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RESEARCH MEMORANDUM

ANALYTICAL STUDY OF BLOCKAGE- AND LIFT-INTERFERENCE CORRECTIONS FOR SLOTTED TUNNELS OBTAINED BY THE SUBSTITUTION OF AN EQUIVALENT HOMOGENEOUS BOUNDARY FOR THE DISCRETE SLOTS

By Don D. Davis, Jr. and Dewey Moore

Langley Aeronautical Laboratory
Langley Field, Va.

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SUMMARY

The solid-blockage interference for a doublet on the tunnel axis and the boundary interference for lifting wings in circular, rectangular, and two-dimensional slotted tunnels have been calculated by substituting an equivalent homogeneous boundary for the physical boundary of discrete slots. In the case of small wings, the interference calculated with the assumption of homogeneity has been found to be consistent with that calculated for the discrete slots for as few as four slots in a circular tunnel.

Furthermore, available experimental results for blockage interference are consistent with the results of the present analysis. As a consequence of the assumption of homogeneity it is possible to express the interference of multislotted tunnels as a function of a single parameter which combines the effects of two physical variables: the ratio of open to total slotted wall perimeter and the number of slots. A curve is presented which permits the rapid evaluation of this parameter and numerical results for lift and blockage interference are plotted against the parameter.

INTRODUCTION

Several investigators have found that in a wind tunnel with boundaries which are partly open and partly closed, the boundary interference on the lift of a wing can be reduced nearly to zero. References 1 to 5 deal with the case of a doublet in the center of the wind tunnel, while references 6, 7, and 8 consider the effects of wing span. There are several reasons why it would be desirable to have a wind tunnel with zero-lift correction. In the first place, the necessity of applying the corrections would be eliminated, although this, in itself, is not of large importance as long as the necessary corrections are known. More important is the fact that the pressure distributions cannot be corrected, and the distortion of both spanwise and chordwise pressure distributions, as a result of boundary interference, places a limitation on the size of model which can be tested in a given wind tunnel.

The primary interest in partly open or slotted wind tunnels, however, is connected with the very different problem of wind-tunnel choking at high subsonic Mach numbers, which is a result of solid blockage interference and which places a very severe limitation on the permissible model size for testing in closed wind tunnels. The blockage correction for a circular slotted tunnel (a wind tunnel in which the open part of the boundary is distributed around the periphery in the form of several longitudinal slots in an otherwise solid boundary) has been considered in reference 9, which indicates theoretically that slot configurations exist for which the blockage correction is greatly reduced. The experimental results that are also included in reference 9 show that slotted tunnels can be used for aerodynamic testing in the transonic speed range. The conventional closed and open tunnels are both unsatisfactory in this speed range, the closed tunnel because of choking and the open tunnel because of the excessive power requirements and the large boundary interference. Much effort has since been expended in the experimental development of transonic slotted tunnels, and several large tunnels of this type are presently in operation (refs. 10 and 11) or in construction. A knowledge of the lift-interference corrections for the slotted tunnels, as well as the blockage corrections, is thus of present interest.

The lift interference in slotted tunnels has been considered in reference 12. This work has been extended in reference 13 to include wings of finite span, and numerical results are presented for several configurations of practical interest.

The calculations required to determine blockage and lift corrections, for a particular slotted-tunnel configuration, by the methods of references 9 and 13 are very laborious especially for tunnels with a large number of slots. In order to obtain a general solution for this problem, Dr. A. Busemann of the Langley Laboratory has suggested that the problems of both lift and blockage interference be treated mathematically from the standpoint of a homogeneous boundary, with the slot effect uniformly distributed over the surface of the boundary. It was reasoned that at some distance from the boundary, in the region of the model, the flow would no longer show the effects of the individual slots, particularly if the actual boundary contained a large number of slots. Furthermore, the wind tunnels which are now in use contain several slots. It is felt that this is desirable in order to increase the uniformity of the interference. This paper will treat the problem in the suggested manner, beginning with the development of a suitable boundary condition to represent mathematically a homogeneous boundary which has the same flow characteristics, at a point in the flow sufficiently removed from the boundary, as the actual physical boundary of alternate open and closed portions of the wall. Numerical results will be presented for circular tunnels with slots uniformly distributed around the circumference, for rectangular tunnels with uniformly distributed slots in the top and bottom walls, for rectangular tunnels with the slot distribution determined by a transformation from a uniformly slotted circular tunnel, and for a two-dimensional tunnel.

The results of this paper are derived on the basis of an incompressible potential flow. The subsonic linearized compressible-flow theory shows that wind-tunnel lift-correction factors are not affected by Mach number; therefore, the lift-correction factors presented in this paper should apply directly to subsonic compressible flows, at least within the range of applicability of the linearized theory. The effect of compressibility upon the blockage interference is to increase the axial-interference velocities with increasing Mach number in proportion to the factor $\frac{1}{\left(1 - M^2\right)^{3/2}}$ (ref. 9).

SYMBOLS

The symbols A_n, B_n, C_n, D_n, C_{1n}, C_{2n}, C_{3n}, C_{4n}, A_k, B_k, A_{st}, B_{st}, C_{st}, D_{st} represent series coefficients.

A area

b semiwidth of rectangular wind tunnel

c nondimensional restriction constant

$\bar{c}$ chord length of a two-dimensional airfoil

c_l section lift coefficient

C cross-sectional area of tunnel

C_L lift coefficient

d slot spacing

h semiheight of rectangular or two-dimensional tunnel

H	stagnation pressure
l	restriction constant
m	doublet strength
N	number of slots
p	static pressure
r_o	open ratio of slotted wall (ratio of slot width to slot spacing)
R	radius of a circular tunnel
S	wing area
t	thickness of slotted wall
u	x-component of additional velocity due to presence of a model in wind tunnel, $U - u_o$
u_o	free-stream velocity at upstream infinity
U	x-component of velocity at any point
v	y-component of additional velocity
v_n	component of additional velocity normal to surface of wall
w	z-component of additional velocity
W	complex velocity in Z-plane
x,y,z	distances in Cartesian coordinate system
Z	complex plane, $x' + iy'$
Γ	circulation around a wing
δ	correction factor due to lift, $\frac{\overline{w}_2}{u_o} \frac{C}{SC_L}$
ζ	complex plane, $\xi + i\eta$
ξ,η	Cartesian coordinates in ζ-plane

θ angle in polar coordinates

λ height-width ratio for rectangular tunnel

λ' constant appearing in transformation from rectangle to circle

μ $\csc \frac{\pi}{2} r_o$

φ incompressible-flow velocity potential

ρ radial distance in polar coordinates

ρ density of air in wind tunnel

σ ratio of vortex span (effective wing span) to diameter of circular tunnel or width of rectangular tunnel

Subscripts:

o free-stream conditions at upstream infinity

1 due to model in free air

2 due to presence of tunnel boundaries

C circular tunnel

n in direction normal to wall surface

R rectangular tunnel

n,k,s,t summation indices

r due to a row of vortices

s signifies additional term required to satisfy boundary condition at slotted walls

T total

BOUNDARY CONDITIONS

Wall with discrete slots.- Longitudinal and transverse cross-sectional views of a slotted wind tunnel are shown in figure 1. In the longitudinal view exaggerated streamlines are drawn to indicate how the air in the vicinity of the wall flows out and in through the slots as it passes the model. The chamber surrounding the tunnel is maintained at a pressure equal to the free-stream static pressure of the flow inside the tunnel. The difference between this pressure and the average local pressure of the flow just inside the wall causes the air to move out and in through the slots. The boundary conditions at the slotted wall for this flow configuration will now be considered.

Consider a set of axes in Cartesian coordinates which are fixed with respect to a model in the wind tunnel, while the air flows by with velocity components U, v, w. Furthermore, let $U = u_o + u$, where u_o is the free-stream velocity at infinity. The pressure at a point inside the tunnel at the wall is given by

$$p = H - \frac{\rho}{2}\left(u_o^2 + 2u_o u + u^2 + v^2 + w^2\right)$$

The free-stream pressure p_o is given by

$$p_o = H - \frac{\rho}{2} u_o^2$$

Assume now that by some means the pressure just outside the wall is maintained at p_o; then, the pressure difference across the wall is

$$\Delta p = p - p_o = -\frac{\rho}{2}\left(2u_o u + u^2 + v^2 + w^2\right)$$

Next, assume that the relationships between the model size and shape and the distance to the wall are such that u, v, w are all much smaller than u_o at the wall. The relation between the pressure difference and the axial velocity inside the tunnel then becomes

$$\Delta p \approx -\rho u_o u \tag{1}$$

This is a known result of the small-disturbance theory. Note that the small-disturbance assumption is required not in the field of flow near the model but only at the walls.

It is now required to find an expression which relates the pressure difference across the wall to the flow through the slotted wall. This expression, combined with equation (1), will establish the relationship between the axial velocity just inside the slotted wall and the flow through the slots. In order to find such an expression it will first be necessary to study the energy in the neighborhood of the slots. This energy is basic to the problem, for the essential mechanism of the slotted wall is that kinetic energy is stored in the air which flows out through the slots. This energy is later returned to the flow when the air flows back through the slots into the tunnel. Consider a thin slotted wall in a field of flow with a uniform velocity normal to the wall at infinity. (See fig. 2.) Because the flow pattern is the same for each slot it is permissible to study a single channel such as the one in which approximate streamlines have been sketched in figure 2. The kinetic energy enclosed in a region of this flow bounded by a transverse plane at $-x_o'$, by the "walls" of the channel, and by the slot is given by

$$\text{Kinetic energy} = \frac{1}{2}\rho \iint_A \varphi \frac{\partial \varphi}{\partial n}\, dA$$

The region of integration may be considered to consist of a surface of unit depth normal to the plane of the page which includes the dashed line shown in figure 2. The component of velocity normal to the closed portion of the slotted wall must be zero. With regard to the open portion, the disturbance potential will be assumed to be zero at the slots, in conformity with previous papers and with the classical practice in treating interference in open tunnels. Because $\frac{\partial \varphi}{\partial n} = 0$ at the channel walls, and because $\varphi = 0$ at the slot, these two regions contribute nothing to the integral. With regard to the transverse plane at $-x_o'$ it is clear that if this plane is sufficiently far away from the slot the potential will have a value $\varphi_{-x_o'}$ which is essentially constant in this plane.

Also,

$$\iint_{-x_o'} \frac{\partial \varphi}{\partial n}\, dA = v_n A$$

the quantity flow. Thus,

$$\text{Kinetic energy} = \frac{1}{2}\rho \varphi_{-x_o'} v_n A$$

In order to complete the evaluation of the kinetic energy it is necessary to determine the value of the potential $\varphi_{-x_o'}$. Let the coordinate origin be taken in the plane of the slotted wall at the center of one of the solid sections and consider the flow to the left of this slotted wall. With the assistance of reference 14, the potential of this flow is found to be

$$\varphi = v_n\left(\frac{d}{\pi} \log_e \csc \frac{\pi}{2} r_o - x' + \frac{d}{\pi} \sum_{s=1}^{\infty} C_s e^{k_s x'} \cos k_s y'\right) \tag{2}$$

where $k_s = \frac{2\pi s}{d}$, d is slot spacing, a is slot width, and $r_o = \frac{a}{d}$. The plane at $-x_o'$ is sufficiently far from the wall that the last term, which falls off exponentially as x' becomes more negative, may be disregarded. Thus,

$$\varphi_{-x_o'} = v_n\left(\frac{d}{\pi} \log_e \csc \frac{\pi}{2} r_o + x_o'\right)$$

The insertion of this value of $\varphi_{-x_o'}$ in the previous kinetic-energy equation results in

$$\text{Kinetic energy} = \frac{1}{2} \rho\left(\frac{d}{\pi} \log_e \csc \frac{\pi}{2} r_o + x_o'\right) v_n^2 A$$

In the absence of the slotted wall, the kinetic energy per unit area of the flow inside the region of integration would be

$$\frac{1}{2} \rho x_o' v_n^2$$

Consequently, the portion of the total energy which may be regarded as being due to the presence of the slotted wall is $\frac{1}{2} \rho l v_n^2$ per unit wall area, where

$$l = \frac{d}{\pi} \log_e \csc \frac{\pi}{2} r_o \tag{3}$$

Note that the quantity l has the dimension of length.

Equivalent homogeneous wall.- Consider, now, the flow field that would result in the region between $-x_0'$ and the wall if the slotted wall were replaced by an imaginary homogeneous wall of zero thickness through which potential flow is possible and which is characterized by the existence in the local flow of an energy $\frac{1}{2}\rho l v_n^2$ per unit wall area associated with a local normal velocity v_n. The velocity of this flow field, at $-x_0'$, will be essentially the same as that of the slotted-wall flow field; furthermore, the total kinetic energies between the planes at $-x_0'$ and at the walls will be the same. For the purpose of calculating the flow to the left of $-x_0'$, therefore, the homogeneous wall is equivalent to the slotted wall.

The result which has been obtained by studying the relatively simple flow from a source at $-\infty$ is that there is associated with the flow through the slots a kinetic energy that is a function of the dimensions of the slotted wall and of the average velocity normal to the wall, and that this energy may be considered to be concentrated at the plane of the wall for the purpose of determining the potential of the flow at a point sufficiently far removed from the wall. If the singularity is located at a finite distance from the wall, or if singularities of types other than sources are introduced, there will be a velocity component parallel to the wall in addition to the normal component v_n. Application of the principle of superposition shows, however, that the energy which is associated with a given flow normal to the wall will not be affected by the presence of additional velocity components parallel to the surface of the wall. It is necessary, though, that the slot spacing be small enough so that the difference in flow through adjacent slots is small. Thus, the analysis will be applicable only to tunnels with several slots. With this qualification the homogeneous wall will be equivalent not only to the slotted wall of figure 2 but also to the slotted wind-tunnel wall of figure 1 (section A-A) insofar as its effect on the model is concerned.

If the slotted wall of figure 1 is replaced by an equivalent homogeneous wall, the energy per unit area at the wall is $\frac{1}{2}\rho l v_n^2$ and the momentum associated with this energy is $\rho l v_n$ per unit wall area. An individual particle of air which follows the outer streamline shown in figure 1 flows out and in through the wall as it passes the model as a result of alternate outward and inward accelerations due to the pressure difference across the slotted wall. The direction of flow of the normal component of velocity does not, in general, correspond to the direction of the pressure difference across the wall. Instead, it is the direction of the normal acceleration which corresponds to the direction of the pressure difference. This pressure difference, which acts in a direction normal to the wall surface must, in the potential flow, be equal to the rate of change of the momentum associated with the presence of the slots. Thus,

$$\Delta p = \frac{D}{Dt}\left(\rho l v_n\right) = \rho \frac{D}{Dt}\left(l v_n\right)$$

Although the quantity l is constant in time at a given point on the wall, it is left under the differentiation sign in order not to exclude the possibility that the slot configuration may vary from point to point on the tunnel wall. The only restriction which is placed on the axial or transverse variations of l is that they must not be too rapid, because the equation for l has been derived on the basis of a two-dimensional flow and a uniformly slotted wall. The derivative is given by

$$\frac{D}{Dt}\left(l v_n\right) = \frac{\partial}{\partial t}\left(l v_n\right) + U \frac{\partial}{\partial x}\left(l v_n\right) + v \frac{\partial}{\partial y}\left(l v_n\right) + w \frac{\partial}{\partial z}\left(l v_n\right)$$

Since the flow is steady and the slot configuration is constant in time when referred to the fixed axes, $\frac{\partial}{\partial t}\left(l v_n\right) = 0$. If now $\frac{\partial}{\partial y}\left(l v_n\right)$ and $\frac{\partial}{\partial z}\left(l v_n\right)$ are assumed to be of the same order as $\frac{\partial}{\partial x}\left(l v_n\right)$ (or of higher order) then the acceleration is given by

$$\frac{D}{Dt}\left(l v_n\right) \approx u_o \frac{\partial}{\partial x}\left(l v_n\right)$$

to the same order of approximation as was used in obtaining equation (1). The pressure difference across the wall is thus related to the velocity through the wall in the following manner:

$$\Delta p = \rho u_o \frac{\partial}{\partial x}\left(l v_n\right) \tag{4}$$

Equating (1) and (4) for the pressure difference results in

$$-u = \frac{\partial}{\partial x}\left(l v_n\right)$$

Let the potential of the flow be given by $\Phi = -u_0 x + \varphi$, where φ is the disturbance potential. In terms of this potential the preceding equation becomes

$$\frac{\partial\varphi}{\partial x} = -\frac{\partial}{\partial x}\left(l\,\frac{\partial\varphi}{\partial n}\right) \quad \text{or} \quad \frac{\partial}{\partial x}\left(\varphi + l\,\frac{\partial\varphi}{\partial n}\right) = 0$$

Integrating this equation in the x-direction along the wall gives $\varphi + l\,\frac{\partial\varphi}{\partial n}$ equal to a constant. One of the boundary conditions which must be satisfied by the wind-tunnel flow is that there be no disturbance at infinity upstream, so the constant must be zero. Thus, the boundary condition at the wall becomes

$$\varphi + l\,\frac{\partial\varphi}{\partial n} = 0 \tag{5}$$

At this point let it be emphasized that, although this condition must be satisfied everywhere on the boundary, all the symbols, including l, refer to only local values of the quantities represented. Thus, l has been left free to vary in both directions on the surface. The boundary condition may therefore be used in the study of wind tunnels in which the slot width is variable in the axial direction. Furthermore, adjacent slots may be of different width or different spacing.

Although only a plane wall was considered in determining the relationship between the wall dimensions and the wall-restriction constant, l, it is possible to show that the same relationship holds for a circular wall. Consider the transformation $\zeta = e^Z$ applied to the region between $y' = d/2$ and $y' = -d/2$ in the flow field shown in figure 2. By using polar coordinates (r,θ) in the ζ-plane, there is obtained $r = e^{x'}$ and $\theta = y'$. Thus, the line $x' = 0$ transforms into an arc of a circle of radius 1. (See fig. 3.) Because of the linear transformation between y' and θ, the open ratio of the wall is unchanged by the transformation. The source of the flow in the Z-plane at $x' = -\infty$ transforms in the ζ-plane into a source at $r = 0$. The line $x' = \infty$ transforms into the arc $r = \infty$. The expression for the potential inside the wall is obtained by applying the transformation to equation 2.

$$\varphi = v_n\left(\frac{d}{\pi}\log_e \csc\frac{\pi}{2}r_o - \log_e r + \frac{d}{\pi}\sum_{s=1}^{\infty} C_s e^{k_s \log_e r}\cos\frac{2\pi s\theta}{d}\right) \quad (r < 1) \tag{6}$$

The potential of the source at the origin in the absence of the wall would be $\varphi = -v_n \log_e r$. Thus, it follows that the additional potential at a point inside the wall, due to the presence of the wall, is nearly $v_n \frac{d}{\pi} \log_e \csc \frac{\pi}{2} r_o$. Consequently, the restriction constant l for a circular tunnel is obtained in the same manner as for a tunnel with plane walls. The function $\log_e \csc \frac{\pi}{2} r_o$ is plotted in figure 4.

Consideration of equation (6) will give some idea of the degree of approximation involved in assuming the additional effective energy due to the presence of the slots to be concentrated in a plane at the wall. As an example, calculations have been made for a circular tunnel with 8 slots, for which $\mu = 7$, where $\mu = \csc \frac{\pi}{2} r_o$. (This gives an open ratio of slightly over 0.09, which is in the range of practical interest.) Figure 5 shows the potential given by equation (6) plotted against the radial distance from the center of the tunnel. The calculations were made along radial lines through the center line of a slot $\left(\theta = \frac{\pi}{8}\right)$ and through the center line of a panel $(\theta = 0)$. Also presented is a curve calculated from the approximate or homogeneous wall equation

$$\varphi = \frac{d}{\pi} \log_e \mu - \log_e r \tag{7}$$

Inspection of figure 5 shows that the approximation is quite satisfactory for the flow in the central part of the tunnel (say $r < 0.6$). For tunnels with more than 8 slots, the region of validity of the approximation will be even larger.

ANALYSIS

Circular Wind Tunnel

Lift interference.- Consider a lifting wing symmetrically located in a circular wind tunnel which has a homogeneous boundary through which potential flow is possible. Let the wing be represented by a single horseshoe vortex. The downwash at the wing will be determined, in the classical manner, by finding the downwash in a tunnel cross section far downstream due to a pair of vortices of opposite sign with circulation $\Gamma/2$, where Γ is the circulation of the horseshoe vortex in the tunnel. The disturbance potential in the plane will be taken as $\varphi_1 + \varphi_2$, where φ_1 is the potential of the vortices in a free field and φ_2 is the interference potential due to the walls. All length dimensions will be made nondimensional by dividing by the tunnel radius R. The boundary condition at the wall ($\rho = 1$) is then

$$\varphi_1 + \varphi_2 + c\left(\frac{\partial\varphi_1}{\partial\rho} + \frac{\partial\varphi_2}{\partial\rho}\right) = 0 \tag{8}$$

where $c = \frac{l}{R}$, the nondimensional restriction constant.

The interference potential φ_2 must satisfy the equation $\nabla^2\varphi_2 = 0$ throughout the interior of the tunnel. In polar coordinates this equation becomes

$$\frac{\partial^2\varphi_2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\varphi_2}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\varphi_2}{\partial\theta^2} = 0$$

By the method of separation of variables, the following family of solutions can be obtained:

$$\varphi_2 = \sum_{n=0}^{\infty}\left(C_{1n} \sin n\theta + C_{2n} \cos n\theta\right)\left(C_{3n}\rho^n + C_{4n}\rho^{-n}\right)$$

Since φ_2 must be finite everywhere inside the tunnel, it must be finite at $\rho = 0$ and, therefore, $C_{4n} = 0$. Let $\theta = 0$ be in the plane of the wing. Then, because of the symmetrical location of the wing, it is apparent that $\varphi_2(\theta) = \varphi_2(\pi - \theta)$. The cosine terms, which do not satisfy this requirement, are omitted by setting $C_{2n} = 0$ (except when $n = 0$). The solution may now be put in the form

$$\varphi_2 = \frac{\Gamma}{4\pi}\left(A_o + \sum_{n=1}^{\infty} A_n\rho^n \sin n\theta\right) \tag{9}$$

The potential of the vortex pair in a free field is, in polar coordinates,

$$\varphi_1 = -\frac{\Gamma}{4\pi}\tan^{-1}\left(\frac{2\rho\sigma \sin\theta}{\rho^2 - \sigma^2}\right) \tag{10a}$$

where σ is the nondimensional semispan of the vortex pair. Also,

$$\frac{\partial\varphi_1}{\partial\rho} = \frac{\Gamma}{4\pi}\frac{2\sigma \sin\theta\left(\rho^2 + \sigma^2\right)}{\left(\rho^2 - \sigma^2\right)^2 + 4\rho^2\sigma^2\sin^2\theta} \tag{10b}$$

In order to find the interference, equations (10a) and (10b) can be expanded in Fourier series at the position of the wall. These series, together with equation (9), can then be inserted in the boundary condition, equation (8), in order to determine the constants A_n in the disturbance potential. The expansions for φ_1 and $\frac{\partial\varphi_1}{\partial\rho}$ at $\rho = 1$ are of the form

$$\frac{4\pi}{\Gamma}\varphi_1 = -\tan^{-1}\frac{2\sigma \sin\theta}{1 - \sigma^2} = \sum_{n=1}^{\infty} B_n \sin n\theta \tag{11}$$

$$\frac{4\pi}{\Gamma}\frac{\partial\varphi_1}{\partial\rho} = \frac{2\sigma\left(1 + \sigma^2\right)\sin\theta}{\left(1 - \sigma^2\right)^2 + 4\rho^2\sin^2\theta} = \sum_{n=1}^{\infty} C_n \sin n\theta \tag{12}$$

No constant terms appear in these expansions so $A_o = 0$. By substituting (9), (11), and (12) in (8) the following equation is obtained for each value of n:

$$A_n \sin n\theta + B_n \sin n\theta + cnA_n \sin n\theta + cC_n \sin n\theta = 0$$

Thus,

$$A_n = -\frac{B_n + cC_n}{1 + cn} \tag{13}$$

After A_n is found, there remains the problem of determining the vertical interference velocity, and from it the interference factor δ. The vertical interference velocity along the line $\theta = 0$ is given by

$$w_2 = -\frac{1}{\rho}\frac{\partial\varphi_2}{\partial\theta} = -\frac{\Gamma}{4\pi}\sum_{n=1}^{\infty} nA_n\rho^{n-1} \tag{14a}$$

The average interference velocity between the origin and the point $(\sigma,0)$ is

$$\overline{w}_2 = -\frac{1}{\sigma}\int_0^{\sigma} \frac{\Gamma}{4\pi}\sum_{n=1}^{\infty} nA_n\rho^{n-1}d\rho$$

$$\overline{w}_2 = -\frac{\Gamma}{4\pi}\sum_{n=1}^{\infty} A_n\sigma^{n-1} \tag{14b}$$

The interference factor δ as given in reference 15 can be written as

$$\delta = \frac{\overline{w}_2 C}{u_o S C_L}$$

where

C cross-sectional area of tunnel (π for the tunnel of unit radius)

S wing area

C_L lift coefficient

u_o tunnel free-stream velocity at upstream infinity

The circulation is related to the lift by

$$SC_L = \frac{4\sigma\Gamma}{u_o}$$

Using this relation there is obtained $\delta = \frac{\pi}{4\sigma\Gamma}\overline{w}_2$. Substituting for $\overline{w}_2$ from equation (14b)

$$\delta = \frac{1}{16}\sum_{n=1}^{\infty} \frac{B_n + cC_n}{1 + cn} \sigma^{n-2} \tag{15}$$

Instead of actually making the expansions indicated in equations (11) and (12), it is possible to infer the values of the constants B_n and C_n from the known corrections for open and closed circular wind tunnels. For a closed circular tunnel Silverstein and White (ref. 15) give the equation

$$\delta = \frac{1}{16\sigma^2} \log_e \frac{1 + \sigma^2}{1 - \sigma^2}$$

Making use of the series expansion (ref. 16)

$$\log_e\left(\frac{x + 1}{x - 1}\right) = \sum_{n=1}^{\infty} \frac{2}{nx^n} \qquad (n = 1,3,5,\ldots)$$

the following equation may be written after setting $\sigma^2 = \frac{1}{x}$:

$$\delta = \frac{1}{16}\sum_{n=1}^{\infty} 2\,\frac{\sigma^n}{n}\,\sigma^{n-2} \qquad (n = 1,3,5,\ldots) \tag{16}$$

But equation (15) gives, for $c = \infty$,

$$\delta = \frac{1}{16}\sum_{n=1}^{\infty} \frac{C_n}{n} \sigma^{n-2} \tag{17}$$

Because equations (16) and (17) must agree, and since they can only agree if the series coefficients are identical, term by term, then

$$\left.\begin{aligned} C_n &= 2\sigma^n \qquad (n = 1,3,5,\ldots) \\ C_n &= 0 \qquad (n = 2,4,6,\ldots) \end{aligned}\right\} \tag{18}$$

The correction for an open circular tunnel is simply the negative of equation (16). For this case (c = 0) equation (15) gives

$$\delta = \frac{1}{16}\sum_{n=1}^{\infty} B_n\sigma^{n-2} \tag{19}$$

Comparison of equations (16) and (19) shows that

$$\left.\begin{aligned} B_n &= -\frac{2\sigma^n}{n} \qquad (n = 1,3,5,\ldots) \\ B_n &= 0 \qquad (n = 2,4,6,\ldots) \end{aligned}\right\} \tag{20}$$

Substituting equations (18) and (20) in (15) results in, finally,

$$\delta = \frac{1}{8}\sum_{n=1}^{\infty}\left(\frac{cn - 1}{cn + 1}\right)\frac{\sigma^{2n-2}}{n} \qquad (n = 1,3,5,\ldots) \tag{21}$$

This correction is plotted as a function of the nondimensional restriction constant in figure 6 for wings of small span.

For the circular tunnel the nondimensional restriction constant is given by $c = \frac{d}{\pi}\log_e \mu$, where d is the angle between two successive slot center lines. Since the number of slots N around the circumference of the tunnel is given by $2\pi/d$, then

$$c = \frac{2}{N}\log_e \mu \tag{22}$$

for a circular wind tunnel.

Solid-blockage interference.- Consider next the problem of solid-blockage interference in the circular wind tunnel. The solid body in the tunnel is represented by a doublet with axis alined with the axis of the tunnel cylinder, and located at the origin of coordinates on the axis of the tunnel. The flow potential is again represented by $\varphi_1 + \varphi_2$, where φ_1 is the free-field potential of the doublet and φ_2 is the interference potential due to the presence of the homogeneous wall. The boundary condition which must be satisfied at the wall ($\rho = 1$), for all values of θ and all values of x, is again given by equation (8). Laplace's equation in cylindrical coordinates, which must be satisfied by φ_2, is

$$\frac{\partial^2\varphi_2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\varphi_2}{\partial\rho} + \frac{\partial^2\varphi_2}{\partial x^2} = 0$$

in the case of circular symmetry about the longitudinal axis x. The solution of this equation which will be used for the problem under consideration is

$$\left.\begin{aligned} \varphi_2 &= \sum_{k=0}^{\infty} A_k I_o\left(\frac{k\pi\rho}{L}\right)\sin\left(\frac{k\pi x}{L}\right) \\ \frac{\partial\varphi_2}{\partial\rho} &= \sum_{k=0}^{\infty} \frac{k\pi}{L} A_k I_1\left(\frac{k\pi\rho}{L}\right)\sin\left(\frac{k\pi x}{L}\right) \end{aligned}\right\} \tag{23}$$

where I_o and I_1 are the modified Bessel functions.

The free-field potential of the doublet is $\varphi_1 = -\frac{x}{\left(x^2 + \rho^2\right)^{3/2}}$.

The value at the wall of this potential and its derivative is now expanded in Fourier series form.

$$\left(\varphi_1\right)_{\rho=1} = -\frac{x}{\left(x^2 + 1\right)^{3/2}} = -\sum_{k=0}^{\infty} B_k \sin\frac{k\pi x}{L} \tag{24}$$

$$B_k = \frac{2}{L}\int_0^L \frac{x}{\left(x^2 + 1\right)^{3/2}} \sin\frac{k\pi x}{L}\, dx \equiv \frac{2}{L} Q_o\left(\frac{k\pi}{L}\right) \tag{25}$$

$$\left(\frac{\partial\varphi_1}{\partial\rho}\right)_{\rho=1} = \frac{3x}{\left(x^2 + 1\right)^{5/2}} = \sum_{k=0}^{\infty} C_k \sin\frac{k\pi x}{L} \tag{26}$$

$$C_k = \frac{2}{L}\int_0^L \frac{3x}{\left(x^2 + 1\right)^{5/2}} \sin\frac{k\pi x}{L}\,dx \equiv \frac{2}{L} Q_1\left(\frac{k\pi}{L}\right) \tag{27}$$

The functions Q_o and Q_1 are defined by equations (25) and (27). Upon substituting the preceding equations into equation (8), the boundary condition becomes

$$-\sum_{k=0}^{\infty} \frac{2}{L} Q_o\left(\frac{k\pi}{L}\right)\sin\frac{k\pi x}{L} + \sum_{k=0}^{\infty} A_k I_o\left(\frac{k\pi}{L}\right)\sin\frac{k\pi x}{L} +$$

$$c\sum_{k=0}^{\infty} \frac{2}{L} Q_1\left(\frac{k\pi}{L}\right)\sin\frac{k\pi x}{L} + c\sum_{k=0}^{\infty} \frac{k\pi}{L} A_k I_1\left(\frac{k\pi}{L}\right)\sin\frac{k\pi x}{L} = 0$$

For each A_k there results an equation

$$A_k = \frac{\frac{2}{L}\left[Q_o\left(\frac{k\pi}{L}\right) - cQ_1\left(\frac{k\pi}{L}\right)\right]}{I_o\left(\frac{k\pi}{L}\right) + c\,\frac{k\pi}{L}\,I_1\left(\frac{k\pi}{L}\right)}$$

Substituting this in the equation for the interference potential gives

$$\varphi_2 = \sum_{k=0}^{\infty} \frac{2}{L}\left[\frac{Q_o\left(\frac{k\pi}{L}\right) - cQ_1\left(\frac{k\pi}{L}\right)}{I_o\left(\frac{k\pi}{L}\right) + c\,\frac{k\pi}{L}\,I_1\left(\frac{k\pi}{L}\right)}\right] I_o\left(\frac{k\pi\rho}{L}\right)\sin\frac{k\pi x}{L} \tag{28}$$

If, now, the fundamental wave length of this expression L is allowed to approach infinity, the summation can be replaced by an integral. The necessary relationship can be obtained as follows:

$$\lim_{L\to\infty} \sum_{k=0}^{\infty} \frac{2}{L} \cdot F\left(\frac{k\pi}{L}\right) = \frac{\displaystyle\sum_{\substack{k=0 \\ L\to\infty}}^{\infty} \frac{2}{L} F\left(\frac{k\pi}{L}\right) d\left(\frac{k\pi}{L}\right)}{d\left(\frac{k\pi}{L}\right)} = \frac{\displaystyle\int_0^{\infty} \frac{2}{L} F\left(\frac{k\pi}{L}\right) d\left(\frac{k\pi}{L}\right)}{d\left(\frac{k\pi}{L}\right)}$$

Since k takes only integer values, the interval $d\left(\frac{k\pi}{L}\right)$ is simply π/L. Thus,

$$\lim_{L\to\infty} \sum_{k=0}^{\infty} \frac{2}{L} F\left(\frac{k\pi}{L}\right) = \int_0^{\infty} \frac{2}{\pi} F\left(\frac{k\pi}{L}\right) d\left(\frac{k\pi}{L}\right)$$

(See, also, ref. 17.) By using this relationship and setting $q = \frac{k\pi}{L}$, equation (28) becomes

$$\varphi_2 = \int_0^{\infty} \frac{2}{\pi}\left[\frac{Q_o(q) - cQ_1(q)}{I_o(q) + cqI_1(q)}\right] I_o(q\rho)\sin qx \, dq \tag{29}$$

The remaining task is to evaluate Q_o and Q_1. If equations (25) and (27) are integrated by parts the results can be brought to the form (see ref. 18)

$$Q_o(q) = qK_o(q)$$

$$Q_1(q) = q^2K_1(q)$$

The final result for φ_2 is, then,

$$\varphi_2 = \frac{2}{\pi}\int_0^\infty \left[\frac{qK_o(q) - cq^2K_1(q)}{I_o(q) + cqI_1(q)}\right] I_o(q\rho)\sin qx\, dq$$

The interference velocity is

$$u_2 = -\frac{\partial\varphi_2}{\partial x} = \frac{2}{\pi}\int_0^\infty q^2\left[\frac{cqK_1(q) - K_o(q)}{cqI_1(q) + I_o(q)}\right] I_o(q\rho)\cos qx\, dq \qquad (30)$$

This equation is for a doublet of strength $\frac{m}{4\pi} = 1$ in a tunnel of unit radius. For the general case, at the origin,

$$u_2 = \frac{m}{2\pi^2R^3}\int_0^\infty \frac{cqK_1(q) - K_o(q)}{cqI_1(q) + I_o(q)}\, q^2 dq \qquad (31)$$

where R is the tunnel radius. The axial interference velocity is plotted as a function of $\left(\frac{1}{c + 1}\right)^{1/2}$ in figure 7. This parameter is chosen because it results in an approximately linear variation of the blockage interference. The values were obtained by mechanical integration of equation (31). Fortunately, the value of the integrand converges rapidly toward zero with increasing q. At $q = 5$, for instance, the value of the integrand was 1/2 percent or less of its maximum value in the calculations which have been made.

Rectangular Wind Tunnel With Top and Bottom Walls Slotted

Lift interference.- Consider a rectangular wind tunnel of semiwidth unity and semiheight λ. Inside the tunnel is a vortex pair of semispan σ located with the span parallel to the width (y) axis of the tunnel and with the center on the center line of the tunnel. The vertical walls at the sides of the tunnel are closed, but the horizontal walls at the top and bottom, with nondimensional restriction constant l, are partially open. The boundary condition at the closed side walls $(y = \pm 1)$

is, of course, $\frac{\partial\varphi}{\partial y} = 0$. This condition is satisfied by a horizontal row of vortices along the plane $z = 0$ (the reflected images, out to $y = \pm\infty$, added to the vortex pair inside the tunnel). The potential of such a row is given by

$$\varphi_r = \frac{\Gamma}{4\pi}\left\{-\tan^{-1}\left[\frac{\tanh\frac{\pi}{2}z}{\tan\frac{\pi}{2}(y - \sigma)}\right] + \tan^{-1}\left[\frac{\tanh\frac{\pi}{2}z}{\tan\frac{\pi}{2}(y + \sigma)}\right]\right\} \tag{32}$$

and the vertical derivative by

$$\frac{\partial\varphi_r}{\partial z} = \frac{\Gamma}{8}\left[-\frac{\sin\pi(y - \sigma)}{\cosh\pi z - \cos\pi(y - \sigma)} + \frac{\sin\pi(y + \sigma)}{\cosh\pi z - \cos\pi(y + \sigma)}\right] \tag{33}$$

To the potential φ_r another potential φ_s is added in order to satisfy the boundary condition at the horizontal walls. The total potential $\varphi_T = \varphi_r + \varphi_s$ must satisfy the conditions

At $y = \pm 1$

$$\frac{\partial\varphi_T}{\partial y} = 0 \tag{34}$$

At $z = \lambda$

$$\varphi_T + l\frac{\partial\varphi_T}{\partial z} = 0 \tag{35}$$

At $z = -\lambda$

$$\varphi_T - l\frac{\partial\varphi_T}{\partial z} = 0 \tag{36}$$

Since φ_r already satisfies equation (34), φ_s must also satisfy (34). A solution of Laplace's equation which meets this requirement is

$$\varphi_s = \frac{\Gamma}{4\pi}\left[\sum_{n=1}^{\infty}\cos n\pi y\left(A_n\sinh n\pi z + B_n\cosh n\pi z\right) + A_o z\right] \tag{37}$$

Because φ_r is an odd function of z and $\frac{\partial \varphi_r}{\partial z}$ is an even function, it is necessary that φ_s be an odd function of z in order that both equations (35) and (36) may be satisfied. Consequently, $B_n = 0$, and

$$\left.\begin{aligned}\varphi_s &= \frac{\Gamma}{4\pi}\left(\sum_{n=1}^{\infty} A_n \cos n\pi y \sinh n\pi z + A_o z\right)\\ \frac{\partial \varphi_s}{\partial z} &= \frac{\Gamma}{4\pi}\left(\sum_{n=1}^{\infty} n\pi A_n \cos n\pi y \cosh n\pi z + A_o\right)\end{aligned}\right\} \tag{38}$$

Now, let equations (32) and (33) be expanded in Fourier cosine series at the boundary $z = \lambda$. Then,

$$\varphi_r = \frac{\Gamma}{4\pi}\sum_{n=0}^{\infty} C_n \cos n\pi y \tag{39a}$$

$$\frac{\partial \varphi_r}{\partial z} = \frac{\Gamma}{4\pi}\sum_{n=1}^{\infty} D_n \cos n\pi y \tag{39b}$$

(It is apparent from the form of equation (33) that there can be no constant term in equation (39b).) If equations (38) and (39) are substituted in equation (35) the following coefficient relationships are found:

$$C_o + A_o\lambda + lA_o = 0 \qquad (n = 0)$$

$$C_n + lD_n + A_n \sinh n\pi\lambda + ln\pi A_n \cosh n\pi\lambda = 0 \qquad (n \neq 0)$$

Thus,

$$\left.\begin{aligned}A_o &= -\frac{C_o}{\lambda + l}\\ A_n &= -\frac{C_n + lD_n}{\sinh n\pi\lambda + ln\pi \cosh n\pi\lambda}\end{aligned}\right\} \tag{40}$$

Let $\varphi_T = \varphi_1 + \varphi_2$ where again φ_1 is the free-field potential of the vortex pair inside the tunnel and φ_2 is the interference potential. From the free-field potential of the vortex pair

$$\frac{4\pi}{\Gamma} w_1 = -\frac{4\pi}{\Gamma}\frac{\partial \varphi_1}{\partial z} = \frac{y - \sigma}{(y - \sigma)^2 + z^2} - \frac{y + \sigma}{(y + \sigma)^2 + z^2}$$

Thus, the interference velocity at any point (y,z) is

$$\frac{4\pi}{\Gamma} w_2 = \frac{\pi}{2}\left[\frac{\sin \pi(y - \sigma)}{\cosh \pi z - \cos \pi(y - \sigma)} - \frac{\sin \pi(y + \sigma)}{\cosh \pi z - \cos \pi(y + \sigma)}\right] - \sum_{n=1}^{\infty} n\pi A_n \cos n\pi y \cosh n\pi z - A_o - \frac{y - \sigma}{(y - \sigma)^2 + z^2} + \frac{y + \sigma}{(y + \sigma)^2 + z^2} \tag{41}$$

The average interference velocity in the plane of the wing $(z = 0)$ is

$$\bar{w}_2 = \frac{1}{2\sigma}\int_{-\sigma}^{\sigma} w_2 \, dy$$

In order to evaluate this integral, obtain first

$$\frac{4\pi}{\Gamma}\int_{-y}^{y} w_2 \, dy = \log_e \frac{1 - \cos \pi(y - \sigma)}{1 - \cos \pi(y + \sigma)} - 2\sum_{n=1}^{\infty} A_n \sin n\pi y - 2A_o y + \log_e \frac{(y + \sigma)^2}{(y - \sigma)^2}$$

As y approaches σ the first and last terms approach minus and plus infinity, respectively. In order to eliminate the indeterminateness, the two terms are combined and the cosine is expanded in series form. Finally, the limit is taken as y approaches σ.

$$\lim_{y \to \sigma} \frac{(y + \sigma)^2}{1 - \cos \pi(y + \sigma)} \frac{1 - \cos \pi(y - \sigma)}{(y - \sigma)^2} =$$

$$\lim_{y \to \sigma} \frac{(y + \sigma)^2}{1 - \cos \pi(y + \sigma)} \left[\frac{1 - 1 + \frac{\pi^2(y - \sigma)^2}{2} - \frac{\pi^4(y - \sigma)^4}{4!} + \dots}{(y - \sigma)^2}\right] =$$

$$\frac{4\sigma^2}{1 - \cos 2\pi\sigma} \frac{\pi^2}{2}$$

By using this limit, the average interference velocity becomes

$$\bar{w}_2 = \frac{\Gamma}{4\pi\sigma}\left[\log_e\left(\frac{\pi\sigma}{\sin \pi\sigma}\right) - \sum_{n=1}^{\infty} A_n \sin n\pi\sigma + \frac{C_o\sigma}{\lambda + l}\right] \tag{42}$$

For this rectangular tunnel $\delta = \frac{\bar{w}_2\lambda}{\Gamma\sigma}$; so

$$\delta = \frac{\lambda}{4\pi\sigma^2}\left[\log_e\left(\frac{\pi\sigma}{\sin \pi\sigma}\right) - \sum_{n=1}^{\infty} A_n \sin n\pi\sigma + \frac{C_o\sigma}{(\lambda + l)}\right] \tag{43}$$

Correction factors are plotted as a function of the restriction constant for several values of σ in figure 8(a) for a square tunnel ($\lambda = 1$) and in figure 8(b) for a tunnel with $\lambda = 0.5$. The nondimensional restriction constant is defined as $c = \frac{l}{h}$, where h is the semi-height of the tunnel. For the tunnel under consideration then, $c = \frac{l}{\lambda}$. The value of c for a rectangular tunnel of height-width ratio λ with N slots in each horizontal wall is given by

$$c = \frac{2}{\pi N\lambda} \ln\mu \tag{44}$$

Solid-blockage interference.- Consider next the problem of solid-blockage interference in the rectangular wind tunnel with top and bottom walls slotted. The solid body in the tunnel is again represented by a doublet with axis alined with axis of the tunnel and located at the origin of coordinates on the tunnel axis. The boundary condition at the closed side wall $(y = \pm 1)$ is $\frac{\partial \varphi_T}{\partial y} = 0$. This condition is satisfied by a horizontal row of doublets with axes alined with the axis of the tunnel and placed along the line $x = 0$, $z = 0$ at $y = 0$, $y = \pm 2$, $y = \pm 4$, . . . $y = \pm \infty$. The potential of such a row is given by

$$\varphi_r = -\frac{m}{4\pi} \sum_{k=-\infty}^{\infty} \frac{x}{\left[(y + 2k)^2 + z^2 + x^2\right]^{3/2}} \tag{45}$$

and the vertical derivative by

$$\frac{\partial \varphi_r}{\partial z} = \frac{m}{4\pi} \sum_{k=-\infty}^{\infty} \frac{3zx}{\left[(y + 2k)^2 + z^2 + x^2\right]^{5/2}} \tag{46}$$

The total potential $\varphi_T = \varphi_r + \varphi_s$ must satisfy equations (34), (35), and (36). As before, φ_s must satisfy equation (34). A solution of Laplace's equation which meets this requirement is

$$\varphi_s = \frac{m}{4\pi} \sum_{s=0}^{\infty} \sum_{t=0}^{\infty} \left\{A_{st} \cosh\left[z \sqrt{(s\pi)^2 + \left(\frac{t\pi}{L}\right)^2}\right] + B_{st} \sinh\left[z \sqrt{(s\pi)^2 + \left(\frac{t\pi}{L}\right)^2}\right]\right\} \cos s\pi y \sin \frac{t\pi x}{L} \tag{47}$$

As in the case of lift interference φ_r is an odd function of z and $\frac{\partial \varphi_r}{\partial z}$ is an even function; therefore, φ_s must be an odd function of z in order that both equations (35) and (36) may be satisfied. Consequently, $B_{st} = 0$, and

$$\varphi_s = \frac{m}{4\pi}\sum_{s=0}^{\infty}\sum_{t=0}^{\infty} A_{st} \cosh\left[z\sqrt{(s\pi)^2 + \left(\frac{t\pi}{L}\right)^2}\right]\cos s\pi y \sin \frac{t\pi x}{L} \tag{48}$$

$$\frac{\partial\varphi_s}{\partial z} = \frac{m}{4\pi}\sum_{s=0}^{\infty}\sum_{t=0}^{\infty} A_{st}\sqrt{(s\pi)^2 + \left(\frac{t\pi}{L}\right)^2}\sinh\left[z\sqrt{(s\pi)^2 + \left(\frac{t\pi}{L}\right)^2}\right]\cos s\pi y \sin \frac{t\pi x}{L} \tag{49}$$

Now let equations (45) and (46) be expanded in Fourier cosine series at the boundary $z = \lambda$. Then,

$$\left(\varphi_r\right)_{z=\lambda} = -\frac{m}{4\pi}\sum_{s=0}^{\infty}\sum_{t=0}^{\infty} C_{st} \cos s\pi y \sin \frac{t\pi x}{L} \tag{50}$$

$$\left(\frac{\partial\varphi_r}{\partial z}\right)_{z=\lambda} = \frac{m}{4\pi}\sum_{s=0}^{\infty}\sum_{t=0}^{\infty} D_{st} \cos s\pi y \sin \frac{t\pi x}{L} \tag{51}$$

where C_{st} and D_{st} are given by

$$C_{st} = \frac{4p}{L}\int_0^1\int_0^L \sum_{k=-\infty}^{\infty} \frac{x \sin \frac{t\pi x}{L} \cos s\pi y}{\left[(y + 2k)^2 + \lambda^2 + x^2\right]^{3/2}} dx\, dy$$

where

$$p = \begin{cases} 1/2 & (s = 0) \\ 1 & (s \neq 0) \end{cases}$$

$$C_{st} = \frac{4p}{\pi}\left(\frac{\pi}{L}\right)\left(\frac{t\pi}{L}\right)\int_0^1\int_0^L \sum_{k=-\infty}^{\infty} \frac{\cos \frac{t\pi x}{L} \cos s\pi y \, dx\, dy}{\sqrt{(y + 2k)^2 + \lambda^2 + x^2}} \tag{52}$$

$$D_{st} = \frac{4p}{L}\int_0^l \int_0^L \sum_{k=-\infty}^{\infty} \frac{3\lambda x \sin \frac{t\pi x}{L} \cos s\pi y}{\sqrt{\left[(y + 2k)^2 + \lambda^2 + x^2\right]^{5/2}}} \, dx \, dy$$

$$= \frac{4p}{\pi}\left(\frac{\pi}{L}\right)\left(\frac{t\pi}{L}\right)\int_0^l \int_0^L \sum_{k=-\infty}^{\infty} \frac{\lambda \cos \frac{t\pi x}{L} \cos s\pi y}{\sqrt{\left[(y + 2k)^2 + \lambda^2 + x^2\right]^{3/2}}} \, dx \, dy \tag{53}$$

By substituting equations (48), (49), (50), and (51) into equation (35), A_{st} can be written as

$$A_{st} = \frac{C_{st} - lD_{st}}{\cosh\left[\lambda\sqrt{(s\pi)^2 + \left(\frac{t\pi}{L}\right)^2}\right] + l\sqrt{(s\pi)^2 + \left(\frac{t\pi}{L}\right)^2} \sinh\left[\lambda\sqrt{(s\pi)^2 + \left(\frac{t\pi}{L}\right)^2}\right]} \tag{54}$$

By substituting the equations (52), (53), and (54) into equation (48), the result is

$$\varphi_s = -\frac{pm}{\pi^2}\left(\frac{\pi}{L}\right)\sum_{s=0}^{\infty}\sum_{t=0}^{\infty}\left(\frac{\frac{t\pi}{L}\sum_{k=-\infty}^{\infty}\int_0^l\int_0^L\left\{\frac{l\lambda \cos\frac{t\pi\beta}{L}\cos s\pi\alpha}{\left[(\alpha + 2k)^2 + \lambda^2 + \beta^2\right]^{3/2}} - \frac{\cos\frac{t\pi\beta}{L}\cos s\pi\alpha}{\left[(\alpha + 2k)^2 + \lambda^2 + \beta^2\right]^{1/2}}\right\}d\beta \, d\alpha}{\cosh\left[\lambda\sqrt{(s\pi)^2 + \left(\frac{t\pi}{L}\right)^2}\right] + l\sqrt{(s\pi)^2 + \left(\frac{t\pi}{L}\right)^2}\sinh\left[\lambda\sqrt{(s\pi)^2 + \left(\frac{t\pi}{L}\right)^2}\right]} \cosh\left[z\sqrt{(s\pi)^2 + \left(\frac{t\pi}{L}\right)^2}\right]\cos s\pi y \sin\frac{t\pi x}{L}\right) \tag{55}$$

As L approaches ∞, the integral with respect to L in equation (55) becomes

$$\int_0^L \left\{ \frac{l\lambda \cos \frac{t\pi\beta}{L} \cos s\pi\alpha}{\left[(\alpha + 2k)^2 + \lambda^2 + \beta^2\right]^{3/2}} - \frac{\cos \frac{t\pi\beta}{L} \cos s\pi\alpha}{\left[(\alpha + 2k)^2 + \lambda^2 + \beta^2\right]^{1/2}} \right\} d\beta =$$

$$\left\{ l\lambda \int_0^\infty \frac{\cos \frac{t\pi\beta}{L}\, d\beta}{\left[(\alpha + 2k)^2 + \lambda^2 + \beta^2\right]^{3/2}} - \int_0^\infty \frac{\cos \frac{t\pi\beta}{L}\, d\beta}{\left[(\alpha + 2k)^2 + \lambda^2 + \beta^2\right]^{1/2}} \right\} \cos s\pi\alpha$$

In reference 19, the first integral on the right is written as

$$\int_0^\infty \frac{\cos q\beta\, d\beta}{\left(\beta^2 + R^2\right)^{3/2}} = \frac{qK_1(Rq)}{R} \tag{56}$$

where $q = \frac{\pi t}{L}$ and $R = \sqrt{(\alpha + 2k)^2 + \lambda^2}$. Multiplying both sides of equation (56) by $-RdR$ and then integrating both sides with respect to R from R to ∞ gives

$$\int_0^\infty \frac{\cos q\beta\, d\beta}{\left(\beta^2 + R^2\right)^{1/2}} = K_o(Rq)$$

Let L approach ∞. Then, the interference potential φ_s and its derivative with respect to z can be written as

$$\varphi_s = -\frac{mp}{\pi^2} \sum_{s=0}^{\infty} \int_0^\infty \left(\frac{q \sum_{k=-\infty}^{\infty} \int_0^1 \left\{ \frac{qc\lambda^2 K_1\left[q\sqrt{(\alpha + 2k)^2 + \lambda^2}\right]}{\sqrt{(\alpha + 2k)^2 + \lambda^2}} - K_o\left[q\sqrt{(\alpha + 2k)^2 + \lambda^2}\right] \right\} \cos s\pi\alpha\, d\alpha}{\cosh\left[\lambda\sqrt{q^2 + (s\pi)^2}\right] + c\lambda\sqrt{q^2 + (s\pi)^2} \sinh\left[\lambda\sqrt{q^2 + (s\pi)^2}\right]} \cosh\left[z\sqrt{q^2 + (s\pi)^2}\right] \sin qx \cos s\pi y \right) dq \tag{57}$$

$$\frac{\partial\varphi_s}{\partial x} = -\frac{mp}{\pi^2}\sum_{s=0}^{\infty}\int_0^{\infty}\left(\frac{q^2\sum_{k=-\infty}^{\infty}\int_0^1\left\{\frac{qc\lambda^2K_1\left[q\sqrt{(\alpha+2k)^2+\lambda^2}\right]}{\sqrt{(\alpha+2k)^2+\lambda^2}} - K_o\left[q\sqrt{(\alpha+2k)^2+\lambda^2}\right]\right\}\cos s\pi\alpha\, d\alpha}{\cosh\left[\lambda\sqrt{q^2+(s\pi)^2}\right] + c\lambda\sqrt{q^2+(s\pi)^2}\sinh\left[\lambda\sqrt{q^2+(s\pi)^2}\right]}\cosh\left[z\sqrt{q^2+(s\pi)^2}\right]\cos qx \cos s\pi y\right) dq$$

$$= -\frac{mp}{\pi^2}\sum_{s=0}^{\infty}\int_0^{\infty}\frac{q^2\left[qc\lambda^2H_1(\lambda,s,q) - H_o(\lambda,s,q)\right]}{C(\lambda,s,q) + cS(\lambda,s,q)}\cosh z\sqrt{q^2+(s\pi)^2}\cos qx \cos s\pi y\, dq \qquad (58)$$

where

$$H_1 = \int_0^1\sum_{k=-\infty}^{\infty}\frac{K_1\left[q\sqrt{(\alpha+2k)^2+\lambda^2}\right]\cos s\pi\alpha\, d\alpha}{\sqrt{(\alpha+2k)^2+\lambda^2}}$$

$$H_o = \int_0^1\sum_{k=-\infty}^{\infty}K_o\left[q\sqrt{(\alpha+2k)^2+\lambda^2}\right]\cos s\pi\alpha\, d\alpha$$

$$C(\lambda,s,q) = \cosh \lambda\sqrt{q^2+(s\pi)^2}$$

$$S(\lambda,s,q) = \lambda\sqrt{q^2+(s\pi)^2}\sinh \lambda\sqrt{q^2+(s\pi)^2}$$

and where l has been replaced by $c\lambda$. The interference velocity u_2 of a doublet symmetrically located in a tunnel with its axis alined with the axis of the tunnel is given by

$$u_2 = u_r + u_s - u_1$$

$$= -\frac{\partial\varphi_s}{\partial x} - \frac{\partial}{\partial x}\left(\varphi_r - \varphi_1\right)$$

$$= -\frac{\partial\varphi_s}{\partial x} + \frac{m}{4\pi}\sum_{k=-\infty}^{\infty}\frac{(y+2k)^2+z^2-2x^2}{\left[(y+2k)^2+z^2+x^2\right]^{5/2}} \qquad (k \neq 0)$$

This is for a doublet of strength $m/4\pi$ in a tunnel of unit semiwidth. For the general case, at the origin,

$$u_2 = -\frac{1}{b^3}\left(\frac{\partial\varphi_s}{\partial x}\right)_{\substack{x=0\\y=0\\z=0}} + \frac{m}{32\pi b^3}\sum_{k=1}^{\infty}\frac{1}{k^3} \tag{59}$$

Equation (59) was solved numerically for the interference velocity u_2 of a doublet in a square tunnel ($\lambda = 1$), and the ratio u_2b^3/m is plotted against $\sqrt{\frac{1}{c + 1}}$ in figure 9.

Lift Interference in Rectangular Tunnel

With All Sides Slotted

Consider the lift interference for a small lifting wing symmetrically located in a rectangular wind tunnel of semiheight λ and unit semiwidth and with all four sides slotted. The widths of the individual slots are determined by calculating the points on the walls of the rectangular tunnel which correspond to the slot edges of a uniformly slotted circular tunnel, with the assistance of a transformation which maps the perimeter of a rectangle of semiheight λ and unit semiwidth into the perimeter of a circle of unit radius. In the analysis of this particular problem the transverse cross-sectional plane of the rectangular tunnel, which has been previously the y,z-plane, will be taken as the complex Z-plane where $Z = x + iy$. The cross-sectional plane of the circular tunnel, which has previously been treated in terms of the polar coordinates (ρ,θ) will here be taken as the complex ζ-plane where $\zeta = \xi + i\eta$. In reference 20 the transformation which maps a rectangle in the Z-plane into a unit circle in the ζ-plane is given as

$$\zeta = \frac{\text{sn }\lambda'Z \text{ dn }\lambda'Z}{\text{cn }\lambda'Z} \tag{60}$$

or

$$\zeta^2 = \frac{1 - \text{cn }2\lambda'Z}{1 + \text{cn }2\lambda'Z} \tag{61}$$

and λ' is determined from

$$\frac{K}{2} = \frac{K'}{2\lambda} = \lambda' \tag{62}$$

where K and iK' are the quarter periods of the preceding Jacobian elliptic functions. A method for calculating λ' and tables of the preceding Jacobian elliptic functions are also given in reference 20. The quadrantal perimeter of a square $(\lambda = 1)$ and the quadrantal perimeter of a rectangle $(\lambda = 1/2)$ were mapped into the quadrantal arc of a unit circle by using equation (61) and the results are given in figure 10.

The lift-correction factor for a small lifting wing symmetrically located in a rectangular wind tunnel may be written as

$$\delta_R = \frac{\lambda \bar{v}_{2R}}{\sigma_R \Gamma} \approx \frac{\lambda \left(v_{2R}\right)_{Z=0}}{\sigma_R \Gamma} \tag{63}$$

since $\bar{v}_{2R} \approx \left(v_{2R}\right)_{Z=0}$ for a small lifting wing.

The complex velocity in the Z-plane at a point on the x-axis is

$$-\frac{dW}{dZ} = -iv_R = -\frac{dW}{d\zeta}\frac{d\zeta}{dZ} = -iv_C\frac{d\zeta}{dZ}$$

or

$$v_R = v_C\frac{d\zeta}{dZ} \tag{64}$$

The interference velocity for the rectangular tunnel is thus

$$v_{2R} = \left(v_{1C} + v_{2C}\right)\frac{d\zeta}{dZ} - v_{1R} \tag{65}$$

In reference (21) $d\zeta/dZ$ is written as

$$\frac{d\zeta}{dZ} = \lambda' \tag{66}$$

at the origin. Evaluation of equation (10b) at the origin gives

$$v_{1C} = -\left(\frac{\partial\varphi_1}{\partial\rho}\right)_{\substack{\rho=0\\ \theta=\frac{\pi}{2}}} = -\frac{\Gamma}{2\pi\sigma_C} \tag{67}$$

From the analysis of lift interference in a circular tunnel (eqs. (14a), (13), (18), and (20))

$$v_{2C} = \frac{\Gamma\sigma_C}{2\pi}\left(\frac{c - 1}{c + 1}\right) \tag{68}$$

at the origin. The vortex semispan in the circular tunnel σ_C is determined from equation (61) by setting $Z = \sigma_R$ $\left(\text{whence, } \zeta = \sigma_C\right)$. The velocity v_{1R} is given by

$$v_{1R} = -\frac{\Gamma}{2\pi\sigma_R} \tag{69}$$

at the origin. This differs from v_{1C} (eq. (67)) only because of the change in vortex span. If, now, equations (66), (67), (68), and (69) are substituted in equation (65) the result is

$$v_{2R} = \frac{\Gamma}{2\pi}\left[\left(-\frac{1}{\sigma_C} + \sigma_C\,\frac{c - 1}{c + 1}\right)\lambda' + \frac{1}{\sigma_R}\right] \tag{70}$$

Substituting this in equation (63) gives, for the lift-correction factor at the center of the vortex span,

$$\delta_R = \frac{\lambda}{2\pi\sigma_R}\left[\lambda'\left(\sigma_C\,\frac{c - 1}{c + 1} - \frac{1}{\sigma_C}\right) + \frac{1}{\sigma_R}\right] \tag{71}$$

In the limit as $\sigma_R \longrightarrow 0$ this becomes

$$\delta_R = \frac{\lambda\lambda'^2}{2\pi}\left[\left(\frac{c - 1}{c + 1}\right) + \left(\frac{1 - 2m}{3}\right)\right] = \frac{\lambda\lambda'^2}{\pi}\left(\frac{2 - m}{3} - \frac{1}{c + 1}\right) \tag{72a}$$

where m is the parameter (see ref. 20) of the elliptic functions of the transformation. The behavior of m is such that if $\lambda < 0.5$ it is sufficient to use

$$\delta_R = \frac{\lambda\lambda'^2}{\pi}\left(\frac{1}{3} - \frac{1}{c + 1}\right) = \frac{\pi}{16\lambda}\left(\frac{1}{3} - \frac{1}{c + 1}\right) \tag{72b}$$

When $\lambda = 1.0$ (square tunnel), $m = 0.5$ and

$$\delta_R = \frac{\lambda\lambda'^2}{\pi}\left(\frac{1}{2} - \frac{1}{c + 1}\right) = 0.274\left(\frac{1}{2} - \frac{1}{c + 1}\right) \tag{72c}$$

When $\lambda > 2.0$, the correction factor is approximately

$$\delta_R = \frac{\lambda\lambda'^2}{\pi}\left(\frac{2}{3} - \frac{1}{c + 1}\right) = \frac{\pi\lambda}{16}\left(\frac{2}{3} - \frac{1}{c + 1}\right) \tag{72d}$$

Equations (72) apply to wings of small span.

The lift correction at the center of the wing has been calculated for a small wing in a square tunnel ($\lambda = 1$) and for a small wing in two rectangular tunnels ($\lambda = 0.5$ and $\lambda = 2$); the lift-correction factor δ_R is plotted against $\frac{1}{c + 1}$ in figure 11.

Two-Dimensional Tunnel

Lift interference.- Consider the problem of the lift interference in a two-dimensional wind tunnel with the top and bottom walls slotted. The wing in the tunnel will be represented by a two-dimensional lifting vortex located at the origin of the coordinate system, which is centered between the slotted walls. The potential of this vortex in a free field is

$$\varphi_1 = \frac{\Gamma}{2\pi} \tan^{-1} \frac{z}{x}$$

and the vertical derivative is

$$\frac{\partial\varphi_1}{\partial z} = \frac{\Gamma}{2\pi} \frac{x}{x^2 + z^2}$$

The interference potential takes the form

$$\varphi_2 = \sum_{n=1}^{\infty} \left(A_n \sin \frac{n\pi x}{L} \sinh \frac{n\pi z}{L}\right) + A_o z \tag{73}$$

There is no axial interference along the tunnel center line $(z = 0)$ because $\frac{\partial\varphi_2}{\partial x} = 0$. The vertical interference at the origin is simply

$$w_2 = - \frac{\partial\varphi_2}{\partial z} = -A_o$$

Inasmuch as the summation does not contribute to the wall-induced velocity at the origin, it is sufficient to study the term $A_o z$ by itself. If the multiple-valued function for φ_1 is set equal to zero at the upstream infinity $(z = -\infty)$ it is apparent that the Fourier series expansion of φ_1 at the top wall will result in a constant term $-\frac{\Gamma}{4}$ plus an odd function of x. The expansion of $\partial\varphi_1/\partial z$ will be an odd function with no constant term. Thus, if the constant terms in φ_1, φ_2, and $\partial\varphi_2/\partial z$ are inserted in the boundary condition at the upper wall $(z = h)$

$$\varphi_T + l \frac{\partial\varphi_T}{\partial z} = 0 \tag{74}$$

the result is

$$-\frac{\Gamma}{4} + A_o h + lA_o = 0$$

$$A_o = \frac{\Gamma}{4h} \frac{1}{c + 1}$$

where $c = \frac{l}{h}$. The vertical interference velocity at the origin is given by

$$w_2 = -\frac{\Gamma}{4h}\frac{1}{c + 1}$$

Thus, the interference is found to be downwash at the origin which varies in magnitude from zero for a closed tunnel to a maximum for the open tunnel. Because

$$\Gamma = \frac{c_l u_o \bar{c}}{2}$$

the induced-flow angle at the origin is

$$\alpha_2 = -\frac{c_l}{8}\frac{\bar{c}}{h}\frac{1}{c + 1} \tag{75}$$

where

$$c = \frac{d}{\pi h}\log_e \csc\frac{\pi}{2}r_o$$

and

d slot spacing on horizontal walls

h half-height of tunnel

r_o open ratio of the uniformly slotted horizontal walls

c_l airfoil-section lift coefficient

$\bar{c}$ airfoil chord

The induced angle is, of course, not constant along the chord of the airfoil; therefore, the wall-induced flow can be considered to have a certain curvature. This curvature has an effect similar to that of camber in an airfoil, and if the length of the chord of the airfoil is sufficiently great compared to the height of the tunnel, a curvature correction is required (ref. 3). This curvature can be determined from a complete solution for φ_2 (eq. (73)) under the boundary condition of equation (74).

Solid-blockage interference.- Consider the problem of solid-blockage interference in a two-dimensional wind tunnel with top and bottom walls slotted. The solid body in the tunnel will be represented by a two-dimensional doublet with axis alined with the tunnel axis, and located at the origin of coordinates on the tunnel axis. The potential of this doublet in a free field is given by

$$\varphi_1 = -\frac{m}{2\pi}\frac{x}{x^2 + z^2} \tag{76}$$

and the vertical derivative by

$$\frac{\partial\varphi_1}{\partial z} = \frac{m}{\pi}\frac{xz}{\left(x^2 + z^2\right)^2} \tag{77}$$

The interference potential φ_2 must satisfy Laplace's equation. Since φ_1 is an odd function of x and an even function of z, φ_2 must also be an odd function of x and an even function of z. Therefore, φ_2 can be written as

$$\varphi_2 = \frac{m}{2\pi}\sum_{n=1}^{\infty} A_k \cosh\frac{k\pi z}{L} \sin\frac{k\pi x}{L} \tag{78}$$

and the normal derivative by

$$\frac{\partial\varphi_2}{\partial z} = \frac{m}{2\pi}\sum_{n=1}^{\infty}\frac{k\pi}{L} A_k \sinh\frac{k\pi z}{L} \sin\frac{k\pi x}{L} \tag{79}$$

The boundary conditions to be satisfied along the slotted walls $(z = \pm h)$ are

at $z = h$

$$\varphi_1 + \varphi_2 + l\frac{\partial}{\partial z}\left(\varphi_1 + \varphi_2\right) = 0 \tag{80a}$$

and

at $z = -h$

$$\varphi_1 + \varphi_2 - l\frac{\partial}{\partial z}\left(\varphi_1 + \varphi_2\right) = 0 \tag{80b}$$

These conditions are satisfied by expanding φ_1 and $\partial\varphi_1/\partial z$ into Fourier sine series in x along the slotted walls and then solving for the coefficients of φ_2 that satisfy these equations. Along the line $z = h$, equations (76) and (77) can be written as

$$\varphi_1 = -\frac{m}{2\pi}\sum_{k=1}^{\infty} B_k \sin\frac{k\pi x}{L} \tag{81}$$

$$\frac{\partial\varphi_1}{\partial z} = \frac{m}{2\pi}\sum_{k=1}^{\infty} C_k \sin\frac{k\pi x}{L} \tag{82}$$

where

$$B_k = \frac{2}{L}\int_0^L \frac{x}{\left(x^2 + h^2\right)} \sin\frac{k\pi x}{L}\,dx \tag{83}$$

and

$$C_k = \frac{2}{L}\int_0^L \frac{hx}{\left(x^2 + h^2\right)^2} \sin\frac{k\pi x}{L}\,dx$$

A single integration by parts results in

$$C_k = \frac{k\pi h}{L}\int_0^L \frac{\cos\frac{k\pi x}{L}}{x^2 + h^2}\,dx \tag{84}$$

By substituting equation (83) for B_k and equation (84) for C_k into equations (81) and (82), respectively, and then solving equations (80), (81), and (82) for A_k, the result is

$$A_k = -\frac{m}{\pi^2}\frac{\pi}{L}\frac{\displaystyle\int_0^L \left(\frac{\frac{hlk\pi}{L}\cos\frac{k\pi x}{L} - \sin\frac{k\pi x}{L}}{x^2 + h^2}\,dx\right)}{\cosh\frac{k\pi h}{L} + l\frac{k\pi}{L}\sinh\frac{k\pi h}{L}} \tag{85}$$

By substituting equation (85) for A_k into equation (78), the result is

$$\varphi_2 = -\frac{m}{\pi^2}\sum_{k=1}^{\infty}\frac{\pi}{L}\frac{\displaystyle\int_0^L\left(\frac{\frac{hlk\pi}{L}\cos\frac{k\pi\alpha}{L} - \alpha\sin\frac{k\pi\alpha}{L}}{\alpha^2 + h^2}\right)d\alpha}{\cosh\frac{k\pi h}{L} + l\frac{k\pi}{L}\sinh\frac{k\pi h}{L}}\cosh\frac{k\pi z}{L}\sin\frac{k\pi x}{L} \tag{86}$$

Let $q = \frac{k\pi}{L}$, $\Delta q = \frac{\pi}{L}$, and let $L \longrightarrow \infty$. Then equation (86) becomes

$$\varphi_2 = -\frac{m}{\pi^2}\int_0^{\infty}\frac{\displaystyle\int_0^{\infty}\left(\frac{\frac{hlk\pi}{L}\cos q\alpha - \alpha\sin q\alpha}{\alpha^2 + h^2}\right)d\alpha}{\cosh qh + lq\sinh qh}\cosh qz\sin qx\,dq$$

$$= -\frac{m}{2\pi}\int_0^{\infty}\left(\frac{lqe^{-qh} - e^{-qh}}{\cosh qh + lq\sinh qh}\right)\cosh qz\sin qx\,dq$$

$$= -\frac{m}{2\pi}\int_0^{\infty}\left[\frac{e^{-qh}(lq - 1)}{\cosh qh + lq\sinh qh}\right]\cosh qz\sin qx\,dq$$

$$= -\frac{m}{2\pi h}\int_0^{\infty}\left[\frac{e^{-q}(cq - 1)}{\cosh q + cq\sinh q}\right]\cosh\frac{qz}{h}\sin\frac{qx}{h}\,dq \tag{87}$$

where qh has been replaced by q and $c = \frac{l}{h}$. The interference velocity u_2 is

$$u_2 = -\frac{\partial\varphi_2}{\partial x} = \frac{m}{2\pi h^2}\int_0^\infty \frac{qe^{-q}(cq - 1)}{\cosh q + cq \sinh q} \cosh\frac{qy}{h} \cos\frac{qx}{h}\, dq \tag{88}$$

At the origin, equation (88) reduces to

$$u_2 = \frac{m}{2\pi h^2}\int_0^\infty \frac{qe^{-q}(cq - 1)}{\cosh q + cq \sinh q}\, dq \tag{89}$$

Equation (89) has been evaluated numerically, and the axial-interference-velocity function u_2h^2/m is plotted against $\sqrt{\frac{1}{c + 1}}$ in figure 12.

DISCUSSION

Application of the Results of the Interference Analysis

Consider the problem of determining, say, the correction factor δ for a particular wing in a square tunnel. The tunnel has four slots in each horizontal wall, with an open ratio of 0.164. The vertical walls are closed. From figure 4, $\log_e \csc \frac{\pi}{2} r_o = 1.37$. After using equation (44) to determine c, the value of $\frac{1}{c + 1}$ is computed to be 0.82. The wing has an effective span of one-half the tunnel width, and for this case, figure 8(a) gives the value of the correction factor as -0.062. If a semispan reflection-plane model of a wing of twice this span is tested, the effective height-width ratio of the tunnel will be 0.5 and the correction factor will be -0.146.

Comparison With Previous Results

A comparison of the results for the lift interference of circular tunnels with the results of reference 13, in which the individual slots are considered, shows striking agreement for wings of small span (fig. 6). In the case of a lifting doublet in the center of the tunnel the results agree within 2 percent for only 4 slots and even more closely for greater numbers of slots. This agreement would seem to be sufficient proof of the justification for the basic assumption underlying the present theory, that the wall can be considered to be homogeneous. This assumption is not made in reference 13. The theory of this reference indicates that for small numbers of slots, say 2 or 4, the lift interference varies appreciably with the orientation of the slots. This variation, of course, cannot be predicted by the present theory.

The results of the analysis of solid blockage interference are compared with the results of the analysis of reference 9 in figure 7. The results of this reference are seen to straddle the curve obtained in the present analysis. The spread in the results of reference 9 may be due to the fact that the series used in the calculations converged very slowly in the case of ten slots.

The present theory will now be compared with the experimental results of reference 9. For the 10-slot circular tunnel the present theory predicts a small negative interference. The magnitude of this predicted interference is about 1/13 that for a closed tunnel and about 1/3 that for an open tunnel. An inspection of figure 7 (ref. 9) shows that the measured interference in the slotted tunnel checks very closely with these values. This is a single-point comparison at the midpoint of the body, however, so it can not be regarded, in itself, as a complete verification of the theory. In fact, this body was so large with respect to the tunnel diameter that the present theory, based on a doublet in the tunnel, could hardly be expected to apply.

A much smaller body was tested in the 8-slot octagonal tunnel of reference 9. For this tunnel the present theory predicts the value of the axial-interference-velocity function to be -0.006. This is less than 5 percent of the magnitude of the interference in a closed circular tunnel. The predicted interference velocity ratio for the $1\frac{1}{3}$-inch model in the slotted tunnel is $\frac{u_2}{u_0} = -0.0002$ for incompressible flow. Even when multiplied by the compressibility factor $\left(1 - M^2\right)^{-3/2}$, it is obvious that this correction is within the usual experimental accuracy for Mach numbers within the range of application of the linearized theory. For instance, at $M = 0.9$, the axial interference velocity is approximately 1/4 of 1 percent of the free-stream velocity. The experimental results in figure 12 of reference 9 show that the interference was negligible along the whole length of the body at all speeds up to a Mach number of about 0.9. Thus, these experimental results are consistent with the present theory with regard to blockage interference.

Application of the Concept of a Restriction Constant

The concept of a restriction constant can be useful in other ways than in the direct solution of problems in slotted wall interference as, for instance, in providing a common basis of comparison for walls with different slot spacings. As an example of this type of problem, consider a circular tunnel with eight evenly spaced slots around the circumference, with an open ratio of 0.15. Suppose it is desired to construct another circular tunnel with the same interference characteristics as this tunnel but that it is also desired to increase the number of slots to 12. For the given 8-slot tunnel $c = \frac{2}{8} \log_e \mu = 0.365$. For the same value of c, the 12-slot tunnel must have an open ratio of 0.071.

Using this concept, it is also possible to determine the effects of changing the shape of the slot cross section. With the realization that the theory depends upon an essentially potential flow in the slots, for instance, the question arises as to whether the sharp-edged slots could not be improved upon. It might be advisable to try smoother shapes in an attempt to obtain a closer approximation to potential flow in the slots. The restriction constant l could be determined for any slot shape either analytically by calculation of the potential flow through a group of such slots in cascade or experimentally by a simple electrical analogy. Once l has been determined the results of the present analysis could be used to predict the interference for such slots.

Experimental Determination of the Restriction Constant

If experimental data of sufficient accuracy to permit the evaluation of the lift- or blockage-correction factor for a particular slotted tunnel are available, the effective value of $\frac{1}{c + 1}$ can be read directly from the curves presented in this paper. Once the value of this parameter is known, corrections may be computed for other wings in the same tunnel.

Limitations of the Theory

Some question arises as to the range of validity of the equation for the restriction constant of a slotted wall, inasmuch as the open ratio required to obtain a given value of c decreases rapidly as the number of slots increases. Since the normal mass flow remains constant with c, this requires a continuously increasing normal velocity in the slots which, although it is permissible in the assumed incompressible flow, is not possible in the actual physical flow. It appears, from these considerations, that the limit of applicability of the restriction-constant equation will be determined by the required normal flow velocities rather than directly by the smallness of the slots. This limitation should be considered in the application of the results of this analysis to the design of wind-tunnel test sections.

There is, also, some question regarding the use of potential-flow theory in the region near the slots. This question arises because the potential theory omits certain flow phenomena, in particular, the presence of a mixing region at the boundary between the moving tunnel air and the essentially static air outside. The adequacy, for the purpose of interference computations, of the potential theory must be determined by experimental studies.

The restriction constant has been determined by assuming the effective free boundary ($\varphi = 0$) to be located along the line between the slot edges. If the boundary were actually located outside of this line, the slot restriction constant would be somewhat larger. If the slots had a finite depth t, instead of being essentially sharp-edged orifices, and if the effective free boundary were at the outside of the slots, the restriction constant would be approximately $l = \frac{d}{\pi} \log_e \mu + \frac{t}{r_o}$. Actually, it is likely that in the case of slots of considerable depth, the location of the effective free boundary would be a function of the local outflow or inflow. Not only would this make any calculation of the interference questionable but it would also present the distinct possibility that the correction factor might be an unpredictable function of model size and angle of attack.

CONCLUDING REMARKS

The solid-blockage interference for a doublet on the tunnel axis and the boundary interference for lifting wings in circular, rectangular, and two-dimensional slotted tunnels have been calculated by substituting an equivalent homogeneous boundary for the physical boundary of discrete slots. In the case of small wings, the results calculated with the assumption of homogeneity have been found to be consistent with those calculated for the discrete slots for as few as four slots in a circular tunnel. Furthermore, available experimental results for blockage interference are consistent with the results of the present analysis.

Through the concept of a wall restriction constant it is possible to reduce the interference due to all different slotted-wall configurations for the same tunnel shape to a single curve. Thus, the number of computations required to describe completely the interference for a given-shape slotted tunnel with different slot spacings and open ratios is greatly reduced.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., May 12, 1953.

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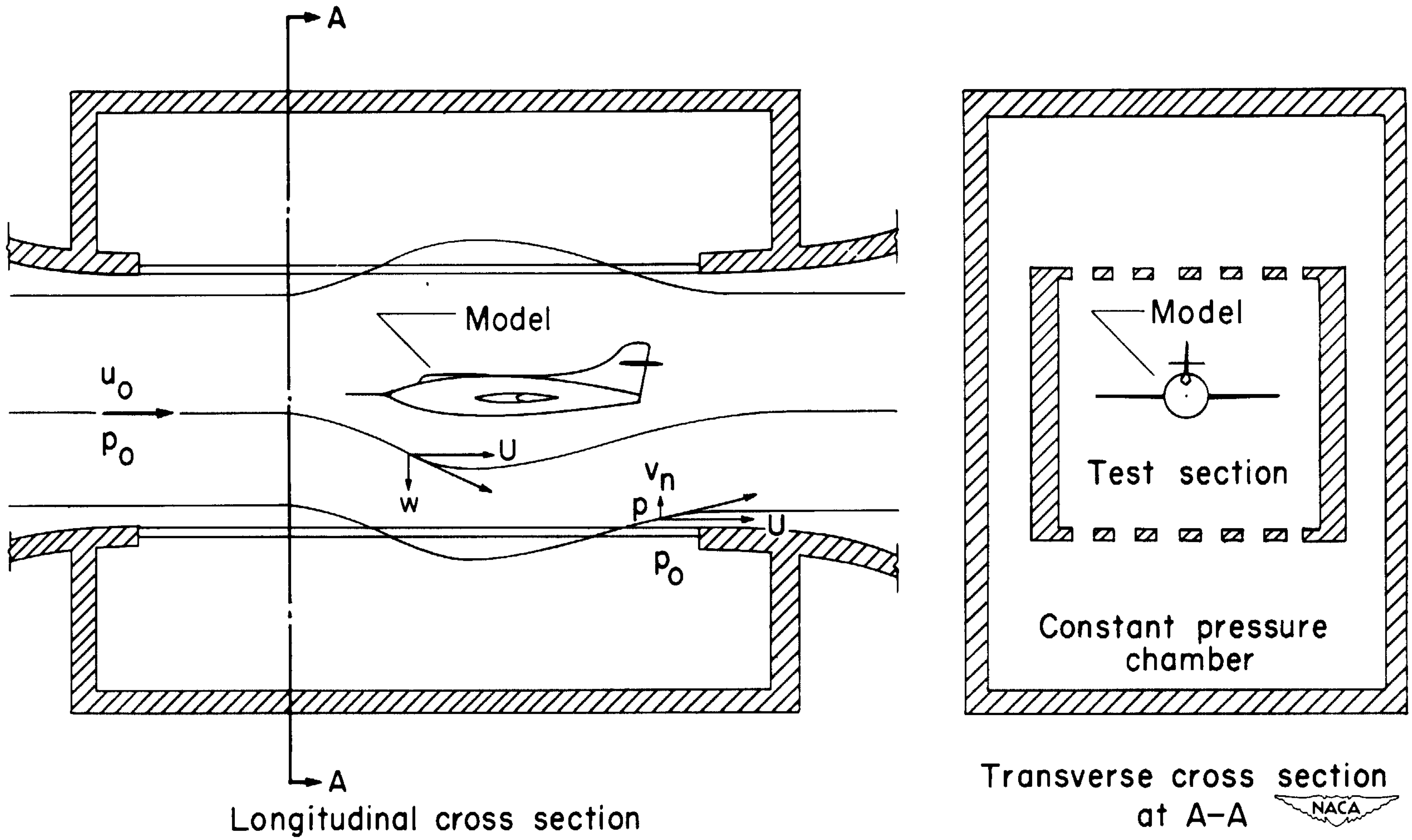


Figure 1.- Schematic longitudinal and transverse cross-sectional views of a slotted tunnel with model installed, showing how air flows out and in through the slots as it passes the model.

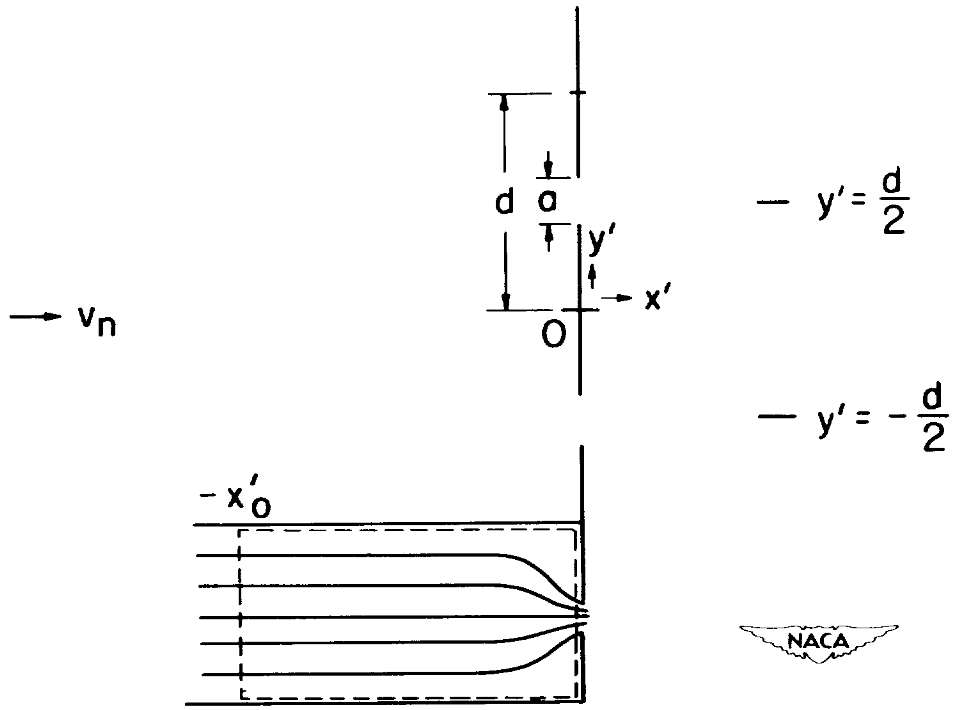


Figure 2.- Sketch showing physical arrangement and nomenclature used in equation for potential difference across a thin, straight slotted wall.

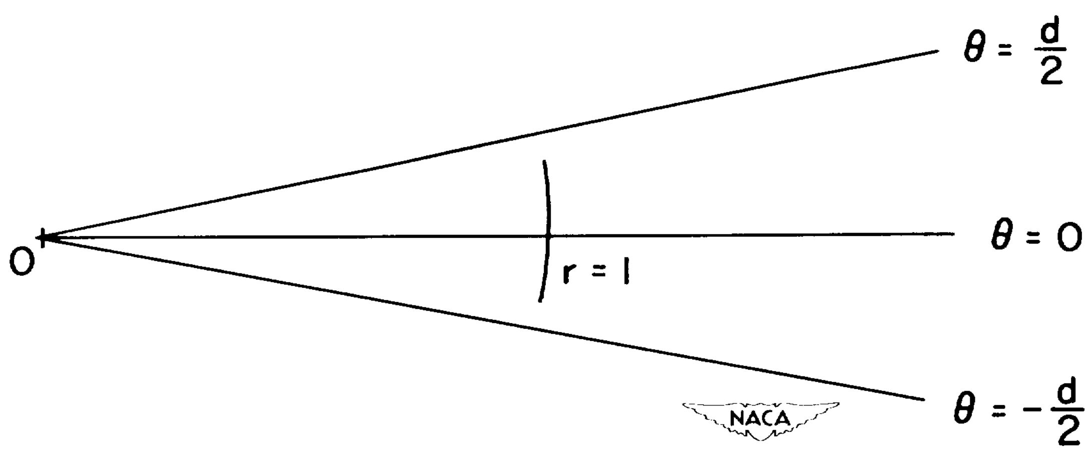


Figure 3.- Sketch of the transformation of the section of the straight slotted wall between $y = \pm\frac{1}{2}d$ into an arc of a circular slotted wall.

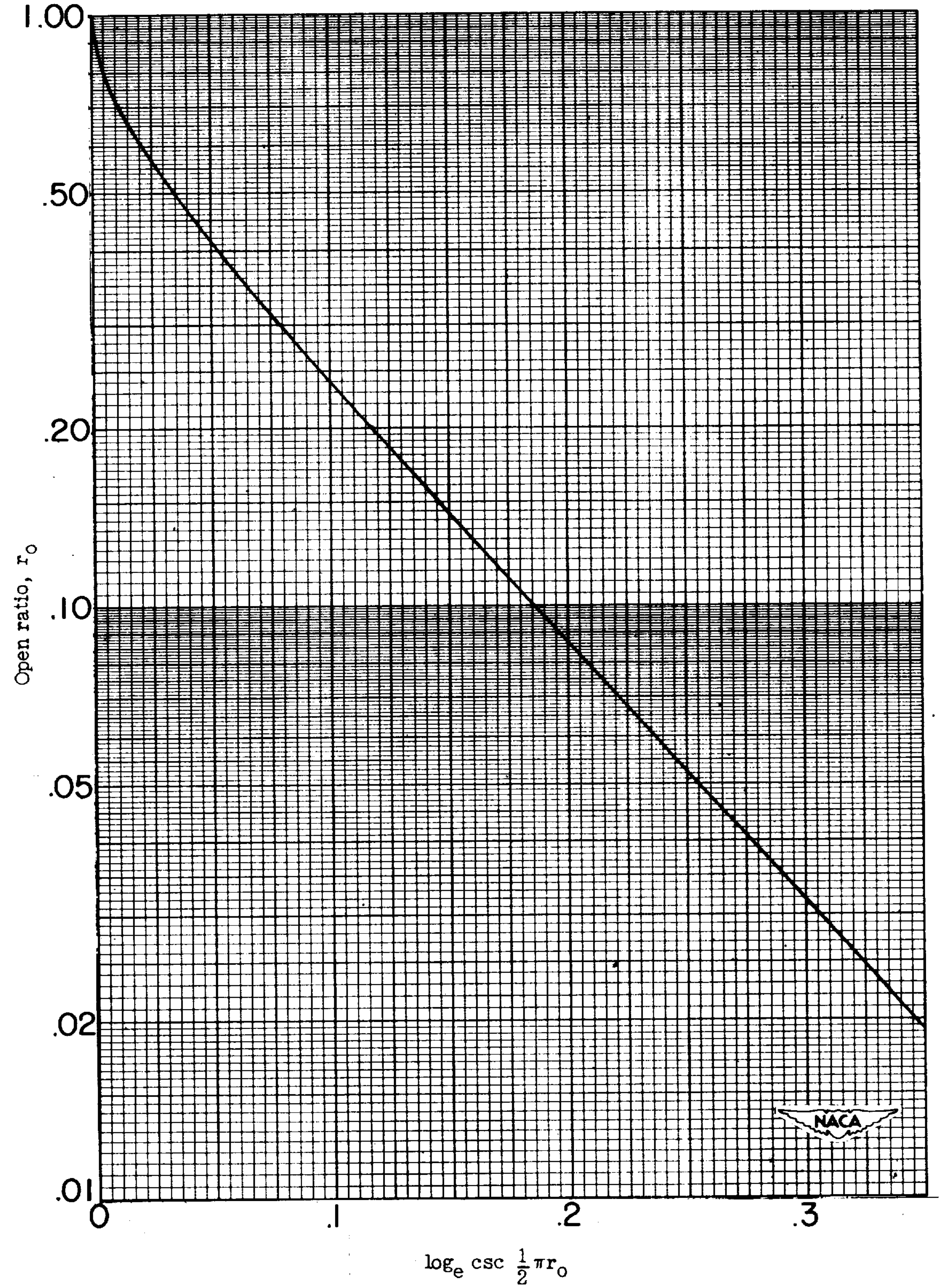


Figure 4.- The function $\log_e \csc \frac{1}{2} \pi r_o$.

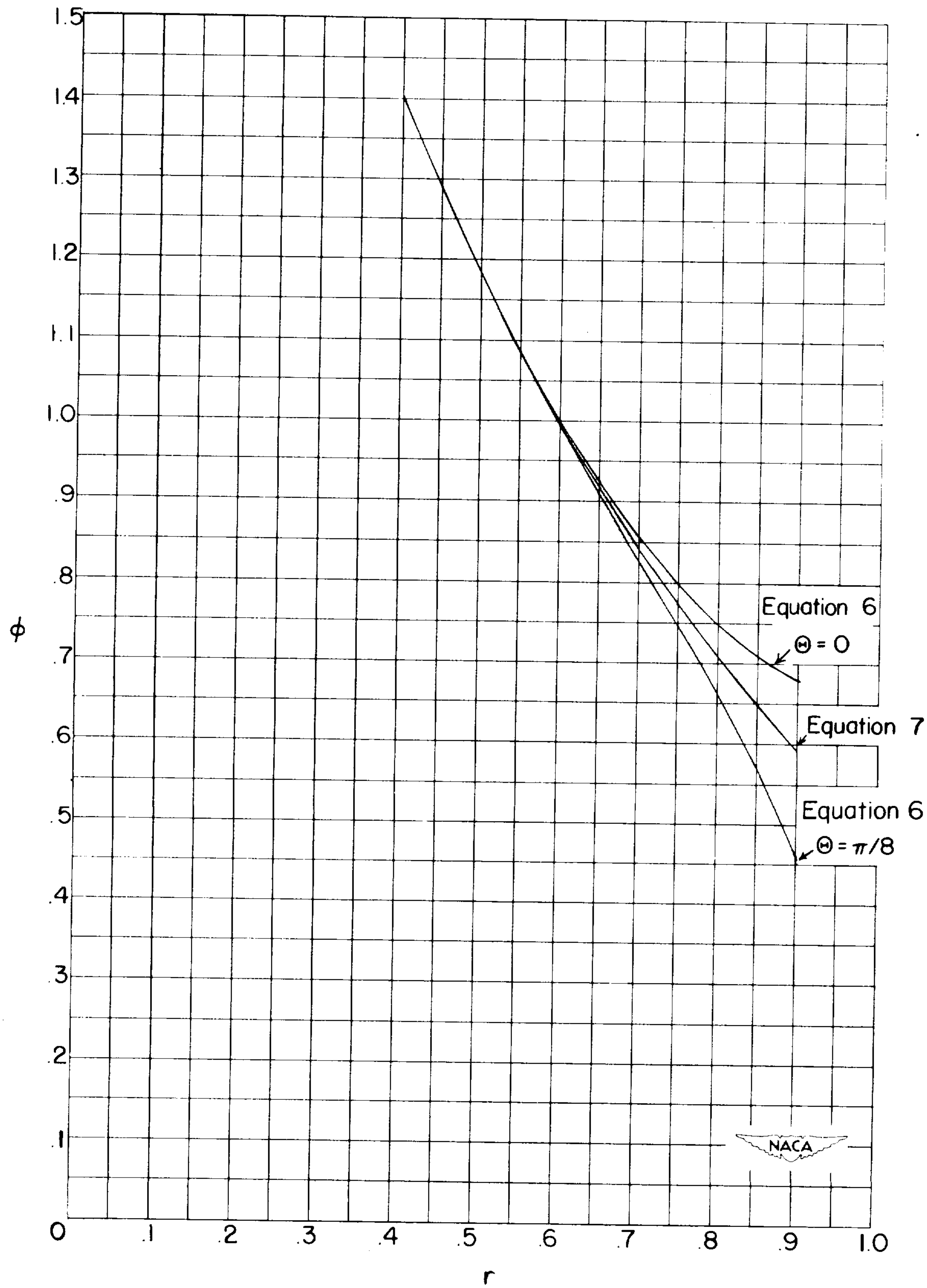


Figure 5.- Comparison of the potential as given by equation (6) and by the approximate equation (7). $d = \pi/4$ (8 slots); $\mu = 7$.

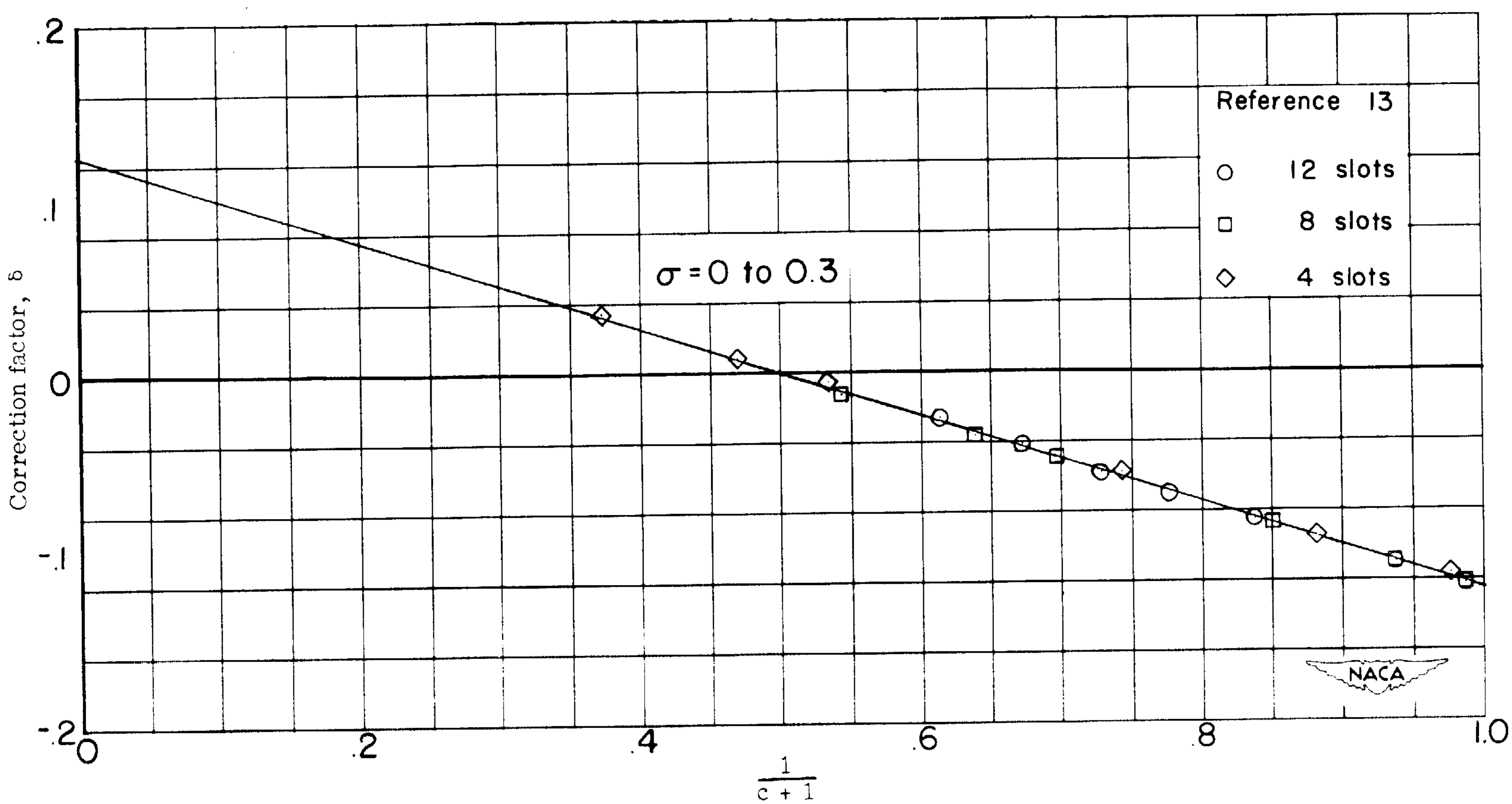


Figure 6.- Boundary-correction factors for lifting wings in circular wind tunnels with slotted walls. For thin slots, $c = \frac{2}{N} \log_e \csc \frac{1}{2} \pi r_o$.

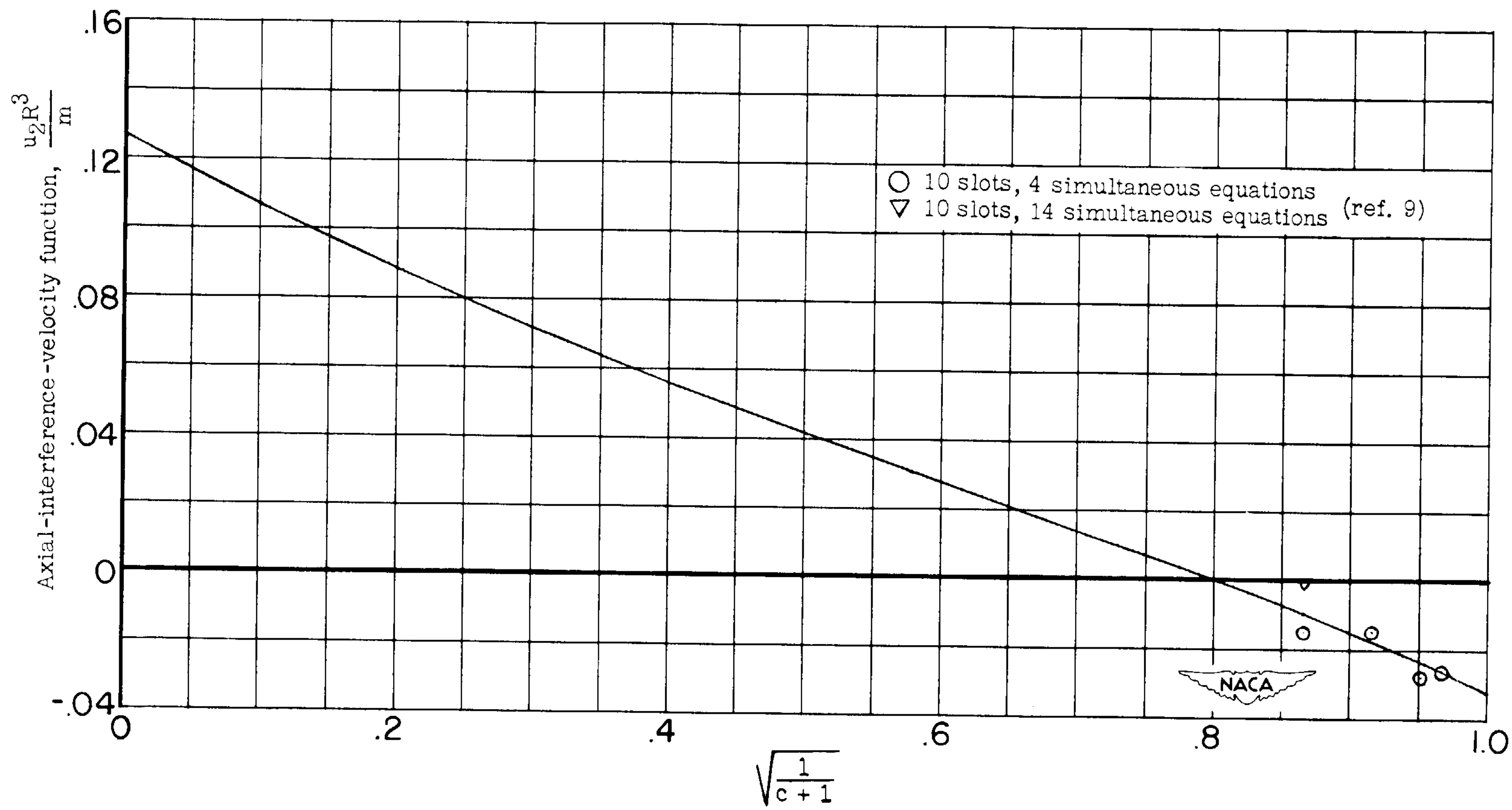
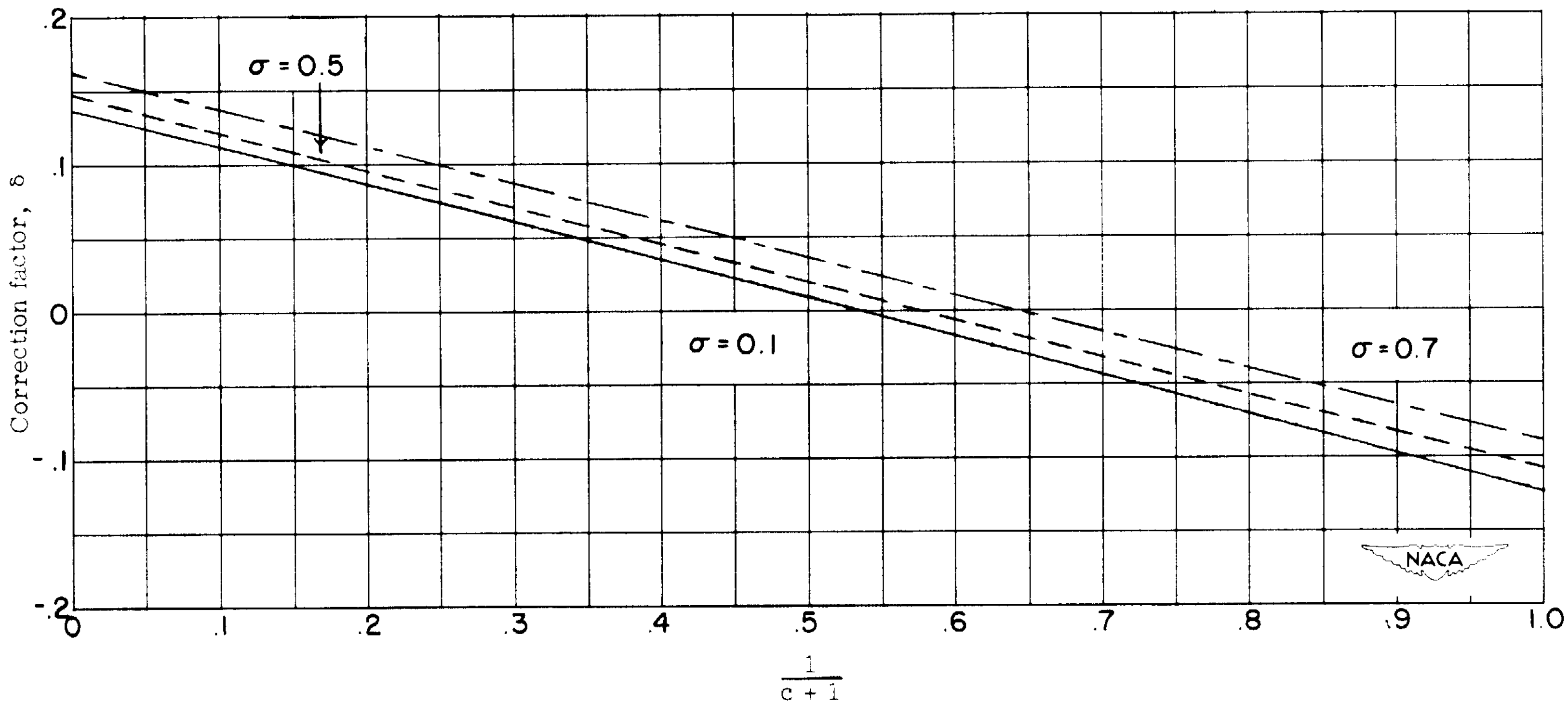
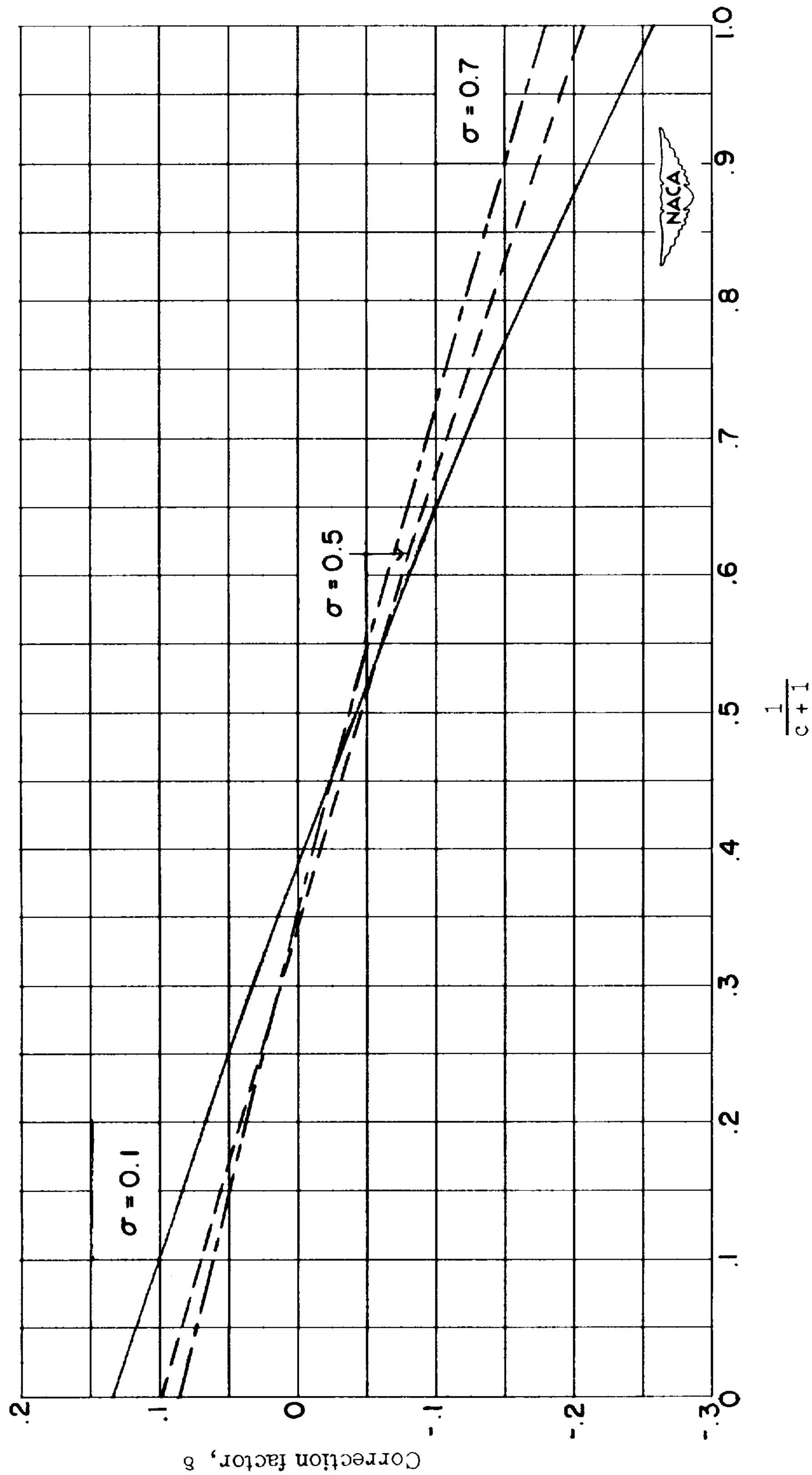


Figure 7.- Solid-blockage interference for circular wind tunnels with slotted walls. For thin slots, $c = \frac{2}{N} \log_e \csc \frac{1}{2} \pi r_o$.



(a) Height-width ratio = 1.0.

Figure 8.- Boundary-correction factors for lifting wings in rectangular wind tunnels with the top and bottom walls slotted. For thin slots, $c = \frac{2}{\pi N\lambda} \log_e \csc \frac{1}{2} \pi r_o$, where N is the number of slots in one horizontal wall, and r_o is the open ratio of the horizontal walls.



(b) Height-width ratio = 0.5.

Figure 8.- Concluded.

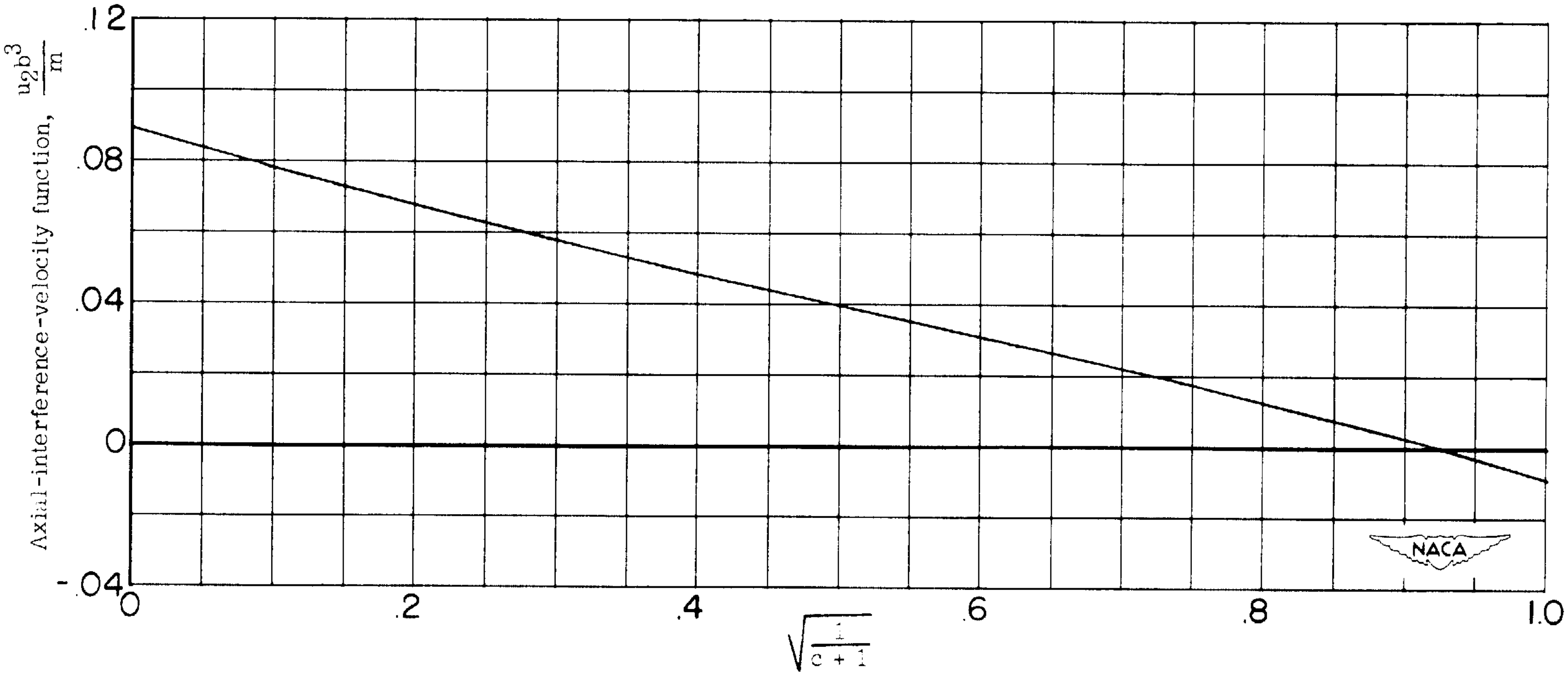


Figure 9.- Solid-blockage interference for rectangular wind tunnels with the top and bottom walls slotted. Height-width ratio = 1.0; for thin slots $c = \frac{2}{\pi N \lambda} \log_e \csc \frac{1}{2} \pi r_o$, where N is the number of slots in one horizontal wall and r_o is the open ratio of the horizontal walls.

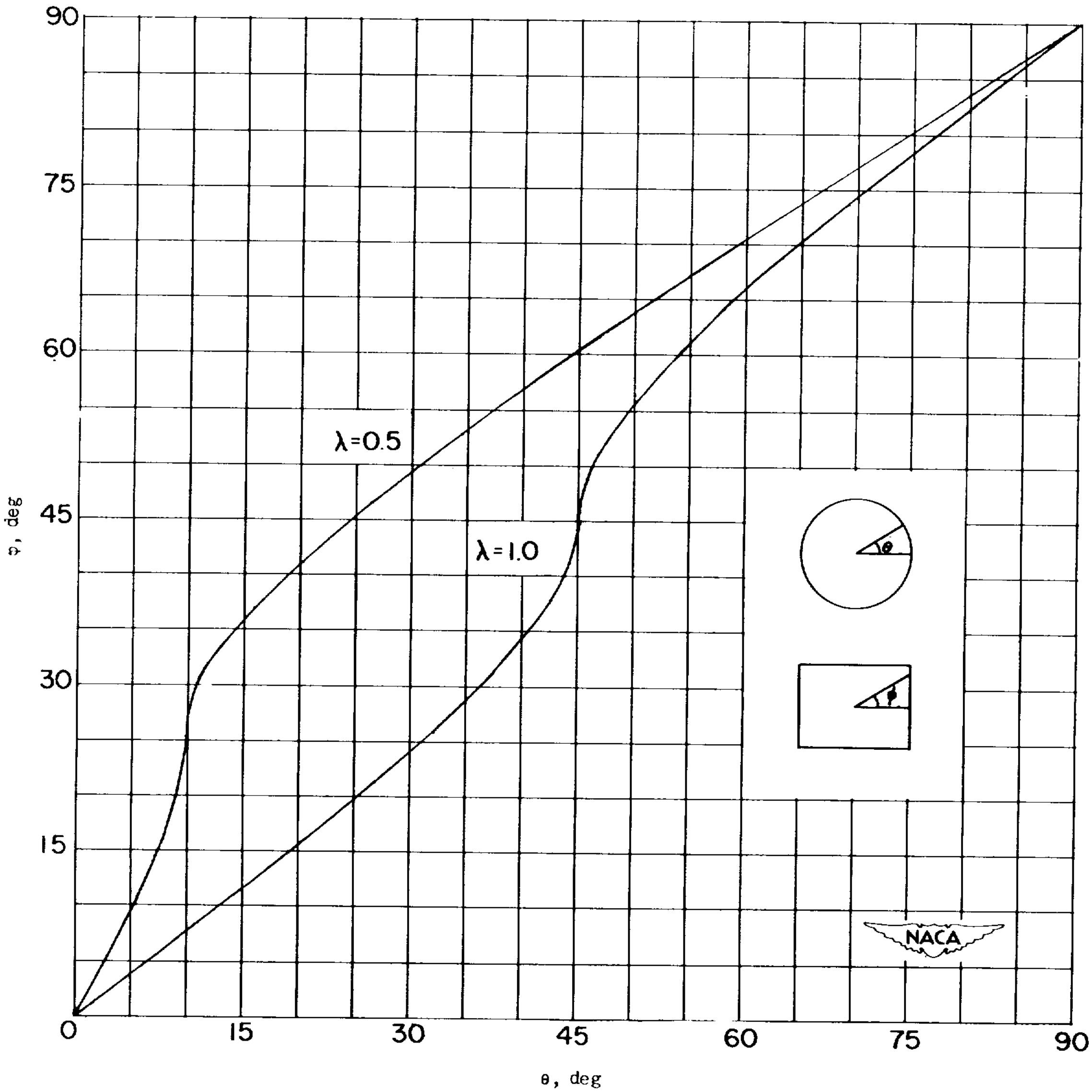


Figure 10.- Transformation of the quadrantal arc of a unit circle into the quadrantal perimeter of a rectangle.

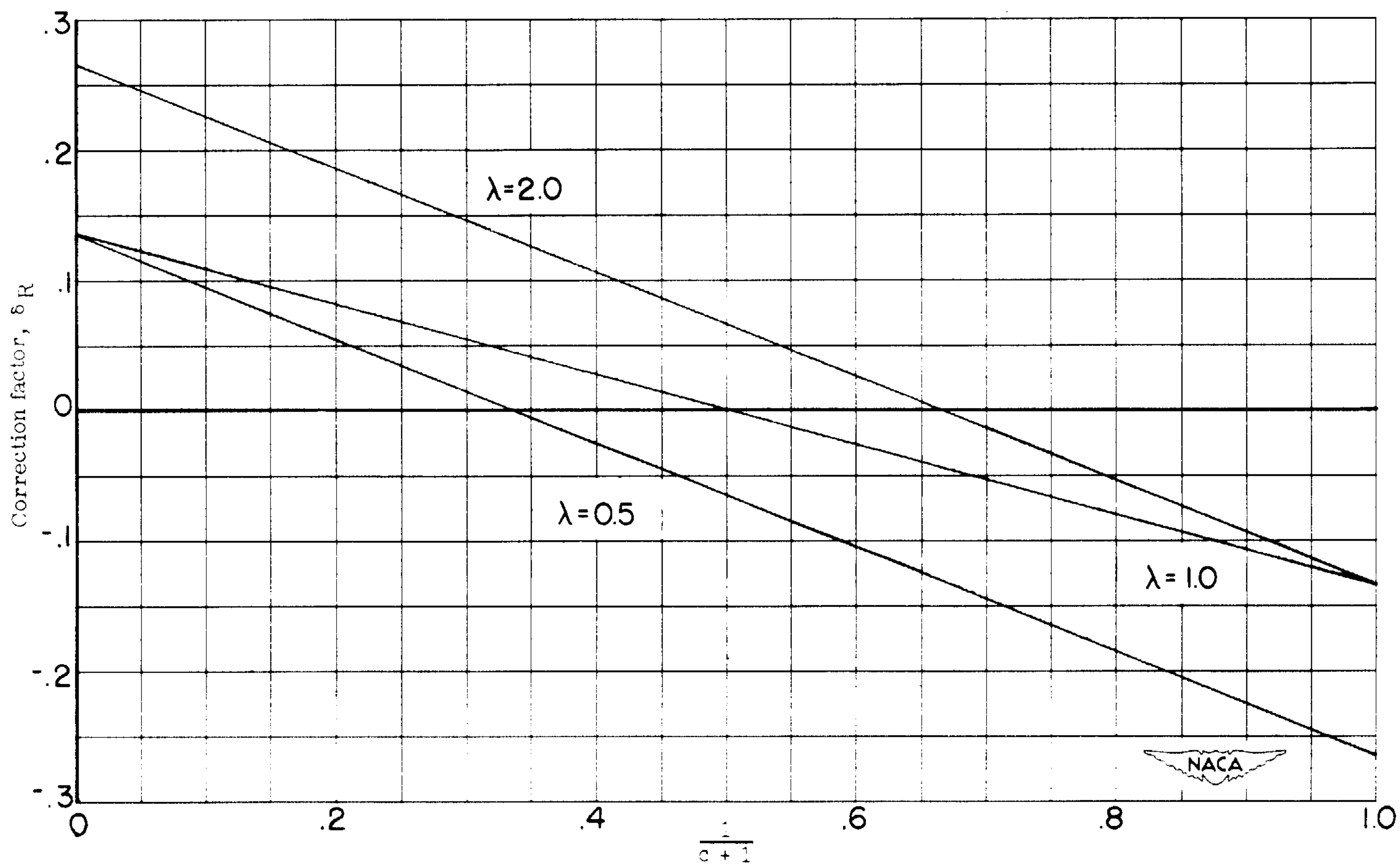


Figure 11.- Boundary-correction factors for small lifting wings in rectangular wind tunnels with all four walls slotted. Slot widths determined by transformation from uniformly slotted circular tunnel. For thin slots, $c = \frac{2}{N} \log_e \csc \frac{1}{2} \pi r_o$, where N is the total number of slots and r_o is the open ratio of the equivalent uniformly slotted circular tunnel.

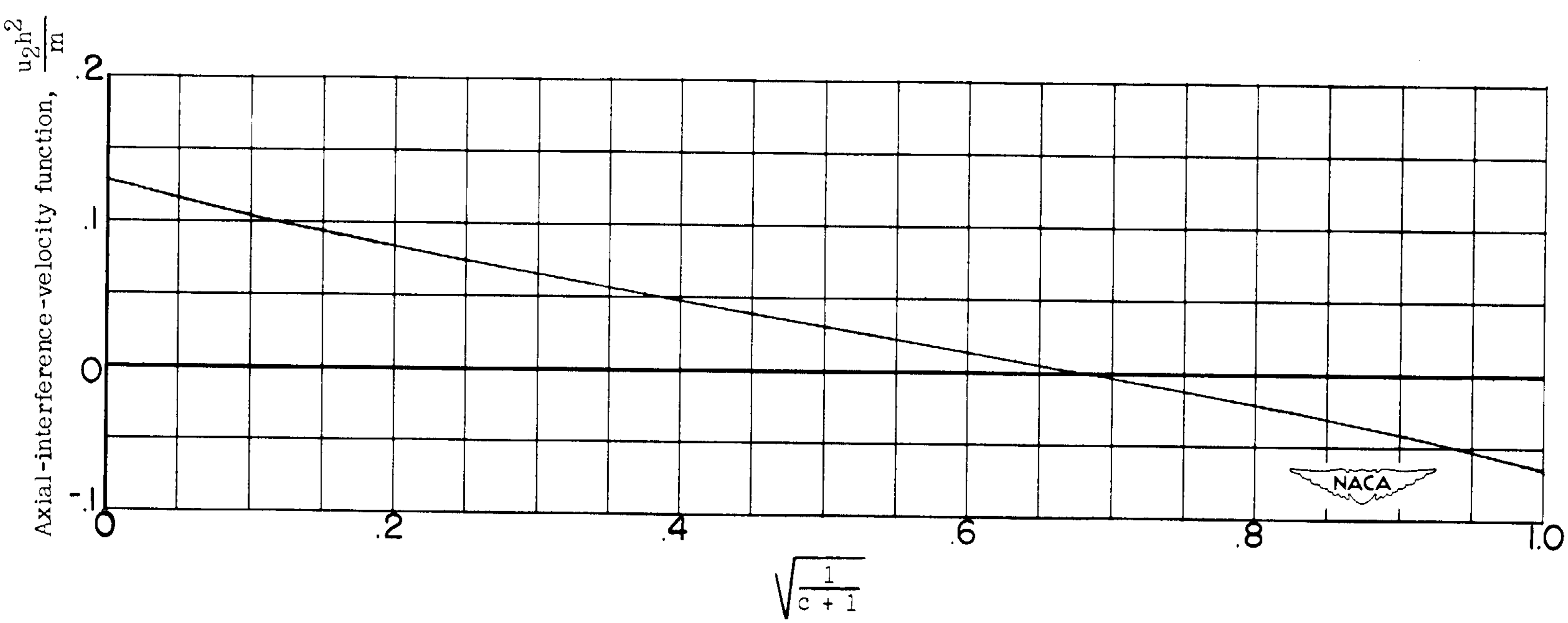


Figure 12.- Solid-blockage interference for two-dimensional slotted wind tunnels. For thin slots, $c = \frac{d}{\pi h} \log_e \csc \frac{1}{2} \pi r_o$, where d is the slot spacing, h is the tunnel semiheight, and r_o is the open ratio of slotted (horizontal) walls.