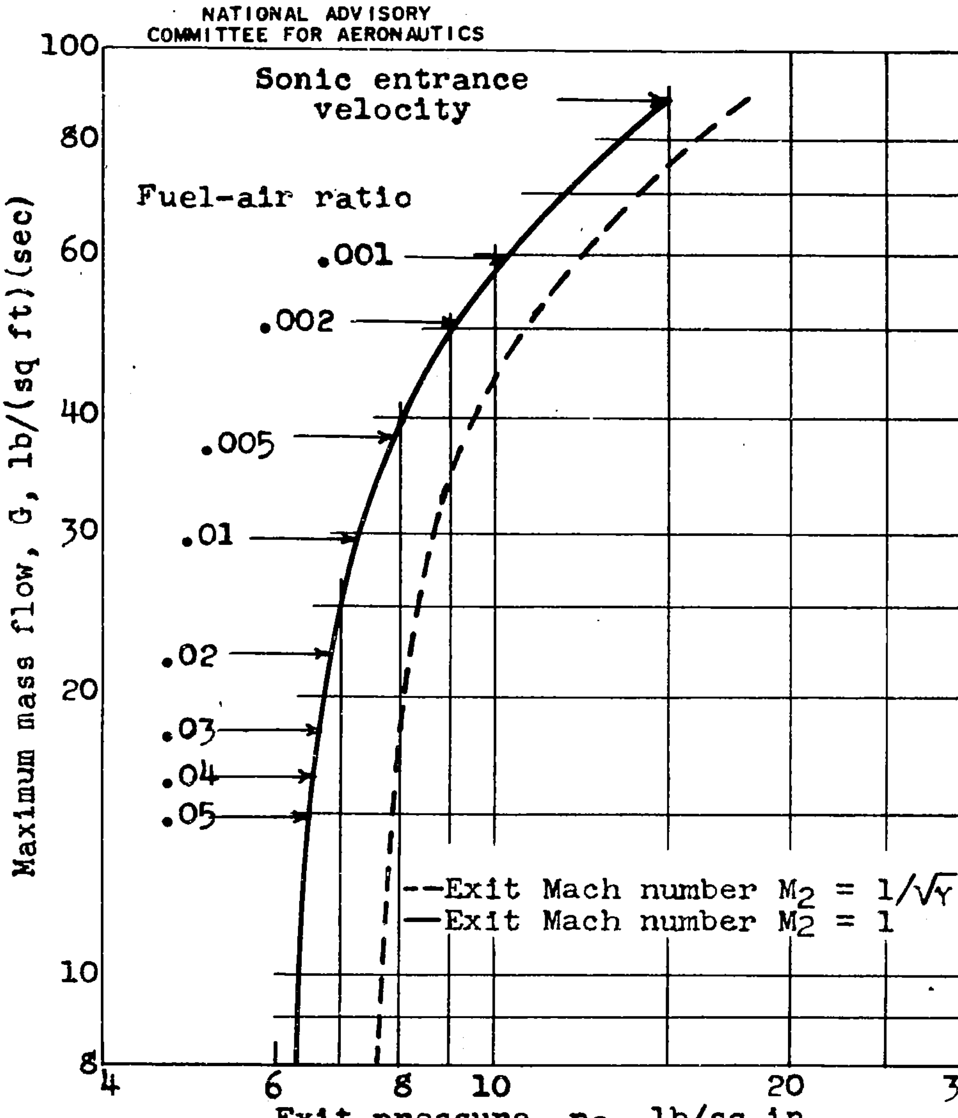

Figure 6. - Relation between maximum mass flow and exit pressure for entrance pressure of 15 pounds per square inch and entrance temperature of 500° F absolute.