THE QUESTION OF SPONTANEOUS WING OSCILLATIONS

(Determination of Critical Velocity through Flight-Oscillation Tests)

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THE QUESTION OF SPONTANEOUS WING OSCILLATIONS*

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SUMMARY

Determinations of the spontaneous oscillations of a wing or tail unit entail many difficulties, both the mathematical determination and the determination by static wing oscillation tests being far from successful and flight tests involving always very great risks. The present paper gives a method developed at the Junkers Airplane Company by which the critical velocity with respect to spontaneous oscillations of increasing amplitude can be ascertained in flight tests without undue risks, the oscillation of the surface being obtained in the tests by the application of an external force. The method was evolved in the wind tunnel but has been successfully tested in actual flight. It is still in course of development.

I. INTRODUCTION

The spontaneous oscillations of wings and tail units have occupied the airplane designers and aeronautical engineers for many years both at home and in foreign countries.

The underlying theory of this oscillation mode is fundamentally known from a multitude of published reports, so as to leave no scientific doubt as to the causes and conditions leading to such oscillations. But the practical solution of the problem is still beset with many difficulties as attested by the fatal accidents still occurring periodically.

Lately the problem of spontaneous oscillations has become particularly acute as a result of the general tendencies in modern airplane design. Thinner profiles, divided tail units, greater number of cutaway sections in the wings, as well as the generally greater weight of the fast airplanes, are all factors which, with the demands for greater speeds reduce the range between critical and maximum velocity and through it promote the danger of oscillation.

Patently the problem cannot be simply brushed aside or its consequences tacitly assumed as inevitable, but rather must be mastered and within the very near future. If not, flying will always be burdened with the danger of spontaneous oscillations or else the actually obtainable speeds must be sacrificed. And, as this is obviously ruled out, there remains but the choice of mastering this oscillation hazard by any and all means.

II. EXISTING METHODS

The methods available at the present time are:

1) Mathematical treatment,

2) Static oscillation test in conjunction with a similarity calculation built up on statistical principles.

The mathematical treatment is practically ruled out in view of the complicated oscillation system with many unknown degrees of freedom as presented by an airplane, and so the profit and the real purpose of the theoretical treatment must be looked for in the fact that the latter defines the important relations and effects of the different quantities on the critical velocities, which is of great importance for the appraisal of new designs. It is important that theory should supply the constructor with directions, how, for instance, to keep the torsional (not the flexural) stiffness at a maximum, to shift the center of gravity as far as possible forward, mass-balance all controls, etc.

But the determination of the critical velocity on the finished airplane itself has remained an experimental problem: the well-known static oscillation test.
airplane is elastically suspended and made to oscillate by means of a centrifugal unbalance. The experimentally obtained resonance figures are written in the formula for the critical velocity

\[ v_K = \frac{t \cdot v}{\omega} \]

where

- \( t \) is wing chord
- \( v \), frequency of amplified oscillation
- \( \omega \), reduced frequency, an empirical factor ranging between 0.89 and 1.15 for wings and at around 0.4 for tail units.

In the absence of the frequency of the amplified oscillation for \( v \) the torsional or natural frequency of the control from the static oscillation test is substituted. Naturally, this method is not perfect because neither the exact depth nor the exact frequency nor the chosen empirical value \( \omega \) itself is known.

To allow for this element of doubt the regulations stipulate 75 percent of the critical velocity as obtained from static oscillation test as the highest permissible velocity. Unfortunately, it is a matter of fact that this demand cannot be complied with in many cases. In cases of this kind where the latter occurs it is required that the oscillation freedom of the airplane be proved by a flight test at 1.5 times the horizontal speed. About the risk of such tests nothing need be said. Speaking from experience, it is not sufficient to reach the required speed but it necessitates flying the entire speed range at different engine r.p.m., and if possible in different flight conditions and gusty weather.

Another drawback of the static oscillation test is that among the numerous oscillation modes it is not always distinctly clear whether one or the other might or might not induce amplified wing oscillations, so that one oscillation mode might be considered harmless when, in fact, it leads to amplification. But more frequently, the opposite is the case. A mode in itself harmless is considered dangerous and, since the natural frequencies already start
at the lowest r.p.m., the result, according to the formula
\[ v_K = \frac{v}{t} \], is a very lowly placed critical velocity. The
consequence is that the proof of the freedom from oscillation rests with the flight test. And there is some apprehension that this will be the general rule for the new high-speed airplanes. All this makes the static oscillation test appear inadequate. For the exact determination of the critical velocity it needs to be supplemented by further experiments which remove the element of doubt from the static test and the risks of the flight test altogether or at least minimize them.

III. EXTENSION OF EXISTING TEST METHODS

Faced by the necessity to obtain reliable data regarding the location of the critical velocity the Junkers company decided to attack the problem theoretically and experimentally. The results of the investigations had not been completed at the time this report was written (February 1935), but the method itself can be described.

It was attempted to combine the static oscillation and the flight test to a single flight oscillation test by setting the airplane into oscillations in flight with an unbalance and then recording the amplitudes. These measurements afford, as will be explained elsewhere, the critical velocity.

IV. MATHEMATICAL-PHYSICAL BASIS OF THE TEST METHOD

A brief description of the theoretical principle of the test method follows.

The spontaneous or amplified oscillations represent a stability problem with all its characteristics. The process is controlled by two or more simultaneous differential equations (depending on number of degrees of freedom) which are homogeneous in the position coordinates. For the latter it affords solutions only for certain values of frequency and velocity at which the amplitudes of the generated oscillations may assume any value; that is, oscillation failure takes place similarly to the flexural
failure of a compressed member. Prior to reaching the values oscillations are absent altogether, which however, in a simple flight test represents the great moment of danger. If the failure of the compressed member were as disastrous as the oscillation failure of an airplane in flight, it would practically prohibit the normal buckling test and the buckling load would have to be arrived at some other way. The procedure could, for instance, be as follows:

The compressive load on the member is applied slightly eccentrically giving it a minor deflection to begin with. As the load is increased the deflection is first proportional to the load, then increases more rapidly until finally the increase is quite severe. Plotting the load-deflection diagram discloses the curve shown in figure 1.

It being known that this represents a stability case, the experimentally established diagram can then be utilized to find the location of the asymptote which the buckling load represents. The estimation is fairly safe to within a few percent of accuracy. In this manner the buckling load of a thin bar can be determined without breaking it.

Now, the case of the spontaneous oscillations is quite similar. To assure regularly measurable deflections calls for an artificial excitation which sets the system in amplified oscillations or, in mathematical terms: the addition of interference functions to the homogeneous differential equations to provide for finite values for the variables. This results in equations for the amplitudes as follows:

\[ A_1^2 = \frac{Z_1}{r(\lambda,\nu)^2 + i(\lambda,\nu)^2} \cdot P^2, \]

\[ A_2^2 = \frac{Z_2}{r(\lambda,\nu)^2 + i(\lambda,\nu)^2} \cdot P^2, \]

\[ A_n^2 = \frac{Z_n}{r(\lambda,\nu)^2 + i(\lambda,\nu)^2} \cdot P^2 \]

where \( P \) is force of excitation,
\( \lambda \), frequency of excitation.

\( v \), flow velocity.

The denominator is the sum of two functions appearing squared and therefore always positive. The numerator \( Z \) is dependent on various quantities, including \( \lambda \) and \( v \) (but only slightly).

Assuming the velocity \( v \) to be constant while steadily changing the exciter frequency \( \lambda \) it affords the amplitude diagram (fig. 2) which for the sake of simplicity illustrates a system having two degrees of freedom.

As the velocity \( v \) changes the resonance points change their position and the resonance amplitudes their magnitude. Proceeding with the velocity from stage to stage and covering the whole frequency range at each stage results in the set of curves, (fig. 3), which were copied from a calculation made under simplified conditions. (The separating vortices and the internal damping are disregarded.)

Two resonance points \( \lambda_1 \) and \( \lambda_2 \) exist at \( v = 0 \), and, the air damping by inferior, the deflections assume great values. The resonance frequencies shift in the airstream and the amplitudes become, at first, smaller, owing to the incipient air damping. The minimum value is reached at a certain velocity after which the resonance amplitudes begin to rise again and assume fairly high values on approaching the critical velocity.

Plotting the maximum resonance values against the velocity results in a curve similar to that for buckling under axial load (fig. 4). Here also the approaching critical velocity must be ascertainable with sufficient accuracy.

V. TEST PROCEDURE

The accuracy of the mathematical findings was checked by wind tunnel test on an airfoil of 60 cm length and 12 cm chord (fig. 5).

The system had two degrees of freedom and was artificially excited by an electric motor. The amplitude, flexural and torsional amplitudes, were registered on a Geiger
recorder. The oscillograms obtained at the various velocities are shown in figure 6.

Plotting the resonance amplitudes against the velocity gives a curve (fig. 7) quite similar to the mathematical curve. From the steeply rising branch of the curve connecting the maximum values the critical velocity of $v_k = 24$ m/s could be estimated which in fact was the case. The test stations revealed a barely noticeable scatter, whence the experiment itself may be considered as being altogether satisfactory. But at the same time it brought out one disagreeable feature, namely, the location of the excitation.

The excitation was initiated, similar to figure 5, by an electric motor with eccentric over a weak spring (fig. 8). The exciter force was located in front.

The test was perfectly satisfactory, i.e., the oscillations set up by the air blast changed to forced oscillations without interfering with each other. The mathematical expectations were fulfilled.

A subsequent test was made with an excitation at the trailing edge, which, however, miscarried, because the amplified oscillations refused to change to forced oscillations. The oscillograms seemed superimposed; the oscillations were restless and jerky. It seemed as if the oscillation mode forced by the excitation did not agree with the amplified mode, as if the ground or basis for the amplified wing oscillations had not been sufficiently prepared. The amplified oscillation then started with jerks. This phenomenon is well known from flight tests. Frequently it is attempted to set up a wing or tail oscillation by all kinds of means without being able to do so. Then all of a sudden at a certain engine r.p.m., flight condition, gust intensity, etc., the oscillation occurs while stubbornly refusing to occur again. Such cases are equally attributable to unfavorable preliminary conditions which prevent the amplitudes from rising in the vicinity of the critical velocity.

Easy realization of a wing oscillation in flight, on the other hand, is obviously bound up with propitious preliminary conditions. The realization of these favorable conditions governs the success or failure of a flight-oscillation test. It is therefore necessary to provide
these propitious conditions for the different oscillation modes in systematic fashion.

The unlike effect of the exciter locations employed in the two experiments is readily explainable for the explored system: If the elastic axis is other than coincident with the axis of gravity it results in combined flexural and torsional oscillations. The system has two natural frequencies and accordingly two oscillation modes. Both are oscillatory motions about two rotatory axes or, as they may be designated, about the two natural axes (fig. 9). When applying the excitation at the leading edge, the force acting downward at a certain moment can be split into its downward components along the natural axes $D_1$ and $D_2$.

$P_1$ produces oscillations about $D_1$, $P_2$ about $D_2$. The path or angular deflection of each oscillatory motion $\beta_1$ or $\beta_2$ is shifted in phase through an amount $\phi_1$ and $\phi_2$ conformable to their particular natural frequencies and the exciter frequency.

If the exciter frequency happens to lie between the two natural frequencies where in fact the frequency of the amplified oscillations is to be expected, the system about $D_1$ oscillates in the supercritical, and about $D_2$ in the subcritical range. (The natural frequency about $D_1$ is lower owing to the mutual locations of the elastic and the gravity axes. It corresponds, so to speak, to the flexural oscillation while that about $D_2$ corresponds to the torsional oscillation.) The vector diagram of the oscillation has the aspect of figure 11.

It is seen that $\beta_2 \beta_1$ is in the lead as manifested by the lead of the nose over the trailing edge. But it is the very motion needed by amplified oscillations to absorb energy. Accordingly the forced oscillation has at this oscillation already the mode of the amplified oscillation.

Take the other case where the excitation starts at the trailing edge. The exciter force can again be divided in its components but they are now in opposite direction (fig. 10b). The vector diagram is as shown in figure 11b. The lead of $\beta_1 \beta_2$ signifies the advance of the trailing over the leading edge. Such an oscillation mode can have only a damping effect, that is, disturb rather than favor the amplified oscillation.
Thus, for the case of combined flexural-torsional oscillations of a wing, the simple rule can be established that the excitation must always act between the natural axes. In practice it will be expedient to locate the excitation near the leading edge so as to assure a propitious condition for amplified oscillations. The proper choice of exciter location will undoubtedly entail a certain difficulty, which, however, is not insurmountable when similar considerations are also made for the otherwise feasible and pertinent modes of oscillation. On a complex system, such as the tail unit represents, the proper location of the excitation could equally be verified by a model test and the latter need not be absolutely dynamically similar, provided the oscillation mode is the same. It is advisable and downright imperative to precede the flight-oscillation test by a static oscillation test, whose sole purpose is now to provide reference points for the natural frequencies and oscillation modes.

 Quite apart from the exciter location the exciter quantity itself is of importance. The amplitudes of the forced oscillations should not fall below a certain magnitude for reasons of instrumental accuracy as well as eventual overcoming of internal friction and air viscosity, which otherwise find an upper limit in the fatigue strength of the structure.

VI. EXPERIMENTS ON AN AIRPLANE

The described test method should have been subjected to further mathematical treatment and wind-tunnel tests, before being tried out on an airplane. But certain circumstances made it imperative to disregard any eventual scruples and to proceed with all speed to a flight-oscillation test.

The experiment, fully meeting the expectations, was carried out on the lateral control surface of the Junkers Ju 86. The centrifugal unbalance was fitted in the fairing of the rear fuselage tip and actuated by electric motor mounted in the fuselage and a flexible shaft. The Geiger recorder installed in the observer's seat was linked with the control surface by means of a thin wire and recorded the torsional oscillations which were coupled with
a translatory motion of the lateral control surface (fig. 12). The rudders were fitted with detachable mass balances. Two series of tests were carried out: one without balances with a view to ascertain the dependability of the test method and the amount of critical velocity without balance, the other with balances fitted, to establish the amount of increase in critical velocity due to the balances and whether it still lies within the range of flight speed. The first test series disclosed results very similar to the wind-tunnel tests, but with minor experimental scatter (fig. 13).

The extrapolation of the curve was indicative of a critical velocity which already had been observed in a previous flight caused by accidental oscillations. This afforded additional proof of the applicability of the flight oscillation test method. The second test series in which the airplane speed reached as high as 400 km/h (248.5 m.p.h.) resulted in a curve which, although evincing increases, disclosed no tendency toward continuous rise at any speed. This proved that the lateral control surfaces with the attached mass balances did no longer tend toward amplified oscillation.

One recommendation resulting from these experiments is the use of open automatic recorders, to assure an uninterrupted check of the existing amplitudes. If other instruments such as the DVL scratch recorder or the optograph are utilized, they should preferably be supplemented by an indicating instrument which keeps the operator advised of the amount of the deflections.

Translation by J. Vanier,
National Advisory Committee for Aeronautics.
Figure 1. - Deflection of eccentrically compressed bar.

Figure 2. - Oscillation made of a system having two degrees of freedom with its two characteristic resonance peaks.

Figure 3. - Typical curve of wing oscillation in air stream. Critical velocity at $v=18\text{m/s}$.

Figure 4. - The resonance deflections of fig.3 plotted against the velocity. The curve in the vicinity of the asymptote is similar to the deflection curve of figure 1.

Figure 5. - Sketch of excitation in model test.
- $E =$ elastic axis
- $S =$ gravity axis

Figure 6. - Resonance amplitudes of fig.6 plotted against velocity. Note the almost complete absence of experimental scatter.
Figure 5.— Model wing mounted for wind-tunnel test. Exciter—electric motor below wing, Geiger recorder on table.

Figure 6.— Oscillograms of model test at various airflow velocities. Flexural frequency: $\lambda_B = 8.5$ Hertz
Torsional " : $\lambda_D = 13.3$ "
Frequency of amplified oscillation : $\lambda = 10.7$ Hertz
Figure 9.- The 4 recorded points of a system having two degrees of freedom - pure sine oscillations may occur only about the natural axis $D_1$ and $D_2$. $\beta_1$ and $\beta_2$ denote the pertinent angular deflections.

Figure 10.- Force $P$ split in its components $P_1$ and $P_2$ acting in the natural axes and exciting pure sine oscillations about the pertinent axis.
(10a) synonymous excitation.
(10b) opposite

Figure 11.- Vector diagram of forced oscillations of system fig. 10a and 10b.
(10a) $\beta_2$ leads.
(10b) $\beta_1$ leads.

Figure 12.- Experimental arrangement on Ju 86.

Figure 13.- Result of flight oscillations test on Ju 86 (no mass balance).
The resonance amplitudes plotted against the flight speed manifest a similar character to the model test and permit of a sufficiently close appraisal of the critical velocity.