

# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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SIDESLIP IN A VISCOUS COMPRESSIBLE GAS

By V. V. Struminsky

Translation

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SIDESLIP IN A VISCOUS COMPRESSIBLE GAS\*

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On the basis of an analysis of the solutions of the equations of Navier-Stokes, it is found that in the slip of a wing in an isothermal or adiabatic flow of a compressible gas, the aerodynamic coefficients  $c_{x1}$ ,  $c_{y1}$ , and  $m_{z1}$  can be accurately determined from the corresponding aerodynamic coefficients of the wing moving without slip. In the general case the aerodynamic coefficients of a sideslipping wing can be determined by the theory, developed in the present paper, of the three-dimensional boundary layer in a compressible gas.

I. In the flow about a rectangular wing of infinite aspect ratio with sideslip, the velocity components  $u$ ,  $v$ , and  $w$ , pressure  $p$ , density  $\rho$ , and temperature  $T$  in a system of coordinates attached to the wing (the  $z$ -axis parallel to the generator of the wing) will depend only on  $x$  and  $y$ . In this system of coordinates, the equations of Navier-Stokes and the boundary conditions can be written in the following form:

$$\left. \begin{aligned} \rho \frac{du}{dt} &= \frac{\partial p_{xx}}{\partial x} + \frac{\partial p_{xy}}{\partial y} \\ \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} &= 0 \\ \rho \frac{dv}{dt} &= \frac{\partial p_{yx}}{\partial x} + \frac{\partial p_{yy}}{\partial y} \\ p &= \rho RT \end{aligned} \right\} \quad (1a)$$

\*"Skolzhenie Kryla v Vyazkom i Szhimaemom Gaze." Doklady Akademii Nauk, SSSR, T. LIV, No. 9, 1946, pp 769-772.

$$\rho \frac{d}{dt} \left( \frac{u^2 + v^2}{2} + I c_p T \right) = \frac{\partial}{\partial x} (u p_{xx} + v p_{xy}) + \frac{\partial}{\partial y} (u p_{yx} + v p_{yy}) +$$

$$I \left\{ \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) \right\} + \mu \left[ \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right] \quad (1b)$$

where  $u = v = 0$  on the surface of the wing;  $u = V_\infty \cos \beta \cos \alpha$  and  $v = V_\infty \cos \beta \sin \alpha$  at infinity;

$$\rho \left( u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} \right) = \frac{\partial}{\partial x} \left( \mu \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial w}{\partial y} \right) \quad (1c)$$

where  $w = 0$  on the surface of the wing;  $w = V_\infty \sin \beta$  at infinity,  $\beta$  is the angle of slip of the wing, and  $\alpha$  is the wing angle of attack.

In the case of isothermal flow ( $T = \text{constant}$ ), the system of equations (1a) can be satisfied by

$$\left. \begin{aligned} u &= V_\infty \cos \beta \bar{u}_0(\bar{x}, \bar{y}, Re \cos \beta, Ma \cos \beta, \alpha) \\ v &= V_\infty \cos \beta \bar{v}_0(\bar{x}, \bar{y}, Re \cos \beta, Ma \cos \beta, \alpha) \\ p &= p_\infty V_\infty^2 \cos^2 \beta \bar{p}_0(\bar{x}, \bar{y}, Re \cos \beta, Ma \cos \beta, \alpha) \\ \rho &= \rho_\infty \bar{\rho}_0(\bar{x}, \bar{y}, Re \cos \beta, Ma \cos \beta, \alpha) \end{aligned} \right\} \quad (2)$$

The nondimensional functions  $\bar{u}_0(\bar{x}, \bar{y}, Re, Ma, \alpha)$ ,  $\bar{v}_0(\bar{x}, \bar{y}, Re, Ma, \alpha)$ ,  $\bar{p}_0(\bar{x}, \bar{y}, Re, Ma)$  and  $\bar{\rho}_0$  are a solution of the corresponding two-dimensional problem. The velocity component  $w(x, y, \beta)$  can be determined from equations (1c) and (2).

In the case of adiabatic flow ( $\mu = \lambda = 0$ ), the system of equations (1a) and (1b) can be satisfied by

$$\left. \begin{aligned} u &= V_\infty \cos \beta \bar{u}_0(\bar{x}, \bar{y}, Ma \cos \beta, \alpha) \\ v &= V_\infty \cos \beta \bar{v}_0(\bar{x}, \bar{y}, Ma \cos \beta, \alpha) \\ p &= p_\infty V_\infty^2 \cos^2 \beta \bar{p}_0(\bar{x}, \bar{y}, Ma \cos \beta, \alpha) \\ \rho &= \rho_\infty \bar{\rho}_0(\bar{x}, \bar{y}, Ma \cos \beta, \alpha) \end{aligned} \right\} \quad (3)$$

In this case the velocity component  $w$  in the entire flow is equal to  $w = V_\infty \sin \beta$ .

II. In order to determine the components along the coordinate axes of the resultant force and resultant moment, the following expressions are used:

$$\left. \begin{aligned} X_1 &= X_1^0(V_\infty \cos \beta, \alpha) \\ Y_1 &= Y_1^0(V_\infty \cos \beta, \alpha) \\ M_{z1} &= M_{z1}^0(V_\infty \cos \beta, \alpha) \end{aligned} \right\} \quad (4)$$

$$\left. \begin{aligned} Z_1 &= \int \mu \omega \, ds \\ M_{x1} &= \int_s \mu \omega \, y \, ds \\ M_{y1} &= - \int_s \mu \omega \, x \, ds \end{aligned} \right\} \quad (5)$$

where  $\omega = \frac{\partial w}{\partial x} \cos nx + \frac{\partial w}{\partial y} \cos ny$ .

Referring the forces and moments to the flow velocity at infinity, for isothermal flow,

$$\left. \begin{aligned} c_{x1} &= \cos^2 \beta c_{x1}^0(\text{Re} \cos \beta, Ma \cos \beta, \alpha) \\ c_{y1} &= \cos^2 \beta c_{y1}^0(\text{Re} \cos \beta, Ma \cos \beta, \alpha) \\ m_{z1} &= \cos^2 \beta m_{z1}^0(\text{Re} \cos \beta, Ma \cos \beta, \alpha) \end{aligned} \right\} \quad (6)$$

For adiabatic flow,

$$\left. \begin{aligned} c_{x1} &= \cos^2 \beta c_{x1}^0 (Ma \cos \beta, \alpha) \\ c_{y1} &= \cos^2 \beta c_{y1}^0 (Ma \cos \beta, \alpha) \\ m_{z1} &= \cos^2 \beta m_{z1}^0 (Ma \cos \beta, \alpha) \end{aligned} \right\} \quad (7)$$

For adiabatic flow moreover,  $c_{z1} = m_{x1} = m_{z1} = 0$ .

Expressions (6) and (7) are direct consequences of the equations of Navier-Stokes.

III. At large Reynolds numbers and at a Prandtl number equal to unity, the following system of equations for the three-dimensional boundary layer in a compressible gas is obtained from equations (1a), (1b), and (1c):

$$\left. \begin{aligned} \rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) &= - \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) \\ \frac{\partial p}{\partial y} &= 0 \\ \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} &= 0 \\ p &= \rho RT \end{aligned} \right\} \quad (8a)$$

$$\rho \left( u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \left( \frac{u^2 + w^2}{2} + I c_p T \right) = \frac{\partial}{\partial y} \mu \frac{\partial}{\partial y} \left( \frac{u^2 + w^2}{2} + I c_p T \right) \quad (8b)$$

$$\rho \left( u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} \right) = \frac{\partial}{\partial y} \mu \frac{\partial w}{\partial y} \quad (8c)$$

where  $u = v = w$  on the wing surface;  $u = U(x, V_\infty \cos \beta)$  and  $w = W = V_\infty \sin \beta$  on the outer limit of the boundary layer.

The term  $U(x, V_\infty \cos \beta)$  is the velocity of the adiabatic flow on the outer limit of the boundary layer. The value of  $U(x, V_\infty)$  can be determined by the method of S. A. Christianovich (reference 1).

In present work, in addition to the simplifying assumption  $Pr = 1$ , it is assumed that there is no heat transfer between the gas and the wing  $(\partial T/\partial y)_0 = 0$ . In this case the simple integral

of equation (8b) may be employed:  $Ic_p T = \frac{u^2 + w^2}{2} = \text{constant} = i_0$ .

By setting  $T_0 = i_0/Ic_p$  as the temperature of stagnation, the relation  $T = T_0 \left(1 - \frac{u^2 + w^2}{2i_0}\right)$  is obtained. By the law of Bernoulli,

$$p = p_0 \left(1 - \frac{U^2 + W^2}{2i_0}\right)^{\frac{k}{k-1}}$$

From the equation of state,

$$\rho = \rho_0 \frac{\left(1 - \frac{U^2 + W^2}{2i_0}\right)^{\frac{k}{k-1}}}{\left(1 - \frac{u^2 + w^2}{2i_0}\right)}$$

For what follows, the following dependence of the viscosity on the temperature is assumed:  $\mu = \mu_0 (T/T_0)^n$ , where  $0.5 \leq n \leq 1.5$ .

We introduce the stream function  $\rho u = \partial \psi / \partial y$ ,  $\rho v = -\partial \psi / \partial x$  and the new independent variables  $\xi$  and  $\eta$ :  $\partial \xi / \partial x = p/p_0$ ,  $\partial \eta / \partial y = \rho/\rho_0$ .

A similar transformation was first applied by A. A. Dorodnitsyn in his work (reference 2). By carrying out the transformation of the variables in equations (8a) and (8c), the following system of equations is finally obtained:

$$\frac{\partial \psi}{\partial \eta} \frac{\partial^2 \psi}{\partial \xi \partial \eta} - \frac{\partial \psi}{\partial \xi} \frac{\partial^2 \psi}{\partial \eta^2} = \rho_0 \frac{1 - \alpha - \beta}{1 - \alpha_0 - \beta_0} U \frac{dU}{d\xi} + \mu_0 \frac{\partial}{\partial \eta} \left[ (1 - \alpha - \beta)^{n-1} \frac{\partial^2 \psi}{\partial \eta^2} \right]$$

$$\frac{\partial \psi}{\partial \eta} \frac{\partial w}{\partial \xi} - \frac{\partial \psi}{\partial \xi} \frac{\partial w}{\partial \eta} = \mu_0 \frac{\partial}{\partial \eta} \left[ (1 - \alpha - \beta)^{n-1} \frac{\partial w}{\partial \eta} \right] \tag{9}$$

where  $\alpha = u^2/2i_0$ ,  $\alpha_0 = U^2/2i_0$ ,  $\beta = w^2/2i_0$ , and  $\beta_0 = W^2/2i_0$ .

IV. The method of Kármán-Pohlhausen is used to solve the system of equations (9). After simple transformations and integration of the first equation with respect to  $\eta$  from 0 to  $\delta$  and the second equation with respect to  $\eta$  from 0 to  $\gamma$ ,

$$\left. \begin{aligned} \frac{d\delta^{**}}{d\xi} + \frac{U'}{U} (\delta^{***} + 2\delta^{**} + \delta^*) &= \frac{\mu_0}{\rho_0 U^2} \left( \frac{\partial u}{\partial \eta} \right)_0 \\ \frac{d\gamma^{**}}{d\xi} + \frac{U'}{U} \gamma^{**} &= \frac{\mu_0}{\rho_0 U W} \left( \frac{\partial w}{\partial \eta} \right)_0 \end{aligned} \right\} \quad (10)$$

where

$$\left. \begin{aligned} \delta^{***} &= \int_0^\delta \left( 1 - \frac{2i_0 - u^2 - w^2}{2i_0 - U^2 - W^2} \right) d\eta \\ \delta^{**} &= \int_0^\delta \frac{u}{U} \left( 1 - \frac{u}{U} \right) d\eta \\ \delta^* &= \int_0^\delta \left( 1 - \frac{u}{U} \right) d\eta \\ \gamma^{**} &= \int_0^\gamma \frac{u}{U} \left( 1 - \frac{w}{W} \right) d\eta \end{aligned} \right\} \quad (11)$$

In these equations,  $\delta$  is the thickness of the boundary layer for the velocity component  $u$ , and  $\gamma$  is the thickness of the layer for the velocity component  $w$ .

By substituting in (11) and (10) the expressions

$$\frac{u}{U} = \frac{12 + \lambda}{6} \left( \frac{\eta}{\delta} \right) - \frac{\lambda}{2} \left( \frac{\eta}{\delta} \right)^2 - \frac{4 - \lambda}{2} \left( \frac{\eta}{\delta} \right)^3 + \frac{6 - \lambda}{6} \left( \frac{\eta}{\delta} \right)^4 \quad (12)$$

$$\frac{w}{W} = 2 \left( \frac{\eta}{\gamma} \right) - 2 \left( \frac{\eta}{\gamma} \right)^3 + \left( \frac{\eta}{\gamma} \right)^4$$

where  $\lambda = \frac{\rho_0}{\mu_0} \frac{\delta^2 dU/d\xi}{1 - \alpha_0 - \beta_0}$ , a system of two ordinary differential equations of the first order is obtained for the determination of the boundary-layer thicknesses  $\delta$  and  $\gamma$ .

It is seen from expression (12) that in a compressible gas the direction of the velocity within the boundary layer does not coincide with the direction on the outer limit of the boundary layer. The flow is as if it were twisted within the boundary layer by a certain angle which changes with the height of the boundary layer and along the wing chord. Near the point of separation in the lower layers of the boundary layer the flow is along the span of the wing.

V. For a flat plate about which there is a flow with sideslip, the system of equations (9) will have the following exact solution:

$$\psi = 2\mu_0 \sqrt{\frac{\rho_0 V_\infty \cos \beta \xi}{\mu_0}} \Psi_0(\tau) \quad (13)$$

$$w = V_\infty \sin \beta \varphi'_0(\tau)$$

where  $\tau = \sqrt{\rho_0 V_\infty \cos \beta / \mu_0} \xi \eta / 2$ . The function  $\varphi_0(\tau)$  satisfies the ordinary differential equation

$$\frac{d}{d\tau} \left\{ \left[ 1 - \alpha_0^2 \varphi_0'^2(\tau) \right]^{n-1} \times \varphi_0''(\tau) \right\} = -2 \varphi_0(\tau) \varphi_0''(\tau) \quad (14)$$

and the boundary conditions  $\eta = 0 \varphi(0) = 0$ ;  $\eta = \infty \varphi'(\infty) = 1$ . For  $\beta = 0$ , equation (13) goes over into the solution of Dorodnitsyn for the flat plate (reference 2).

On the basis of the expressions here given,

$$c_{x1} = \frac{4 \varphi_0''(0)}{\sqrt{\text{Re}_\infty \cos \beta}} \left( 1 + \frac{k-1}{2} \text{Ma}^2 \right)^{\frac{n-1}{2}} \cos^2 \beta$$

$$c_{z1} = \frac{4 \varphi_0''(0)}{\sqrt{\text{Re}_\infty \cos \beta}} \left( 1 + \frac{k-1}{2} \text{Ma}^2 \right)^{\frac{n-1}{2}} \sin \beta \cos \beta$$

where  $\psi''_0(0)$ , as follows from equation (14), depends on

$$a_0^2 = \frac{1/2(k-1)Ma^2}{1 - 1/2(k-1)Ma^2}$$

The value of the drag coefficient of a flat plate is determined from the expression

$$c_x = c_{x1} \cos \beta + c_{z1} \sin \beta = \frac{4\psi''_0(0)}{\sqrt{Re_\infty \cos \beta}} \left(1 + \frac{k-1}{2} Ma^2\right)^{\frac{n-1}{2}} \cos \beta$$

#### REFERENCES

1. Christianovich, S. A.: Rep. No. 481, CAHI, 1940.
2. Dorodnitsyn, A. A.: Prik. Mat. i Mek., vol. 6, 1942.

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