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Luftfahrtforschung
Vol. 17, No. 7, July 20, 1940
Verlag von R. Oldenbourg, München und Berlin

Washington
December 1940

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# RECTANGULAR SHELL PLATING UNDER UNIFORMLY DISTRIBUTED HYDROSTATIC PRESSURE*

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## SUMMARY

A check of the calculation methods used by Föppl and Hencky for investigating the reliability of shell plating under hydrostatic pressure has proved that the formulas yield practical results within the elastic range of the material. Föppl's approximate calculation leaves one on the safe side. It further was found on the basis of the marked ductility of the shell plating under tensile stress that the strength is from 50 to 100 percent higher in the elastic range than expected by either method.

## A. INTRODUCTION

Knowledge of the stresses in rectangular shell plating under hydrostatic pressure is important to the airplane designer since he frequently has to deal with such structural-parts. This applies above all to hulls and floats which at take-off and landing or during handling in rough sea are subject to enormous water pressure by the waves. But then it also includes gasoline tanks and gasoline chambers which undergo considerable internal pressure because of high accelerations. Concerning the calculation there are the reports by Föppl (reference 1) and Hencky (reference 2), the practicability and reliability of which are checked against various experimental results.

## B. CALCULATION OF RECTANGULAR SHELL PLATING UNDER HYDROSTATIC PRESSURE

Both authors proceed from the assumption that the

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*"Rechteckige Blechhaut unter gleichmässig verteiltem Flüssigkeitsdruck." Luftfahrtforschung, vol. 17, no. 7, July 20, 1940, pp. 207-210.

skin possesses no flexural stiffness ($J = 0$) and is not strained in the relaxed state. On these premises Föppl has set up differential equations for thin plates with great deflections. His approximate solution published in 1920 contained, however, only the calculation for the square skin. A year later Hencky gave a solution of the differential equations with the help of the method of difference.

## 1. Föppl's Method

The evaluation of the approximate solution for rectangular shell panels (fig. 1) leads to the simple, practical formulas below. Föppl's condition for the validity of the formulas, that the deflection of the plate must be considerably greater than the sheet thickness is probably always complied with by the sheet thicknesses employed in airplane design.

Deflection in panel center (point m):

$$f = n_1 \, a \sqrt[3]{\frac{pa}{Es}}$$

stresses in panel center (point m):

$$\sigma_{z_m} = n_2 \sqrt[3]{p^2 \, E \, \frac{a^2}{s^2}}$$

$$\sigma_{y_m} = n_3 \sqrt[3]{p^2 \, E \, \frac{a^2}{s^2}}$$

stresses on the edge in the center of the short side b of the rectangle (point $r_b$):

$$\sigma_{z_{r_b}} = n_4 \sqrt[3]{p^2 \, E \, \frac{a^2}{s^2}}$$

$$\sigma_{y_{r_b}} = n_5 \sqrt[3]{p^2 \, E \, \frac{a^2}{s^2}}$$

stresses on the long side of the rectangle, a (point $r_a$):

$$\sigma_{z_{r_a}} = n_6 \sqrt[3]{p^2 \, E \, \frac{a^2}{s^2}}$$

$$\sigma_{y_{r_a}} = n_7 \sqrt[3]{p^2 \, E \, \frac{a^2}{s^2}}$$

where $p$ is uniformly distributed hydrostatic pressure, $kg/cm^2$

$E$ modulus of elasticity, $kg/cm^2$

$a$ and $b$ half rectangle sides, cm

$s$ sheet thickness, cm

$n_1$ to $n_7$ coefficients

The coefficients $n_1$ to $n_7$ were also determined for rectangular plates and can be read from figure 2 for any aspect ratio of the rectangle $\lambda = \frac{b}{a}$.

The maximum stresses occur accordingly in the center of the edge of the long rectangle side in z direction. Even the square plate is stressed highest in edge center and not in plate center, as claimed by Föppl in his volume "Drang und Zwang." A new edition is to carry this correction. In order to convey a clear picture of the stress distribution over the whole plate, the stress coefficients $n$ for the square plate are shown in figures 3 and 4 plotted against the corresponding plate points.

Example:

Dimensions of plate, 60 × 60 cm

Sheet thickness, $s = 1.4$ mm

$E = 740{,}000$ $kg/cm^2$

Load, $p = 20$ $t/m^2$

Compute the deflection $f$ in plate center and the maximum stress $\sigma$ on the edge.

$$f = 0.8\, a \sqrt[3]{\frac{pa}{Es}} = 0.8 \times 30 \times \sqrt[3]{\frac{2 \times 30}{740000 \times 0.14}} = 1.99 \text{ cm}$$

$$\sigma_{z_{r_a}} = \sigma_{y_{r_b}} = 0.56 \sqrt[3]{E \left(\frac{pa}{s}\right)^2}$$

$$= 0.56 \sqrt[3]{740000 \left(\frac{2\times30}{0.14}\right)^2} = 2700 \text{ kg/cm}^2$$

2. Hencky's Method

This method was published in the Zeitschrift für angewandte Mathematik und Mechanik (1921), pp. 81 and 423 in the report entitled: "The Calculation of Thin Rectangular Plates with Vanishing Flexural Stiffness." For the square plate the difference method affords:

$$\text{Deflection } f = n_1 \, a \sqrt[3]{\frac{pa}{Es}}$$

The $n_1$ factors can be taken from figure 5.

Stresses:

$$\sigma_z = n_2 \sqrt[3]{E \left(\frac{pa}{s}\right)^2} \qquad \sigma_y = n_3 \sqrt[3]{E \left(\frac{pa}{s}\right)^2}$$

The factors $n_2$ and $n_3$ are plotted in figures 6 and 7. The determination of the factors for rectangular plates entails considerable paper work and has therefore not been made so far.

For the same example as below 1 the deflection is

$$f = 1.99 \frac{0.72}{0.80} = 1.8 \text{ cm}$$

$$\sigma_{z_{r_a}} = \sigma_{y_{r_b}} = 2700 \frac{0.436}{0.56} = 2100 \text{ kg/cm}^2$$

## C. COMPARISON OF EXPERIMENTAL RESULTS WITH THE CALCULATION METHOD

For purposes of checking the cited calculation methods as to practicability and reliability various 60×60 cm

plates of different thicknesses were stressed to failure under hydrostatic pressure. The employed test set-up is shown in figures 8 and 9.

Tank and cover were of rugged construction in order to keep interference of skin deflection in consequence of tank deformation to a minimum.

The measurements disclosed a good agreement between the computed stresses and deflections and the experimental results so long as pure elastic behavior of the material prevails. The principles of the theory are herewith realized. This stress range is decisive for our designs, because according to the design specifications the strains under service loads ($j = 1.0$) must lie within the elastic limit and the permanent deformations therefore remain unimportant (5 percent).

The experiments further proved pure elastic behavior to be tied to relatively narrow limits of the stress. The plastic form changes start very soon. In this range the calculation gives, of course, erroneous values, deflections too small, stresses too high. Sheets under tensile stresses, as we have here, are capable of very pronounced plastic form changes and considerable permanent strains. Through this plasticization of the sections the plate bulges out more, which tends to effect a reduction in the stresses. The bearing strength can therefore be raised considerably above the mathematical values. The plastics theory lately has been concerned with the stresses in the plastic range, but the theory is still in its initial stage.

For confirmation of the foregoing arguments the test data of a 60×60 cm, 1.4 mm thick shell plating are included. Figure 10 shows the experimental deflections in the center of the shell panel plotted against the load in comparison with the mathematical values, along with the permanent deformations from the different loads after relaxation to zero. It is seen that the plastic deformations already start between 1 and 2 atmospheres and that in this area the theoretical and experimental deflection curves also disperse considerably. The plate does not fail until at 11.2 atmospheres; hence the elastic range up to 2 atmospheres at most is quite small compared with the ultimate load.

According to calculation the following ultimate loads would have been afforded for $\sigma_2 = 4400$ kg/cm²:

Föppl:

$$p = \sqrt{\frac{\sigma^3 \ s^2}{n^3 \ E \ a^2}} = \sqrt{\frac{4400^3 \times 0.14^2}{0.56^3 \times 740000 \times 30^2}} = 3.8 \text{ atm}$$

Hencky:

$$p = \sqrt{\frac{4400^3 \times 0.14^2}{0.44^3 \times 740000 \times 30^2}} = 7.14 \text{ atm}$$

The ultimate loads obtained are substantially higher for the stated reasons. Föppl's approximate solution compared to Hencky's gives substantially lower values in the elastic range; hence it leaves one on the safe side. In figure 11 Hencky's computed elastic line in panel center is compared with the experimental for 2 atmospheres.

The difference in this load stage is small because only minor permanent deformations are present, according to figure 10 (in the vicinity of the elastic range).

The edge stresses recorded with tensiometer showed themselves too small, since in consequence of the not altogether negligible flexural stiffness of the sheet the fixed end moment was still effective on the test station and to a lesser extent because the tensiometer recorded merely the length change of the chord but not that due to sheet curvature (fig. 12). The load recorded at 2 atmospheres in center of plate edge was 1700 kg/cm², i.e., much lower than computed previously.

When the stresses are computed from the measured deformations, they are in better agreement with the calculation. The elastic line (fig. 11) satisfies the parabola equation very satisfactorily

$$x = \frac{4f}{l^2} y(l - y)$$

so that the arc length can be ascertained according to the relation (fig. 13)

$$b = l + \frac{8}{3}\frac{f^2}{l}$$

The total strain on length $l$ is:

$$\epsilon_y = \frac{\Delta l}{l} = \frac{b - l}{l} = \frac{8}{3}\left(\frac{f}{l}\right)^2$$

On the assumption that the stresses $\sigma_y$ and $\sigma_z$ over the axis of symmetry of the square are constant and that $\sigma_z$ on the average is $\frac{3}{4}\sigma_y$ (which is approximately justified on the basis of the $n$ values of figures 6 and 7) the elastic range follows Hooke's law:

$$\epsilon_y = \frac{1}{E}\left(\sigma_y - \frac{\sigma_z}{m}\right) = \frac{13}{16}\frac{\sigma_y}{E}$$

With $f = 2.0$ cm recorded in the present example, we have

$$\sigma_y = \frac{16}{13} E \epsilon_y = \frac{16}{13} 740000 \frac{8}{3}\left(\frac{f}{l}\right)^2 = 2700 \text{ kg/cm}^2$$

The agreement with the computed stress according to Föppl is perhaps accidentally so good. The failure occurred in the center of the sheet edge, where the stresses in the elastic range also are maximum (fig. 14).

## REFERENCES

1. Föppl, A. and L.: Drang und Zwang.

2. Hencky, H.: Die Berechnung dünner rechteckiger Platten mit verschwindender Biegungsteifigkeit. Z.f.a.M.M., Bd. 1, Heft 2, April 21, 1921, pp. 81-89.

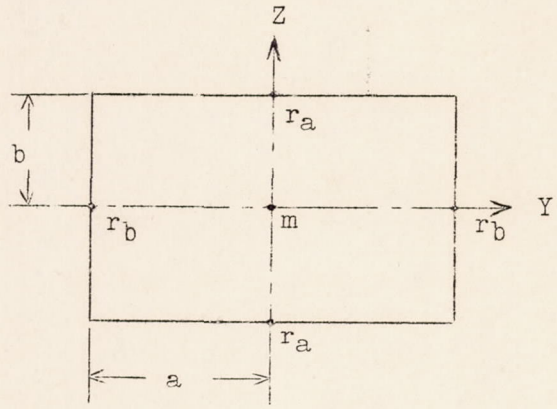


Figure 1.- Notation of rectangular skin plating.

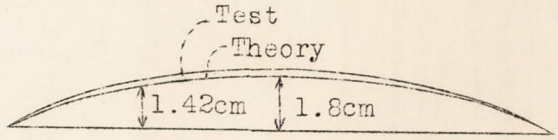


Figure 11.- Computed and experimental elastic lines at 2 at. in skin panel center.

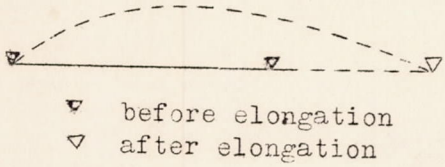


Figure 12.- Tensiometer knife edge.

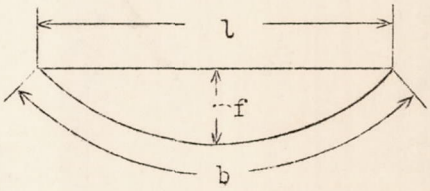


Figure 13.- Parabola notation.

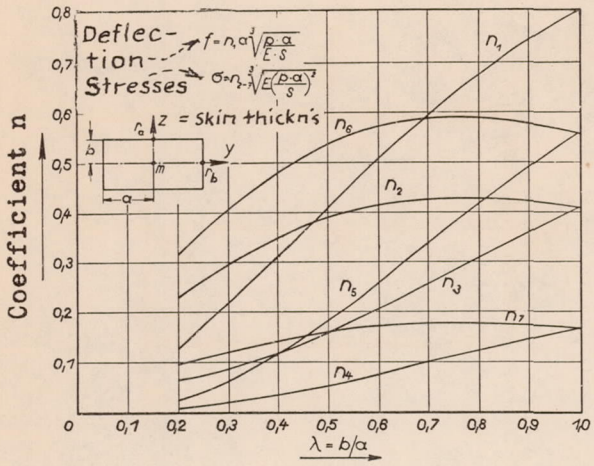


Figure 2.- n values for any aspect ratio $\frac{b}{s}$ of rectangle.

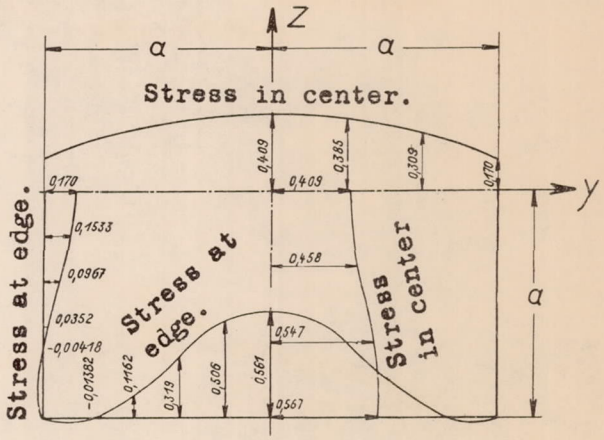


Figure 3.- Stress factors n for $\sigma_2$.

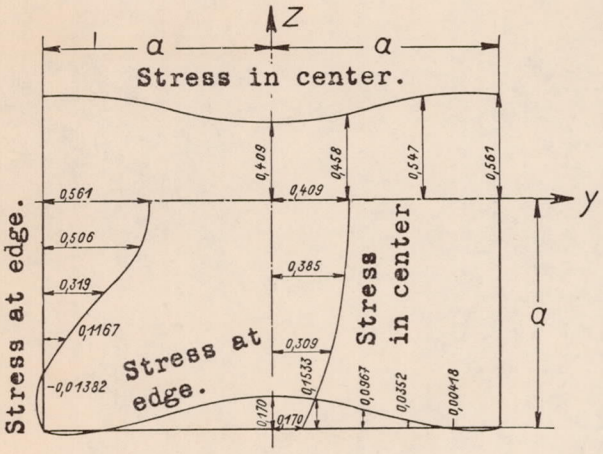


Figure 4.- Stress factors n for $\sigma_y$.

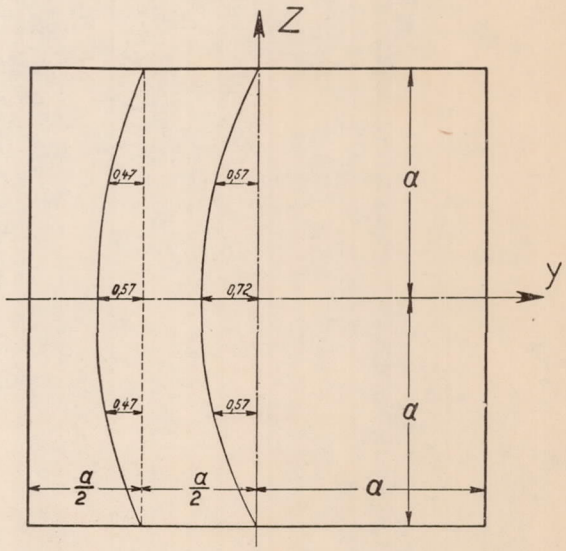

Figure 5.- Hencky's values $n_1$.

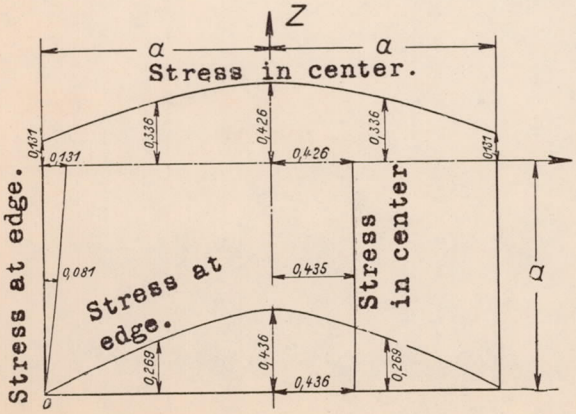


Figure 6.- Stress factors $n_3$ for $\sigma_2$.

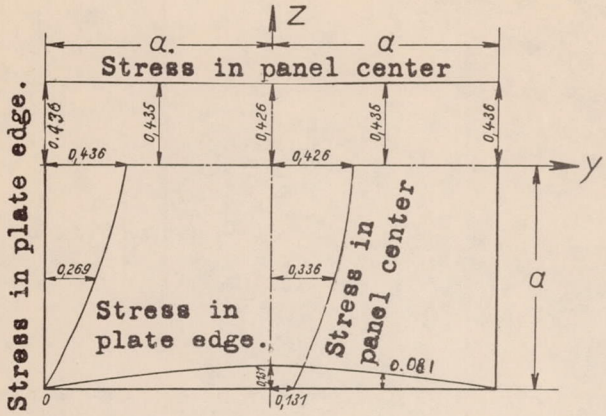


Figure 7.- Stress factors $n_z$ for $\sigma_y$

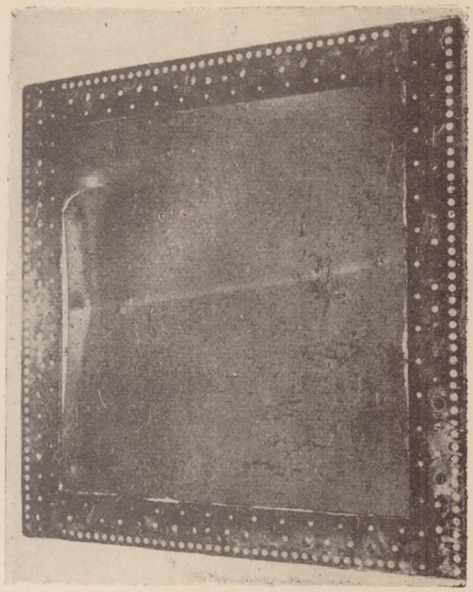

Figure 14.- View of break of 600x600x1.4 mm sheet at 11,2 at.

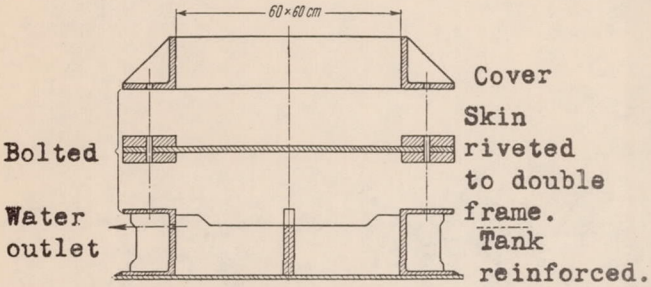


Figure 8.- Diagrammatic sketch of test set-up.

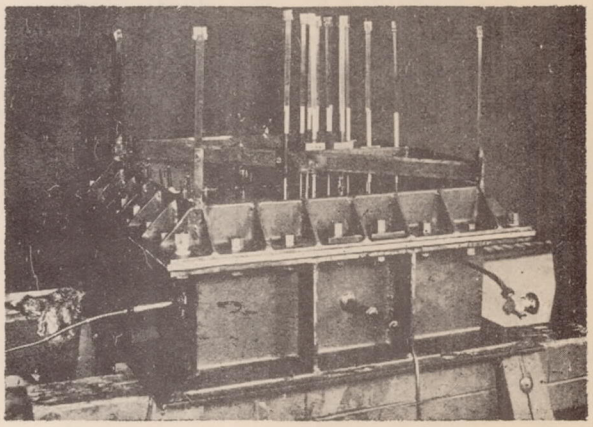

Figure 9.- Testing device.

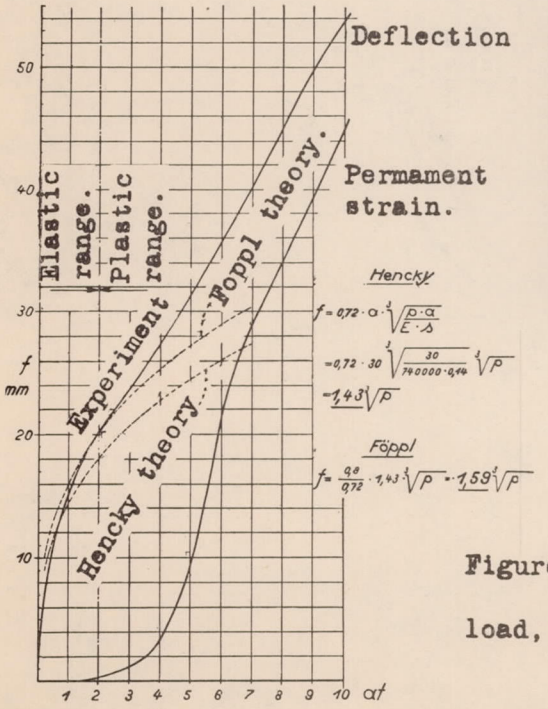


Figure 10.- Deflection in panel center plotted against load, skin panel: 600x600x1.4 mm.