RELATIONS BETWEEN THE MODULUS OF ELASTICITY
OF BINARY ALLOYS AND THEIR STRUCTURE

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OF BINARY ALLOYS AND THEIR STRUCTURE*

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(Elastic modulus as a function of concentration in binary systems -
Continuous and limited solid solution series - Eutectic
series - Alloys with intermetallic phases)

INTRODUCTION

The present report represents a widening of our knowledge concern-
ing the elastic modulus of binary alloys as a function of the concen-
tration. The extent of measurements published so far is quite small as
can best be seen from the fact that they have been to the greatest part
incorporated in this report. Investigation had been limited to the
alloys of the greatest technical importance, thus predominantly to ductile
alloys. Only a very few data refer to brittle alloys. In contrast, this
investigation took the intermetallic compounds into special consideration.
This paper is a continuation of previous publications reporting on the
elastic modulus of ordered intermediary phases of various systems.2 to 6

The elastic modulus there was determined from bending vibration
tests with the test setup indicated by F. Förster7. Ductile alloys were
examined mostly in the form of drawn and annealed rods, all others in
the form of cast square bars which had been heat-treated suitably for
the adjustment of equilibrium. The rods had a length of 150 to 200
millimeters and a diameter of 6 to 10 millimeters.

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1*Köster, W: "Beziehungen zwischen dem Elastizitätsmodul von Zweistoffle-
gierungen und ihrem Aufbau." Zeitschrift für Metallkunde, Bd. 39, 1948,
Köster, W., and Schneider, A: Zeitschr. f. Metallkunde, Bd. 32,
1940, p. 156.
Köster, W. and Rosenthal, K: Zeitschr. f. Metallkunde, Bd. 32,
1940, p. 163.
Since the brittle intermetallic compound types are inclined to crack formation, one cannot forthwith rely on the measured values of the elastic modulus being correct. Thus, as a rule, alloys in the two-phase regions adjoining on both sides have been investigated at the same time in order to check the value measured on the homogeneous phase from the variation of the modulus in these fields. If one failed to produce a flawless homogeneous specimen of a crystal type, one had to extrapolate its modulus from the dependence on concentration in the two adjacent fields. The test results show this method to be sufficiently reliable.

The control just mentioned is desirable also in order to eliminate the influence of contingent severe anisotropy which may exist, particularly for cast bars. The possibility of this influence being effective is considerably smaller in a heterogeneous alloy, in which the crystal type occurs, for instance, only in an eutectoid or as a primarily hardened crystal type, than in the case of a homogeneously hardening crystal type. Even though one may be inclined to think, due to the experience with the test findings, that all specimens investigated must be regarded as quasi-isotropic, one must not forget that no check has been made in this direction.

CONTINUOUS SOLID SOLUTION SERIES

Ag-Au. - The values of the elastic modulus of silver-gold alloys are plotted for various measuring temperatures in figure 2. Values calculated by A. Röhl on the basis of single crystal measurements fit well into the trend. For room temperature, they lie on a curve slightly bent upward. With rising temperature, the curvature decreases until its sense finally changes to the opposite. This fact is based on the different temperature variation of the elastic modulus in case of pure metals and of alloys. This temperature variation is represented for the four alloys investigated in figure 1. In a temperature range between 400° and 600°, the curves of the alloys show considerably more curvature than those of the two pure metals. Thereby, the temperature coefficient of the modulus of the alloys becomes noticeably larger than that of the metals at higher temperatures whereas it drops at room temperature slowly from the value of silver to that of gold. The small figure at the left of figure 1 indicates the dependence on the concentration of the mean temperature coefficient for three temperature ranges. The data regarding the dependence on the concentration of the elastic modulus in solid solution series are therefore not always unequivocal; they may depend on the temperature considered.

Au-Cu. - Three values for the disordered state of multicrystalline alloys measured by A. Rohl, at room temperature lie approximately on the connecting line between gold and copper (fig. 3). The value measured by W. Koster on the thoroughly investigated test bar of the alloy AuCu is, for reasons unknown, probably somewhat too low. On a second test bar, the value 10,200 kilograms per millimeter was found. In figure 3 and in all other figures, the elastic modulus for copper has been established as 13,300 kilograms per millimeter. This value is regarded as the one valid for purest copper as expounded in more detail below.

Ag-Pd. - Starting out from silver, the measurements of the authors show good agreement with those of A. Rohl up to 75 At.-percent Pd (fig. 4). The addition of palladium increases the elastic modulus of the silver considerably more than the straight connecting line silver palladium indicates. The maximum value of the modulus lies at 50 At.-percent Pd. For higher palladium contents, the test values of A. Rohl are so peculiar that defects in the specimens seem to be the only explanation.

Au-Pd. - Figure 4 also contains the test result obtained on the gold-palladium system. What is valid for silver palladium applies here, too. The greatest deviation from the rule of mixtures observed so far occurs in these two systems.

Cu-Pd. - The values of the elastic modulus lie on a curve of a slightly convex curvature with respect to the axis of concentration. This is true for room temperature as well as for higher temperatures as can be seen from figure 5. In the first case, the modulus of copper is somewhat larger; in the second case, noticeably smaller than that of palladium.

Cu-Pt. - At room temperature, the values of three alloys are arranged (according to A. Rohl (footnote 10), W. Köster (footnote 2), and new tests) about the straight connecting line between copper and platinum. At 850°, however, the values lie on a curve with a curvature tending toward higher values.

Newly investigated were the alloys with 74 and 82 percent by weight or 48 and 60 At.-percent, respectively. Both alloys have in the ordered state a face-centered cubic lattice of slightly trigonal distortion, according to the investigations of J. O. Linde. The trigonal distortion is considerably smaller for the second than for the first alloy. Figure 6 shows the dependence on the temperature of the elastic modulus of both

Although the transition to the ordered state involves a change of lattice for these alloys, they do not noticeably differ from alloys such as AuCu₃ or PtCu₃ (which make the transition to the ordered state without a change of lattice (see footnote 2)) as far as the variation of the modulus is concerned. The reason is that the elastic modulus increases more and more rapidly below the transition temperature. The position of the breaks on the curves at 810° and 820° shows good agreement with the results of previous tests; also there is nothing extraordinary in the behavior of quenched alloys. When they are heated, the ordered state sets in; the elastic modulus increases. The slower the temperature rise, the sooner the heating-up curve approaches the cooling-off curve. For the alloy with 74 percent by weight Pt, the temperature was kept constant at 410° for 3 hours. Hence, the steep rise of the corresponding curve at this point. Figure 8 supplements the report mentioned before, footnote 2.

Pt-Ir, Pt-Rh, Pd-Ir. - The three systems, platinum with iridium, platinum with rhodium, and palladium with iridium were investigated only for slight iridium and rhodium content. The measurements on the two first-named systems were made by E. Guillet.¹² According to figure 7, the values in the first system lie somewhat below the connecting straight line, whereas they are arranged approximately on it for the two others.

Cu-Ni. - Figure 9 shows, according to data of Z. Nishiyama¹³ and E. Grüneisen¹⁴ and to additional measurements by the author, the elastic modulus of the copper-nickel alloys as a function of the concentration. The measured values lie approximately on the connecting straight line between copper and nickel, if for nickel the value in the magnetically saturated state is chosen. Likewise, the values of the magnetically saturated alloys in the ferromagnetic field lie on this straight line, whereas the values for the demagnetized state lie to a rising degree below this straight line due to the additional magnetostrictive extension occurring with increasing nickel content.

Co-Pt. - The elastic modulus of the alloy CoPt here in the face-centered cubic γ state lies far below the connecting straight line between cobalt and platinum (fig. 10). At 800° and 900° also the modulus is still smaller than corresponds to the rule of mixtures but the deviation is considerably less than at room temperature.¹⁵

Ni-Mn. - Nickel and manganese form below the solidus curve a continuous series of solid solutions. In the solid state, the ordered phases Ni₃Mn and NiMn are formed. A new contribution to the development of the system by W. Köster and W. Rauscher⁶ will be published in this journal in the near future. In that report, the variation of the concentration with its peculiarities will be described in detail.

Mo-W. - The elastic modulus of the molybdenum-tungsten mixed crystals lies, according to figure 11, on the connecting straight lines of the values for the two pure metals.

LIMITED SOLID SOLUTION SERIES

Alloying elements of limited solubility in the base metal generally reduce the elastic modulus of the latter. According to the present state of knowledge, there exist only a few noteworthy exceptions. Figures 12 to 16 represent the variation of the elastic modulus in the limited mixed crystal series investigated of silver, copper, gold, magnesium, and aluminum.

In the case of silver, antimony exerts the strongest influence on the modulus with arsenic, tin, zinc, and aluminum following; still smaller is the influence of magnesium, manganese, and copper. For 10 At-percent of the admixture, the reduction amounts to 1 to 10 percent.

In the case of copper, a peculiar phenomenon is observed. The elastic modulus rises before it decreases. This observation has already been made by Z. Nishiyama¹³ who examined the effect of aluminum, zinc, manganese, and tin as admixture; his results are in excellent agreement with those presented in this report. The maximum value is attained mostly with a very low content of admixed metal, sometimes (for instance, for aluminum or zinc) with a slightly higher content. An examining glance at figure 13 shows that all the downward branches of the curves tend toward one point of the ordinate axis, roughly estimated as 13,300 kilograms per millimeter². This value is regarded as the elastic modulus of purest copper. The lower value of about 12,500 kilograms per millimeter² usually measured is caused by impurities, most probably by the content of oxygen in commercially pure copper. Its influence is compensated by the deoxidizing effect of the other additions. It seems that the oxygen absorbed by copper in solid solution has a particularly reducing effect on the modulus which is lowered further by the oxygen admixed.

heterogeneously as cuprous oxide. One report\textsuperscript{17} points out a remarkable peculiarity in the temperature variation of the elastic modulus of copper which is eliminated by the additions of zinc, tin, magnesium, beryllium, and the like. This phenomenon and the obviously reduced value of the modulus of commercially pure copper are certainly most closely related. The indentation in the temperature curve of the elastic modulus of commercially pure copper had disappeared when 5 percent Zn, 1 percent Sn, 0.8 percent Mg, and 0.5 percent Be had been added. The modulus values pertaining to these alloys, 13,000, 13,200, 13,280, and 12,950 kilograms per millimeter\textsuperscript{2}, lie in every case noticeably above the value for so-called pure copper.

According to figure 13, the sequence of effect of the admixed elements is about antimony, tin, magnesium, beryllium, silicon, silver, manganese, zinc, and aluminum. For 10 At.-percent, the reduction of the modulus amounts to 3 to 30 percent.

Figures 14 to 15 show the influence of the admixed elements on the elastic modulus of magnesium and aluminum. The sequence of effect is, in the first case, tin, lead, bismuth, thallium, aluminum, silver, and zinc; in the second, copper, silicon, zinc, and magnesium. The order of magnitude of effectiveness is about the same as for copper. Furthermore, the present report offers, for the statement that the elastic modulus with elements of limited solubility decreases, the example of mutual absorption of tin in antimony and antimony in tin (fig. 56); also, of zinc in cadmium (fig. 22).

\textbf{Ni-Be.} - By absorption of beryllium in solid solution the elastic modulus of nickel is reduced, according to G. Masing and O. Dahl.\textsuperscript{18}

In two cases, an increase of the elastic modulus was observed. The first is that of admixture of lithium to metals as, for instance, aluminum and lead (figs. 50 and 51). The effect is quite considerable.

Second, the modulus of gold may be increased by partially soluble admixtures. This applies, according to figure 16, for cadmium, iron, and magnesium. Zinc hardly changes the modulus while addition of tin lowers it.

\textsuperscript{17}Koster, W: Zeitschr. f. Metallkunde, Bd. 39, 1948, p. 9.

\textsuperscript{18}Masing, G., and Dahl, O: Wissenschaftliche Veröffentlichungen (Scientific Publications) Siemens-Konzern 8, 1929, p. 211.
EUTECTIC SERIES OF ALLOYS

In figures 17 to 24, the elastic modulus as a function of the concentration is represented in a number of eutectic alloy series. In the heterogeneous state, the elastic modulus varies according to the rule of mixtures. The test points lie on the connecting straight line of the values of the two limiting concentrations. The scatter is generally slight. The experience with the alloys reported on below also amounts always to the fact that in heterogeneous regions the modulus varies linearly with the concentration. So far, the system lead-tin (fig. 24) is the one exception not clarified. The authors checked a finding of S. Aoyama and T. Fukuroi\(^1\) which seemed incredible and obtained almost the same values. An influence of structure or texture might be the cause.

The systems silver-copper, aluminum-tin, aluminum-zinc, cadmium-zinc, lead-antimony, and lead-tin were investigated throughout. The alloys copper-chrome and aluminum-silicon were tested partially. From the last measured series, the value for chrome and silicon may be extrapolated to about 18,500 and 11,500 kilograms per millimeter\(^2\), respectively.

ALLOY SERIES OF COPPER, SILVER, AND GOLD WITH METALS OF SECOND PERIODIC GROUP

Cu-Zn, Ag-Zn, Au-Zn, Ag-Cd, Au-Cd. - The alloys of copper, silver, and gold with zinc and cadmium were investigated by W. Köster (footnote 3). The dependence on the concentration was discussed separately (footnote 5). In this report, it is represented graphically in figures 25 to 29; supplements and measured results published meanwhile by other authors have been included. The variation of the modulus in the \(\alpha\) solid solutions series has been discussed before. The compounds in this region, Au\(_3\)Cd, are distinguished by high modulus values (footnote 4).

From the boundary of the solid solution region onward the modulus drops to the value of the ordered \(\beta'\) phases which show the lowest value in all systems. The \(\xi\) forms which these alloys are able to assume have an elastic modulus higher by far than that of the \(\beta'\) structures. Then the modulus increases at transition to the \(\gamma\) fields. In the series with copper and silver, the \(\gamma\) phase has the highest value, but in the

gold systems, too, it exceeds that of the $\beta$ and $\xi$ forms. Toward the $\epsilon$ crystal types the modulus again decreases. It is larger than that of zinc or cadmium. A detailed comparison of the systems may be found in the report mentioned in footnote 5.

The system copper-zinc has been investigated also by L. Guillet (footnote 12), furthermore, by M. J. Druyvesteyn and J. L. Meyering. As can be seen from figure 25, the dependence on the concentration is essentially the same in all three series investigated. Two deviations are to be pointed out briefly. According to L. Guillet, the measured values in the $\alpha$ solid solution series are somewhat lower than those indicated by W. Köster. The latter, on the other hand, are in perfect agreement with the data of Z. Nishiyama, (footnote 13); however, although it cannot be inferred from L. Guillet's findings, the initial increase of the modulus of copper due to admixture of zinc cannot be doubted, particularly in view of the regular recurrence of the phenomena in other alloys discussed above.

On the other hand, M. J. Druyvesteyn and J. L. Meyering justly pointed out that the $\beta$ phase value given by W. Köster is probably too low for an isotropic specimen to which the data were supposed to refer. As a criterion for their judgment, Druyvesteyn and Meyering used the Poisson ratio and compressibility which are to be calculated from the elastic and transverse elastic modulus measured on the same specimen; probable values must result. The two authors have shown on numerous specimens of different types of manufacture that this condition is rarely satisfied by the $\beta$ alloy and that its most probable value in case it is fulfilled is about 9300 kilograms per millimeter. The reason lies in the unusually large elastic anisotropy of the crystal type measured by W. Webb and by J. S. Rinehart. According to M. J. Druyvesteyn and J. L. Meyering, manufacture of a quasi-isotropic specimen of the alloy is difficult since it is disposed to formation of textures. Thus, commercial products made of the alloy will predominantly show the low value named. For a physical evaluation of its elastic behavior, however, one will use the value valid for the quasi-isotropic state.

Cu-Be. - This system has been treated by L. Guillet (footnote 12) (fig. 30). The modulus of copper is increased by addition of beryllium. Thus the body-centered cubic phase CuBe with cesium chloride superstructure has, in this case, a higher modulus than copper. It lies, however, below the connecting straight line whereas the value for the crystal type CuBe$_2$ of the Cl5 type approximately coincides with it. In the solid solutions state, beryllium lowers the modulus of copper as can

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be seen from a series of measurements of G. Masing and O. Dahl\textsuperscript{23} (fig. 13). In comparison, the same alloys showed after annealing at 350° values increasing with growing copper content (fig. 30).

**Ag-Mg.** - In this system, the rule is confirmed that the Hume-Rothery \( \beta' \) phase has a pronouncedly low, the hexagonal \( \epsilon \) phase a remarkably high modulus of elasticity (fig. 31).

**Ag-Al.** - In the region between the \( \alpha \) solid solutions and the hexagonal \( \epsilon \) phase which again shows an increased elastic modulus, the modulus increases uniformly (fig. 33). The crystal type Ag\textsubscript{3}Al with \( \beta\)-Mn structure is not distinguished by a high elastic modulus. The alloy with 22 atom-percent Al was quenched from 660°. It then consisted of a mixture of \( \alpha \) and \( \beta \) crystals, and its modulus lay at 7300 compared to 8200 kilograms per millimeter\textsuperscript{2} in the slowly cooled off state. By annealing to 500°, the modulus again rose to the former value. This shows that the body-centered cubic \( \beta \) phase has a low value of elastic modulus in this system, too.

**Cu-Al.** - In the system copper-aluminum, the \( \gamma \) phase appears immediately at room temperature, along with the \( \alpha \) solid solution phase. Consequently, the modulus rises sharply from the boundary of the solid solution region. At higher temperatures, the \( \beta \) phase is stable and may be obtained in a modified form by quenching to room temperature. L. Guillet (footnote 12) determined the elastic modulus on a specimen treated in this manner; the modulus again lies far below the connecting straight line between the two metals. It is noteworthy that the modulus considerably decreases in the region of the \( \gamma \) mixed crystal, the structural peculiarities of which were described in detail by A. J. Bradley, H. J. Goldschmidt, and H. Lipson.\textsuperscript{24} The \( \eta \) phase\textsuperscript{25} which is structurally related to the \( \gamma \) phase also still has a rather high elastic modulus. Then it decreases sharply to the compound Al\textsubscript{2}Cu from which the transition to aluminum is obtained. In the range of the \( \delta \) and \( \eta \) crystals, the alloys were cracked; thus, no reliable values could be measured. L. Guillet's (footnote 12) values agree to a great extent with those of the author.

**Cu-Si.** - The elastic modulus of the intermediary phases of \( \beta\)-Mn and \( \gamma \) like structure lies far above the connecting straight line and

\begin{footnotesize}
\begin{itemize}
\item \textsuperscript{25}Köster, W., and Moeller, K: Zeitschr. f. Metallkunde, Bd. 33, 1941, p. 278.
\end{itemize}
\end{footnotesize}
is the higher the greater the silicon content. The alloys were readily prepared. The practically homogeneous crystal types were investigated (fig. 34).

Cu-Sn. - With 13,800 kilograms per millimeter\(^2\), the 8 phase has the highest value in this system; it lies far above the connecting line. The hexagonal ε' phase has a remarkably low value; it corresponds approximately to the rule of mixtures. The elastic modulus of η' was measured as 9350 kilograms per millimeter\(^2\). In the region ε + η' the value given by the plotted curve ought to appear; however, according to the plotted curve, one of the measured values is too high (fig. 35). The values given by L. Guillet (footnote 12) fit in well with the course of the curves.

Ag-Sn. - The differences in this system are slight (fig. 36). The hexagonal phase Ag\(_5\)Sn(ε) has the lowest value, the rhombic phase Ag\(_3\)Sn(ε\(_1\)) the highest.

Cu-Sb. - The elastic modulus of copper is sharply reduced by absorption of antimony in solid solution. Starting from the boundary of the solid solution region, the modulus then drops almost linearly toward the antimony. The two intermetallic compound types Cu\(_3\)Sb and Cu\(_2\)Sb are hardly distinguishable against the pattern of concentration (fig. 37).

Ag-Sb. - The two phases ε and ε\(_1\) which show few structural differences also have almost the same modulus of elasticity. In the eutectic regions of the systems copper-antimony and silver-antimony, the values lie satisfactorily on straight lines; however, the straight lines do not end at 5600 kilograms per millimeter\(^2\) on the antimony side, the value customarily assumed for quasi-isotropic antimony specimens, but at about 7000 kilograms per millimeter\(^2\) (fig. 38) in both cases.

ALLOYS WITH MAGNESIUM

Mg-Cd. - Cadmium and magnesium form onward from 33 atom-percent Cd up to magnesium a solid solution series in which lie two superstructure phases CdMg and CdMg\(_2\). As was to be expected, these two alloys are distinguished by peaks in the concentration curve (fig. 39). The third ordered phase CdMg\(_3\) becomes noticeable in the same manner. The elastic modulus of cadmium is reduced by absorption of magnesium in solid solution.

Mg-Zn. - The crystal types MgZn, MgZn\(_2\) (C14 types) and MgZn5 stand out little in the concentration curve. It can roughly be described as a curve slightly bent downward (fig. 40).
Mg-Al. - Whereas magnesium affects the elastic modulus of aluminum only a little in the solid solution, it lowers it considerably when it occurs in the form of the crystal type $\text{Al}_2\text{Mg}_2$ which is in equilibrium with it. Further addition of magnesium leads to another reduction of the modulus in the range of the crystal types between $\text{Al}_2\text{Mg}_2$ and $\text{Al}_3\text{Mg}_4$. With the appearance of the last named in the structure, the modulus is increased again. At the concentration of the compound ($\alpha$-Mn structure) it reaches a peak value; then it decreases to the value of the saturated magnesium solid solution. The elastic modulus of $\text{Al}_3\text{Mg}_2$ lies considerably below that of $\text{Al}_3\text{Mg}_4$, almost on the connecting straight line between the values of the two elements (fig. 41).

Mg-Tl. - The elastic modulus of the crystal types $\text{Mg}_3\text{Tl}$ and $\text{Mg}_2\text{Tl}$ lies approximately on the connecting straight line (fig. 42). Thus, the modulus of the magnesium is little reduced up to 33 atom-percent Tl. As soon as the phase $\text{MgTl}$ with cesium chloride structure appears, however, the modulus is lowered sharply. $\text{MgTl}$ has a modulus only a very little higher than thallium with which it is in equilibrium.

Mg-Ca. - Magnesium and calcium each form an eutectic series with the crystal type $\text{CaMg}_2$ of the Cl4 type of the Laves phases. The latter’s elastic modulus lies above the connecting straight line between magnesium and calcium; the modulus increases, starting from both pure metals, to the value of the compound (fig. 43).

Mg-Cu. - The system is made up of three eutectic partial series. The dependence on the concentration of the elastic modulus may be described as a straight line leading from the magnesium to the saturated copper solid solution. The two crystal types $\text{Cu}_2\text{Mg}$ (Cl5 type) and $\text{CuMg}_2$ therefore do not stand out particularly (fig. 44).

Mg-Ni. - In this system which contains the two crystal types $\text{Mg}_2\text{Ni}$ and $\text{MgNi}_2$ (C36 type), too, the modulus increases on three straight lines which are only slightly mutually inclined toward the nickel. The values for the compounds are somewhat lower than corresponds to the rule of mixtures (fig. 45).

Mg-Si, Mg-Sn, Mg-Pb. - In all three systems, the elastic modulus of the compound with fluorite structure, $\text{Mg}_2\text{Si}$, $\text{Mg}_2\text{Sn}$, and $\text{Mg}_2\text{Pb}$, lies above the connecting straight line between the elements. The modulus of the magnesium is increased in every case by accession of the compound in the structure, in sequence increasingly so from lead over tin to silicon. The moduli of lead and tin also are increased, the modulus of silicon, however, reduced (fig. 46).

Mg-Sb, Mg-Bi. - The systems of magnesium with tin, lead, antimony, and bismuth have already been described by W. Köster and K. Rosenthal (footnote 6). In contrast to the systems with elements of the fourth
group, the modulus of the compounds of lanthanum oxide structure formed by magnesium with elements of the fifth group, Mg₃Sb₂, Mg₃Bi₂, is smaller than corresponds to the rule of mixtures. Hence, the modulus of magnesium as well as of antimony and bismuth is lowered by alloying (fig. 47).

**ALLOYS WITH ALKALI METALS AND OF THE ALKALINE EARTHS**

**Na-Cd.** - The two intermediary crystal types of the system Cd₆Na and Cd₂Na melt without decomposition. They form with cadmium and mutually among themselves an eutectic series in each case. The elastic modulus decreases, over Cd₆Na down to Cd₂Na, more or less linearly (fig. 48). Since the compound Cd₂Na is adjoined by a two-phase field with sodium as the second crystal type, one may expect further on a variation of the modulus as represented by the dashed curve. Accordingly, the modulus of the two compounds lies only a little below the connecting straight line.

**Li-Cd.** - The hexagonal phase Cd₃Li in the solid state has about the same elastic modulus as cadmium. Similarly, the modulus of the face-centered cubic crystal type CdLi₃ which likewise develops in the solid state is only little higher than that of lithium. Between Cd₃Li and CdLi₃ the modulus decreases linearly over the wide static field of the phase CdLi of the NaCl type (fig. 49).

**Li-Al.** - As mentioned before, the modulus increases in the solid solution region of the aluminum side. Likewise, it rises in the adjoining heterogeneous static field. Thus the modulus of the crystal type AlLi of the NaCl type is higher than corresponds to the rule of mixtures (fig. 50).

**Li-Pb.** - The system has been investigated on the lead side up to the crystal type LiPb with cesium chloride structure which is poorest in lithium. The modulus of lead is increased by absorption of lithium. The modulus of the compound is smaller than corresponds to the rule of mixtures. In the heterogeneous region, it therefore decreases (fig. 51).

**Li-Bi.** - The body centered tetragonal crystal type BiLi has an elastic modulus somewhat smaller than corresponds to the rule of mixtures (fig. 52).

**Al-Ca.** - The elastic modulus sharply decreases toward the compound Al₄Ca which has a layer lattice and increases again toward the crystal type Al₂Ca of the Laves C15 type. The value of the latter lies slightly below the connecting straight line (fig. 53).
ALLOYS WITH ANTIMONY

Al-Sb. - In the system synthesized from two eutectic series, the high melting crystal type AlSb with zinc blend lattice occurs. It has doubtlessly a very low elastic modulus which, however, can be indicated only approximately by extrapolation since the bars of the pure crystal type could not be prepared free from cracks (fig. 54).

Sb-Zn. - Similarly, the elastic modulus of the rhombic crystal type SbZn is pronouncedly low. The value for the neighboring crystal type Sb$_3$Zn$_4$ is not much higher. In contrast, the modulus of the phase Sb$_2$Zn$_3$ following next is considerably higher than that of the compounds richer in antimony. While the latter lie below the straight line between the values of the two metals, the former lies approximately on the straight line (fig. 55).

Sb-Sn. - The measured results of the authors agree perfectly with those of S. Aoyama and T. Fukuroi (footnote 19). The modulus of antimony is somewhat reduced by absorption of tin and vice versa. The modulus of the cubic compound SbSn of the NaCl type lies approximately on the connecting straight line. Starting from antimony and from tin, it slightly increases toward the heterogeneous static fields (fig. 56).

CARBIDES, NITRIDES

Ti-C, V-C, Cb-C, Ta-C. - The four carbides crystallize in the same form according to the NaCl type. Their elastic modulus lies, according to figure 57, somewhat below the connecting straight line between the modulus of the metal and that of the carbon which has been estimated by E. O. Bernhardt\textsuperscript{26} to be, in the diamond form, about 80,000 kilograms per millimeter\textsuperscript{2}. The values for the carbides are TiC 32,200 kilograms per millimeter\textsuperscript{2}, VC 27,600 kilograms per millimeter\textsuperscript{2}, CbC 34,500 kilograms per millimeter\textsuperscript{2}, and TaC 29,100 kilograms per millimeter\textsuperscript{2}.

Mo-C, W-C, Fe-C. - The elastic modulus of Mo$_2$C is smaller, 22,100 kilograms per millimeter\textsuperscript{2}, than that of molybdenum as was determined on two separately prepared specimens (fig. 58).

In the system tungsten-carbon, the elastic modulus of the carbides W$_2$C and WC was measured. The modulus of W$_2$C, 42,800 kilograms per

\textsuperscript{26}Bernhardt, E. O: Zeitschr. f. Metallkunde, Bd. 33, 1941, p. 135.
millimeter$^2$, lies only slightly higher than that of tungsten. WC, in contrast, shows the high value of 72,200 kilograms per millimeter$^2$ therewith approaching that of carbon. The modulus of both carbides as a function of the temperature can be seen in figure 59.

The elastic modulus of iron is lowered by alloying with carbon in the form of iron carbide. From the reduction in the range to 1.7 percent, the value for the carbide Fe$_3$C may be extrapolated to approximately 18,500 kilograms per millimeter$^2$.\textsuperscript{27}

WC-Co. - Tungsten carbide bonded with cobalt is widely known as hard metal (widia). The addition of cobalt sharply reduces the elastic modulus of the carbide as is to be expected according to the rule of mixtures in view of the heterogeneous structure. For 6 percent Co, the modulus lay at 55,100; for 10 percent Co at 52,500, and for 20 percent Co at 42,300 kilograms per millimeter$^2$. Because of the technical importance of the material, the dependence on temperature of the modulus was determined for the specimen with 6 percent Co. It is represented in figure 59.

TiN. - The elastic modulus of TiN was measured as 8060 kilograms per millimeter$^2$. The specimen had the density 3.03 compared to the calculated value 4.81. Since the density of the nitride specimen was very small compared to the theoretical value, the elastic modulus actually is probably higher than the value given here. The modulus of TiN is only slightly lowered up to 600\textdegree (fig. 59).

SUMMARY

A comprehensive survey of the elastic modulus of binary alloys as a function of the concentration is presented. Alloys that form continuous solid solutions, limited solid solutions, eutectic alloys, and alloys with intermetallic phases are investigated. Systems having the most important structure have been singled out for examination in order to obtain fundamental criteria for the relation between lattice structure, type of binding, and elastic behavior. A report concerning the evaluation of the present test findings in the indicated direction will be published in this journal in the near future.

We express our gratitude to the platinum foundry, W. C. Heraeus, Hanau, for supporting our work by preparing the precious metal specimens.

\textsuperscript{27}Köster, W: Arch. für das Eisenhüttenwesen, ud. 14, 1941/42, p. 271.
and to both the metal works, Plansee, Reutte, and the Osram AG., Berlin, for letting us have the carbide and nitride bars.

We express our deep appreciation to Miss Emmi Amrhein for her aid in the preparation of the specimens and the measurement of the elastic modulus.

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