TURBULENCE AND MECHANISM OF RESISTANCE
ON SPHERES AND CYLINDERS

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Turbulence in Pipes, around Obstacles, and through Screens

Turbulence is generally expressed as a haphazard, irregular maze of eddying motions whose shape, direction, and intensity are variable and indefinite. Now, it is generally conceded that all flows of water and air are turbulent, with the exception of Poiseuille's laminar or parallel flow in pipes. But even this becomes turbulent as soon as a certain critical velocity, dependent upon the test conditions, has been reached. Our photographic methods have made the turbulence now accessible to visual observation. The turbulence in pipes consists of annular vortices whose rotation in the axis of the pipes is toward the exit. The vortices lie irregularly in normal or oblique transverse planes and can be connected to one another by branching. In the eccentric cyclic motions the axes of rotation are near the walls which are shrouded by a fine, consistently laminar boundary layer. The rows of the vortices are in sections, are wholly enveloped by jointed circulations, and form gyromas, compound eddies of different degrees wherein they can rotate around each other.

The turbulence of flow around obstacles and, in general, the dynamic turbulence of free flows set up by contact with solid or fluid boundaries, evinces the same essential characteristics. This, of course, does not include the thermal turbulence, the turbulence of thermal convection, which is of different type and outside the scope of this paper.

The turbulence of straighteners in wind tunnels is of particular interest. The wind artificially produced by a fan contains strong, irregular vorticities which, when passing through the straighteners, are largely removed, leaving behind the metal screen a much finer, regular, evenly distributed screen turbulence.

The structure of such turbulence may be seen in Figure 1. It is the flow through a screen of parallel metal strips 10 cm wide set 2 cm apart. Whereas the camera is stationary, the screen moves, traversing during the 1/5 second time of exposure the distance shown by a light reflection as a fine white line next to the rear ends of the plates. The flanks of the plates are covered by a chain of fine vortices which at 6-10 cm back of the plate converge into larger vortices under violent motions and are similar in shape and position to "Karmans vortex street." Some distance away from the screen the rows break up, the vortices of the individual streets diverge and make room for the advancing suction flow which soon joins up with the adjacent streets in a labyrinthine maze of partial flows. The form of this transition is illustrated in Figure 2. The inflow denotes the loss in velocity which an originally parallel streamline flow incurs in each point by passing through the field. One of those turbulent relative movements is given in Figure 3.

To visualize the effect of a plate screen on a turbulent flow we utilized the coarse turbulence of a wide-meshed bar screen as shown in Figure 4. This screen was mounted some distance ahead of the plate screen and both moved through the quiescent fluid. The stationary camera recorded the picture. (Fig. 5.) The vortices visible at the upper margin of the photograph are split up when passing through the plate screen and partially destroyed, while at the same time interrupting the fine, uniform formation of the vortex streets and leaving a maze of vortices and vortex groups behind which are enclosed by joint circulations; the motions are predominantly forward, but also in any other direction with fluid almost at rest in between. Even such a narrow plate screen is unable to change a turbulent into a uniform parallel flow; it merely refines the turbulence, the vortices, turning partly to the right and partly to the left, approximately equal in size, the spacing of the screen plates, but otherwise are irregular in every respect.
The turbulence cannot be equalized except by viscosity and will therefore persist for some time after passing through the screen so long as the width of the screen is as large as that for our models.

The straighteners in wind tunnels usually consist of square cells of 10 cm zinc strip. Consequently, the vortices set up on the walls inside of the cells gyrate partly around vertical, and partly about horizontal axes which are cyclically connected to each other. The double layers of the opposingly directed vortices impress the same cell-like structure on the air stream as the straightener, and the orbital fluid columns of the vortices investing each cell space in close sequence, form a kind of skeleton in the stream which offers a certain resistance against deformation by outside forces.

B. Turbulence and Resistance on Spheres

According to a statement of G. Eiffel the translatory velocities measured by Pitot tubes in the cross section of the wind tunnel evince an average variation of around 0.5 to 1.1 per cent, whereas without straightener, the resistance cannot be measured at all because of the turbulence. As a result it was to be assumed that the turbulence, left in the stream, would have no marked bearing on the resistance.

But subsequent measurements on spheres carried out at Göttingen and Paris disclosed the surprising fact that under certain conditions even a very inferior turbulence can reduce the resistance considerably, in direct contrast to what would be expected from many other experiences.

Professor L. Prandtl* has published a very comprehensive paper on this subject, from which we quote:

According to the well-known resistance formula

\[ W = \kappa \rho F v^2 \]

the coefficient \( \kappa \) for spheres was defined in Göttingen at \( \kappa = 0.22 \) against Eiffel's 0.088. A subsequent check in Paris revealed that \( \kappa = 0.22 \) occurs at low velocities and then drops in a transition zone at increasing velocity to this small figure. Eiffel also emphasized that both figures for \( \kappa \) pertain to different

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forms of flow. The discrepancy in the test data was explained by the difference in velocity, which in the Gottingen tunnel was from 4 to 8 m/s against Eiffel's 30 m/s.

Prandtl then installed a jet between straightener and test station which raised the velocity to 23 m/s and made the flow very uniform and nonvortical. Now a repetition of the tests likewise disclosed the marked drop in k but at decidedly higher velocities and values of Reynolds Numbers.

According to the Reynolds law of similitude for fluids of constant volume, the flow on geometrically similar bodies is also geometrically and dynamically similar, if they occur at equal values of $R$. Thus the resistance measurements on spheres did not agree with the law of dimensions and made the extrapolation of the wind tunnel data appear questionable.

Now Reynolds himself postulates that in pipes the transition from laminar to turbulent flow attitude does set in at a predetermined figure, $R = 2000$, but at a much lower figure if the flow already contained previously existing vortices. From this one may assume that the divergence of the resistance of spheres from the law is traceable to some effect of turbulence.

In the Eiffel tunnel the flow had to be turbulent because the test station was directly behind the straightener. So the previously uniform flow in the Gottingen tunnel was also made turbulent by means of a screen made from 1 mm binding twine into a 5 cm mesh screen which was mounted close in front of the sphere. The result was a drop of more than half the resistance in the erstwhile transition zone and a drop in k at practically the same values of Reynolds Numbers as in Eiffel's tests.

This proved experimentally that the cause of the reduced resistance lay in the turbulence set up by the straightener, which was lowered considerably when installing the nozzle and raised again by the screen made of binding twine.

*The $R$ numbers result from $R = \frac{\nu d}{\mu}$, where $\nu = \text{flow velocity}$, $d = \text{dimension of object}$ and $\nu = \text{kinematic viscosity} = \frac{\mu}{\rho}$, $\mu = \text{viscosity}$, $\rho = \text{fluid density}$. 
The significance of the mechanical relationship between turbulence and resistance now came into the foreground. The smoke photographs revealed for the large and small value of $k$ the forms reproduced in Figures 6 A and 6 B. For high resistance, case A, the turbulent body behind the sphere with its waivering front edge $c$ extended about 5-8° forward beyond the equator, whereas in case B, the separating line was 10-25° to the rear. The more intimate bond of the flow around the sphere and the diminution of the eddying body bound up with it, is therefore an effect of the flow turbulence, although this itself was not discernible in the smoke. Consequently, the test failed to disclose the manner in which the turbulence sets up the profound upheaval in the entire flow pattern.

Prandtl attempted to solve this problem by theoretical considerations. The sudden change in the flow and in the resistance of the sphere occurs at Reynolds Number $R \frac{v_d}{D} = 130,000$. The Reynolds Number denotes the ratio of the inertia to the frictional forces of the fluid. Now it seems startling that a reversal of the flow induced by the reciprocal effect of friction and inertia does not set in until at such a high number of this ratio, particularly since the friction appears to play an altogether subordinate role. On the other hand, this ratio is valid for the free fluid only, not for the thin boundary layer in which the velocity of the free flow drops to zero on the surface of the body. Friction and inertia are here of the same order of magnitude and "one is justified to a certain extent in assuming that the starting-point of the sudden resistance change is to be expected here."

This assumption established the connection with the boundary layer theory. But how could the screen turbulence of the flow react on the boundary layer? Probably to the extent of making the otherwise laminar boundary layer turbulent also. If this was the case then the turbulence of the boundary layer was the real cause of the lower resistance and of the reversal of the total flow, and this effect was bound to occur also whenever the boundary layers became turbulent for any other reason.

To produce turbulence a wire ring 1 mm thick was mounted 15° in front of the equator of a 28 cm sphere. According to calculation the boundary layer is 1 mm thick and it was assumed to have been made turbulent by the vortices set up on the ring. The result of the measurement
confirmed the expectation. The ring reduced the resistance from 0.24 to 0.058 and 0.09 for the entire velocity range.

Prandtl calls this interesting experiment an "experimnetum crucis," which allegedly proves that the screen turbulence of the flow, quite like the wire ring, makes the boundary layers turbulent and thus lowers the resistance. He also opines that the low resistance of slender bodies in general, such as airships, were due to turbulent boundary layers, whereas the high resistances of wide bodies such as of the transverse plate, were due to laminar boundary layers.

C. Critical Considerations

Resistance measurements, unfortunately, yield nothing as to the mechanism of the resistance; hence it is not surprising when the pertinent analogical deductions and heuristic assumptions do not stand up under closer examination, particularly as regards the last method by means of which Prandtl believes to be able to explain the reversal of the resistance law on spheres and, in general, the low resistance coefficients on other bodies by turbulence in the boundary layer.

According to Reynolds, the laminar flow through pipes is connected with the low resistance in proportion to \( v \), the turbulent flow with the high resistance, in proportion to \( v^2 \); so by analogy, the low resistance on the sphere, after the reversal, would have to be traceable, if not to laminar, at least to materially reduced turbulence. And this, in fact, agrees with Prandtl's flow for low resistances which presents a decidedly diminished vortex field back of the sphere. But the analogy is not in keeping with the assumption that the low resistance is due to increased friction and turbulence by virtue of the turbulence in the boundary layer. It would explain an increase but not a decrease in the resistance.

Besides, the boundary layer is, by nature and concept, always laminar and nonvortical. Consequently, the term "turbulent boundary layer" is formally "contradictory in adjetcto," and is evidently meant to allude to that turbulent layer which accompanies the boundary layer in the ambit of negative pressure in all flows around solid bod-
ies without altering their laminar motion. But these vortices are in nowise restricted to critical $R$, but rather occur at the very instant of incipient motion on the sides of the body, from where they quickly detach and soon settle in the space at the rear vacated by the flow where they form the large eddying body of the gyroma. The very surprising result of the "experimentum crucis" was that the interference ring did reduce the resistance. It also is to be assumed that the vortices back of the ring had some bearing on it by locally augmenting the normal vortex formation and circulation, but it does not prove that now the whole reversal is a result of the "turbulent boundary layer," because the ring consigns positive forces into the fluid from its front end also, which alter the entire pressure and flow system of the spheres, and the remaining laminar boundary layer hereby is of secondary significance. As matters stand, it is impossible to maintain the hypothesis of turbulent boundary layers as suitable basis for explaining the strange resistance phenomena. For that reason, we shall investigate what mechanism changes the flow of the high resistances into the form coordinated to the small coefficients:

1) On Prandtl's wire ring,
2) By screen turbulence, and
3) In nonvortical flow.

D. Mechanism of Reversal

1. On wire ring

The flow on spheres can be represented by stereoscopic photography under water, but in order to maintain the clearness of the field, coarser sight corpuscles must be used so that the very fineness of the motion cannot always be brought out as distinctly as on the surface of the water. For that reason I used immersed circular cylinders instead of spheres, assuming that on the same profile, if not the same, at least similar phenomena would occur.

Back in 1918 I had made various experiments on a cylinder on whose side lines I had placed two $1 \text{ mm}$ thick wires $15^\circ$ ahead of the diameter called equator. The profile corresponded with a meridian cut of the Prandtl sphere with wire ring in effective position. The experi-
ments terminated negatively, the wire revealed no essential change in flow; that is, it acted like the wire ring on the equator of the sphere. More about the reason of the difference follows elsewhere.

At the suggestion of Professor O. Krell, Berlin, I recently resumed the experiments on cylinders of 9 cm diameter. In place of the wires we recessed small strips of 0.8 mm zinc in radial position into the surface of the cylinder, so that they protruded about 2 mm. (See fig. 7.)

In normal flow around the cylinder without strips (fig. 7) the visible forward boundary of the gyroma extended 20-25° beyond the equator, or 15-20° farther than on the sphere. At the same point lies, according to Eisner's tests* for velocities of from \( v = 40 \) to \( 60 \) cm/s, the pressure minimum of roughly \(-1.35\) of the dynamic pressure in the forward flow separation point. Even the zero pressure, the forward boundary of the negative pressure, is at 58° on the cylinder, or 15° farther ahead than on the sphere.

This difference explains why my experiments with strips 15° ahead of the equator were just as ineffective as the wire ring at 0° on the sphere.

But if the strips are mounted at about 22.5° of the width in the forward acute angle of the gyroma where they push the strong retrograde motion on the cylinder wall and together with the side flows toward the outside, the flow picture (fig. 8) manifests in contrast with Figure 7 a decided widening out of the gyroma whose maximum width has shifted to the rear. These are signs of increased negative pressure on the rear end of the cylinder and thus of the resistance.

Figure 9 shows a different aspect. Here the strips are shoved forward from without the ambit of the gyroma to 45° width. The flow hugs the cylinder wall to far behind the equator, and the inward pushing side flows contract the gyroma which in moments of fluctuation can almost lead to complete displacement.

To bring out the effect of the strips more forcefully, I increased them to 7 mm. Now, it will be seen (fig. 10) how they catch and dam up the flow on the front and spill it at much higher velocity over the edges. The strong curvature at which the streamlines now proceed to the rear suggests that the strongest centrifugal forces of the whole periphery of the cylinder exist at this point behind the strips and that, therefore, the negative pressure in the axis of the adjacent vortex and on the wall must also pass through a minimum. From the slope of the side flows toward the median line and the contraction of the gyroma, we then can infer as to the drop in negative pressure on the rear end, and reduced resistance.

Shifting the strips to 67.5° width on the forward pole, still manifests a marked diminution of the gyroma.

A remarkable verification and extension of the present results is found in the pressure measurements on a sphere, conducted by Professor O. Krell* in the tunnel of the Berlin Technical Institute, and of which Figure 11 is one example. These measurements were made at wind velocities ranging from \( v = 6 \) to \( 52.5 \) m/s. The dashed curve is the pressure distribution over an 80 mm smooth sphere, the full line the pressure over the same sphere but with 2 mm ring mounted at 43°, similar to Prandtl's wire ring.

The smooth sphere shows a dynamic or positive pressure over the forward calotte which drops laterally depending upon the proportion of the radial ordinates, and oversteps the zero limit at 45°. The whole other surface is under negative pressure or suction. The asymmetry of the curve illustrates the pronounced variations of the pressure which go hand in hand with those of the flow.

The effect of the ring is astonishing. Where elsewhere the pressure approaches zero, in front of the ring is a pressure rise which even exceeds half the dynamic pressure in the forward pole. Directly behind is the sudden drop to negative pressure of like amount. This absolute minimum experiences between 50 and 60° from the pole a replenishing up to one-half to one-third its amount more-

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ly to drop to a second equatorial minimum. At the back of the sphere the ring has lowered the suction from one-fourth to one-half of that on the smooth sphere.

Professor Krell explains the drop of the suction curve at around 60° from the pole and the further relationship between pressure and flow as follows: "The air stream spills over the interference ring as over a weir, strikes the sphere at a tangent of about 60°, rebounds and shoots on with its kinetic energy to form a new suction maximum at 90°. Under the action of this suction the flow is pulled toward the surface of the sphere and hem in the dead water spatially as well as in suction effect, thus reducing the air resistance of the sphere."

The relations between pressure distribution and resistance are particularly illuminating. Krell points out how the effect of the ring resulted in forward displacement of the center of gravity of the negative pressure field and how the pressure minima back of the ring produced an extraordinary increase over the front hemisphere and a decrease over the rear. Now the resistance is the vector sum of the components of the pressure forces in the direction of the progressing motion. The suction forces over the rear half act positive in the sense of the resistance, as the dynamic forces over the front half; they are in forward direction over the side area of the front half, hence reduce the resistance. The remarkable result of the integration was that the forward directed suction forces were much greater than the total resistance of the rear half.

Professor Krell concludes his report with the derivation of the flow reversal from the pressure variations on the ring as shown in the smoke pictures of Prandtl and in a number of my own streamline photographs.

2. Effect of screen turbulence on the flow around a cylinder

The primary object now is to ascertain whether and to what extent the theory of the turbulent boundary layer holds true in the other case, where the reversal on plain, smooth spheres or cylinders is caused by the turbulence of the straightenonr or any other screen and shifted to slower velocities and Reynolds Numbers.
The screen turbulence prevalent in a flow cannot change the laminar structure of the boundary layer, but if it has any effect on the normal vortex formation it results in either an enlargement or a diminution of the vortices, depending on whether the effect emanates from turbulence vortices having the same or inverse sense of rotation as the vortices on the body.

To check the behavior, we made a series of tests in which a 1 cm wide bar was mounted 2 cm ahead of the center of the cylinder. The small bar manifests two rows of vortices, rolling to the right and left over the surface of the cylinder and having the same sense of rotation as the normal vortices engendered here.

The result of these tests was negative. The instantaneous photographs of both series showed wholly similar flows and irregularities in streamlines as variations in the gyroma formation, and failed to reveal any tendency toward increase or decrease in the normal vortices on the cylinder. The theoretical assumption that the action of the screen turbulence is toward vorticity and greater friction of the boundary layer is, therefore, not substantiated by these tests.

However, the objection might be raised that a different result was hardly to be expected because in the wind tunnel measurements the reversal of the flow, through screen turbulence, did not occur until at much higher velocities than used in the photographic experiments.

As a matter of fact, at these velocities even a complete screen of parallel metal plates (straightener) or binding string failed to bring on the reversal of flow on the cylinder. On the other hand, the wind tunnel measurements likewise revealed that the reversal is speeded up as the screen turbulence becomes more intense. Consequently, it was legitimate to assume that the same result would occur even at slow photographic velocities when employing the bar screen instead of the plate screen with its much inferior turbulence.

Such a bar screen consisted of thin pieces of wood, 1 cm wide, and spaced 1 cm apart. It was twice as wide as the cylinder and was mounted 2 cm distant, so that in one test series a bar, in the other a gap, forces the center of the cylinder. The turbulence vortices next to
the surface of the cylinder had, in the first test series, 
the same sense of rotation as the vortices on the cylin-
der; in the second series, the opposite sense.

The result was clear and definite. All photographs 
exhibited the complete reversal. (Figs. 12 and 13.) The 
lateral flows envelop the cylinder in an unsettled manner 
far beyond the equator and strikingly contract the area 
of the gyromas which never exceed the width of the cylin-
der. Their shape is fairly tapered, their length varies 
and at times is not greater than the diameter of the cyl-
der. In one case the space of the gyroma was filled up 
by a one-sided transverse flow leaving only traces of sta-
tionary vortices. The experiments prove that on the cylin-
der as on the sphere, the position of the critical Rey-
nolds Numbers and the flow reversal depend upon the degree 
of screen turbulence contained within the flow and that, 
given sufficiently strong turbulence, the form of flow 
conjugated to the small resistance coefficients exists 
from the beginning even at very small velocities.

It follows herefrom, that it was no fault of the ve-
locities when the reversal failed to materialize either 
with one or two bars. The turbulence vortices passing 
over the surface of the cylinder were of the same type 
and intensity as in the test series with complete bar 
screen. If they were contingent upon the "turbulence of 
boundary layer," the latter would have had to become tur-
bulent in all these cases in the same way, and reversal 
would have had to occur aft of the separate bars as well. 
The reversal here is not attributable to greater friction 
through vorticity in the boundary layer, as Prandtl postu-
lates, but to a direct action of the turbulence of the 
complete screen enveloping the whole field of the inertia 
forces.

An illuminating insight into the finer structure of 
the flow and the correlations of this mechanism is afford-
ed by the fields (of force) (of the stationary camera). 
They illustrate the motions set up in the quiescent fluid 
by the conveyance of the solid body.

The normal field of force of a cylinder in parallel 
unlimited flow agrees in its front half fairly closely 
with the known absolute streamlines of the theory. The 
lines radiate divergently forward and in well-shaped, uni-
form curves turn sidewise and rearward.
In the field of the bar screen (fig. 2) the row of white rectangles denotes the path traversed by the screen bars during the exposure (1/5 second). The maze-like turbulent flow with its ramifications is directed toward the rear of the bars and embraces the engendered vortex streets, which become more and more distorted and undiscernable the more they participate in the forward motion. As previously stated, this motion denotes the velocity losses which the corresponding streamline motion (fig. 3) sustains because of the friction and vortex formation on the cylinder.

Field of force of cylinder in the ambit of the screen (fig. 14): In the combined field the total flow appears as a circulatory motion emanating from the face of the screen, escaping to the sides and rear, then with branches between the vortices advancing inward, gradually passing into the inflow which extends as far as the screen bars and attaining their full velocity in them.

With this circulation around the screen, the lines of force discharging from the cylinder are so interwoven that the regular shapes of the absolute streamlines are no longer recognizable. Because of the proximity of the screen, they diverge sharply to the side quite close to their source, advance in bunches in the direction of the vortex streets and into the spaces back of the individual bars, and pass in this way into the general cyclic flow.* The displacement of the fluid in front of the cylinder is therefore, essentially different from the normal flow, by predominantly outward directed transverse forces without contemporary acceleration to the rear; and similar inward directed forces act on the filling of the space back of the cylinder.

The effect of the field of force on the fluid masses passing through it is portrayed in the streamline photographs. The repellent transverse forces set up a marked sinking in the water level extending far forward, and of the pressure on the sides of the cylinder, whereas the bunched lines of force pushing in from the sides increase the pressure behind the cylinder and in that way keep the

*Likewise, in cases where the cylinder is mounted far behind a plate screen (straightener), the bunched forward deflection of the lines of force - instead of rearward - and their advance in the direction of the vortex streets, has a very peculiar aspect.
vortex formation of the gyroma within moderate limits.

The effect of a bar screen described here differs from that of a straightener or a binding twine screen only quantitatively. Although, according to the resistance measurements, screens of this type can produce the same effect when their turbulence reaches the same intensity as on the bar screen by sufficient increase in flow velocity.

3. Reversal of Resistance Law in Nonvortical Flow

Accordingly, it might appear as if the reversal of the resistance law observed in the wind tunnels was not generally attributable to the effect of the screen turbulence of the universally used straighteners, and could not therefore, occur in a nonvortical flow. But this surmise is erroneous. Captain G. Costanzi* measured the resistance of spheres by moving them through the calm water of a large test channel. On one sphere of 100 mm diameter the resistance curve revealed the transition from a higher to a lower coefficient at a mean speed of 3 m/s, which is equivalent to a wind velocity of \( v = 42 \text{ m/s} \) and a critical Reynolds Number which appears to accord with the Göttingen measurements, without wire ring and thread screen. This proves the occurrence of reversal in a homogeneous flow even without screen turbulence, and the release with turbulence or ring at low velocities and Reynolds Numbers. Now that Prandtl's theory is untenable, the question as to the real, always effective cause of the reversal remains an unsolved riddle, which has not been touched in the preceding investigations. In connection with the resistance measurements it was noted at the range of the critical and supercritical Reynolds Numbers - the small resistance coefficients - that the spheres were exposed to very violent lateral pressure variations. The manometric pressure readings yielded the strongly forward enlarged negative pressure lobes alternating on both sides. In consequence thereof, the circulations must have been impressed by very strong, oscillatory fluctuations.

Now, flows of this kind can be observed at moderate speeds on thin cylinders in water. Their development is

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as follows: Disregarding the first stages of the incipient motion the flow is always symmetrical. This is seen in the uneven structure and size of the two halves of the gyroma. After the escape of the first vortex doublet, the other wake eddies follow obliquely in succession, because the contraction occurs always on the greater half of the gyroma. A part of this half remains temporarily on the rear end of the cylinder and forms the nucleus of the succeeding vortex. But since the number of the vortices increases with the velocity, the separations occur more rapidly until at last one side of the gyroma is lifted off from the cylinder, whereas the other side spreads out over the rear end of the cylinder and at the same time advances on the side surface. In the meantime the vortex formation is almost completely suppressed on the opposite side, the stagnation point has perceptibly shifted toward this side, more and more fluid continues to spill over the side of the cylinder on which the vortex formation is strongest and the negative pressure therefore lowest, while higher pressure momentarily suppresses or restricts the vortex formation on the opposite side.

A fully developed flow is shown in Figure 15. It is the field of force of a cylinder of 22 mm diameter moving at \( v = 97.4 \text{ cm/s} \) through calm water, copied from the photograph of a test series made at Adlershof, but for a different purpose. On account of the superposed translation there is a zigzag flow (instead of vortex rows) which is the bond here of the motion pushing forward between the vortices. As to the vortex formation, we note only a vortex adhering to the cylinder on the left side.

In the field of force this vortex appears far advanced in development and consisting of numerous small vortices coiled up in the developed vortex. On the right side the vortex formation is temporarily interrupted. There the lines of force swinging in from the front meet the stronger lines from the opposite side and together form on the side of the cylinder an interference and a pressure which is opposite to the lowest negative pressure on the vortex source of the left side.

The developed vortices of the two rows stand united on the outside by flow bridges and on the inside evidence common forward zigzag flow.

The speed \( v \) together with the spacing of the flow
bridges and the scale of the picture reveal that 11 dou-
blets per second have been produced on the cylinder, and
that is also the number of pressure vibrations.

The water temperature in the test was 17.5°C, the
estimated kinematic viscosity \( v = 0.01 \) cm\(^2\) s\(^{-1}\), so for \( v = 100 \) cm and \( d = 2.2 \) cm the Reynolds Number was \( R_e = \frac{v d}{\nu} = 22000 \). At \( v = 0.14 \) viscosity of the air the same Rey-
olds Number is obtained in the experiment in air by 14-
fold velocity \( v = 14 \) m/s, or, in other words, the same
flow prevails in the same cylinder, according to the law
of similitude, but with the difference that the vortex
sequence and the number of pressure vibrations, propor-
tionally to the velocity have increased from 11 to 11 ×
14 = 154 per second. The pressure vibrations on cylinders
of different diameter are then easily determined for the
same Reynolds Number.

Unfortunately, the Adlershof test program contains
only a few photographs for \( v = 150 \) m/s, which do not show
essential differences against \( v = 100 \) cm/s in the flow.
I am, therefore, unable to give any more details on the
flow at different Reynolds Numbers nor on the development
of the phenomena from the initially symmetrical form of
flow. Neither am I in a position to repeat and extend the
experiments which might reveal a preferable basis for
checking the similitude law, with my limited test equip-
ment.

There can be no doubt that this particular form of
flow is in fact a problem of supercritical attitude of mo-
tion because it is only through it that the strong later-
al pressure vibrations which are assigned to this atti-
tude and to the small resistance coefficients can occur.

But then the critical question arises: To what cause
is the transition of the symmetrical flow and pressure
distribution into its asymmetrical, oscillatory form to be
attributed?

The answer to this is: The arrangement of the vor-
tices in the double row behind the cylinder corresponds
to that of a "vortex street" which, according to Karman's
experiments, is stable when the ratio of the distance of
both rows to the distance of two vortices of one row =
0.283. This is plainly the case here. The stable arrange-
ment first develops under the vortices left in the dead space by the cylinder, then advances with increasing velocity toward the cylinder and finally predominates the whole field enveloping the cylinder.

Consequently, the cause of the reversal lies in the fact that the fairly symmetrical circulation and vortex formation on the cylinder becomes unstable at the critical velocities and changes into the stable but asymmetrical form of flow of the vortex street. The one-sided vortex formation, still remaining, makes the marked decrease in resistance forthwith comprehensible.

Any turbulence, particularly of screens and straighteners, as well as the strips on the cylinder in our tests or the wire ring on the sphere, endangers the continuance of the initially symmetrical flow, makes it unstable and thus permits the transition into the stable, asymmetrical form even at slower velocities than in homogeneous flow.

Inasmuch as the stability of the vortices is formed in the dead space behind the cylinder, it plainly is founded on the equilibrium of pure forces of inertia which are bound up with the orbital motion of the vortices and thus proportional to \( u^2 : r \), where \( u = \text{peripheral velocity} \) and \( r = \text{radius of vortex} \). Then since \( u \) is proportional to velocity \( v \) of the flow and \( r \) is proportional to diameter \( d \) of the cylinder, these forces on cylinders of different diameters must have the same ratio of \( v^2 : d \), if similitude is to prevail. That is, in other words, the Froude law of similitude. The legitimacy of this conclusion awaits experimental proof which I am unfortunately, not in position to supply. On the other hand, experience teaches that the regular arrangement of the vortex street on thin cylinders or narrow bodies occurs at very much slower velocities than on wide bodies, whereas the opposite should be expected according to the Reynolds law. But this, evidently, cannot prevent the fact that the full reversal of the flow and of the resistance nevertheless follows the latter law.

The oscillatory vortex formation on the sphere in uniform flow is difficult to visualize other than that the point of origin rotates to right and left around the sphere and the produced endless vortex, wound tightly in a spiral, is left behind the sphere like a single-thread screw. In longitudinal section then appears the well-
known picture of the vortex street. In this fictitious system the revolution speed of the spiral at large R in the air would become so great and the period of the lateral pressure variations so short as to fail to show up in manometric pressure readings and hardly discernible alongside the mean values. Besides, it was assumed in this consideration that the sphere remained perfectly stationary in the flow and that the holders necessary to accomplish this, do not disturb the flow.

These conditions are, unhappily, never encountered in a real flow. And so it leads in the not perfectly uniform flow to the violent, irregular pressure variations, which Prandtl attributes to the tongue-shaped motions of the separation line, or to Krell's sudden reversal of the pressure extremes in his measurements. Here the strongest pressure differences remained steady above and below, then suddenly and in wholly irregular periods, reversed their positions. In this arrangement the flow at times was in equilibrium, sometimes for such a protracted period that "the wait for the reversal had to be given up for lack of time." In my opinion the cause lies with the inevitable variations in the air stream, which was not exactly symmetrical and had a 4.5° upward slope, even when the sphere was set at the same angle. Besides, the 6 mm pipe through the center of the sphere used as manometer lead may have helped to bring on the phenomena. The irregular variations in flow and pressure on the sphere must, obviously, prevail upon the form and set-up of the vortices in the dead water.

As on circular cylinder and sphere, so also on bodies with elliptic profiles, the augmented vortex formation can approach at critical velocities the forward limit of the negative pressure field and thus alter the entire pressure distribution and resistance coefficient. On bodies such as airships or plates in longitudinal position the vortices always begin to form in front at the zero limit of the pressure. But the resistance of airships as of vessels, is materially affected by the body shape, which also defines the form of the total field of force. Lines of force discharging forward and returning rearward on a ship set up the bow and stern wave. The stern wave balances the resistance of the dynamic pressure to the extent of increasing the forward directed pressure on the stern, and this effect is so much smaller as the stern wave is farther behind the ship. The speed
of such ships can be increased by lengthening the hull.

The high resistance on the transverse plate is not due to the laminar boundary layer, but rather to the fact that the lines of force do not converge with the stern wave until aft of the gyroIX, and the rear end, reached by the suction only, is deeply imbedded in the negative pressure. But even on the sphere and on the cylinder the transition of the resistance to lower coefficients is closely bound up with the proximity of the returning lines of force at the rear end of the body.

Summary

The nature of turbulent flow through pipes and around obstacles is analyzed and illustrated by photographs of turbulence on screens and straighteners. It is shown that the reversal of flow and of the resistance law on spheres is not explainable by Prandtl's turbulence in the boundary layer. The investigation of the analogous phenomena on the cylinder yields a reversal of the total field of flow. The very pronounced changes in pressure distribution connected with it were affirmed by manometric measurements on spheres by Professor O. Krell. The reversal in a homogeneous nonvortical flow is brought about by the advance of the stable arrangement of Karman's dead air vortices toward the test object and by the substitution of an alternatingly one-sided or rotating but stable vortex formation in place of the initially symmetrical formation. This also explains the marked variations of the models.

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Fig. 1 Field of force of a plate screen. The surface of the plates is covered with fine vortex layers, which some distance behind the model, develop into larger vortices of the vortex streets.

Fig. 2 Field of force of a narrow bar screen.

Fig. 3 Field of flow of a narrow bar screen.

Fig. 4 Field of force of a wide bar screen.

Fig. 5 Effect of plate screen on the flow of Fig. 4

Fig. 7 Normal field of flow of a smooth cylinder.
Fig. 6 Flow round a sphere according to Prandtl.
A at large, B at small resistance coefficients.

Fig. 11 Pressure distribution on sphere with interference ring at \( v = 12 \text{ m/sec.} \) according to Krell. The dotted curve is the pressure distribution on the smooth sphere without ring.

Fig. 15 Oscillatory vortex formation in the field of force of a cylinder for homogenous flow. Copy of an under-water photograph.
Fig. 8 Circulation around a cylinder with strips 22½ radians ahead of equator.

Fig. 9 Circulation around cylinder with 3 mm strips at 45° ahead of the equator.

Fig. 10 Flow as in Fig. 9, but with 7 mm strips, the gyroma is seen to extend to the strips.

Fig. 12 Field of flow of cylinder behind bar screen, one bar in center. The gyroma is materially smaller (compare Fig. 7.)

Fig. 13 Flow as in Fig. 11, but with gap in front of center.

Fig. 14 Field of force of a cylinder in the field of a bar screen.