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THE EFFICIENCY OF COMBUSTION TURBINES WITH
CONSTANT-PRESSURE COMBUSTION

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The paper here presented was made available to a restricted group in 1937. Since interest in the gas turbine has risen considerably in recent years and some possibilities also have been indicated of the application of this type of engine to the airplane, the paper with some modifications introduced by the developments of recent years is now made more generally available.

Of the two fundamental cycles employed in combustion turbines, namely, the explosion (or constant-volume) cycle and the constant-pressure cycle, the latter is considered more in detail and its efficiency is derived with the aid of the cycle diagrams for the several cases with adiabatic and isothermal compression and expansion strokes and with and without utilization of the exhaust heat. Account is also taken of the separate efficiencies of the turbine and compressor and of the pressure losses and heat transfer in the piping. The results show that without the utilization of the exhaust heat the efficiencies for the two cases of adiabatic and isothermal compression, respectively, differ only slightly. The advantage of the decrease in the work expenditure with isothermal compression is thus offset by the increase in the heat supplied. The heating of the compressed air by the exhaust gases is of advantage, however, particularly for isothermal changes of state. Of great importance are the separate efficiencies of the turbine and compressor. It may be seen from the curves that it is necessary to attain separate efficiencies of at least 80 percent in order that useful results may be obtained. There is further shown the considerable effect on the efficiency of pressure losses in piping or heat exchangers.

INTRODUCTION

The favorable efficiency of its theoretical cycle and its simplicity have been a constant inspiration to inventors and designers to realize a practical design of the gas turbine. The direct conversion into rotational energy made possible with this type of engine led also to the expectation of a considerable reduction in weight and size as compared with the reciprocating type engines.

From the invention of John Barber in 1791 to the present time, proposals for the construction of gas turbines have been numerous.

From reference 1 it is apparent that it has been possible only recently to obtain results which in certain special cases appear to permit the application of combustion turbine installations. Thus, the Thyssen firm, for example, has ordered the installation of a large Holzwarth gas turbine, which is intended to replace the waste gas engines heretofore employed. Turbine installations have also been made by the firms of Brown, Bovery & Company and Escher-Wyss for exhaust gas utilization in the chemical industry and for emergency current supply. The economy of steam installations is not, however, as yet attained with these turbines.

In the following, with the aid of theoretical considerations, a brief derivation is given of the attainable efficiencies, practical points of view being considered only insofar as they enable an estimate to be made of the actual results that may be expected. The reasons for the difficulties met with in the development of the combustion gas turbine and an explanation of the many failures encountered will appear from the equations derived. These equations furthermore indicate the methods to be followed for obtaining improved economy of the combustion turbine to an extent where it may be employed for general engineering purposes and also in aviation.

FUNDAMENTAL WORK CYCLES

As in the case of reciprocating engines, two types of combustion turbines may be distinguished:
a) The constant-pressure turbine corresponding to the Diesel engine.

b) The explosion turbine corresponding to the spark-ignition engine.

Since the constant-pressure process can be realized with particularly simple and reliable means and promises a weight to power ratio and size that are small enough to be of advantage in the field of airplane design, this particular work process will be treated in detail.

2. THE ADIABATIC CONSTANT-PRESSURE CYCLE

a) Without Utilization of the Exhaust Heat

Figure 1 shows the theoretical PV and TS diagrams of a cycle with adiabatic compression and expansion strokes between the pressures $p$ and $p_0$.

At state 1 air under pressure $p_0$ is drawn in and compressed to pressure $p$ (stroke 1-2). The air is then heated along the isobar 2-3 and then expands adiabatically in a turbine along 3-4. At 4 heat is being removed and the volume decreases at constant pressure $p_0$ to the value at 1 (stroke 4-1). A similar closed cycle during which the air remains within the machine also lies at the basis of the work process of hot and cold air machines.

The actual process deviates from the one described because of a number of losses that occur and also because of the following conditions:

1. The heat supplied is by combustion in the air and the gas constant, the exponent $\kappa$, and the specific heat vary.

2. The air after expansion exhausts to the atmosphere and fresh air is drawn in (open process).

If $L_t$ is the power of the turbine and $L_v$ that of the compressor, the power delivered is

$$L = L_t - L_v$$  \hspace{1cm} (1)
When the heat required to increase the temperature from $t_2$ to $t_3$ is denoted by $Q_z$, the efficiency is

$$\eta = \frac{A(L_t - L_f)}{Q_z} \quad (2)$$

where $A$ is the mechanical equivalent of heat. For adiabatic compression and expansion, we may write

$$\eta = \frac{c_p[(T_3 - T_4) - (T_2 - T_1)]}{c_p(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad (3)$$

a result which is also directly obtained from the TS diagram. Since the compression and expansion occur between the same pressure limits, it is also possible from the equation for the adiabatic to express the temperature ratio in terms of the pressure ratio. We thus have

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \left(\frac{P}{P_0}\right)^{\frac{K-1}{K}} = \alpha \quad (4)$$

from which are derived the equations

$$T_1 = \frac{1}{\alpha} T_2 \quad \text{and} \quad T_4 = \frac{1}{\alpha} T_3$$

There is thus obtained for the efficiency of the adiabatic process without losses

$$\eta_a = 1 - \frac{1}{\alpha} = 1 - \left(\frac{P_0}{P}\right)^{\frac{K-1}{K}} \quad (5)$$

A table for the value of $\alpha$ is found in "Hütte" I (26th ed.) p. 519.

It may be pointed out that the theoretical efficiency of a cycle for gas or Otto engines according to figure 2 has the same value.

The theoretical efficiency of a combustion turbine, according to equation (5), depends only on the pressure ratio and thus not on the absolute values of the pressures and the temperatures.

In figure 3 curve a shows the variation of $\eta_a$ with the pressure ratio. The theoretically attainable
efficiencies must therefore be considered as favorable. In the same figure is plotted the efficiency of a theoretical process with the steam cycle shown in figure 4 between the same pressure limits, a feed water temperature of 150°C and steam temperature of 450°C commonly used (curve b). It may be seen that the theoretical efficiency of the combustion turbine cycle is considerably higher than that of the steam cycle. Even with a pressure ratio of 50, only about 26 percent of the heat is utilized with the steam cycle. This fact, which is not often considered, is to be ascribed mainly to the large latent heat of the water necessary for the vaporization and pressure generation but only a small part of which is converted into mechanical work. The generation of pressure in the steam boiler is thus a process involving large losses, since the required quantities of heat are largely lost through the nature of the work process.

In spite of this, the efficiency of steam installations is still better than that of the combustion turbines built thus far, and the reason is to be found in the fact, as will be explained more in detail later, that with these turbines the losses through the auxiliary machines are in far greater proportion than in the case with steam installations.

b) With Full Exhaust Heat Utilization

In the process thus far considered, the exhaust gases left the turbine with temperature $T_4$ (fig. 1). If this temperature lies above $T_2$, the air after compression may be preheated with the aid of the exhaust heat and the efficiency thus improved. The heat supplied $Q_2$ will then be reduced by the amount $c_p(T_4 - T_2)$. There is thus obtained corresponding to equation (3)

$$\eta = \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_2) - (T_4 - T_2)} = 1 - \frac{T_2 - T_1}{T_3 - T_4} \quad (6)$$

By use of equation (4), there is obtained

$$\eta_a' = 1 - \frac{T_2}{T_3} = 1 - \alpha \frac{T_1}{T_3} \quad (7)$$

This efficiency depends on the pressure ratio and on the
temperature limits within which the process takes place. In figure 3 \( \eta_a' \) is plotted for the temperatures \( t_3 = 600^\circ, 700^\circ, \) and \( 800^\circ \) C and 15\(^\circ\) initial temperature (curves c, d, and e). The curves run partly above and partly below a, which they intersect at the points I, II, and III. With increasing pressure ratio, \( \eta_a' \) decreases since the temperature at the end of compression increases and the gain in heat from the exhaust is reduced. Finally \( T_4 \) drops below \( T_2 \) and the heat transfer becomes disadvantageous because the exhaust gas is then heated by the compressed air.

As may be seen, considerably large efficiencies may be attained in this manner. With smaller pressure ratios even better heat utilization is possible than is attainable with the normal explosion process. With the Diesel engine such large efficiencies are obtained only with very high compression ratios.

In the case of the steam cycle, such efficiencies can not be attained even with ideal assumptions because only a small amount of heat can be regained from the exhaust, and thereby the theoretical process improved.

c) Losses Taken into Account

The actual cycle deviates from the ideal because of the following factors:

a) The compression does not take place adiabatically but is associated with losses by heat transfer and throttling. If \( \eta_V \) is the efficiency of the compressor and \( L_V \) the theoretical work without taking account of the bearing losses, the work done is

\[
L_V' = \frac{1}{\eta_V} L_V
\]  \hspace{1cm} (8)

b) The expansion similarly does not occur without losses. The power developed by the turbine is therefore

\[
L_t' = \eta_t L_t
\]  \hspace{1cm} (9)
c) The utilization of the available exhaust heat is not complete because of the imperfection of the heater, the efficiency of which is denoted by $\eta_w$.

d) On account of defective combustion and radiation, the fuel energy is not completely utilized by the working gas. If the combustion-chamber efficiency is denoted by $\eta_b$, an amount of heat $Q_z' = \frac{1}{\eta_b} Q_z$ must be produced ($Q_z$ is useful heat).

e) There are losses in the drive, taken care of by an efficiency $\eta_g$; the losses through the bearing friction in the turbine and the compressor being conveniently taken into account by an added term.

f) On account of the flow losses, the pressure drop to be delivered by the compressor is increased while the drop producing useful work is decreased.

g) The specific heats depend on the temperature and, as previously mentioned, on the amount of fuel supplied, so that, strictly speaking, $c_p$ does not drop out from equations (3) and (6). Moreover, it is to be noted that the gas constant $R$ and also the exponent of the working gas are changed by the combustion.

If, for the moment, the effects of points f) and g) are neglected, the efficiency of the actual cycle, with the magnitudes introduced above, will be

$$\eta = \eta_g \left( \frac{\eta_t \ A \ L_t - \frac{1}{\eta_v} A \ L_v}{\frac{1}{\eta_b} Q_z} \right)$$

In the computation of $Q_z$, account is to be taken of the fact that because of the incomplete compression the temperature of the air rises above the temperature $T_2$ at the end of compression and that corresponding to the reduced utilization of the heat in the turbine, the temperature $T_4$ after the expansion increases.
If \( T_4' \) is the temperature at the end of the actual expansion, its value may be computed from the equation

\[
\frac{T_3 - T_4'}{T_3 - T_4} = \eta_t
\]

from which is obtained

\[
T_4' = T_3 - \eta_t (T_3 - T_4)
\]  \( (11a) \)

Making use of equation (4), we obtain

\[
T_4' = T_3 \left[ 1 - \eta_t \left( 1 - \frac{1}{\alpha} \right) \right] = T_3 (1 - \eta_a \eta_t)
\]  \( (12) \)

In a similar manner there is found

\[
T_2' = T_2 \left( \frac{1}{\alpha} + \frac{\eta_a}{\eta_v} \right)
\]  \( (13) \)

With the above values \( Q_z \) in equation (10) becomes, on introducing the efficiency of the exhaust heat utilization

\[
Q_z = c_p \left[ (T_3 - T_2') - \eta_w (T_4' - T_2') \right]
\]

\[
= c_p \left[ T_3 - T_2' (1 - \eta_w) - \eta_w T_4' \right]
\]  \( (14) \)

In the above equation 1 - \( \eta_w \) is the loss \( V \) of the exhaust heater. By substitution of \( T_2' \) and \( T_4' \) there is obtained finally

\[
\omega_z = c_p \left[ T_3 (1 - \eta_w (1 - \eta_t \eta_a)) - T_2 V \left( \frac{1}{\alpha} + \frac{\eta_a}{\eta_v} \right) \right]
\]

\[
= c_p \left( T_3 u - T_2 w \right)
\]  \( (15) \)

where

\[
u = 1 - \eta_w (1 - \eta_t \eta_a) \quad \text{and} \quad w = V \left( \frac{1}{\alpha} + \frac{\eta_a}{\eta_v} \right)
\]  \( (16) \)

For the numerator of equation (10) there is obtained

\[
c_p \left[ \eta_t (T_3 - T_4) - \frac{1}{\eta_v} (T_2 - T_1) \right] = c_p \eta_a (\eta_t T_3 - \frac{1}{\eta_v} T_2)
\]  \( (17) \)

With the above values there is found
where the effects of \( \eta_t, \eta_v, \) and \( \eta_w \) are combined in the magnitude \( \gamma_a \). For \( \eta_w = 0 \), there is obtained

\[
\eta = \eta_g \eta_b \eta_a \frac{T_3 \eta_t - T_2 \frac{1}{\eta_v}}{T_3 - T_2 \left( \frac{1}{\alpha} + \frac{1}{\eta_v} \right)}
\]  

(19)

and for \( \eta_w = 1 \)

\[
\eta = \eta_g \eta_b \eta_a \frac{T_3 \eta_t - T_2 \frac{1}{\eta_v}}{T_3 \eta_t \eta_a}
\]  

\[
= \eta_g \eta_b \left( 1 - \frac{T_2}{\eta_t \eta_v \eta_3} \right)
\]  

(20)

It is possible, of course, to determine \( \eta \) also with the aid of an IS diagram. Such a method has, however, the disadvantage of having the dependence of the efficiency on each of the magnitudes less clearly brought out than is the case with equation (18).

Because of the large number of variables \( \eta \) cannot be represented on a single curve sheet. In figures 5 to 9 \( \eta \) is plotted as a function of the inlet temperature in the turbine for various values of \( \eta_w \) and an air temperature of 150°C. For the sake of simplicity \( \eta_b \) and \( \eta_g \) have here been set equal to 1, and for the same reason the efficiency of the compression and expansion are assumed to be the same. The curves show that the behavior of the actual gas turbine deviates considerably from that of the theoretical. For example, for

\[
\eta_t T_3 = \frac{1}{\eta_v} T_2 \quad \text{or} \quad \eta_t \eta_v = \frac{T_2}{T_3}
\]

we have \( \eta = 0 \). For low efficiencies \( \eta_t \) and \( \eta_v \), a high gas temperature is therefore necessary even for idling independently of the efficiency of the heat exchanger.

It is seen also that \( \eta \) considerably increases with
the gas temperature and that for each value of $t_3$ and $\eta_t$ and $\eta_V$, for a definite pressure ratio that need not correspond to the maximum, there is an optimum value. The value of $\eta$ like that of $\eta_a$ depends only on the pressure ratio and not on the absolute pressure. For $\eta_W = 1$, $\eta$ decreases with increasing pressure ratio. In this case therefore, the smallest possible pressure ratio should be striven for although this will be associated with increased size of the machine.

The value of $\eta$ depends naturally very much on that of $\eta_W$. The greatest effect, however, is that of the losses in compression and expansion. Without a heat exchanger a very large operating temperature must be reckoned with and particularly good efficiencies must be obtained to attain the economy of a modern steam installation. It should be clear at any rate from figure 9 why it has not been possible to attain a satisfactory fuel consumption with most gas turbines built thus far and why only very recently has any improvement been obtained.

To bring out more clearly the effect of the individual efficiencies $\eta_t$, $\eta_V$, and $\eta_W$ on $\eta$, the magnitude $\gamma_a$, which depends only slightly on $\eta_a$ for the previous assumptions, was plotted against $t_3$ in figures 10 to 12. A value of $\gamma_a > 1$ means improvement of $\eta$ over $\eta_a$ through utilization of the exhaust, a gain which with smaller values of $\eta_t$ and $\eta_V$ is first attained at higher values of $\eta_W$. Particularly to be noted is the drop in $\gamma_a$, i.e., the increase of the losses with increasing pressure ratio. This explains the intersecting of the $\eta$-curves in figures 5 to 9, since $\gamma_a$ becomes smaller with increasing pressure ratio while $\eta_a$ improves.

In the computation no account was taken of the pressure losses to the heat exchanger. These losses, which affect the picture considerably, will be discussed later.

The pressure losses in the combustion chamber, the piping, the heat exchangers, etc., for a fixed theoretical pressure ratio lead to an increase in the compressor power and a decrease in the turbine power. This effect is best taken into account by a corresponding impairment in $\eta_t$ or $\eta_V$, these losses being included in the work cycle of each machine. If the pressure ratio of the cycle is, for example, $p/p_0$, the compressor work without pressure loss is
L = \frac{1}{\eta_V} \frac{G \cdot R \cdot T}{p_0} (\frac{p^{K-1}}{p_o^K} - 1)

and the actual work

L' = \frac{1}{\eta_V} \frac{G \cdot R \cdot T}{p_0} (\frac{p^{K-1}}{p_o^K} - 1)

which can also be written

L' = \frac{1}{\eta_V} \frac{G \cdot R \cdot T}{p_0} (\frac{p^{K-1}}{p_o^K} - 1)

There is then obtained

\frac{\eta_V'}{\eta_V} = \frac{1}{\frac{p}{p_0}} \frac{p^{K-1}}{p_o^K} - 1

(21)

where \ p' = p + \Delta p; \ \Delta p \ denotes \ the \ pressure \ loss. \ Setting

\frac{\Delta p}{p} = a

we obtain

p' = p(1 + a)

(22)

The corresponding relation holds for the turbine efficiency.

In figure 13 \ \eta'/\eta \ is plotted against \ a \ for \ various \ pressure \ ratios, \ so \ that \ the \ impairment \ of \ \eta_t \ and \ \eta_V \ due \ to \ the \ pressure \ losses \ may \ be \ read \ off \ from \ the \ diagram. \ The \ higher \ the \ pressure \ ratio \ the \ smaller \ is \ the \ effect \ of \ the \ flow \ losses, \ although \ it \ should \ be \ noted \ that \ these \ losses \ increase \ with \ increasing \ absolute \ pressure. \ The \ value \ of \ a \ must \ be \ determined \ from \ conditions \ applying \ in \ each \ case.

To bring out the significance of the pressure losses for the efficiency of the gas turbine, \ \eta \ was plotted as a function of \ \Delta p \ in figure 14 for the following assumptions:
Curve a: \( \eta_t = 0.8, \eta_v = 0.72, \eta_w = 0.75, \eta_b = 0.95 \)
\( \eta_g = 0.98, \frac{P}{P_0} = 4, t_3 = 650^\circ C, t_1 = 15^\circ C \)

Curve b: \( \eta_t = 0.85, \eta_v = 0.82, \eta_w = 0.75, \eta_b = 0.95 \)
\( \eta_g = 0.98, \frac{P}{P_0} = 4, t_3 = 650^\circ C, t_1 = 15^\circ C \)

Curve a corresponds approximately to the efficiencies generally attained at the present time, whereas for curve b values of \( \eta_t \) and \( \eta_v \) were assumed that may be considered as attainable in the near future. The point A corresponds to a value of \( \eta \) without pressure losses and \( \eta_w = 0 \). It may be seen that even with common present-day efficiencies the effect of the pressure losses is very great. For a flow resistance of 0.175 atm - corresponding to 129 mm Hg or 4.4 percent of the pressure ratio - the advantage of the heat exchanger is lost while for \( \Delta p \sim 0.55 \text{ atm} \) - corresponding to 405 mm Hg or 13.7 percent of the pressure ratio - the output of the system becomes zero. The relations are very much better for curve b where a greater efficiency is assumed. It follows that the use of a heat exchanger promises to be of advantage only for high individual efficiencies of turbine and compressor because of the unavoidable pressure losses.

In the case of a steam plant, however, the pressure losses in the feed water circuit are of secondary importance as regards economy. With the efficiencies of the present-day auxiliary machines an entirely reasonable fuel consumption is therefore attainable.

As mentioned under point g) in section 2 c, the change in the gas constant through the combustion and the dependence of the specific heats on the temperature also have an effect on the actual cycle. This effect will now be briefly considered.

The gas constant and the volume to be handled by the turbine are slightly increased by the combustion products. In addition, the specific heat of the gases is increased, so that for heating to a definite temperature a large amount of heat is required. It may be said that to a first approximation the two effects counterbalance each other.
While the specific heats of the working gas at the pressures occurring in a combustion turbine may be assumed independent of the pressure, it is necessary to take account of their change with temperature. Since the initial and final temperatures are generally not very different, this effect is largely eliminated and need not be taken into account in the computation.

In order to check the admissibility of all the simplifications, the efficiencies of a cycle with and without exhaust heat utilization will be determined by computation and by the use of the IS tables of Pflaum and compared with each other.

Example a):

\[ \eta_w = 0, \quad \eta_t = 0.85, \quad \eta_v = 0.82, \quad \eta_b = 0.95 \]
\[ \eta_e = 0.98, \quad t_3 = 650^\circ C, \quad \frac{P}{P_0} = 4 \]

Pressure loss between compressor and turbine, 0.1 atm
Pressure loss between turbine and gas outlet negligible.

With the aid of figure 13, there is obtained \( \eta_v = 0.802 \).

From equation (18) there is computed \( \eta = 16.7 \) percent while from the IS tables (larger scale diagrams were used), there is similarly found the value \( \eta = 16.7 \) percent.

Example b):

\[ \eta_w = 0.75 \]

Pressure loss between compressor and turbine, 0.2 atm
Pressure loss between turbine and gas outlet, 0.1 atm

There is then obtained \( \eta_t = 0.83 \) percent and \( \eta_v = 0.785 \) percent.

With these values there is obtained through computation 22.2 percent and from the chart 22.3 percent.

The accuracy of the computation is thus entirely satisfactory.
3. The Constant-Pressure Cycle with Isothermal Compression and Adiabatic Expansion

a) Without Exhaust Heat Utilization

Figure 15 shows the PV and TS diagrams of this cycle. The compression from $p_0$ to $p$ occurs at the constant temperature $T_o$ along the curve 1-2. The remaining working strokes are similar to those described in section 2a. The efficiency is

$$
\eta_{is,a} = \frac{A(L_a - L_{is})}{c_p(T_3 - T_4) - AR T_o \ln \frac{p}{p_0}}
$$

Making use of equation (4) and the relation

$$
\frac{AR}{c_p} = \frac{c_p - c_v}{c_p} = \frac{k - 1}{k}
$$

we obtain

$$
\eta_{is,a} = \frac{T_3 \eta_a - T_o \ln \alpha}{(T_3 - T_0)}
$$

In contrast to $\eta_a$ (equation (5)) $\eta_{is,a}$ depends on the pressure ratio and on the temperature limits within which the cycle operates. Whereas $\eta_a$ steadily increases with increasing $p/p_0$, the value of $\eta_{is,a}$ possesses a maximum at

$$
\frac{p}{p_0} = \left(\frac{T_3}{T_0}\right)^{\frac{k}{k-1}}
$$

where it assumes the value

$$
1 - \frac{T_o \ln \frac{T_3}{T_o}}{T_3 - T_o}
$$

The values of $\eta_{is,a}$ are plotted in figure 16 against the pressure ratio for the temperatures $T_3 = 500^\circ$, $600^\circ$, and $700^\circ$ C and $T_o = 15^\circ$ C. For comparison there were
added the curves for $\eta_a$ and for the efficiency of a steam turbine working on the cycle of figure 3. As may be seen, the theoretical efficiency of this process in spite of the reduced work of compression is smaller than that of the adiabatic process, the reason for which is found in the heat abstraction during the compression.

b) With Complete Heat Utilization

In the most favorable case the amount of heat removed from the exhaust gas and supplied to the air after compression is $c_P(T_4 - T_0)$. We then have

$$Q_z' = c_P(T_3 - T_4)$$

and

$$\eta_{is,a'} = 1 - \frac{T_0}{T_3} \ln \alpha$$

For $\frac{P}{P_0} = 1$ there is obtained

$$\eta_{is,a'} = 1 - \frac{T_0}{T_3}$$

This value corresponds to the efficiency of the Carnot cycle between the temperatures $T_0$ and $T_3$.

The values of $\eta_{is,a'}$ are similarly plotted in figure 16 for the same temperatures. For $\frac{P}{P_0} = 1$ we have $\eta_{is,a'} = \eta_a'$. The curves $\eta_{is,a'}$ drop much more slowly with increasing pressure ratio than the curves $\eta_a'$ in figure 3, so that any harmful effect of the heat exchange which occurs when $T_4$ drops below $T_0$ becomes appreciable only at higher pressure ratios.

c) With the Losses Taken into Account

The losses arising in the actual cycle are the same as those already described in section 2c. For an incomplete isothermal compression two cases are to be distinguished:
1. The temperature remains unchanged, i.e., the heat produced through the non-reversible changes in state is conducted away.

2. A temperature rise occurs during the compression.

In the first case there is obtained with the notation of equations (10) to (20) the relation

\[
\eta = \eta_g \eta_b \frac{\eta_t A L_a - \frac{1}{\eta_v} A L_i s}{Q_z - \eta_w c_p (T_4' - T_0)}
\]

\[
= \frac{T_3 \eta_t \eta_a - \frac{1}{\eta_v} T_0 \ln \alpha}{T_3 u - T_0 V}
\]

(31)

where the heat losses through incomplete combustion and radiation are combined in \( \eta_b \) and the mechanical losses referred to the power delivered combined in \( \eta_g \); \( u \) is equal to \( 1 - \eta_w (1 - \eta_t \eta_a) \) and \( V \) is equal to \( (1 - \eta_w) \).

Without exhaust heat utilization (\( \eta_w = 0 \)) there is obtained

\[
\eta = \eta_g \eta_b \frac{T_3 \eta_t \eta_a - \frac{1}{\eta_v} T_0 \ln \alpha}{T_3 - T_0}
\]

(32)

and with complete exhaust heat utilization (\( \eta_w = 1 \))

\[
\eta = \eta_g \eta_b \left( 1 - \frac{T_0 \ln \alpha}{T_3 \eta_t \eta_a \eta_v} \right)
\]

(33)

With the notation and assumptions of figures 5 to 9 (\( \eta_g, \eta_b = 1; \eta_t = \eta_v \)) the values for \( \eta \) computed from equations (31) to (33) were plotted in figures 17 to 21.

For

\[
T_3 \eta_t \eta_a = \frac{1}{\eta_v} T_0 \ln \alpha
\]

or

\[
T_3 = \frac{1}{\eta_v \eta_t \eta_a} T_0 \ln \alpha
\]

(34)

we have \( \eta = 0 \).
Comparison of figures 17 to 21 with figures 5 to 9 gives the following results:

1. The combustion turbine with isothermal compression is less sensitive as regards the operating temperature since the start of useful output \((\eta = 0)\) occurs at smaller values of \(t_3\).

2. Since the curves \(p = \text{const}\) lie closer together it follows that the effect of the pressure ratio is also smaller.

3. The efficiencies are generally somewhat better.

4. The increase in \(\eta\) through utilization of the exhaust heat is greater.

Summarizing, we may say that the cycle with isothermal compression would give more favorable fuel consumptions than that with adiabatic compression.

Figures 22 to 24 show \(\gamma_{is,a} = \frac{\eta}{\eta_{is,a}}\) as a function of \(t_3\) and \(\frac{P}{P_o}\).

A comparison with the corresponding curves of the adiabatic process shows that \(\gamma_{is,a} > \gamma_a\), i.e., that with isothermal compression the exhaust heat is better utilized since the temperature difference between \(t_3\) and \(t_0\) is greater.

The effect of pressure losses on \(\eta\) is taken into account, as explained in section 2c, through a corresponding decrease in \(\eta_t\) and \(\eta_V\). The value \(\eta'/\eta\) for an isothermal process is obtained in the following manner. We have

\[ L_{is} = R T_0 \ln \frac{P}{P_o} \text{ mkg/kg} \]

Again denoting by \(\eta'\) the efficiency with the losses included, we may set up a relation corresponding to equation (21)

\[ \frac{\eta'}{\eta} = \frac{\ln \frac{P}{P_o}}{\ln \frac{P'}{P_o}} = \frac{1g \frac{P}{P_o}}{1g \frac{P'}{P_o}} \]

where \(P' = P(1 + a)\).
In figure 25 this value is plotted against $a$ for various pressure ratios. Comparison with figure 13 shows that in the case of the isothermal process the effect of pressure losses on $\eta_v$ and $\eta_t$ is smaller.

To bring out the importance of the pressure losses in piping, heat exchangers, etc., the curve $\eta = f(\Delta p)$ was drawn in figure 26 for the following assumptions:

Curve a: $\eta_t = 0.8$, $\eta_v = 0.72$, $\eta_w = 0.75$, $\eta_b = 0.95$

$$\frac{P}{P_0} = 4$$

$t_3 = 650^\circ C$, $t_1 = 15^\circ C$

Curve b: $\eta_t = 0.85$, $\eta_v = 0.82$, $\eta_w$, etc., as with a.

(The same values were assumed as in section 2c.)

At point A the value of $\eta$ is the same as would be obtained for a work cycle without heat exchanger and without pressure losses. In this case therefore the improvement of the efficiency through exhaust-heat utilization is of the same magnitude as its impairment through the pressure losses. As shown by a comparison with the curves c and d of the adiabatic process, the pressure losses arising from the use of a heat exchanger are less serious with isothermal than with adiabatic compression.

The computed example a) in section 2c gives for isothermal compression by the use of IS tables an efficiency of 17.2 percent while the value obtained by computation is 16.9 percent. For example b) the corresponding figures are 29.4 percent and 28.8 percent. The deviations arise mainly from the inaccurate determination of the isothermal work $T_0 \Delta S$ from the IS diagram owing to the sharp angles of intersection of the curves $p = \text{const}$ and $T = \text{const}$. The examples show, however, that the computation of the efficiencies by equation (31) in spite of factors neglected ($c_p$ not constant, effect of combustion) can be carried out with sufficient accuracy.

The underlying assumption in the derivation of equations (31) to (35) was that the compression stroke was carried out under constant temperature. In case the temperature at the end of compression assumes, because of insufficient cooling, a value $T_0' > T_0$, there is obtained in place of equation (31) the relation
4. THE CONSTANT-PRESSURE CYCLE WITH ISOTHERMAL COMPRESSION AND EXPANSION

The cycle with isothermal expansion has frequently been suggested in the literature. To bring about such a change of state, it is necessary to supply the gas with a quantity of heat equal to that removed in doing work and this may be conveniently effected by the burning of fuel during the expansion. This process can be carried out with relative ease, particularly at high temperatures, because in this case the fuel would be burned without any special ignition. Other authors (reference 2) propose arranging a combustion chamber behind each ring of blades of the turbine whereby in place of the isotherms a tooth-shaped broken curve will be obtained consisting of pieces of adiabatics and isobars.

Although this cycle is more difficult to realize than those discussed previously, a brief discussion will be given since the cycle promises notable advantages. Figure 27 shows that work cycle in the TS diagram. (The cycle with adiabatic compression and isothermal expansion will not be considered further, since no gain in information is to be expected.)

The efficiency without losses and without exhaust-heat utilization is

\[ \eta_{is} = \frac{A(L_{is,t} - L_{is,v})}{Q_z} \]

Since the quantity of heat to be added during the expansion is equal to \( A L_{is,t} \), we have \( Q_z = A L_{is,t} + c_p(T - T_0) \)

There is then obtained
With complete exhaust-heat utilization, we have
\[ \eta_{is} = 1 - \frac{T_0}{T} \]  \hspace{1cm} (38)

The efficiency of such a process is thus equal to that of a Carnot cycle between the same temperature limits.

With the losses taken into account, we have
\[ \eta = \eta_g \eta_b \frac{\eta_t A L_{is,t} - \frac{1}{\eta_v} A L_{is,v}}{\eta_t A L_{is,t} + (1 - \eta_w) c_p(T - T_0)} \]

In computing the heat added, we take account of the fact that for an isothermal change of state the heat added is reduced in proportion to the losses that occur. There is obtained, finally, for the efficiency
\[ \eta = \eta_g \eta_b \frac{\eta_t T - \frac{1}{\eta_v} T_0}{\eta_t T + V \frac{\kappa}{\kappa - 1} \ln \frac{P}{P_0}} \]  \hspace{1cm} (39)

In figures 28 and 29 \( \eta \) was plotted for \( \eta_w = 0 \) and 1 under the same assumptions as the other plotted curves. It may be seen that the maximum attainable efficiencies lie somewhat above those for the process last discussed. Particularly to be noted is the considerably less dependence of turbine and compressors on the separate efficiencies. For \( \eta_w = 1 \), \( \eta \) does not depend on the pressure; for \( \eta_w = 0 \), \( \eta \) improves with increasing pressure. The start of delivery of external work (\( \eta = 0 \)) has shifted considerably toward the lower temperatures. The efficiency for \( \eta_w = 0 \) is only slightly smaller than for the previous
process. This may be explained by the fact that with the isothermal process the heat supplied is converted into mechanical work at the highest thermal state, that is, under the most favorable relations. In utilizing the exhaust heat, the purely isothermal process is definitely superior to the one previously discussed.

The effect of pressure losses on \( \eta_t \) and \( \eta_v \) can be determined with the aid of figure 25. A curve chart for this work process corresponding to figure 26 would show a still smaller drop of the efficiency due to pressure losses.

The following may be said in summary on the processes discussed. The dependence of the efficiencies of all the processes on the separate efficiencies is considerable. The use of heat exchangers is of advantage only if no larger pressure losses are thereby introduced. The impairment in \( \eta \) through pressure losses with the various cycles described decreases in the following order: adiabatic, adiabatic-isothermal, isothermal. The idling temperature, that is, the temperature at which the mechanical work begins to be developed decreases in the same order. With increasing work drop, the efficiency is partly improved and partly impaired.

The work relations with other constant-pressure cycles proposed, for example, with adiabatic expansion down to below atmospheric, then cooling and compression of the gases to atmospheric pressure may be rapidly computed without any difficulty from the examples given, and no discussion of these will therefore be given.

5. THE WORK CYCLE OF AN EXPLOSION TURBINE WITH
ISOThERMAL COMPRESSION AND ADIABATIC EXPANSION
WITH AND WITHOUT EXHAUST HEAT UTILIZATION

For comparison with the constant-pressure turbine two work cycles of the explosion turbine will be discussed. Only the process most generally applied of isothermal compression with complete removal of the heat arising through the losses will be considered so that the temperature remains constant in spite of the work losses during compression.
The efficiency is most readily derived from a TS diagram as shown in figure 30. The expansion in the turbine with losses taken into account is generally represented by the line 3-4'. The lost work in the isothermal compression can be brought out in the diagram as follows: The work of compression which is increased by the lost work is first assumed to be performed without losses so that a pressure $p_2'$ higher than the originally attained end pressure $p_2$ is obtained. The gas pressure is then allowed to drop by a throttling process to the original pressure. The higher compression is represented on the TS diagram by the line 2-2', which is at the same time a throttle curve from 2' to 2. The area under the curve 1-2' is therefore equal to the work in an isothermal compression with the losses taken into account, and at the same time it represents the quantity of heat to be removed. The efficiency, as is known, is equal to

$$\eta = 1 - \frac{Q_a}{Q_z}$$

where $Q_a$ is the area under the curve 2'-1-4' while the quantity of heat added $Q_z$ is represented by the area under curve 3-2. The efficiency is obtained from the equation

$$\eta_{is, a} = \eta_g \eta_b \left[ 1 - \frac{\kappa - 1}{\eta_v} \frac{T_0 \ln \frac{p_2}{p_1} + \kappa (T_4' - T_0)}{T_3 - T_0} \right]$$

which for the pressure ratio $p_2 : p_1 = 1$ simplifies to

$$\eta_{is, a} = \eta_g \eta_b \left[ 1 - \kappa \frac{T_4' - T_0}{T_3 - T_0} \right]$$

In figure 31 the curves are again plotted for various separate efficiencies of turbine and compressor for $\eta_g$ and $\eta_b = 1$ and $T_0 = 288$. Comparison with the corresponding constant-pressure cycle (fig. 21) shows that the efficiencies come out less favorable, and only at temperatures of 2000° do they assume values such as those attained at 800° with the constant-pressure cycle.

With full utilization of the exhaust heat there is obtained the TS diagram shown in figure 32. It is here assumed that the transfer of the heat from the temperature
to the working gas of temperatures $t_2$ and $t_0$, respectively, occurs in a heat exchanger at constant pressure. From the diagram the following equation is obtained

$$\eta_{i,s,a} = \eta_g \eta_b \left[ 1 - \frac{\kappa - 1}{\eta_v} \frac{T_0 \ln \frac{P_2}{P_1}}{(T_3 - T_4')} \right]$$  \hspace{1cm} (43)

where the temperature $T_4'$ is obtained from the equations

$$\frac{T_4'}{T_3} = \frac{P_2}{P_3}$$  \hspace{1cm} (44)

and

$$T_4' = T_3 \left[ 1 - \eta_t \left( 1 - \frac{P_1}{P_3} \right) \frac{\kappa - 1}{\kappa} \right]$$  \hspace{1cm} (45)

Figure 33 shows the curves for $\eta$. Comparison with the corresponding curves of the constant-pressure process (fig. 17) similarly shows the advantage of the latter.

It is to be noted, however, that the possibilities of cooling the blades with the periodic explosion process permit higher blade temperatures than can be attained generally in the constant-pressure turbine. The turbine efficiencies should be considerably lower, however, with the rapidly alternating gas velocities than the efficiencies attained with the constant-pressure operation. Aside from this, the periodic operation involves an increase in the dimensions of the turbine since the gas volumes are cut off at certain times, that is, are at rest, whereas with the constant-pressure turbine the gases flow through the entire system with high velocity. For installations where low weight is essential, the explosion-type turbine should therefore be less suitable. As shown by the operation results of the Holzwarth gas turbine, it is also of little advantage as regards efficiency.

6. GENERAL PRACTICAL CONSIDERATIONS

In the derivation of the equations above and the discussion of the efficiencies, no discussion was given of the possibility of the practical design of a combustion turbine. Such possibility depends at the present time on the extent of further development in engineering and
materials. With the aid of the curves shown, a continuous check can be made on the attainable efficiencies and possible fuel consumptions.

With non-stationary power plants not only the efficiencies but also the weight-to-power ratios of the turbines are of importance. As follows from the equations below, the outputs of the turbine with compressor are generally large in comparison with the output of the turbine, so that small weights must be striven for to obtain a favorable over-all weight. If the power delivered by the system is denoted by \( L \) and the power of the compressor by \( L_v \), then the power \( L_{s,v} \) referred to the compressor is given by the equation

\[
L_{s,v} = \frac{L}{L_v} = \frac{\eta_t \frac{L_t}{\eta_v} - \frac{1}{\eta_v} L_v}{\frac{1}{\eta_v} L_v} = \eta_t \frac{L_t}{\eta_v} \frac{L_t}{L_v} - 1 \quad (46)
\]

With adiabatic compression and expansion, there is obtained the equation

\[
L_{s,v} = \frac{T_3}{T_1} \frac{\eta_t \eta_v}{\alpha} - 1 \quad (47)
\]

Similarly there is obtained for the power \( L_{s,t} \) referred to the turbine the expression

\[
L_{s,t} = \frac{L}{L_t} = 1 - \frac{T_1}{T_3} \frac{\alpha}{\eta_t \eta_v} \quad (48)
\]

As follows from the formulas, for a given output of the system the output of turbine and compressor increases with decreasing temperature, lower separate efficiencies, and increasing pressure ratio. For the numerical example a) given previously, there is obtained, for example, for \( L_{s,v} \) the value 0.25 and for \( L_{s,t} \) 0.20. For 1 horsepower delivered output, it is therefore necessary with the chosen relations that the turbine develop 5 horsepower and the compressor, 4 horsepower with the corresponding weights.

For the cycle with isothermal compression and adiabatic expansion, there are obtained in a similar manner these equations:
from which for the above example $L_{s,v}$ is found to have the value 0.53 and $L_{s,t}$ the value 0.346. The isothermal compression therefore has a favorable effect on the weight-to-power ratio although the weight of the heat exchanger must still be taken into account.

Compressors of the centrifugal type are those that will enter mainly into consideration since the reciprocating-type blowers are too heavy. At the present time the most widely used compressors are the centrifugal superchargers, which are not cooled at the lower pressure ratios and at the higher ratios are designed with jacket, blade, or intercooling. The cooling through water injection occasionally employed cannot, in general, be considered for movable power plants on account of the water loss. The delivery pressure of an impeller in commercial compressors of this type is about 1.2 atmospheres. For larger pressures a multistage design must be resorted to with a corresponding increase in structural length and weight. The weight-to-power ratio of large fixed compressors with cooling amounts at the present time to about 10 kg/hp and over, so that the compressor for the example considered would, according to equation (47) weigh about 40 kilograms per horsepower delivered by the combustion turbine. The best efficiencies attained with such machines referred to the isothermal process lie at 70 percent.

With single-stage compressors as applied to scavenging or charging blowers, it was possible recently to obtain more favorable relations, particularly in the aviation industry. Thus, for example, the delivery pressure of a DVL compressor for a peripheral speed of the impeller of about 400 meters per second has been raised to 3 atmospheres and above with measured adiabatic efficiencies of 70 percent. With pressures of about 1.4 atmospheres efficiencies of 80 percent were attained. The unit weight of such machines lies far below 1 kg/hp. Multistage compressors of this type of construction have not yet been tested. It may be doubted whether such good efficiencies will be obtained with these.
With the Velox boiler power plants of the BBC firm and also with their combustion turbines axial compressors are employed, such as are built by the Escher-Wyss firm. The adiabatic efficiencies attained amount to over 80 percent. The delivery pressure of a single stage is restricted, however, at the present state of development and, in general, amounts to about 1.05 to 1.07 atmospheres, so that to attain a definite pressure drop, it is necessary to use many blade wheels, a condition which leads to increased structural length. The development of this compressor is still in a fluid stage. It is to be expected that the application of higher peripheral speeds and a close investigation of the flow at the blades will lead to still greater improvements in the output pressure and efficiency and hence to smaller size and weight. It should be possible in axial compressors to convert such amounts of energy that their unit weights in larger designs amount to only a few hundred grams per horsepower. The possibilities here indicated at any rate point to the axial compressor as the type most suitable for the combustion turbine.

Because of the great effect of the operating temperature on the efficiency of combustion turbines, the latter must be built for as high gas temperatures as possible. These turbines do not differ essentially in their construction from steam turbines. With larger units dual-flow turbines will be found convenient, since the temperatures in these are more uniformly distributed and therefore smaller heat stresses arise. The temperatures of 450° to 500° C used in recent steam plants are sufficient for obtaining satisfactory economy. For the combustion turbine with constant-pressure combustion a turbine temperature of at least 600° will be required. The newer blade materials already possess a sufficiently high endurance strength to make possible operating temperatures of 600° and more, particularly when only short-time operation is required. Development is still in a fluid stage and a further increase in the admissible temperatures may be expected. It is also conceivable that in the course of further progress ceramic materials will be brought to a state of development where they can be applied to high-stressed turbine blades. It will then be possible to raise the operating temperatures considerably. Cooling the blades is difficult with constant-pressure turbines of good efficiency. From the curves given, it may be decided whether a cooling effect at the expense of an impairment of the turbine efficiency promises a gain in
economy or whether it appears more suitable to adopt the explosion-type turbines that are easier to cool.

The weight of the housing can be kept to a lower value as compared with that of the steam turbine, since the gas pressures can be chosen considerably lower than those usual with steam turbines. A special type of construction is necessary, however, to prevent warping. Particularly with turbines which must quickly be put into operation, a piling-on of material such as that represented by the usual flanges of the steam turbine must be avoided as far as possible. It is to be expected that further structural improvements will be obtained through separation of the heat-stressed from the mechanically stressed parts.

The weight-power ratio of large steam turbines of light construction is about 1.5 kg/hp. Since in the case of gas turbines the heavy low-pressure part can, in general, drop out and the weight of the housing on account of the lower gas pressure can be kept down, a ratio of 1 kg/hp may be expected.

Turbines with isothermal expansion have up to the present not been designed. It is to be assumed that a supply of heat by combustion within the blade ring leads to a flow disturbance with a consequent drop in the efficiency. It is not certain, moreover, whether, with the irregular temperature distribution to be expected, the blade materials can withstand the stresses. The mounting of combustion chambers behind each ring of blades should lead to inadmissible pressure losses and operating difficulties. A realization of this type of turbine design, which is particularly favorable from the point of view of efficiency, is therefore as yet impossible.

No essential difficulties are encountered in the design of a reliably operating combustion chamber with the heat-resisting materials at present available. Even disregarding the usual application of ceramic materials as an outer covering and cooling the combustion chamber walls only with air, continuous operation should be possible with proper design. Because of the large effect of pressure losses care is to be taken to obtain as good a flow as is possible in the combustion chamber.

The same is true for the heat exchanger. With the present designs of combustion turbines with heat exchangers this part takes up considerable space, so that with movable
power plants, particularly in the case of airplanes, the required space cannot be made available. In this field further investigation is necessary before heat exchangers are built that also satisfy the weight and size requirements.

The piping of gas-turbine power plants is subject to unusually high temperatures, especially since because of the required small pressure loss and the large gas volumes involved a relatively large diameter is required. Compared with a steam turbine, there is the advantage, however, that the working pressure can be chosen very much smaller and the pipes need be only very short, since the combustion chamber can be arranged near the turbine.

The number of auxiliary machines of such a combustion turbine installation is small. In general, pumps will be required only for the oil circulation and for the fuel. A particular advantage is that, at least with adiabatic compression, no cooling water or cooling air is required except for oil cooling.

SUMMARY

The theoretical and true efficiencies of a combustion turbine cycle with constant-pressure combustion are computed and discussed for the three cases of

1. Adiabatic compression and expansion
2. Isothermal compression and adiabatic expansion
3. Isothermal compression and expansion

The result obtained is that the efficiency depends essentially on the separate efficiencies of the compressor and turbine. Comparison with the type of turbine with explosion combustion shows the advantage lies with the constant-pressure combustion turbine. A brief discussion is given finally of the present state of development of compressor, turbine, and accessories in relation to particular features of the combustion turbine process.

Translation by S. Reiss,
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REFERENCES


Figure 1.- Theoretical PV and TS diagrams of the cycle.

Figure 2.- Theoretical PV diagram of the Otto engine.

Figure 3.- Efficiency of a constant-pressure combustion turbine and of a steam turbine as a function of the pressure ratio.

Figure 4.- TS-diagram of a steam cycle.

Curve a. Theoretical efficiency of the turbine without heat exchanger.
Curve b. Theoretical efficiency of the steam turbine.
Curve c. Theoretical efficiency of the turbine with complete heat exchange at 800° operating temperature.
Curve d. Theoretical efficiency of the turbine with complete heat exchange at 700° operating temperature.
Curve e. Theoretical efficiency of the turbine with complete heat exchange at 600° operating temperature.
Figures 5 to 9.- Efficiency of the combustion turbine as a function of the operating temperature for various degrees of exhaust heat utilization for various pressure ratios and separate efficiencies of compressor and turbine.
Figures 10 to 12:

Plot of $\gamma_a$ as a function of the operating temperature for various degrees of exhaust heat utilizations. Notation the same as for figs. 5-9.
Figure 13.- Variation of $\eta'/\eta$ with $\Delta p$, pressure loss, for various pressure ratios for adiabatic compression or expansion.

Figure 14.- Effect of the pressure losses on the efficiency of the combustion turbine.

Figure 15.- Theoretical PV and TS diagrams of a cycle with isothermal compression and adiabatic expansion.

Figure 16.- Efficiency of a combustion turbine with isothermal compression and adiabatic expansion.
Figures 17 to 21. - Efficiency of a combustion turbine with isothermal compression and adiabatic expansion as a function of the operating temperature for various degrees of exhaust heat utilization for various pressure ratios and separate efficiencies. Notation as for figs. 5-9.
Figures 22 to 24.
Variation of $\gamma$ as a function of the operating temperature for various degrees of exhaust heat utilization.

Figure 32.
TS-diagram of a turbine with explosion cycle with exhaust heat utilization.

Figure 31.
Efficiency of a turbine with explosion cycle with isothermal compression and adiabatic expansion without exhaust heat utilization. Notation of curves same as for figs. 5-9.

Figure 33.
Efficiencies of a turbine with explosion cycle with isothermal compression and adiabatic expansion with heat utilization.
Curve a for example a. " b " " b.
Curves c and d for adiabatic compression.

Figure 25.- Variation of $\eta'/\eta$ with pressure loss $a$ for various pressure ratios for isothermal compression or expansion.

Figure 26.- Effect of pressure losses on the efficiency of the gas turbine with isothermal compression.

Figure 27.- Theoretical TS- diagram of a cycle with isothermal compression and expansion.
Figure 28 and 29. Efficiency of a combustion turbine with isothermal compression as a function of the operating temperature for various degrees of exhaust heat utilization for various pressure ratios and separate efficiencies. Notation the same as for figs. 17-24.

Figure 29.

Figure 30. TS-diagram of a turbine with explosion cycle. = heat added. // heat removed.