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TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 937

STRESSES IN SINGLE-SPAR WING CONSTRUCTIONS WITH INCOMPLETELY BUILT-UP RIBS

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STRESSES IN SINGLE-SPAR WING CONSTRUCTIONS WITH INCOMPLETELY BUILT-UP RIBS*

By F. Reinitzhuber

I. INTRODUCTION

The constructional elements of single-spar wing structures are main spars, torsion tubes, ribs, and auxiliary spars. The ribs, which generally extend over the entire wing section, are for the purpose of stiffening the torsion tubes and taking up the forces in the torsion tubes and in the main spar. Mounted structures in the wing, engine, and landing gear often make it necessary to build up only one-half of the wing cross section completely with flanges and webs as a flexural supporting member. The other half of section is then omitted entirely or only the flanges are retained. As long as only small forces are to be taken up by such incompletely built-up ribs and the distances of the latter from the neighboring fully built-up ribs are not too large (fig. 1), an accurate investigation is not necessary. The effect of the incomplete rib may then be neglected and the applied loads distributed over the neighboring complete ribs. If, however, large forces due to the engine or landing gear are to be applied to the incomplete ribs, it is necessary to investigate the force distribution in detail, as will be done below.

II. LOAD ASSUMPTIONS

A single-spar wing structure is investigated, having two auxiliary spars (front and rear), several fully built-up ribs, and two neighboring incompletely built-up ribs.

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*"Über die Krafteinleitung in einholmige Flügeltragwerke durch unvollkommen ausgebildete Querwände." Luftfahrtforschung, vol. 16, no. 7, July 20, 1939, pp. 349-354.

In the rear torsion tubes of the latter, the webbing is missing so that only the flanges remain. Of the several complete ribs only the two are considered that are on each side of the incomplete ribs. This is sufficient to determine the qualitative effect of the disturbance due to the incomplete ribs; the quantitative disturbance will also be small. The investigation is therefore extended over the portion of the wing shown in figure 2, the wing being attached with no deformation restraint at rib 5.

The above wing portion is loaded by normal and tangential transverse forces and torsional moments which arise at the outer wing and act on rib 0. Furthermore, local forces from the engine and the landing gear are applied at the front half of ribs 2 and 3. The forces due to the outer part of the wing are only slightly changed by the incomplete construction of ribs 2 and 3 as far as their distribution over the wing section is concerned. This type of loading will therefore not further be investigated and only the force distribution due to the applied loads at ribs 2 and 3 will be considered. Since the computation for applied tangential or normal forces or torsion moments is fundamentally the same but the torsion moments for the shear in the wing covering are of chief importance the present study will be restricted to the investigation of the torsion moments only.

III. COMPUTATION OF THE WING PORTION INVESTIGATED BY THE "SHEAR FIELD METHOD"

The computation of systems that consist of shear skins, flanges, and ribs is most suitably carried out by the "shear field method" often employed. This method is characterized by the assumption of a pure shear condition in the sheet panels between the flanges and the stiffeners while all the axial forces are assigned to the spars and stiffeners through the use of an "effective width." * A computation based on these assumptions follows the methods that are employed for statically indeterminate computations.

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*A detailed description of the shear field method is found, for example, in reference 1.

The number of static indeterminates for the wing portion shown in figure 2 fixed at rib 5 with no deformation restraint (with 3 longitudinal ties) is $4 \times 3 + 5 \times 1 - 2 = 15$, since the double cells between two neighboring ribs are connected each by three redundant longitudinal ties; furthermore, each double cell is statically indeterminate with the exception of the cells 1-2 and 3-4 for which through the omission of the web sheet at each half of the ribs 2 and 3 the number of static redundancies is decreased by two.

The computation of highly redundant statically indeterminate systems, which consist of double cells, has been carried out for space frameworks in a work of H. Ebner and H. Köller (reference 2). Three forces groups, obtained by disconnecting the three redundant longitudinal ties at each rib, and a suitable shear at each double cell (in the case of torsion loading, the shear at the center spar) are introduced as the static indeterminates. In the region of the ribs 1 to 4, it is to be noted that by the omission of the web sheets in the rear half of ribs 2 and 3 there are two static redundancies fewer than in the other double cells. For this reason, in this portion of the double cells, instead of three undetermined shears, only one need be introduced, most suitably chosen in the web of the rear spar.*

After this choice of the static redundancies, the 15 elasticity equations may be set up whose solution then gives the values of the static indeterminates, which may be slightly or not at all coupled. The work of computation still remains extremely large. If the portion of the wing under consideration were extended by using more double cells, the number of static indeterminates and hence the work of computation would still further increase considerably. It is therefore necessary to make simplifying assumptions that will enable us to obtain a general idea of the force distribution in wings resulting from the application of forces at incomplete ribs without the necessity for carrying out the large computation work. The simplifications refer to the manner of loading the system and the choice of the static indeterminates.

*Another method of computation for the problem under consideration is presented in a paper by H. Köller (reference 3). The method there indicated is conveniently applied if the web sheet in the rear portion of the incomplete ribs is removed after a statically indeterminate computation had already been carried out for complete ribs.

The forces in cells 0-1 and 4-5 are not of decided importance for the distribution of the forces in the torsion tubes. They are neglected by disconnecting longitudinal ties at ribs 1 and 4. Considering the ribs 1 and 4 as stiff, then cell 0-1 is without stress and the cell 4-5 distributes the applied torsional moment in proportion to the stiffness of the forward and rear torsion tubes. (See reference 4.) The cells 0-1 and 4-5 may therefore be omitted. There then remains to investigate the sevenfold statically indeterminate W-system of figure 3a. In most practically occuring cases, the system will be symmetrical with respect to a plane parallel to the ribs through MM or may be assumed symmetrical through suitable averaging. In symmetrical systems, it is convenient to decompose the load into symmetric and skew-symmetric groups. The moments M_2 and M_3 applied in the W system are split up into three load cases, W_1, W_2, and W_3, represented in figures 3b, 3c, and 3d. Of the three cases, only the W_1 and W_3 systems are of significance for the statically indeterminate computation. The W_2 system distributes the moment $\underline{M}_1 = \frac{M_2 + M_3}{2}$ in proportion to the stiffness of the forward and rear torsion tubes and needs no further investigation.

It is now necessary to choose the 7 static redundancies. By cutting through 5 sheets, namely, those bounded by the center and rear spars and the ribs 1, 2, and 3, 4, respectively, and the web sheet of the rear spar the sevenfold statically indeterminate system is reduced to a fivefold statically indeterminate system and a twofold statically indeterminate base system. The 5 static indeterminates, X_1, ... X_5, which are made up of the unknown shears of the 5 sheets cut through are shown in figures 4a, b, c, d.* They are so chosen as to be only slightly coupled with one another. The equation coefficients corresponding to the elasticity equations therefore become

$$\delta^x_{1.3} = \delta^x_{1.4} = \delta^x_{1.5} = 0 \quad \text{and} \quad \delta^x_{2.3} = \delta^x_{2.4} = \delta^x_{2.5} = 0$$

The remaining coupling members, however, $\delta^x_{1.2}$, $\delta^x_{3.4}$, $\delta^x_{4.5}$, are also small in comparison with $\delta^x_{1.1}$, ... $\delta^x_{5.5}$. The

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*The use of shears as static indeterminates is also found in reference 5.

$X_1 \ldots X_4$ systems can also be represented as force groups by loosening the longitudinal ties in the rear spar at ribs 10 and 11 or by cutting the flanges of the ribs at the center spar.

The twofold statically indeterminate base system, which consists of a closed torsion tube extending from ribs 1 to 4, is converted into the statically determinate null system by loosening a longitudinal tie at ribs 2 and 3. The force groups Y_1 and Y_2 in figure 5 are taken as the static indeterminates. In this manner, the seven static indeterminates are chosen. Y_1 and Y_2 are not coupled ($\delta^y_{1.2} = 0$). In the X_1 and X_2 systems $Y_2 = 0$, in the X_3, X_4, and X_5 systems $Y_1 = 0$. For the W_1 loading case $Y_2 = 0$ and $X_3 = X_4 = X_5 = 0$, for the W_3 loading case $Y_1 = 0$ and $X_1 = X_2 = 0$. The elasticity equations for the loading group W_1 are therefore

$$\delta^x_{1.1} X_1 + \delta^x_{1.2} X_2 + \delta^x_{1.0} = 0$$

$$\delta^x_{2.1} X_1 + \delta^x_{2.2} X_2 + \delta^x_{2.0} = 0$$

and for the loading group W_3:

$$\delta^x_{3.3} X_3 + \delta^x_{3.4} X_4 + \delta^x_{3.5} X_5 + \delta^x_{3.0} = 0$$

$$\delta^x_{4.3} X_3 + \delta^x_{4.4} X_4 + \delta^x_{4.5} X_5 + \delta^x_{4.0} = 0$$

$$\delta^x_{5.3} X_3 + \delta^x_{5.4} X_4 + \delta^x_{5.5} X_5 + \delta^x_{5.0} = 0$$

where the $\delta^x_{i,k}$ and $\delta^x_{i,o}$ values are to be determined from a simple statically indeterminate base system. In the loading case W_1 and the X_1, X_2 systems, the elasticity equation of the statically indeterminate base system is

$$\delta^y_{1.1} Y_1 + \delta^y_{1.0} = 0$$

similarly in the loading case W_3 and the X_3, X_4, X_5 systems.

$$\delta^y_{2.2} Y_2 + \delta^y_{2.0} = 0$$

The solution of the sevenfold statically indeterminate problem is thus reduced to a twofold (W_1) and a threefold (W_3) statically indeterminate problem each with a simply indeterminate base system.

For approximate computations, the static redundancies Y_1 and Y_2 of the base system may be neglected. The static indeterminate X_2 for the loading case W_1 is small and may be set equal to zero. There is then obtained

$$X_1 \cong - \frac{\delta^x_{1.0}}{\delta^x_{1.1}}$$

an approximate value which gives useful results for the most important loading case W_1. If, instead of the torsion moment M_1 of the W_1 loading case, two tangential forces act as external load on ribs 2 and 3, there is obtained as approximate value, since $X_1 \cong 0$

$$X_2 = - \frac{\delta^x_{2.0}}{\delta^x_{2.2}}$$

Having determined the force distribution in the W system, the characteristic force groups which arise from the deformation restraint at ribs 1 and 4 and were up to now neglected, may be determined in a supplementary computation. Their effect on the distribution of the applied forces in the front and rear tubes is small, however, as will appear from the numerical example in the following section.

IV. ILLUSTRATIVE EXAMPLE

The numerical computation will be carried out for the wing portion shown in figure 6a. Tables Ia to Id give the thickness of the sheets and stiffener cross sections. The covering and spar webs were assumed as rigid in shear while the webs of the ribs were considered as tension fields. Deviating from the system shown in figure 3, the covering between ribs 2 and 3 in the rear wing portion is not part of a cylindrical shell corresponding to the engine nacelle but has the shape shown in figure 7. In this case, too, under transverse loading a pure shear stress condition is obtained. The shear flow is

$t_x = t_o \left(\frac{h_o}{h_x}\right)^2$. The virtual work obtained is thus

$$\int \frac{t_x^{(n)}\ t_x^{(m)}\ d\ F_x}{s\ G} = t_o^{(n)}\ t_o^{(m)}\ \frac{F_{red}}{s\ G} \left(F_{red} = l_o H,\right.$$

$$\left. \text{where } H = \int \left(\frac{h_o}{h_x}\right)^3 d\ s\right)$$

an expression which formally agrees with that for parts of cylindrical shells. Instead of the entire area that must be substituted in cylindrical shells, there enters a reduced area with which the computation is carried out as with cylindrical shells.

For the numerical computation, various assumptions were made with reference to the stiffness of the ribs and the reduction of the static redundancies and these are given in tables IIa, b. If a value $\underline{M}_1$ = 1,000 kg m is chosen for loading case W_1 (fig. 3b), then the shears of the covering and spar webs and the flange forces of the spars are those given in tables IIIa, b.

A clear picture of the effect of the various assumptions of table II is obtained from table IV by comparing the horizontal components P_o and P_u of the flange forces of ribs 2 and 3 at the center spar. The table also shows the moment contributions M_h of the rear tubes obtained from $M_h = \frac{P_o + P_u}{2} h$, where h = 69.41 cm is the distance between the rib flanges at the center spar. If $P_o \cong P_u$, the force distribution in the covering of the rear tubes is essentially given through M_h. It may be seen that the flange forces and the moment in the rear tubes depend on the stiffness of the ribs, particularly of the flanges in the incompletely built-up rear part of the ribs 2 and 3. For this reason, it is convenient in the case of curved sheet to place the flanges in the rear part of ribs 2 and 3 outside of the wing profile in order to keep the bending moments in the flanges small so that the flanges become stiffer. If the latter are assumed as rigid (case 1), the moment contribution of the rear tubes is 0.364 $\underline{M}_1$ (case A, complete restraint against deformation) and 0.343 $\underline{M}_1$ (case B, no restraint). If they are taken to be elastic (case

4), the moment drops to 0.126 $\underline{M}_1$ and 0.125 $\underline{M}_1$, respectively. If, in addition to the flanges in the rear part of ribs 2 and 3, the front parts and ribs 1 and 4 are assumed elastic, the moment increases to 0.185 $\underline{M}_1$ (case 3) and 0.320 $\underline{M}_1$ (case 2) but is essentially dependent on the manner of force application. If ribs 2 and 3 were made complete over the entire cross section of the torsion tubes then, for ribs assumed stiff without taking account of the deformation forces, the moment of the rear tubes would be 0.472 $\underline{M}_1$.

The deformation restraint at ribs 1 and 4 leads in case 1 (sixfold statically indeterminate) and in case 4 (threefold statically indeterminate) to only small changes in the moments (from 0.364 $\underline{M}_1$ to 0.343 $\underline{M}_1$ and from 0.126 $\underline{M}_1$ to 0.125 $\underline{M}_1$), so that the neglect of cells 0-1 and 4-5 appears justified. Good approximate solutions are given by cases C (twofold statically indeterminate) and D (simply statically indeterminate) for the shears of the covering and the spar webs but not for the flange forces which chiefly depend on the neglected deformation restraint. In an approximate computation, however, the latter are of no particular importance, since they play the part of additional forces to the main shear stresses of the covering. Case C is then favorably applied by determining P_o and P_u in addition to the moment. If a knowledge of the moment, however, is sufficient, the simply statically indeterminate computation of case D will yield useful approximate values.

In loading case W_3 (fig. 3d), M_2 was chosen equal to 1,000 kgm. The assumptions of loading case W_1 (table 2) were also investigated with the exception of cases 2, 3, and D. The shears of the covering and the spar webs and the flange forces of the spars are given in tables Va, b. The forces P_o and P_u are given in table VI, which also gives the moment M_h computed from $\frac{P_o + P_u}{2} h$. Only rough approximations are obtained since $P_o \neq P_u$ and therefore in the rear tubes no pure torsion moment is applied. As in the loading case W_1, there here also appears a large dependence of the forces P_o and P_u and hence the moment M_h on the stiffness of the flanges in the rear part of ribs 2 and 3. For rigid flanges (case 1), $P_o = \pm 579.5$ kg, $P_u = \mp 352.4$ kg (case A, complete restraint against deformation) and $P_o = \pm 550.5$ kg, $P_u = \mp 367.2$ kg (case B, no restraint) and for elastic flanges (case 4)

$P_o = \pm 104.2$ kg, $P_u = \mp 85.7$ kg, and $P_o = \pm 101.3$ kg and $P_u = \mp 68.2$ kg, respectively. From these values, it may be seen that in loading case W_2, the restraint against deformation at ribs 1 and 4 leads to a relatively greater change in the forces P_o and P_u and hence in the effect of the rear tubes than in loading case W_1. The fourfold, statically indeterminate case B (no restraint against deformation) still gives sufficiently accurate values, since there will generally be no complete restraint and loading case W_3 is much less important than case W_1 for the total force distribution. The further reduction of the static indeterminates in case C (threefold statically indeterminate) by neglecting the deformation forces in the front tubes leads to useful approximate results for P_o and P_u and for the shears of the covering and spar webs.

V. SUMMARY

It is shown that the force distribution resulting from incomplete ribs in single spar wing structures may be determined with the aid of the shear field method by a statically indeterminate computation. A numerical computation is given of the force distribution of a wing structure whose two neighboring incomplete ribs with web missing in half the section are torsionally loaded. In spite of the incomplete ribs, there is obtained a distribution of the applied forces over the entire wing cross section which depends considerably on the stiffness of the ribs, particularly of the flanges in the rear part of the tubes and of the manner of application of the forces on the rib. Through the use of simplifying assumptions, it is possible to reduce the number of static redundanices and thus the computation work without introducing much error in the results. If two equal and opposite moments are applied at the incomplete ribs, a onefold statically indeterminate computation is already sufficient for obtaining a useful approximate value of the moment contribution of the rear tubes.

Translation by S. Reiss,
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Table Ia. Ratio s_c/s of Thickness of Sheet of Covering and Spar Web (see fig. 6a)

Cell	Covering				Spar Web		
	a	b	c	d	v	m	h
1-2 3-4	1.0	0.930	1.0	0.833	0.909	1.0	1.0
2-3	1.0	.833	1.0	.833	.833	1.0	1.0

Table Ib. Thickness Ratio s_c/F (cm^{-1}) of Cross Sections of the Spar Flanges (see fig. 6a)

Cell	Front spar		Center spar		Rear spar	
	I	II	III	IV	V	VI
1-2 3-4	0.0406	0.0348	0.0031	0.0038	0.0367	0.0360
2-3	.0406	.0348	.0031	.0038	.0435	.0435

Table Ic. Ratio s_c/s of Thickness of Sheet of the Webbing of ribs 1, 4 and 2,3 (see fig. 6b,c)

Rib	Panel							
	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8
1 and 4	0.500	1.00	1.00	1.00	0.500	0.833	0.833	0.833
2 and 3	.313	.625	.625	.625	.313	-	-	-

Table Id. Thickness Ratio s_c/F (cm^{-1}) of Cross Sections of Flanges of Ribs 1, 4, and 2,3 (see fig. 6b,c)

Rib		Panel								
		0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9
1 and 4	O.G.[a]	0.0261	0.0261	0.0167	0.0167	0.0153	0.0148	0.0193	0.0260	0.0476
	U.G.[b]						.0162	.0253	.0481	.0865
2 and 3	O.G. U.G.	.0270					0.0238 ($s_c/J = 0.0588$ cm^{-3}) .0520			

[a]Upper flange [b]Lower flange

Table IIa. Assumptions Made in Numerical Example on the Elastic Behavior of the Ribs and on the Manner of Load Application

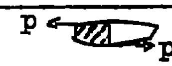
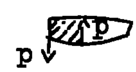
Case	Ribs 1, 4 and front half of ribs 2, 3.	Flanges in rear part of ribs 2, 3.	Moment application at ribs 2, 3
1	Rigid	Rigid	-
2	Elastic	Elastic	
3	Elastic	Elastic	
4	Rigid	Elastic	-

Table IIb. Assumptions Made in Numerical Example on Deformability of Double Cells and on the Number of Static Indeterminates

Case	Fixity conditions at ribs 1 and 4	Assumptions on the deformation forces in front tubes and ribs 2 and 3	Number of static indeterminates W_1	W_3
A	Restraint against deformation by assumption of inelastic cells 0-1 and 4-5	Deformation forces in front tubes at ribs 2 and 3 taken into account	6	7
B			3	4
C		Deformation forces in front tubes and ribs 2 and 3 are neglected (Y_1 = 0, Y_2 = 0) (Case D, X_2 = 0)	2	3
D	No restraint against deformation at ribs 1 and 4		1	-

Table IIIa. Shear Flows (kg/cm) of Covering and Spar Webs in Symmetric Loading Case W_1*
(see fig. 3b)

Cell	Case		Covering a	Covering b	Covering c	Covering d	Spar web v	Spar web m	Spar web h
1-2 and 3-4	1	A	∓2.89	∓3.39	±3.25	±3.66	∓6.77	±3.16	0
		B	∓3.55	∓3.20	±3.91	±3.44	∓5.84	±2.97	0
		C	∓4.21	∓3.24	±4.52	±3.53	∓4.36	±2.28	0
		D	∓4.36	∓3.12	±4.36	±3.68	∓4.36	±2.32	0
	2	B	∓6.41	∓1.12	±1.91	±5.40	∓5.46	±3.68	0
		C	∓6.90	∓1.04	±2.34	±5.36	∓4.62	±3.38	0
		D	∓4.71	∓2.66	±4.71	±3.13	∓4.71	±2.97	0
	3	B	∓4.51	∓1.66	±4.54	±1.92	∓7.47	±5.25	0
		C	∓5.54	∓1.57	±5.50	±1.88	∓5.52	±4.49	0
		D	∓5.52	∓1.58	±5.52	±1.86	∓5.52	±4.49	0
	4	A	∓4.49	∓1.10	±4.43	±1.35	∓8.84	±6.40	0
		B	∓5.10	∓1.09	±5.05	±1.33	∓7.56	±5.88	0
		C	∓5.89	∓1.11	±5.82	±1.38	∓5.86	±5.12	0
		D	∓5.86	∓1.14	±5.86	±1.34	∓5.86	±5.12	0
2-3	1-4	A-L	0	0	0	0	0	0	0

Upper sign applies to cell 1-2; lower to cell 3-4.

*Shears in the covering on the fuselage side of a section normal to the wing axis directed opposite to the flight direction are positive.

Shears in the covering on the fuselage side of a section normal to the wing axis directed upward are positive.

Table IIIb. Flange Forces (kg) of Spars in Symmetric Loading Case W_1* (see fig. 3b)

Rib	Case		Front spar I	Front spar II	Center spar III	Center spar IV	Rear spar V	Rear spar VI
1 and 4	1	A	+106.6	-96.8	-100.5	+98.2	+93.1	-100.7
	1	B-D						
	2,3	B-D	0	0	0	0	0	0
	4	B-D						
	4	A	+119.5	-121.1	-83.0	+91.3	+30.3	-37.0
2 and 3	1	A	-39.1	+35.4	+36.6	-35.8	-34.0	+36.9
		B	-86.1	+72.5	+98.2	-93.8	-119.9	+129.0
		C	-5.8	-5.8	+49.3	-48.9	-121.5	+132.6
		D	0	0	+40.6	-61.5	-117.1	+138.0
	2	B	+35.9	+132.9	-60.9	-268.5	-41.8	+202.5
		C	+85.5	+85.5	-92.7	-240.3	-39.1	+201.2
		D	0	0	+34.6	-52.4	-99.8	+117.6
	3	B	-111.0	+109.7	+89.7	-98.4	-62.1	+72.0
		C	+.6	+.6	+19.6	-32.6	-58.8	+70.5
		D	0	0	+20.5	-31.1	-59.2	+69.8
	4	A	-43.6	+44.2	+30.2	-33.3	-11.1	+13.6
		B	-92.3	+94.4	+69.7	-81.1	-40.9	+50.1
		C	+1.4	+1.4	+12.6	-25.6	-41.6	+51.7
		D	0	0	+14.8	-22.4	-42.6	+50.2

*Tensions in the flanges are positive.

Table IV. Horizontal Components P_o and P_u of Flange Forces of Ribs 2 and 3 at Center Spar and Moment M_h of Rear Tubes in Symmetric Case W_1

Case	A		B		C		D	
	P_o / P_u	M_h	P_o / P_u	M_h	P_o / P_u	M_h	P_o / P_u	M_h
1	+548.7 / -500.9	0.364 M_1	+517.5 / -472.3	0.343 M_1	+524.1 / -485.6	0.350 M_1	+505.4 / -505.4	0.351 M_1
2	- / -	-	+180.5 / -741.6	0.320 M_1	+168.7 / -736.8	0.314 M_1	+430.7 / -430.7	0.299 M_1
3	- / -	-	+268.0 / -263.7	0.185 M_1	+253.7 / -258.1	0.178 M_1	+255.7 / -255.7	0.178 M_1
4	+178.3 / -185.5	0.126 M_1	+176.4 / -183.4	0.125 M_1	+179.6 / -189.1	0.128 M_1	+183.8 / -183.8	0.128 M_1

Table Va. Shear Flows (kg/cm) of Covering and Spar Webs in Antisymmetrical Loading Case W_3*
(see fig. 3d)

Cell	Case		Covering a	Covering b	Covering c	Covering d	Spar web v	Spar web m	Spar web h
1-2 and 3-4	1	A	-0.08	-0.70	+0.74	+1.08	-1.07	+2.10	-4.80
		B	+.35	-.44	+.71	+.19	-1.70	+2.16	-3.50
		C	-.12	-.49	+1.16	+.27	-.64	+1.68	-3.56
	4	A	-.99	+.83	+.99	-.32	-2.11	+3.48	-3.65
		B	-.65	+.96	+.77	-.74	-2.61	+3.49	-2.81
		C	-1.27	+.96	+1.38	-.72	-1.32	+2.95	-2.85
2-3	1	A	+2.63	+2.88	-3.79	-1.49	+5.40	-2.94	-.07
		B	+2.96	+2.96	-4.18	-1.76	+5.32	-3.66	+1.84
		C	+3.48	+3.01	-4.68	-1.82	+4.08	-3.10	+1.93
	4	A	+3.98	+1.47	-4.13	-.94	+6.74	-4.54	-.85
		B	+4.22	+1.59	-4.36	-1.24	+6.49	-4.94	+.68
		C	+4.91	+1.61	-5.03	-1.27	+4.97	-4.28	+.71

*Shears in the covering on the fuselage side of a section normal to the wing axis directed opposite to the flight direction are positive.

Shears in the covering on the fuselage side of a section normal to the wing axis directed upward are positive.

Table Vb. Flange Forces (kg) of Spars in Antisymmetrical Loading Case W_3* (see fig. 3d)

Rib	Case		Front spar I	Front spar II	Center spar III	Center spar IV	Rear spar V	Rear spar VI
1 and 4	1	A	∓53.1	±40.3	∓6.2	±52.5	±122.9	∓156.4
		B-C	0	0	0	0	0	0
	4	A	∓47.7	±42.7	∓0.5	±26.1	±89.8	∓110.4
		B-C	0	0	0	0	0	0
2 and 3	1	A	∓90.2	±52.5	±96.0	∓38.9	∓83.4	±64.0
		B	∓76.9	±37.3	±110.8	∓61.6	∓147.8	±138.2
		C	∓19.5	∓19.5	±77.1	∓29.7	∓152.2	±143.8
	4	A	∓89.5	±84.5	±61.8	∓55.2	∓16.1	±14.5
		B	∓73.6	±69.0	±70.5	∓73.9	∓69.2	±77.3
		C	∓2.0	∓2.0	±27.1	∓31.9	∓70.8	±79.7

*Tensions in the flanges are positive.

Table VI. Horizontal Components P_o and P_u of Flange Forces of Ribs 2 and 3 at Center Spar and Moment M_h of Rear Tubes in Antisymmetrical Case W_3.

Case	A P_o / P_u	A M_h	B P_o / P_u	B M_h	C P_o / P_u	C M_h
1	±579.5	0.323 M_a	±550.5	0.284 M_a	±566.7	0.296 M_a
	∓352.4		∓267.2		∓287.4	
4	±104.2	0.066 M_a	±101.3	0.059 M_a	±104.8	0.063 M_a
	∓85.7		∓68.2		∓75.6	

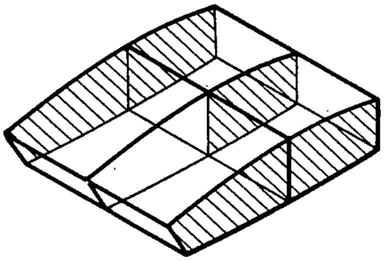


Figure 1.- Portion of a single-spar wing with incompletely formed rib.

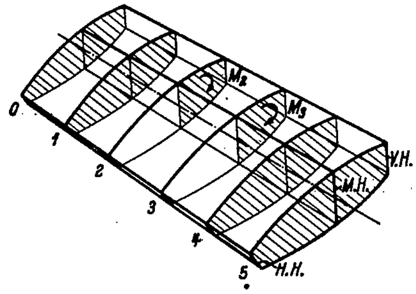
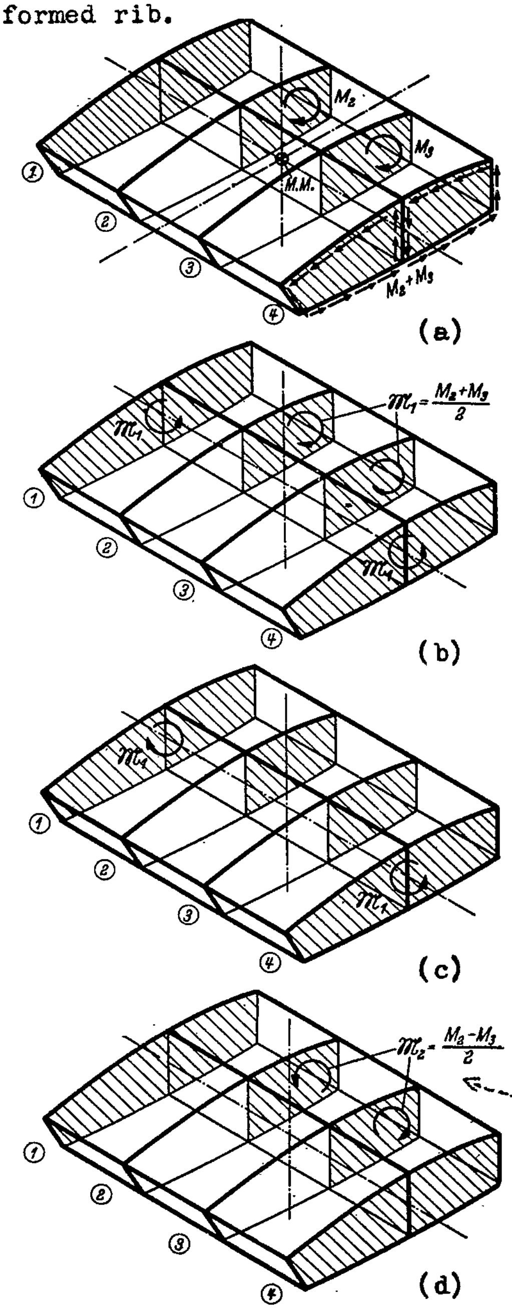


Figure 2.- Portion of a single-spar wing with two auxiliary spars and two incompletely formed ribs.



(a)

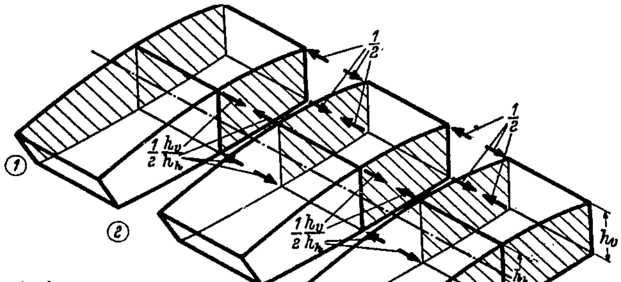
(b)

(c)

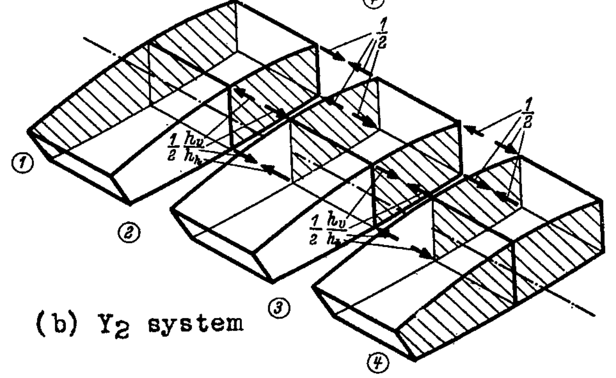
(d)

a, W-system with loading M_2 and M_3
b, Loading case W_1
c, " " W_2
d, " " W_3

Figure 3a to d.- Decomposing of the loading M_2 and M_3 in the W-system into the loading cases W_1,W_2,W_3 ($W=W_1+W_2+W_3$)

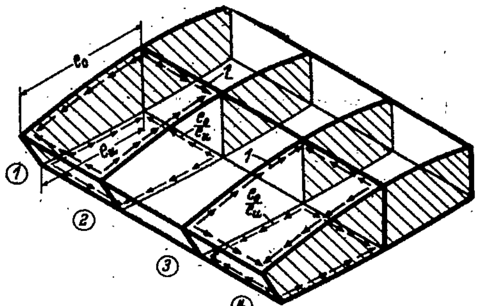


(a) Y_1 system

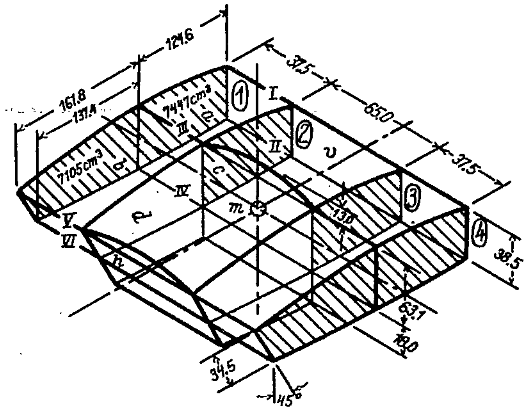


(b) Y_2 system

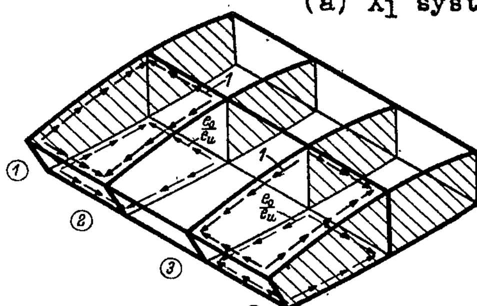
Figure 5a,b.- Static indeterminates Y_1 and Y_2 of the base system.



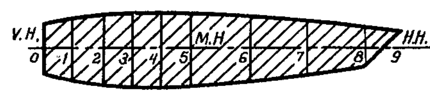
(a) X_1 system



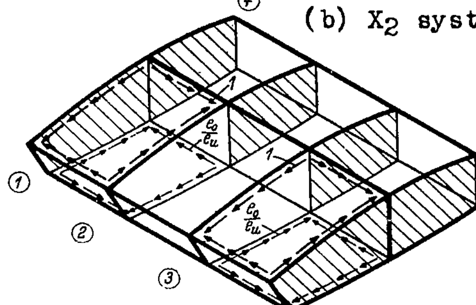
(a) Portion of a single-spar wing with two auxiliary spars and two incompletely formed ribs. a,b, upper covering; c,d, lower covering; v,m,h, spar webs; I, II, III, IV, V, VI, spar flanges.



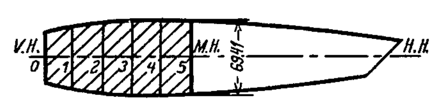
(b) X_2 system



(b) Ribs 1 and 4.

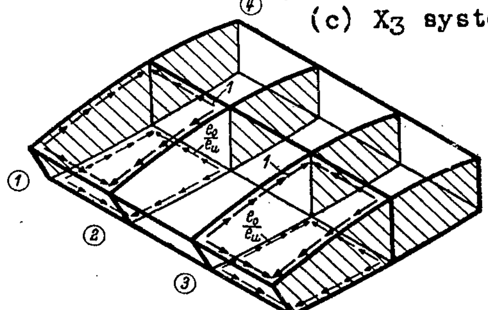


(c) X_3 system



(c) Incomplete ribs 2 and 3.

Figure 6 a to c



(d) X_4 system

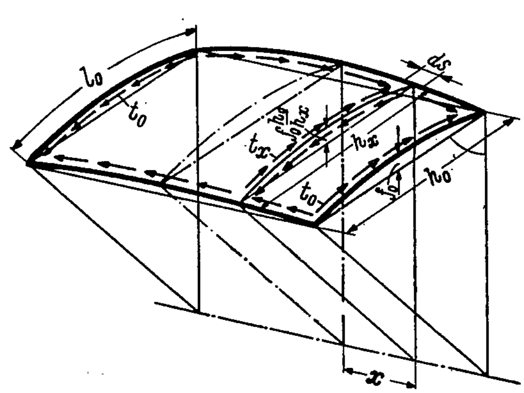
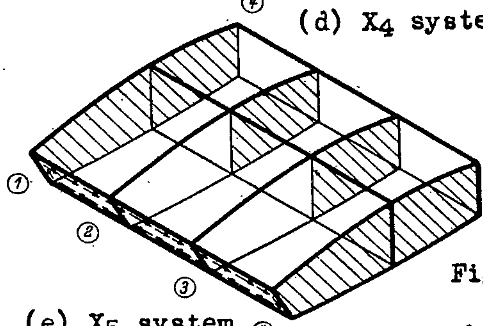


Figure 7.- Portion of covering sheet.



(e) X_5 system

Figure 4a to e.- Static indeterminates X_1 to X_5 (There are shown only the shears at an edge).