PRELIMINARY RESULTS FROM FATIGUE TESTS WITH
REFERENCE TO OPERATIONAL STATISTICS

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Aim of the Test

If one admits the possibility of performing tests with reference to operational statistics, an answer is required for the following, frequently-raised question:

What tolerable total numbers of flying hours result for our airplane constructions if they are dimensioned in regard to presently valid strength specifications or, respectively, how are our constructions to be dimensioned if a definite requirement is made regarding their life?

A reply to this question is the main goal of our strength (fatigue) tests; however, it must be pointed out that, at first, one can attempt only answers indicating a direction for further research, and that numerical particulars and rules for dimensioning can be given only after the performance of systematic tests.

Test Method Selected

Before presenting a few partial results from tests with reference to operational statistics, the handling of our tests up to date is briefly described. The tests were based on the initial statistics represented in figure 1; one point of the summation curve indicates the number of times a certain load range is exceeded during 3000 flying hours ($v_R = 350$ km/h). In order to make these statistics clear, it is pointed out that they are the gust statistics given by Kaul for airplanes of construction classes 2 and 3; they contain as maximum load that caused by a 12 m/second-gust in cruising flight, thus for a ratio of $v_R/v_h = 0.85$ approximately the "safe gust load" (load for 10 m/second-gust in horizontal flight at maximum speed).

Various possibilities exist for interpreting these statistics for a practical test. Since according to Kaul's statistics positive and negative gusts occur with approximately equal frequency, it suggested itself always to group one positive and one negative gust of equal absolute magnitude together to a "gust load cycle" about the mean load of level flight (1-g load) (cf. fig. 2). On the other hand, it is also possible to interpret a positive and a negative gust each by itself as alternating load with the 1-g load as limiting value. This interpretation involves a variation during the test not only of the upper load limits but also of the mean load and a doubling of the number of load reversals in comparison to the interpretation described before. In order to save time, we selected at first the interpretation as gust load reversal. It remains for future tests to clarify the problem whether the types of interpretation described here sufficiently resemble the actual loading process or whether a further approximation according to Professor Teichmann's expositions is necessary.

Figure 3 shows the course of a strength test with reference to operational statistics. The total load is divided into several load ranges (for instance 9), and the entire load sequence into several partial sequences (for 3000 flying hours, for instance, into about 12 partial sequences of 250 flying hours each). In order to obtain a mixture of high and low loads simulating that of actual operating conditions, one runs through each partial sequence once in increasing, once in decreasing direction.

Test series being conducted at present will show how much further the sequence selected here must be approximated to the "idealized sequence" in order to evaluate correctly the influence of the sequence in determining the "tolerable magnitude" (or "tolerable scale") for statistics.

Presentation of the Test Results

The result of an individual test is, following the statistical way of thinking, represented as summation curve. Figure 4 shows, on the example of a duralumin tube (with the burrs not removed from the holes), the result of a strength test with reference to operational statistics for a total number of $10.8 \times 10^6$ stress reversals (corresponding to 3000 flying hours with $v_r = 350$ km/h). The test is selected in such a manner that the summation curve of the test, the middle stepped line $k_2$, agrees as well as possible with the summation curve of the operational loads, the line curve $k_1$. The stepped lines drawn additionally (left and right of $k_2$) indicate the scatter band of the frequencies which results from tests with several identical test specimens for the same stress value $O_{bol}$ (lower scatter limit: 2000 h; upper scatter limit: 4500 h). One can see that the scatter band of the tests with reference to operational statistics does not essentially
differ from the scatter band of the Wohler line. As a result, we note for future reference: the tolerable number of flying hours is 2000 h, when the stress scale is 19.3 kg/mm$^2$.

The repetition of such tests with differing stress scales of the initial statistics yields a relation between these "stress scales" and the number of flying hours in the form of "lines of tolerable maximum stress", also called "scale curves". The tests represented in figure 5 were performed with constant loading ratio $V$ between tolerable maximum stress and its coordinated mean stress ($V = 3.4$). Regarding the scatter of the tests, it should be noted that, throughout, test points at the upper scatter limit corresponded to test specimens with low yield point.

### Influence of Various Parameters

**Influence of the sequence.** - If one runs through a total loading sequence of $10.8 \times 10^6$ stress reversals (about 3000 flying hours at $v_r = 350$ km/h), beginning with the maximum load, monotonically decreasing, a "tolerable scale" $> 10.5$ kg/mm$^2$ and $< 13$ kg/mm$^2$ results for the duralumin tubing mentioned. If, on the other hand, one runs through the same loading sequence monotonically increasing, starting with the minimum load, a "tolerable scale" $> 18.5$ kg/mm$^2$ and $< 19.5$ kg/mm$^2$ results. In comparison, the lower scatter limit of the "tolerable scale" for loading in individual partial sequences is 17 kg/mm$^2$. After spot-check tests with a still smaller subdivision than the one selected here, the "tolerable scale" increases to still more than 17 kg/mm$^2$; accordingly it may be assumed that the "tolerable scale" for an "idealized sequence" probably lies between 17 and 19.5 kg/mm$^2$. Comparative tests on Cr-Mo-steel tubes with the burrs not removed from the holes led to fundamentally similar conditions (cf. fig. 6).

**Influence of relaxation periods.** - If after each partial sequence relaxation periods of 48 hours are inserted, there results for statistics with large tolerable scales and small numbers of flying hours a reduction of the tolerable scale, compared to tests without relaxation periods; in contrast, the tolerable scale increases for statistics with small tolerable scale and large numbers of flying hours (cf. fig. 7) (This result agrees with the results known from one-step tests; there one obtains according to Bollenrath for duralumin in the range of limited-life strength a reduction of the tolerable frequencies and according to American investigators in the neighborhood of the endurance limit an increase in tolerable frequencies).
Various Forms of Statistics

Influence of the loading ratio.- In order to give some indication of the influence of various load factors on the tolerable scale, a series of tests based on the same statistics but with different ratios of maximum stress to mean stress have been performed. This ratio corresponds to the ratio of safe load to loading in level rectilinear flight, thus directly to the safe load factor.

The tests so far did not lead to a clear result; they are reproduced in figure 8 only for the sake of completeness. In the transition from load factors of the order of magnitude 3.5 to load factors of the order of magnitude 2 an increase of the tolerable scales in the region of high numbers of flying hours seems possible (it is emphasized at this point that these test results may yet undergo essential changes in case of closer approximation to the actual loading process).

In order to obtain some idea of the effects of variations of the summation curve on the tolerable scale in the range of high stresses, a series of tests with one other statistic were performed. Figure 9 shows the comparison of the initial statistic (gust statistic according to Kaul) with the other statistic. It might be mentioned that this other statistic is the gust + pull-up statistic for airplanes of construction class 4.

Figure 10 shows the result of the tests. In the range of high tolerable scales and numbers of flying hours between 1000 and 2000 one may expect for the gust + pull-up statistic tolerable scales about 10 percent smaller than those for the initial statistic. The difference between the tolerable scales for different statistics must become less and less in the direction toward a so-called "endurance scale."

Application of the Results to Airplane Construction

The permissible stresses indicated in the strength specifications (pt. 1008) may be regarded as "tolerable scales" of summation curves of the operational stresses. The comparison of a summation curve for 3000 flying hours \( v_T = 350 \text{ km/h} \) in the permissible scale with a summation curve for the same number of flying hours in the tolerable scale shows that, for the example selected here (duralumin tube), the requirements of the strength specifications lie on the unsafe side (cf. fig. 11).

In addition to the summation curves for tolerable or, according to the BVF, permissible scale mentioned earlier, a requirement of the
approving agency has been plotted. It has been interpreted here in a particular manner. The PfL circular letter Nr. 328 requires that the following fraction of the safe gust load be endured 10^5 times before failure:

\[ n_{Br} = 1.35 \left[ 1 \pm 0.4(n_{safe} - 1) \right] \]

or in stresses

\[ \sigma_{Br} = 1.35 \left[ \sigma^V \pm 0.4(\sigma_{safe} - \sigma^V) \right] \]

If for \( \sigma_{Br} \) the Wöhler value \( \sigma^0_{cf} \) is substituted for \( 10^5 \) at a reduced 1-g load ratio \( V_{cf} = 0.6 + 0.4n_{safe} \) a stress value may be given for \( \sigma_{safe} = \sigma_{perm}^0 \) that can be regarded as permissible scale of the summation curve, namely

\[ \sigma_{perm}^0 = \frac{\sigma^0_{cf} \times n_{safe}}{0.61 + 0.54n_{safe}} \]

For the example considered herein, the PfL requirement lies well on the safe side.

Figure 12 shows the comparison of lines of tolerable maximum stresses with lines of permissible maximum stresses for the case that the gust statistics are decisive.

For a ratio of \( v_T/v_h = 0.85 \) the stress in entering the 12m/second gust at cruising speed may be assumed to be about equal to, for instance, the stress in entering the 10m/second gust at maximum horizontal velocity, so that the tolerable maximum stresses of the statistics may be compared directly to the stresses for safe load. It is found that - if the airplane characteristics of figure 12 are taken into consideration - the dimensioning conforms to both gust statistics and strength specifications only for the case of about 2000 flying hours. In contrast, with the PfL requirement taken into consideration, about 3750 flying-hours would be tolerable. In the range of high numbers of flying hours the strength specifications as well as the PfL requirement lie on the unsafe side. In the right-hand ordinate scale the stresses in entering the 10m/second gust at cruising speed are given for comparison.

A thorough study of figure 12 might lead to the objection that the recently published result of an industrial test does not fit in here although the structural element investigated there and the tubing investigated in the present report were manufactured of approximately the same material (duralumin with \( \sigma_B = 42 \, \text{kg/mm}^2 \)) and about equal notch
effects existed. The following statement may be made: The number of flying hours of 12,000 \( (v_T = 240 \text{ km/h}) \) endured in this industrial test comes about, first, because a summation curve of the operational stresses other than the one in the tests described here was taken as a basis - the frequency ratio was about 1 to 1.5 - so that a comparison should be made only on the basis of about \( 8000 \times 240/300 = 6400 \) instead of 12,000 flying hours. This number would probably be reduced still further if the calculation were based on the same load factor (for the industrial test about 2.5, for DVL tests 3.4) so that the result is in good agreement with the test results shown above.

Subsequent to various discussions at the Dornier-Flugzeugwerke the problem was raised how drilled flat bars of DM 31 behave under stresses with reference to operational statistics, and how the result varies when the hole is filled first by an unstressed bolt, then by a stressed one. (The flat bars and the holes were very carefully finished). Figure 13 shows the results of the tests performed so far. For all forms of bars investigated the tolerable maximum stresses of the statistics up to 10,000 flying hours and more are above the permissible stresses according to BVF. In the transition to construction elements not especially manufactured for test purposes, too, the curve of permissible stresses (according to BVF) is probably exceeded.

Conditions similar to those for the drilled flat bars of DM 31 result for tubes of Cr-Mo-steel with holes. Here again the tolerable stresses of operational statistics throughout exceed, up to 10,000 flying hours and more, the requirement of the strength specifications; on the other hand, only about 2000 flying hours result in this example if the requirement of the approving agency is taken into consideration (cf. fig. 14).

According to the results described so far the question is justified how the ratio of tolerable maximum stress to permissible stress according to BVF changes with transition to structural elements with smaller notch effect.

For instance, the result for a test bar of Cr-Mo-steel with M 10-thread was that even for the 1.15-fold amount of the maximum stresses permissible according to the BVF no failure occurs during 6000 flying hours \( (v_T = 350 \text{ km/h}) \) (cf. fig. 15). A comparison of the summation curve of this test with the corresponding Wöhler line for similar test specimens shows that probably during the following 15,000 flying hours no failure would have occurred, either.

In case of a screw coupling, for instance, as it is applied on a larger scale occasionally as screw spar coupling, the result becomes considerably worse. The number of flying hours endured up to failure at the same tolerable maximum stress is here only 1500 (cf. fig. 16).
A lowering of the maximum stress to the amount permissible according to strength specifications cannot appreciably increase the number of flying hours - the corresponding Wöhler line would cross well over the summation curve of the test in the region of the mean load ranges. The example shows that in constructions of this type the problem of strength at a load with reference to operational statistics is of increased significance.

**Strength Test with a Spar Connection**

A welded spar connection of the form A of Cr-Mo-steel (cf. fig. 17) was presented, dimensioned according to specifications for a safe load of 7.8 t ($n_{\text{safe}} \approx 4.6$); its effective ultimate load was $15.9 \text{ t} \approx 7.8 \text{ t} \times 1.8 \times 1.15$ ($j = 1.8$ according to point 1008 of the BVF, $j = 1.15$ according to point 1035 of the BVF).

It was to be proved that this connection had sufficient fatigue strength. From the various defects existing in the construction type A it was to be expected that the connection A would not satisfy the requirement of its operational statistics. The test result confirmed this conjecture (fig. 17, line k2). In a one-step test according to PfL-circular letter 328 the connection endured only the total number of all load cycles to be tolerated in one load range of the statistics.

After a re-design of the connection to make it safer against fatigue together with an increase of the safety factor from $1.8 \times 1.15$ to about $1.8 \times 1.3$, the connection finally satisfied its operational statistics for a period of 5000 hours ($v = 175 \text{ km/h}$).

**Influence of the Ratio $v_r/v_h$**

In a discussion which took place at the DVL a few months ago the question was raised, what effect an increase in cruising speed, for instance by installation of more powerful engines, would have on the life of airplane constructions. The following comment may be made: If the cruising speed is increased (unfavorable case) to that amount of the maximum horizontal velocity on which the original dimensioning had been based, the tolerable total number of flying hours is lowered for two reasons:

1. Because the maximum stress of the statistics increases - which signifies that in case of the same dimensioning it is necessary to change for instance from the curve $v_r/v_h = 0.85$ to the curve $v_r/v_h = 1$, 

2. Because now a greater number of gusts per flying hour is encountered - which signifies that now the flying-hour scale corresponding to \( v_r = v_h = 350 \text{ km/h} \) is decisive.

Thus the effect of such a measure is a reduction of the tolerable total number of flying hours from 5000 to about 3500 (cf. fig. 18).

SUMMARY

Figure 19 shows a synopsis of the results gained so far from strength tests with reference to the operational statistics.

Since the initial statistics themselves as well as their prevailing interpretation and application to the practical test may be regarded as unconditionally lying on the safe side, it may be stated that no immediate cause for apprehension exists, particularly so when the newest directives for interpretation of the points 1015 and 1035 of the BVF are taken into consideration in the dimensioning and shaping of structural elements of airplanes.

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Figure 1.- Initial statistics for tests 1938. (Summation curve of the gust stress for airplanes of construction classes 2 and 3 in cruising flight with 350 km/h.) $H_{67.5}$ = frequency of exceeding load ranges beyond "67.5." $A_{67.5}$ = load range of magnitude 67.5 percent.

Figure 2.- Interpretations of the initial statistics.
Figure 3.- Course of a strength test with reference to operational statistics.

Test specimen: tube 50 × 1 of duralumin, Flieg 3115.5, with 5 mm hole in the maximum-stressed fibre

\[ \sigma_B = 46 \text{ kg/mm}^2 \]

\[ \sigma_{0.2} = 34 \text{ kg/mm}^2 \]

(Permissible minimum values:

\[ \sigma_B = 40 \text{ kg/mm}^2 \]

\[ \sigma_{0.2} = 28 \text{ kg/mm}^2 \])

Figure 4.- Summary representation of a strength test with reference to operational statistics (bending).
Permissible minimum values: \( \sigma_B \) flieg. - 40 kg/mm\(^2\), \( \sigma_{0.2} \) flieg. - 28 kg/mm\(^2\)

Values determined from 18 test bars: \( \sigma_B \) - 40 kg/mm\(^2\), \( \sigma_{0.2} \) - 28 kg/mm\(^2\)

Type of stress: normal bending

Figure 5.- Curve of tolerable scales for duralumin tubes (about Flieg. 3115.5) with three holes of 5 millimeter diameter in the maximum-stressed fibre.

Figure 6.- Influence of the sequence.
Without relaxation periods

With relaxation periods (48 hours after every 0.9 \times 10^6 stress reversals)

Figure 7. - Influence of relaxation periods.

Permissible minimum values: \( \sigma_B \text{ flieg } = 40 \text{ kg/mm}^2 \), \( \sigma_{0.2} \text{ flieg } = 28 \text{ kg/mm}^2 \)

Type of stress: normal bending

Figure 8. - Influence of the 1-g load ratio (load factor).
Initial statistic (gust statistics for airplanes of construction classes 2 and 3, 3000h, V = 350 km/h)
Other statistic (gusts plus pull-up statistic for airplanes of construction class 4, 3000h, V = 350 km/h)

Figure 9.- Various forms of the statistics.

Test specimen: tubes of duralumin (about Flieg. 3115.5) with three holes of 5 mm diameter in the maximum-stressed fibre
Permissible minimum values: \( \sigma_B \) flieg = 10 kg/mm\(^2\), \( \sigma_{0.2} \) flieg = 28 kg/mm\(^2\)
Type of stress: normal bending
Initial-load ratio: \( v = \frac{\sigma_0}{\sigma_c} = 3.4 \)

Figure 10.- Influence of various statistics.
According to PfL circulatory letter No. 328

\[ \sigma_0^0 = \frac{\sigma_{\text{cf}} \text{safe}}{\sigma_{\text{perm}}} = 0.31 + 0.54n_{\text{safe}} \]

\[ \sigma_{\text{cf}} = \text{Wöhler value for } 10^5 \text{ stress reversals at } V_{\text{cf}} \]

\[ V_{\text{cf}} = 0.6 + 0.4n_{\text{safe}} \]

Wöhler line with \( V_{\text{cf}} \).

**Figure 11.** - Summation curves in tolerable and permissible scale.

Characteristics of the airplane: \( n(115) \approx 3.4 \), \( \frac{V}{V_h} = 0.85 \), \( V_h = 350 \text{ km/h} \), \( n_{\text{cruise}} = 3 \)

Characteristics of the structural element: tubes of duralumin (about Flieg. 3115.5) and three holes of 5 mm diameter in the maximum-stressed fibre

Permissible minimum values: \( \sigma_B \text{flieg} = 40 \text{ kg/mm}^2 \), \( \sigma_{0.2} \text{flieg} = 28 \text{ kg/mm}^2 \)

Type of stress: normal bending

Initial-load ratio: \( \frac{\sigma_0}{\sigma_t} = 3.4 \)

**Figure 12.** - Example for gust statistics.
Permissible minimum values: $\sigma_{B \text{ flieg}} = 43 \text{ kg/mm}^2$, $\sigma_{0.2 \text{ flieg}} = 27 \text{ kg/mm}^2$

Values determined on test bars: $\sigma_B = 48 \text{ kg/mm}^2$, $\sigma_{0.2} = 30 \text{ kg/mm}^2$

Type of stress: tension - compression

Initial-load ratio: $\varphi = \frac{\sigma_0}{\sigma_T} = 3.4$

Figure 13.- Curves of tolerable scales for flat bars of DM 31 (Flieg. 3125.4).

Permissible minimum values: $\sigma_{B \text{ flieg}} = 60 \text{ kg/mm}^2$, $\sigma_{0.2 \text{ flieg}} = 48 \text{ kg/mm}^2$

Type of stress: normal bending

Initial-load ratio: $\varphi = \frac{\sigma_0}{\sigma_T} = 3.4$

Figure 14.- Curve of tolerable scales for tubes of Cr-Mo-steel, Flieg. 1452.9, with three holes in the maximum-stressed fibre.
Permissible minimum values: $\sigma_B^{\text{flieg}} = 90 \text{ kg/mm}^2$, $\sigma_{0.2}^{\text{flieg}} = 79 \text{ kg/mm}^2$

Values determined on the test bar: $\sigma_B = 99 \text{ kg/mm}^2$, $\sigma_{0.2} = 89 \text{ kg/mm}^2$

Type of stress: tension - compression

Figure 15. - Individual test with a threaded shaft of Cr-Mo-steel (Flieg. 1452.5).

Permissible minimum values: $\sigma_B^{\text{flieg}} = 90 \text{ kg/mm}^2$, $\sigma_{0.2}^{\text{flieg}} = 65 \text{ kg/mm}^2$

Values determined on the test bar: $\sigma_B = 99 \text{ kg/mm}^2$, $\sigma_{0.2} = 89 \text{ kg/mm}^2$

Type of stress: tension - compression

Figure 16. - Individual test with a screw-coupling of Cr-Mo-steel (Flieg. 1452.5).
Nonconcentric; relief of the minimum cross section too small

Lap fillet joint (unfavorable), nonuniform stress distribution (points of maximum stress)

Figure 17.- Individual tests with spar connections.

Improved connection B

Maximum load value (l)

K₁ = summation curve of a test with reference to flight statistics, just withstood by connection B

K₂ = test which led to failure of the connection A

Figure 18.- Influence of the ratio \( \frac{v_{\text{cruise}}}{v_{h}} \).
Figure 19.- Synopsis of the results obtained so far.