EXPERIMENTS WITH NEEDLE BEARINGS

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In recent years there has been introduced into the construction of engines, particularly in aircraft engines, a new type of bearing which presents notable characteristics of efficiency and reliability in comparison with those previously used. This is the needle bearing (fig. 1), comparable in design to a roller bearing without cage, with rollers of very small diameter, 2 to 4 mm (0.08 to 0.16 in.), in proportion to their length, but differing greatly from the latter in its manner of functioning and therefore in its supporting ability and resistance to motion.

In roller bearings, as in ball bearings, there is a tendency to substitute rolling friction for sliding friction, thus leaving the lubricating oil with only a secondary function. In needle bearings, on the contrary, the lubricant has its characteristic function of putting itself automatically in pressure between the surfaces so as to avoid direct contact. This function is greatly facilitated by the needles, and hence the supporting capacity is increased beyond the customary limits of ordinary bearings.

To the consequent reduction in the coefficient of sliding friction is due the ability to function with high ratios between the length and diameter of the needles (8 to 10), an ability not possessed by roller bearings which are therefore incapable of functioning without cages. In fact, if a roller gets out of parallel with its journal while the engine is running, it cannot return to its correct position without sliding, which is opposed by the considerable sliding friction in the case of roller bearings where the lubricant acts only by its greasiness, as distinguished from needle bearings in which everything cooperates to reduce the coefficient of friction to a minimum.

"Esperienze su cuscinetti ad aghi." Rivista Aeronautica, October, 1932, pp. 47-71.
To the smallness of the diameter of the needles, as well as to the increase in the number of generatrices of contact with the journal and the especially favorable behavior of the lubrication, is due the ability to function for long periods without the addition of oil, which is held in sufficient quantity by the capillary effect of the narrow spaces between the needles.

Up to a certain limited value of the specific pressure, contrary to what may seem to be the case at first view, the needles do not rotate about their own axes, as in roller bearings, but revolve as a whole about the journal, dragged along by the latter and sliding on the journal and on the bushing like a ring slipped over the journal itself (reference 1).

These recall the functioning of the rings of the Parsons bearings used in steam turbines and the reduction in the relative velocity of the contact surfaces (between the journal and the ring and between the ring and the bushing). The consequent reduction of the ratio between the calories corresponding to the work of friction and the surface area would explain the ability of such bearings to support greater loads.

Under such conditions there would be realized the equilibrium between the forces of friction exerted on the needles by the film of oil set in rotation by the journal and the forces of friction exerted by the lubricant which fills the spaces between the bushing and the contiguous needles. With such equilibrium the needles would not rotate.

However, it is recalled that, after exceeding a limiting value of the specific pressure and after the failure of the oil film between the needles and the journal and the consequent considerable increase in the friction between the journal and the needles, the latter are forced to rotate like the rollers of roller bearings and thus to restore the continuity of the lubricating film.

Hence the needles would rotate under abnormal conditions and would represent a safety device against the danger of seizing, the effect being to restore the oil film destroyed by the excessive pressure. In reality, on a more accurate examination of the problem, the behavior of the needles appears considerably different. This different behavior, rather than the generally accepted hypothe-
ses, would seem to explain the brilliant results which characterize this type of bearing. The experimental results constitute the best confirmation of our conclusions.

Let us consider a journal supported by a certain number of needles and the load $p$, which rests on the bearing, distributed between them in such manner that the wear of the journal and of the bearing, in harmony with Rey's hypothesis, is proportional to the work of friction. The wear being necessarily the same at all points, the normal pressure exerted by the journal on each needle (fig. 2) is represented by

$$\frac{p}{\cos \varphi} = \text{constant}$$

and, calling $p_{\text{max}}$ the value of $\frac{p}{\cos \varphi}$ which belongs to the lower needle for $\varphi = 0$, we have

$$p = p_{\text{max}} \cos \varphi \quad (1)$$

We can consider the two components of $p$: namely, $p/cos \varphi$ which, according to formula (1), is equal to $p_{\text{max}}$ for each needle and which acts on the bearing, and $p \tan \varphi$ which tends to make each needle climb on the bearing till it presses against the next needle.

Through the symmetry of the bearing and through the clearance between the needles* it happens therefore that all the needles on both sides of the lowest needle tend to move away from the latter, as indicated in figure 3, with increasing pressure on each side up to the two needles at the ends of the horizontal diameter, and with uniform pressure for all the needles of the upper half of the bearing.

The value $N$ of this pressure of the angle $\varphi$ is measured by the sum of the components of $p \tan \varphi$ in the direction of $N$ (forming the angle $\varphi_1$ with the horizontal). These components (fig. 4) are represented by

*A radial clearance of not less than 0.02 to 0.04 mm (0.0008 to 0.0016 in.) is prescribed for the regular functioning of the bearings.
AB = \( p \tan \varphi \cos (\varphi_1 - \varphi) \)

and hence, considering the small diameter of the needles,

\[
N = \int_0^{\varphi_1} p \tan \varphi \cos (\varphi_1 - \varphi) \, d\varphi
= p_{\text{max}} \int_0^{\varphi_1} \sin \varphi \cos (\varphi_1 - \varphi) \, d\varphi.
\]

By integrating we obtain

\[
N = p_{\text{max}} \left[ \frac{1}{2} \cos \varphi_1 \sin^2 \varphi + \frac{1}{2} \varphi \sin \varphi_1 - \frac{1}{4} \sin \varphi_1 \sin^2 \varphi \right]_0^{\varphi_1}
\]

which, by successive developments, is reduced to

\[
N = p_{\text{max}} \frac{1}{2} \varphi_1 \sin \varphi_1
\]

According to this formula, \( N \) increases (as is natural) along with \( \varphi_1 \) and, in the special case of \( \varphi_1 = \pi/2 \) (needle situated on the horizontal diameter of the journal), we have the maximum value \( N_{\text{max}} \) measured by

\[
N_{\text{max}} = \frac{\pi}{4} p_{\text{max}}
\]

The conditions of a needle vary therefore with its position on the journal. In general, for a given position of the needle, as defined by the angle \( \varphi \), we have to consider the following forces (fig. 5):

- The normal pressure \( p \) exerted on the journal and the normal reaction \( p \) exerted on the bearing;
- The friction \( f \) produced by the journal and the bearing (or by the interposed lubricant);
- The normal pressure \( N \) exerted by the next lower needle and the normal reaction (which, considering the smallness of the needles, is assumed to be equivalent to the preceding) exerted by the next higher needle;
- The friction \( 2f \) produced by two adjacent needles;
- The friction, whose moment is \( M_1 \), produced by the lubricant on the remaining wetted surface of the needle.

All together we must therefore consider: the moment which tends to make the needle rotate
\[ M' = 2 \, f \, p \, r \]  
(4)

and the moment which tends to oppose the rotation
\[ M'' = 2 \, f \, N \, r + M_l. \]

For all the needles of the upper half of the bearing, \( p \) is zero and therefore the needles have no tendency to rotate.* For all the needles of the lower half of the bearing, \( p \) increases, with the diminution of \( \varphi_1 \), from zero to \( \varphi_{max} \) according to formula (1), and \( N \) decreases, according to formula (2), from the value
\[
\frac{\pi}{4} \, D_{\text{max}},
\]
indicated by formula (3), to zero. Figure 6 represents the laws of variation of these forces.

For a double reason therefore, with the diminution of \( \varphi_1 \), there is an increasing tendency of the needle to rotate under the action of the applied forces. If \( M_l \) be disregarded, it is easy to determine the value \( \varphi_0 \) of \( \varphi_1 \) corresponding to the condition of equilibrium between the moment which tends to produce rotation and the moment which tends to prevent it.

It suffices to write \( M' = M'' \) or \( 2 \, f \, p \, r = 2 \, f \, N \, r \) from which \( p = N \). On substituting the values of \( p \) and \( N \) from formulas (1) and (2), we obtain
\[
\cos \varphi_0 = \frac{1}{2} \, \varphi_0 \, \sin \varphi_0
\]
or
\[
\varphi_0 = 2 \, \cot \varphi_0 \quad (5)
\]
which is satisfied by the value**
\[
\varphi_0 = 61^\circ 30' \sim
\]

If no friction were produced by the lubricant on the wetted surface of the needles, the moment \( M \) would pre-

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*If the radial components of the normal reactions of the two laterally contiguous needles are disregarded.

**The diagrams in figure 6 confirm this result in the point of intersection of the two curves.
vail for all values of the angle under about 60°. In reality this limit is lower and depends on the nature and temperature of the lubricant, but in any case the tendency to rotate increases with the proximity of the needles to their lowest position.

Since, when a needle is at its lowest point, it is detached from the contiguous needles, as already demonstrated, and therefore $N$ becomes zero, the action opposed to rotation, as represented by the friction of the reactions of the contiguous needles, becomes zero (fig. 6), while the moment tending to produce rotation reaches its maximum value, which increases with the load.

It follows that, when the needle reaches the lowest point of the bearing, it will have the greatest tendency to rotate and will, in fact, rotate with a peripheral velocity equal to that of the journal, i.e., without slipping on the latter, provided the effect of the engine moment $M'$ on the resisting moment $M''$ overcomes the inertia offered by the needle to the motion of rotation.

If the angular velocity of the needle about its own axis, in the hypothesis of zero slipping, is very high (e.g., 15 times that of the journal), its moment of inertia being minimum, this condition is satisfied by the normal revolution speed of the engine.*

On rotating the needle at the same peripheral velocity as that of the journal, the needle will continue to rotate and, in rotating, to push the whole system of needles before it until it reaches a position corresponding to a pressure of the contiguous needles sufficient to develop a frictional resistance, which, taking into account

*We must have

$$\frac{1}{2} i \omega^2 = \int_0^a (M' - M'') \, da$$

from which, with obvious substitutions, we obtain

$$\frac{1}{2} i \omega^2 (\frac{R^2}{r^2}) = \frac{1}{2} p_{\text{max}} R$$

or

$$p_{\text{max}} = i \frac{R}{r^2} \omega^2.$$
the inertia of the needle, will stop the rotational motion. After this the needle will continue to revolve integrally with all the others about the axis of the journal, but slipping without rotating with respect to the journal and bushing.

It is easy to calculate the angular velocity of the ring equivalent to all of the needles. Given the mechanism of the phenomenon, this is represented by the angular velocity of the gear casing in the formula of Willis

\[ \Omega = \frac{\omega_c + \frac{d\omega_a}{d\omega_c}}{1 + \frac{d\omega_a}{d\omega_c}} \]

in which \( \omega_c \) must be zero and \( \omega_a \) represents the rotational velocity of the journal. We can then write

\[ \Omega = \frac{\omega_a}{\frac{d\omega_a}{d\omega_c} + 1} \]

On substituting the ratio of the diameters corresponding to the case under consideration, e.g., \( d_c = 51 \text{ mm} \) (2.01 in.) and \( d_a = 45 \text{ mm} \) (1.77 in.), we have

\[ \Omega = 0.47 \omega_a \]  \hspace{1cm} (6)

The system of needles, which we can consider all together as a ring slipped over the journal, thus acquires a rotational velocity, if there is no slipping,* equal to 0.47 of the velocity of the journal, its motion being insured by the needles in the lowest position, which function as true rollers propelled by the whole system of needles.

* * *

Under these conditions the functioning of the needle

*This slipping occurs when the resistance offered by the needles as a whole to the motion of rotation about the axis of the journal is excessive and impedes the functioning of the propelling needles, thus forcing them to slide. This case corresponds to the case of insufficient clearance between the needles, or of sudden acceleration of the angular velocity of the journal.
bearing is truly interesting and rational. In all the parts of the bearing surface, corresponding to limiting values of the specific pressure, the system of needles behaves like a ring rotating at a velocity intermediate between the velocity of the journal and that of the bushing. The conditions of lubrication are therefore particularly favorable, not only because, due to capillarity, the oil cannot be taken away (the surface endowed with relative motion being doubled, a fact favorable to the removal of heat which affects the limiting value of the load), but still more because of the particular form of the system which reproduces in the most efficacious manner the type of bearing proposed and tested by Brillie with such excellent results (reference 2).

It consists of slender strips of bearing surface represented by the generatrices of the needles in contact (mediate) with the journal, and of slender "reservoirs" of oil interposed between them and represented by prisms of triangular mixtilinear section between the contiguous needles. In the relative motion between the journal and the ring and between the ring and the bushing, the oil is dragged along by the effect of viscosity according to the theory of Reynolds and put under pressure in correspondence with the generatrices of the needles. Passing then to the next reservoir, the latter is also put under pressure, always so that the charge corresponding to the flow of the oil by hydrodynamic action, is sufficient to offset the lateral leakage due to the shortness of the needles and fortunately opposed by the capillary form of the spaces between the needles.*

As follows from the experiments of Brillie, the oil reservoirs interposed between the elements of the plane surface acquire a supporting ability similar to that of the surfaces themselves in such manner that the effective surface of the bearing is not diminished by them. Let us consider the motion of the oil in one of those reservoirs (fig. 8). The film of oil interposed between the surfaces is drawn from the journal without mixing with the oil of the reservoir. The latter is set in motion by this film through the effect of viscosity, and the positive and negative velocities are stabilized at the various points ac-

*It is only necessary to recall the formula of Poiseuille which gives the volume of the capillary tubes

\[ Q = \frac{\pi \cdot r^4}{8 \cdot \mu \cdot l} \]
According to a parabolic law as shown in the diagram. There is thus created a closed circuit of the liquid filaments in which they behave like an element of a roller bearing, acquiring the ability to support the load, and the corresponding frictional resistance is reduced to a minimum in consequence of the low relative velocity of the liquid filaments.

In this way the best functioning conditions are realized for all parts of the bearing surface on which the needles simply have the motion of sliding on the journal and on the bushing.

On all the other parts of the bearing surface, on which the pressure is the greatest and where there would not be sufficient oil drawn from the journal alone (or held by the bushing) to support the load and maintain the continuity of the oil film, the process of lubrication is supplemented by the rotation of the needles. Thus rolling friction replaces sliding friction at the points of maximum specific pressure and insures the continuity of the oil film, notwithstanding the increased load, because the oil is drawn in larger quantity between the bearing surfaces not only by the action of the journal (or of the bushing), but also the action of the needle which rolls on the bushing. From the preceding it follows that needle bearings constitute an important device for realizing the best functioning conditions.

In the low-pressure zones the lubrication is left to the hydrodynamic action of the surfaces in motion by reducing the friction to the minimum through the effect of the oil reservoirs conceived by Brillie. In high-pressure zones, the conservation of the oil film is insured by the rotation of the needles, which replaces sliding friction by rolling friction, as in roller bearings.

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It was considered of sufficient interest to undertake experimental researches for the purpose of confirming the above results and for determining at the same time the coefficients of friction for the various loading conditions.

The test apparatus shown in figure 9 was installed in the engine laboratory of the Naples School of Engineering. A shaft with rotating steel disks of suitable moment of inertia (1 = 0.2 kgm s\(^2\)) and weight Q was supported on two needle bearings of the type shown in figure 1. Between
these two supports was placed a collar supported by two other needle bearings like the first. A weight $P$ could be suspended from this collar by means of a suitable lever.

The load supported by the bearings (fig. 9a) was thus measured by $P/2$ for the central bearings and by $(P+Q)/2$ for the lateral bearings. Due to the smallness of the weight $Q$ ($53 \text{ kg} = 117 \text{ lb.}$) in comparison with the loads $P$ used in the tests and the relatively small effect of the load on the coefficient of friction, we may consider the total load $2P + Q$ as equally distributed between the four bearings.

By means of a belt drive the system was set in rotation at a suitable angular velocity, and the rate of retardation was recorded after eliminating the belt. It was thus tested with specific pressures increasing from 7.3 to 101 kg/cm² (104 to 1,437 lb./sq. in.). Greater pressures could not be employed without introducing causes of error due to the bending of the shaft by the load $P+Q$, on account of the particular method of installation which did not permit the spherical adjustment of the bearings. Table I and figure 10 contain the recorded angular velocities and accordingly represent the curve of retardation. From these values it was possible to calculate the moments of the inertia forces $I \frac{d\omega}{dt}$ by introducing the expression

$$I \frac{d\omega}{dt} = M_a + M_v$$

into the equation of motion. In this equation $M_a$ represents the moment of the frictional resistance of the bearings and $M_v$ the moment of the ventilating effect. The ventilating effect was calculated by Stodola's formula (reference 3):

$$N = 75 \times 10^{-6} \beta D^2 v^3 \gamma$$

in which $N$ is the power in kgm/s absorbed by a disk of diameter $D$ rotating with a peripheral velocity of $v$ m/s in a medium having a specific weight of $\gamma$ kg/m³.

Table II and figure 11 contain, in terms of the revolution speed, the values of $M_v$ thus calculated and, by difference with the values of $I \frac{d\omega}{dt}$ already found, the corresponding values of $M_a$. Being able to write

$$M_a = f (2P + Q) r$$
in which \( 2P + Q \) is the total load distributed between the four bearings and \( r \) the radius of the needles, we were able to obtain the values of the coefficient of friction \( f \) corresponding to the various angular velocities, as given in Table I, Figure 12.

As shown in the diagrams, for every value of the specific pressure, the coefficient of friction decreases with increase of the angular velocity up to an optimum value of the latter beyond which it begins to increase. This optimum value increases with the specific pressure.

This corresponds perfectly to the physical interpretation of the phenomenon, that is, as the angular velocity increases, the efficacy of the lubrication increases, due to the hydrodynamic action of drawing the oil by the surfaces in motion. However, above a certain limited value of the angular velocity, the viscosity of the lubricating oil is diminished by the increased temperature due to the increased friction, which is a function of the revolution speed. Obviously, therefore, this velocity limit increases with the increase in the specific pressure, because a more abundant drawing of the oil is necessary in order to maintain the same temperature of the surfaces and therefore the same viscosity of the oil.

It follows, however, from the mean values recorded in Figure 12 that, for every value of the angular velocity, the coefficient of friction diminishes as the specific pressure increases up to an optimum value, beyond which the coefficient of friction begins to increase. The optimum value of the specific pressure increases with the angular velocity.

The results are fully confirmed by the following facts. For small specific pressures, so long as the oil remains interposed between the surfaces, the frictional resistance does not increase with the load, because it concerns the internal friction of the oil which is independent of the pressure.* Hence the coefficient of friction, understood as the ratio between the frictional resistance and the load, diminishes as the specific pressure increases. However, above a certain optimum value of the

*This follows Newton's law

\[
R = \mu \Omega \frac{dy}{dy}
\]
specific pressure (beyond which, for the angular velocity and for the viscosity of the oil to which the experiment is referred, there begins to be manifest the inability of the oil to remain between the surfaces), any increase in the specific pressure tends to worsen the conditions of lubrication and therefore to increase the coefficient of friction.

This optimum value of the specific pressure logically increases with the angular velocity, because, for the greater quantity of oil set in rotation by the journal, the bearing capacity of the surfaces is increased.

In order to have an experimental confirmation of our explanation of the functioning of needle bearings, it is first necessary to determine whether the coefficients of friction remain unchanged when it is made impossible for the needles to rotate in the bearing.

Due to the small diameter of the needles, such a realization offers great practical difficulties. Nevertheless, after a long series of attempts, it was successfully accomplished by making, with a special machine, a small groove in each needle (fig. 13), thus rendering it possible to apply a slender steel ring to prevent all the needles from rotating, while retaining sufficient clearance between the steel ring and the bearing.

With the bearings thus modified, the experiments were repeated and yielded the new retardation curves (table III and fig. 14) making it possible to obtain by the method already indicated (table IV and fig. 15) the new values of the coefficient of friction. These new values, even for small specific pressures, differ too much from those based on normal functioning conditions to make it possible to retain, as acceptable, the hypothesis that the needles do not rotate. On the other hand, the rotation was rendered visible by observing the needles with a specially adapted stroboscopic device.

On removing the protecting flange of the outside casing of the bearing, it was possible to solder to a needle a fine wire in extension of its axis. To the end of this wire there was attached a paper disk (normal to its axis) of 5 mm (0.2 in.) diameter, half white and half black. On illuminating the bearing with a neon lamp controlled by a suitable interrupter operated by an electric motor running at a suitable speed, it was possible to observe the rota-
tion of the needle about its axis when it passed near the most heavily loaded generatrix of the bearing. It was not possible, however, by means of this device (as would have been desirable) to measure the angular velocity of rotation, because it was so irregular and so easily affected by vibrations and load variations.

With this stroboscopic device it was, however, possible to measure the angular velocity of the rotation of the needle about the axis of the journal, with the results shown in figure 16, which agree almost perfectly with our theory (and therefore confirm the rotation of the needles), indicating by dotted lines the values corresponding to the formula of Willis.

It was therefore necessary to invent another device for recording the rotary motion of the needles, in order to obtain a complete experimental demonstration of the principles previously enunciated. This was accomplished with two different devices, one electric and the other mechanical, the results of which serve to check one another.

The electric device corresponded to figure 17 with a Rumkorff coil and Wehnelt interrupter. The tip of the wire soldered to the needle was bent at right angles, so that, during the rotation of the needle, it passed within 5 mm (0.2 in.) of a metal ring mounted on the same axis, and the inner surface of the bearing was electrically insulated from the journal on which it was placed. When, with the rotation of the journal, the needle passed the most heavily loaded generatrix, the metal tip passed within 5 mm of the ring and thus caused the passage of an electric spark. This spark was easily recorded, because, as follows from figure 17, it also passed to a metal drum around which was wound a sheet of suitably prepared black paper. The passage of the spark made a small hole in the sheet of paper, and, the drum having a known rotational velocity, it was thus possible to record the complete revolutions of the needle by the effect of the rotation in varying the time.

Figure 18 contains an example of the records. It follows first of all that the motion of the needles is not strictly uniform, as is natural since it depends on the condition expressed by equation (4), a single vibration or a local variation in the lubrication being sufficient to accelerate or retard the determination of the rotation of the needles.
The sparks occur more frequently in correspondence with the passage of the needles through the most heavily loaded zone of the bearing and are lacking altogether while the needle is passing over the upper half of the journal, in harmony with what has already been said.

It sometimes happened that the sparks continued even when the rotation was considered finished and showed a much greater frequency under these conditions. This was due to the fact that, at the termination of the rotation, the tip of the wire remained sufficiently near the ring to allow (through the ionization of the air from the preceding discharges) the passage of a spark every time the Wehnelt interrupter broke the circuit. The increased frequency of the perforations corresponded, in fact, to the frequency of the Wehnelt interruptions.

The mechanical device for checking the results obtained by the electric method, rendered it possible to obtain directly the diagram of the motion of the needle, due to the low angular velocity of the shaft. The metal point attached to one of the needles was slightly eccentric and, with the rotation of the journal, described a path in a plane normal to its axis. With a suitable micro-metric device it was possible to apply to the point a sheet of smoked paper in the plane of its path.

Figure 19 shows two of the diagrams thus obtained in which is clearly recorded the rotational motion of the needle which occurred when the friction produced by the load prevailed over the other forces applied to the needle. The rotating needle made one or more rotations according to the conditions of loading, lubrication, velocity, etc., even a single vibration sufficing to alter the law of motion. The result was that the generatrix of the journal, to which corresponded the manifestation of the rotary motion, was not the lower, more heavily loaded one, but, in every instance, was shifted forward in the direction of motion with respect to that one, and the angle of advance increased with the velocity. There was thus rendered manifest, in perfect harmony with the considerations enunciated, the effect of the inertia of a needle on its rotary motion about its own axis, which motion must logically increase with the angular velocity of the journal.

The experimental investigations have thus afforded complete confirmation of our theory on the functioning of needle bearings in contrast with the theory generally accepted.
From this it appears that needle bearings represent an interesting realization of the best functioning conditions of bearing surfaces, that from this is derived the justification of the very high loading values - up to 250 kg/cm² (3,556 lb./sq.in.) in current practice - and that these bearings have very favorable coefficients of friction and conditions of safety.

**TABLE I**

<table>
<thead>
<tr>
<th>P = load on lever, kg</th>
<th>p = specific pressure on bearings, kg/cm²</th>
<th>t = interval between readings, sec.</th>
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</thead>
<tbody>
<tr>
<td>P = 0</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>p = 7.3</td>
<td>20</td>
<td>33.7</td>
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<tr>
<td>t = 5</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>1,450</td>
<td>500</td>
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</tr>
<tr>
<td>1,380</td>
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<tr>
<td>1,310</td>
<td>460</td>
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<tr>
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TABLE II

<table>
<thead>
<tr>
<th>Total load (2P + Q), kg</th>
<th>205</th>
<th>555</th>
<th>1080</th>
<th>1955</th>
<th>2825</th>
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<tbody>
<tr>
<td>Specific pressure, kg/cm²</td>
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<td>20</td>
<td>38.7</td>
<td>70</td>
<td>101</td>
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<tr>
<td>n=1250</td>
<td>( M_\text{v} = 0.107 )</td>
<td>( M_\alpha = 0.19 )</td>
<td>( M_\alpha = 0.29 )</td>
<td>( M_\alpha = 0.45 )</td>
<td>( M_\alpha = 0.59 )</td>
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TABLE III

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<th>P = load on lever, kg</th>
<th>p = specific pressure on bearings, kg/cm²</th>
<th>p' = corrected specific pressure (to take account of reduction in area due to arresting ring), kg/cm²</th>
<th>t = interval between readings, sec.</th>
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<td>580</td>
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<td>1,100</td>
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<td>850</td>
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N.A.C.A. Technical Memorandum No. 707

17
TABLE IV

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<th>Total load (2P+Q), kg</th>
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<th>451</th>
<th>835</th>
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<td>Specific pressure, kg/cm²</td>
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<td>16.5</td>
<td>30</td>
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<td>Corrected specific pressure, kg/cm²</td>
<td>9.3</td>
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<table>
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<th>n = 1,150</th>
<th>M_v = 0.107</th>
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<th>f = 0.57</th>
<th>M_a = 0.95</th>
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<th>M_a = 2.29</th>
<th>f = 0.107</th>
<th>M_a = 5.89</th>
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Translation by Dwight U. Niner, National Advisory Committee for Aeronautics.

REFERENCES

1. Pitner: Journal de la Société des Ingenieurs de l'automobile, June, 1930; and Génie Civil, Oct. 11, 1930.


Figure 4.

Figure 5.


Figure 6.

\[ W = \frac{1}{2} p_{\text{max}} \sin \varphi \]

\[ p = p_{\text{max}} \cos \varphi \]

Figure 7.

Figure 8.
\[
\begin{align*}
\frac{P}{2} & \quad \frac{P}{2} \\
\downarrow & \quad \downarrow \\
\frac{P+Q}{2} & \quad \varphi & \quad \frac{P+Q}{2}
\end{align*}
\]

Figure 9a

Figure 9.
1, \( p_l = 7.35 \text{ kg/cm}^2 \)
2, " = 20.00 "
3, " = 33.70 "
4, " = 68.00 "
5, " = 101.00 "

Figure 10.-Retardation curves.
1. $P_m = 7.35\text{ kg/cm}^2$
2. " = 20.00 "
3. " = 38.70 "
4. " = 68.00 "
5. " = 101.00 "

Figure 11.
1, \( p_m = 7.35 \text{ kg/cm}^2 \)  \( \mu_m = 9.35 \text{ kg/cm}^2 \)

2, \( " = 16.50 " \)  \( " = 21.00 " \)

3, \( " = 30.00 " \)  \( " = 38.00 " \)

4, \( " = 55.00 " \)  \( " = 70.50 " \)

Figure 14.—With needles immobilized (or arrested)
N.A.C.A. Technical Memorandum No. 707

Figs. 15, 16

Figure 15. - With needles immobilized (or arrested)

Figure 16.
Figure 17.

\[ \Omega = 0.415 \omega \rightarrow \Omega = 0.275 \omega \rightarrow \Omega = 0.415 \omega \rightarrow \Omega = 0.416 \omega \]

Time scale 30mm = 1 sec.

Figure 18. Weight of lever, 25 kg
r.p.m. of shaft, 500 (rev. = 10mm)
Mean pressure, 25.7 kg/cm²

Figure 19.