LOW-SPEED PITCHING DERIVATIVES OF LOW-ASPECT-RATIO WINGS
OF TRIANGULAR AND MODIFIED TRIANGULAR PLAN FORMS

By Alex Goodman and Byron M. Jaquet

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SUMMARY

A low-speed investigation was made in the 6- by 6-foot curved-flow test section of the Langley stability tunnel to determine the effects of change in profile and aspect ratio on the pitching derivatives of triangular wings. The effects of aspect ratio on the pitching derivatives of a series of modified triangular wings, obtained by cutting various portions from the tips of a basic triangle, also were determined.

The results of the investigation indicated that the values of the damping-in-pitch parameter $C_{m_q}$ obtained for the triangular and modified triangular wings were about one-fifth to one-tenth as large as the value that might be expected for a typical airplane, having a conventional wing and horizontal-tail arrangement, but were nearly the same as the values of $C_{m_q}$ for unswept wings. It should be realized, however, that values of the nondimensional parameter $C_{m_q}$ are not necessarily indicative of the actual damping in pitch. Of the three profiles investigated (flat plate, 12-percent-thick biconvex, and NACA 0012), the results obtained for the NACA 0012 section showed the smallest variation of the damping-in-pitch parameter and the lift due to pitching over the greater part of the lift-coefficient range. The flat-plate profile had the largest values of the damping-in-pitch parameter and the lift due to pitching.

Comparison of the experimental values of the damping-in-pitch parameter and the lift due to pitching at zero lift coefficient, obtained for the triangular and modified triangular wings, with untapered swept-wing theory indicated very good agreement when experimental values of the lift-curve slope and the static margin ($X/c$) were used in the theoretical relations. The low-aspect-ratio triangular-wing theory seemed to be applicable only up to aspect ratios of 0.5. For higher aspect ratios, the theoretical values diverged rapidly from the experimental results.
A systematic program has been initiated in the Langley stability tunnel in order to determine, experimentally, the static and rotary stability derivatives of various wings and complete airplane configurations. The rolling-flow and curved-flow equipment (references 1 and 2) is being used to determine the rotary derivatives.

As part of this systematic program, a series of triangular-wing models is being investigated. The static and rolling characteristics of several triangular and modified triangular wings are reported in reference 3.

The present investigation was made in order to determine the pitching derivatives of the triangular wings and modified triangular wings of reference 3.

This investigation deals with the effects of profile for one triangular plan form, the effect of aspect ratio of triangular wings for one profile, and the effect of variation of aspect ratio of a modified triangular plan form, which is obtained by cutting portions from the tips of a basic triangular wing.

The experimental values of the pitching derivatives for these wings are compared with available theory.

**SYMBOLS**

The data presented herein are in the form of standard NACA symbols and coefficients of forces and moments which are referred to the stability system of axes with the origin at the quarter-chord point of the mean aerodynamic chord. Positive forces, moments, and angular displacements are shown in figure 1. The coefficients and symbols used herein are defined as follows:

- \( C_L \): lift coefficient
  \[
  C_L = \frac{Lift}{\frac{1}{2} \rho v^2 S}
  \]

- \( C_D \): drag coefficient
  \[
  C_D = \frac{Drag}{\frac{1}{2} \rho v^2 S}
  \]
\( C_m \) pitching-moment coefficient \( \left( \frac{\text{Pitching moment}}{1/2 \rho V^2 S c^2} \right) \)

\( C_{L\alpha} = \frac{\partial C_L}{\partial \alpha} \)

\( C_{Lq} = \frac{\partial C_L}{\partial q \bar{c}} \frac{q \bar{c}}{2V} \)

\( C_{Dq} = \frac{\partial C_D}{\partial q \bar{c}} \frac{q \bar{c}}{2V} \)

\( C_{m_q} = \frac{\partial C_m}{\partial q \bar{c}} \frac{q \bar{c}}{2V} \)

\( A \quad \text{aspect ratio} \left( \frac{b^2}{S} \right) \)

\( b \quad \text{wing span} \)

\( S \quad \text{wing area} \)

\( c \quad \text{local chord parallel to plane of symmetry} \)

\( \bar{c} \quad \text{mean aerodynamic chord} \left( \frac{2}{S} \int_0^{b/2} c^2dy \right) \)

\( c_r \quad \text{root chord} \)

\( \lambda \quad \text{taper ratio} \left( \frac{\text{Tip chord}}{\text{Root chord}} \right) \)

\( x \quad \text{longitudinal distance rearward from apex of triangle to quarter-chord point of any chordwise station} \)

\( x' \quad \text{longitudinal distance rearward from apex of triangle to quarter-chord point of mean aerodynamic chord} \left( \frac{2}{S} \int_0^{b/2} cx \, dy \right) \)
The present investigation was conducted in the 6-by-6-foot curved-flow test section of the Langley stability tunnel in which pitching flow is simulated by mounting the model rigidly on a support strut and curving the air stream. A discussion of this procedure is given in reference 2.

All tests were made with the models mounted on a six-component balance system at the quarter-chord point of the mean aerodynamic chord. Model dimensions and the test conditions are presented in table I. The models tested herein are those used for the tests given in reference 3, with the exception of models 5 and 6, data for which are not presented herein.

The modified triangular wings (models 8, 9, and 10) were formed by cutting portions from the tips of a basic triangular wing (model 7) and adding tips of revolution. Photographs of some of the models are presented as figure 2.

Each of the models listed in table I (with the exception of models 4 and 10) was tested through an angle-of-attack range from $\alpha = -2^\circ$ through the stall at the values of $q\bar{C}/2V$ given in table I.
All tests were made at a dynamic pressure of 24.9 pounds per square foot. The Reynolds number of each test, based on the mean aerodynamic chord of the model, is given in table I. The test Mach number was 0.13.

CORRECTIONS

The angle of attack and the drag coefficient were corrected for the effects of the jet boundaries by methods derived for unswept wings. (See reference 2.) The lift coefficient was corrected for the cross-tunnel pressure gradient which is associated with pitching flow.

Corrections were not applied to the data to account for blocking or support strut tares.

RESULTS AND DISCUSSION

Presentation of Results

The variation of lift coefficient with angle of attack for the models reported herein are presented in figure 3. The complete static and rolling characteristics of the models are given in reference 3. The pitching derivatives of the models investigated are presented as follows:

Effect of profile of triangular wings
Effect of aspect ratio of triangular wings
Effect of aspect ratio of modified triangular wings

In figure 7 a comparison of the experimental values of the damping in pitch and the lift due to pitching at zero lift coefficient with the values given by the low-aspect-ratio triangular-wing theory of reference 4 and the untapered-swept-wing theory of reference 5 is presented.

Lift Characteristics

A comparison of the variation of lift coefficient with angle of attack as presented in figure 3 with the results given in reference 3 indicates that at low and moderate angles of attack the slopes of the lift curves, presented in the present paper, are very nearly the same.
as the slopes presented for the same wings in reference 3. However, the values of maximum lift coefficients obtained in the present tests are as much as 11 percent lower than those of reference 3. The differences possibly are caused by the differences in support strut tares and also the fact that canopies were not used for the models of the present tests. (See reference 3.)

Pitching Derivatives

Effects of profile.—A comparison of the values of the damping-in-pitch parameter $C_{mq}$ obtained for the low-aspect-ratio triangular and modified triangular wings (figs. 4, 5, and 6) with values reported for a conventional airplane with horizontal tail (reference 6) indicates that the values obtained for the triangular wings are relatively small (about the same as for an unswept wing). The values of $C_{mq}$ for the conventional airplane of reference 6 are approximately 5 to 10 times greater than the values obtained for the present wings. It should be realized, however, that according to usual practice, both the coefficient $C_m$ and the nondimensional angular-velocity parameter $q\sqrt{V}$ are in terms of the wing mean aerodynamic chord. When comparing wings having the same area, but different aspect ratios, therefore, the derivative $C_{mq}$ is not necessarily indicative of the actual damping in pitch.

In general, the pitching derivatives for the triangular wings show rather small variations over the greater part of the lift-coefficient range. The results presented in figure 4 for a triangular wing of $A = 2.31$ and $\Lambda_c/\alpha = 52.2^\circ$ indicate that the effects of profile are quite small at low and moderate lift coefficients but become more important at the higher lift coefficients. Of the three profiles investigated, the results obtained for the wing with NACA 0012 section showed the smallest variation over the greater part of the lift-coefficient range and the flat-plate profile had the largest values of the derivatives.

A comparison between available theory (references 4 and 5) and the experimental results for $C_{Lq}$ and $C_{mq}$ at $C_L = 0$ is given in table II. The theories considered will be explained in some detail in the following section; however, only the theory of reference 5 is in a form suitable for predicting effects of changes in profile through use of experimental values of the lift-curve slope and the static margin ($\overline{X}/\overline{S}$). Using experimental values in the theory of reference 5 results in a good prediction of the trend of $C_{Lq}$ and $C_{mq}$ as affected by profile. (See table II.)
Effects of aspect ratio.—In figure 7 the experimental variations with aspect ratio of $C_L$ and $C_m$ obtained for the triangular and modified triangular wings at $C_L = 0$ are compared with values obtained from the methods based on the theory of low-aspect-ratio triangular wings (reference 4) and the untapered swept-wing theory of reference 5.

According to the triangular-wing theory of reference 4, the variation of $C_L$ and $C_m$ with aspect ratio can be expressed as

$$C_L = \frac{\pi A}{2} + \pi A \frac{\bar{x}}{c}$$  \(1\)

$$C_m = -\frac{3}{16} \pi A - \frac{\pi A}{2} \frac{\bar{x}}{c} - \pi A \left(\frac{\bar{x}}{c}\right)^2$$  \(2\)

where the aerodynamic-center location is considered to be at the $\frac{2}{3}$-root-chord point. The present triangular wings were mounted at the $\frac{1}{2}$-root-chord point (quarter-chord point of mean aerodynamic chord) and, therefore, the value of $\bar{x}/c$ in equations (1) and (2) is $1/4$. Applicability of equations (1) and (2) decreases with increasing aspect ratios, and an aspect ratio of 0.5 was estimated as the upper limit of utility in reference 4. The comparison made in figure 7(a) of the experimental values of $C_L$ and $C_m$ with values given by equations (1) and (2) verifies the aforementioned statement. It can be seen that if the experimental data are extrapolated to lower aspect ratios, reasonably good agreement with theory might be expected at an aspect ratio of about 0.5.

The untapered swept-wing theory of reference 5 presents the variation of $C_L$ and $C_m$ as

$$C_L = \left(\frac{1}{2} + 2 \frac{\bar{x}}{c}\right) C_{L\alpha}$$  \(3\)

$$C_m = -C_{L\alpha} \left[2 \left(\frac{\bar{x}}{c}\right)^2 + \frac{1}{2} \frac{\bar{x}}{c}\right] - \frac{1}{8} a_0 \cos \Lambda_c/4 - \frac{1}{24} \frac{A^3 a_0 \cos \Lambda_c/4}{A + 6 \cos \Lambda_c/4} \tan^2 \Lambda_c/4$$  \(4\)
where the aerodynamic center is assumed to be at the quarter-chord point of the mean aerodynamic chord.

Charts, based on equations (3) and (4), with theoretical values of \( \frac{X}{C} \) and \( C_{Lq} \) are presented in reference 5. The use of the chart values of \( C_{mq} \) and \( C_{Lq} \) yields results which are somewhat smaller than the experimental values (fig. 7) but indicates the trend for both the triangular and modified triangular wings. When experimental values of \( C_{Lq} \) and \( \frac{X}{C} \), as given in reference 3, are inserted in equations (3) and (4), very good agreement with experiment is obtained for the triangular wings, and fair agreement is obtained for the modified triangular wings. (See fig. 7.)

CONCLUSIONS

An investigation conducted in the 6- by 6-foot curved-flow test section of the Langley stability tunnel in order to determine the effects of a number of geometric variables on the low-speed pitching derivatives of triangular and modified triangular wings (obtained by cutting portions from the tips of a basic triangle) indicates the following conclusions:

1. For the triangular and modified triangular wings investigated, values of the damping-in-pitch parameter \( C_{mq} \) were about one-fifth to one-tenth as large as the value that might be expected for a typical airplane, having a conventional wing and horizontal-tail arrangement, but were nearly the same as the values of \( C_{mq} \) for unswept wings. It should be realized, however, that values of the nondimensional parameter \( C_{mq} \) are not necessarily indicative of the actual damping in pitch.

2. For triangular wings of the same plan form, both the damping-in-pitch parameter and the lift due to pitching showed the smallest variation over the greater part of the lift-coefficient range when the NACA 0012 airfoil was used than when either a flat plate or a 12-percent-thick biconvex airfoil was used. The flat-plate profile had the largest values of the damping-in-pitch parameter and the lift due to pitching.

3. Comparison of the experimental values of the damping-in-pitch parameter and the lift due to pitching at zero lift coefficient, obtained for the triangular and modified triangular wings, with untapered swept-wing theory indicated very good agreement when experimental values of the lift-curve slope and the static margin (\( \frac{X}{C} \)) were used in the
theoretical relations. The low-aspect-ratio triangular-wing theory seemed to be applicable only up to aspect ratios of 0.5. For higher aspect ratios, the theoretical values diverged rapidly from the experimental results.

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REFERENCES


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**Table I. Pertinent Model Dimensions and Test Conditions**
TABLE II. COMPARISON OF EXPERIMENTAL AND THEORETICAL VALUES
OF $C_{lq}$ AND $C_{mq}$ AT $C_L = 0$ FOR THREE PROFILES

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a) Low-aspect-ratio triangular-wing theory.
b) Untapered swept-wing theory; based on theoretical $C_{lq}$ and $X/\alpha$.
c) Untapered swept-wing theory; based on an experimental $C_{lq}$ and $X/\alpha$. 
Figure 1.- System of stability axes. Positive forces, moments, angles, and velocities are indicated.
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Figure 2. - Model mounted in 6-foot curved-flow test section of Langley stability tunnel.
Figure 3.- Effect of profile and aspect ratio of triangular wings and aspect ratio of modified triangular wings on variation of $C_L$ with $\alpha$. 
Figure 4. - Effect of profile of a triangular wing of aspect ratio 2.31 on the variation of $C_{mq}$, $C_{Dq}$, and $C_{Lq}$ with $C_L$. $\Lambda_{c/4} = 52.2^\circ$. 
Figure 5.- Effect of aspect ratio of a triangular wing of NACA 0012 profile on the variation of $C_{mq}$, $C_{Dq}$, and $C_{Lq}$ with $C_L$. 
Figure 6.- Effect of aspect ratio of a triangular wing of NACA 0012 profile on the variation of $C_{m_{q}}$, $C_{D_{q}}$, and $C_{L_{q}}$ with $C_{L}$. $A_{c}/4 = 36.9^\circ$. 
Figure 7.- Comparison of experimental and theoretical values of $C_{m_q}$ and $C_{L_q}$. NACA 0012 profiles parallel to plane of symmetry.

$C_L = 0$. 

(a) Triangular wings. 

(b) Modified triangular wings.