NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1313

ON THE RECORDING OF TURBULENT LONGITUDINAL AND TRANSVERSE FLUCTUATIONS

By H. Reichardt

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1. ON THE SIGNIFICANCE OF FLUCTUATION MEASUREMENTS

A thorough understanding of the turbulent flow movements cannot be arrived at from investigations of the temporal mean values of the flows alone. Study of the fluctuation phenomena themselves is indispensable. Thus turbulence research entered a new promising stage when investigators started performing fluctuation measurements and basing theories on those measurements.

The investigations of fluctuations carried out so far refer almost exclusively to the so-called isotropic turbulence. It represents a damping phenomenon; it is the simplest type of turbulence where merely the longitudinal fluctuations need to be measured. However, it is precisely nonisotropic turbulence which involves the most essential problem of the turbulent exchange movement and the turbulent apparent friction. Investigation of nonisotropic turbulence requires measurement of longitudinal and transverse fluctuations.

2. DIRECTIONAL PROBES FOR FLUCTUATION MEASUREMENTS

Reliable measurements of quadratic mean values (as are required for fluctuation research) are practically feasible only with hot wires. Any hot-wire directional probe that can be manufactured in sufficiently small dimensions is usuable for simultaneous recording of longitudinal and transverse fluctuations.

Directional probes consist of at least two hot wires of the same type in an arrangement which is sensitive to angle changes. The difference in voltages over these wires is a measure for the angle deviation and the transverse velocity v' of the flow; the variation of the voltage sum is a measure for the variation u' of the longitudinal velocity.

*"Über das Messen turbulenter Längs- und Querschwankungen." Zeitschrift für angewandte Mathematik und Mechanik, Band 18, Heft 6, December 1938, pp. 358-361.

In the arrangement of Simmons and Bailey (fig. 1a), the directional sensitivity of the hot wires is based on their being placed in oblique flow. In case of the two parallel hot wires of Burgers (cross section represented in fig. 1b), the directional sensitivity depends on the mutual influencing of the temperature and velocity fields of the wires.

In the three-wire arrangement of the author (fig. 1c), the velocity and temperature gradients in the wake of the front (third) hot wire are made use of. The front wire may serve for the measurement of u' but also for the measurement of temperature fluctuations T'. Thus it is possible to determine not only the turbulent apparent friction $\rho\overline{u'v'}$, but also the turbulent heat transfer $\rho c_p\overline{T'v'}$ directly from the fluctuations (ρ = air density, c_p = specific heat).

3. GENERAL RELATIONS

Quite independently of the manner in which the directional sensitivity is attained, the following general considerations apply to directional probes used for the measurement of fluctuations.

The "effective cooling velocity" [1] w on a hot wire is a function of the angle α (fig. 1) and of the velocity u of the undisturbed flow. The $w(\alpha)$-curves of a hot wire for the separate velocities are symmetrical to the axis $\alpha = 0$ on which lie the minimum values of w. The maximum values of w are generally identical with the undisturbed velocity u.

For fluctuation measurements, the probe must be set up in such a manner that the hot wires form fixed angles α, or $-\alpha$, respectively, with the main flow direction x. Then the flow flows against both hot wires with the same velocity w if the flow vector lies in the x-direction and the velocity of the oncoming flow u is locally constant.

If the flow varies its direction with respect to the x-axis by the small angle φ and its intensity by Δu, and if there exists a velocity gradient $\frac{\Delta u_1 - \Delta u_2}{\Delta y}$ in y-direction vertically to x, one obtains for the modifications of the cooling velocities Δw_1 and Δw_2 approximately

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[1]According to definition, w includes also the effects of an altered temperature of the air.

$$\Delta w_1 = \frac{\partial w}{\partial \alpha}\varphi + \frac{\partial w}{\partial u}\Delta u_1 \tag{1}$$

$$\Delta w_2 = -\frac{\partial w}{\partial \alpha}\varphi + \frac{\partial w}{\partial u}\Delta u_2 \tag{2}$$

The voltage E over a hot wire is a function of w. This function $f(w)$ depends in the individual case on the chosen connection. Generally one may state that f decreases with increasing w to the asymptotic value E_0 of the cold wire.

For small variations, Δw, one may write

$$E = f(w) + \frac{df}{dw}\Delta w \tag{3}$$

or, respectively, for the voltage difference measured in the Wheatstone bridge

$$E_1 - E_2 = e_v = \frac{df}{dw}\left(2\frac{\partial w}{\partial \alpha}\varphi + \frac{\partial w}{\partial u}\frac{du}{dy}\Delta y\right) \tag{4}$$

(Δy = distance of the hot wires). If one further introduces the transverse fluctuation $v' = \varphi u$ and denotes $\eta = \frac{w}{u}$, one obtains

$$e_v = b\left(v' + \varphi_0 u + c\,\frac{du}{dy}\right) \tag{5}$$

Here φ_0 signifies a small angle of deviation which may very easily appear in the setting up of the probe

$$b = 2\,\frac{df}{dw}\frac{\partial \eta}{\partial \alpha} \tag{6}$$

$$c = \frac{u \frac{\partial \eta}{\partial u} + \eta}{2 \frac{\partial \eta}{\partial \alpha}} \Delta y \sim \frac{\eta}{2 \frac{\partial \eta}{\partial \alpha}} \Delta y \tag{7}$$

The quantity b is a measure for the sensitivity of the v'-measurement while c represents the influence of the velocity gradient on the v'-measurement.

The quantity b is, in general, dependent on the velocity. In case of the three-wire probe, however, b may be made constant within the range of small velocities since for the small Reynolds numbers of the front wire (of about 0.01-mm diameter) two effects act against one another: While df/dw decreases with increasing w, the dent in the velocity curve behind the front wire gradually deepens whereby $\partial\eta/\partial\alpha$ increases; however, when the velocity is increased far beyond 1 m/s, the dent variation no longer progresses so rapidly, and the damping of df/dw predominates.

Since the turbulent fluctuations in velocity are not always small, the dependence of b on the velocity makes a correction necessary. One may write as an approximation

$$b = b(\bar{u}) + \frac{db}{d\bar{u}} u' \tag{8}$$

where $\bar{u}$ signifies the mean velocity and u' the longitudinal fluctuation. After introducing the dimensionless number

$$\kappa = \frac{\bar{u}}{b} \frac{db}{d\bar{u}} \tag{9}$$

one obtains, neglecting the correction terms of second degree, instead of equation (5)

$$\underline{e_v/b = v' + \varphi_0 u + \kappa \frac{u'v'}{\bar{u}} + c \frac{du}{dy}} \tag{10}$$

4. FORMATION OF VARIOUS MEAN VALUES

In general, one does not measure the bridge voltage itself but a current caused by the bridge voltage. If this current is proportional to the bridge voltage, the current mean values also are proportional to the corresponding voltage mean values.

For the linear voltage mean, one obtains

$$\bar{e}_v/b = \varphi_0 \bar{u} + \kappa \frac{\overline{u'v'}}{\bar{u}} + c \frac{d\bar{u}}{dy} \tag{11}$$

This equation is to be used for the experimental determination of c.

In order to find the quadratic mean value, the equation

$$e_v'/b = v' + \varphi_0 u' + c \frac{du'}{dy} \kappa\left(\frac{u'v'}{\bar{u}} - \frac{\overline{u'v'}}{\bar{u}}\right) \tag{12}$$

must be squared. One obtains

$$\overline{e_v'^2}/b^2 = \overline{v'^2} + 2\varphi_0 \overline{u'v'} + 2c \frac{d}{dy}(\overline{u'v'}) + 2\kappa \frac{\overline{u'v'^2}}{\bar{u}} \tag{13}$$

The assumption that $\overline{u' \frac{\partial w'}{\partial z}}$ vanishes because of lacking correlation is taken into account in the c-term of this equation.

If the equation

$$e_u/a = u' \tag{14}$$

exists for the longitudinal fluctuations u', one obtains according to equation (12) for the mixed product $\overline{u'v'}$ which is proportional to the turbulent apparent friction

$$\frac{\overline{e_{u}'e_{v}'}}{ab} = \overline{u'v'} + \varphi_{O}\overline{u'^{2}} + \frac{c}{2}\frac{d\overline{u'^{2}}}{dy} + \kappa\frac{\overline{u'^{2}v'}}{\bar{u}} \tag{15}$$

The terms with κ in the equations (13) and (15) are small and vanish, except in case $\kappa = 0$, even for symmetrical $u'v'$ distribution.

5. EXAMPLE

Figures 2 and 3 show $\sqrt{\overline{v'^2}}$ and $\overline{u'v'}$ measurements with the three-wire probe in a fully developed tunnel flow as functions of the wall distance y. The distances of the hot wires from one another were of the order of magnitudes 0.1 millimeter. The distance of the tunnel walls was 24.4 centimeters, and the mean maximum velocity was $U_m = 100$ centimeters per second.

In the present velocity range, for the three-wire probe, b = constant and thus $\kappa = 0$. The κ-terms would have been eliminated anyway since the $u'v'$ distribution had been found to be symmetrical.[2]

The dashed curve in figure 2 connects the $\sqrt{\overline{v'^2}}$ measuring points. The solid curve represents the $\sqrt{\overline{v'^2}}$ values improved with consideration of the c-term.

The $\overline{u'v'}$-curve of figure 3 passing through the zero point corresponds to the turbulent apparent friction

$$\rho\overline{u'v'} = \tau_{tot} - \mu\frac{d\bar{u}}{dy} \tag{16}$$

The total shearing stress τ_{tot} corresponds to the straight line passing through the points $y = 12.2$ and $10^4\frac{\overline{u'v'}}{U_m^{\,2}} = 25.8$. The position of

[2] Reichardt, H.: Messungen turbulenter Schwankungen (Measurements of turbulent fluctuations). Naturwissenschaften 26, 1938, p. 404.

this straight line was determined from the pressure drop. Thus the measuring points obtained with the three-wire probe lie, on the average, with satisfactory accuracy on the $\overline{u'v'}$-curve required by the pressure gradient and the velocity gradient. Without consideration of the c-term, the mean measured values would come to lie on the lightly drawn curve underneath.

Translated by Mary L. Mahler
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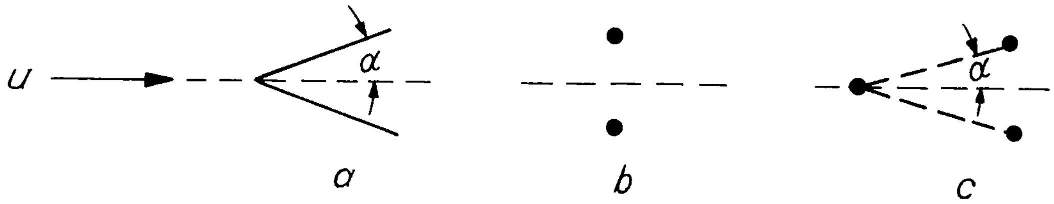

Figure 1.

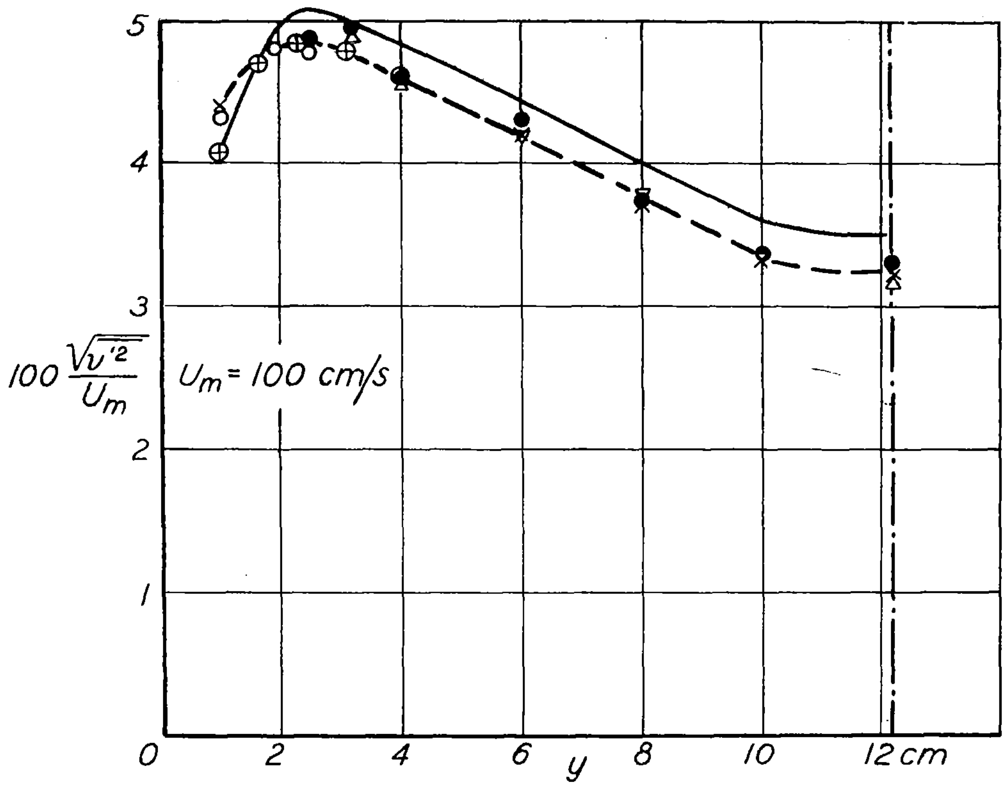

Figure 2.

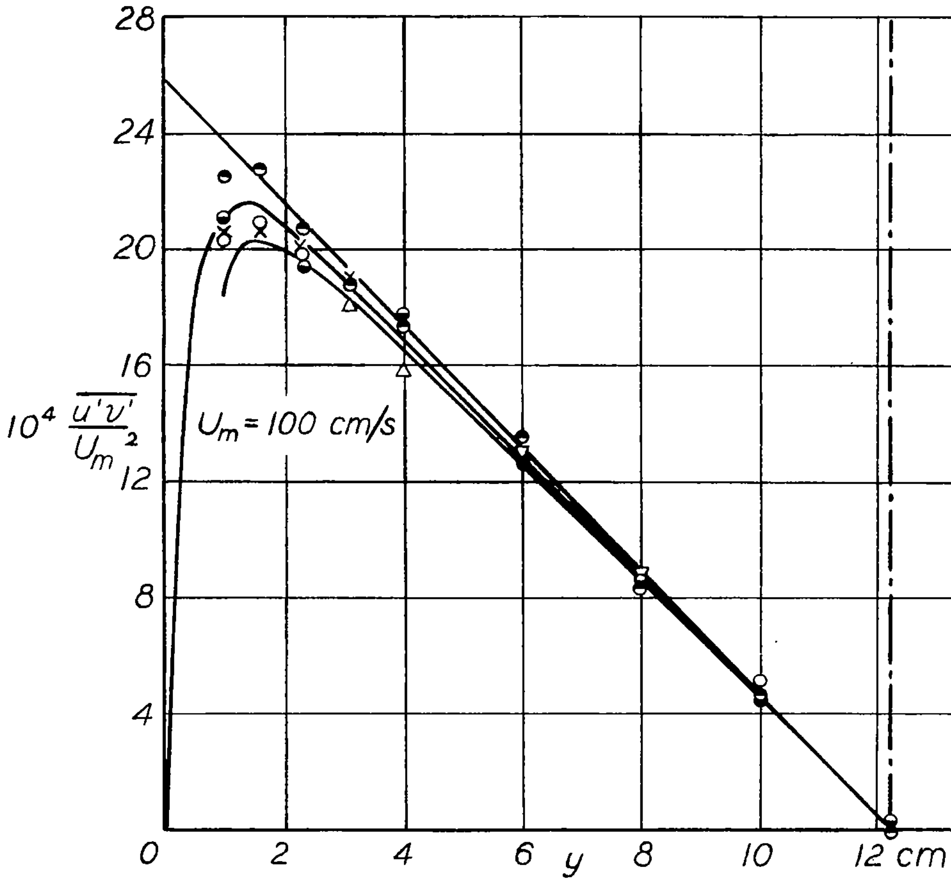
28
24
20
16
$10^4 \frac{\overline{u'v'}}{U_m^2}$
12
8
4
0
U_m = 100 cm/s
2
4
6
y
8
10
12 cm

Figure 3.