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614

THE TRANSFERENCE OF HEAT FROM A HOT PLATE TO AN AIR STREAM

By Franz Elias

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TECHNICAL MEMORANDUM NO. 614

THE TRANSFERENCE OF HEAT FROM A HOT PLATE TO AN AIR STREAM*

By Franz Éliás

The latest researches in theoretical and experimental hydrodynamics reveal that the flow phenomena in fluids and in gases past a solid body are restricted to a relatively thin layer - the "hydrodynamical boundary layer." These investigations advanced our knowledge on the friction of fluids and gases in motion,** and evolves new premises for the study of heat transference in flow phenomena.

The transference of heat in moving fluids is by conduction and convection. Whereas the fluid cleaves to the wall, the heat must obviously pass between this layer and the solid body analogous to that between two solids. This postulate is confirmed by the complete absence of temperature jumps at the boundary separating the fluid from the solid body.*** Even the propagation of heat in fluids reveals a certain type of conduction. In the physical representation and in the mathematical formulation we distinguish between heat conduction by "molecular" transference and heat transmission by "molar" motion.

*"Die Wärmeübertragung einer geheizten Platte an strömende Luft," from Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule Aachen, No. 9, 1930, pp. 10-39.

**L. Prandtl, Reports of the Third International Congress for Mathematicians, 1904, p. 484.

Th. v. Kármán, "Laminar and Turbulent Friction," Abh. d. Aerodyn. Inst. d. Techn. Hochschule Aachen, No. 1, p. 1; also see Z. angew. Mathem. Mechan., Vol. 1, 1921, p. 233.

***W. Nusselt, "The Transmission of Heat in Pipes," Habilitationsschrift Dresden, Berlin, 1909, p. 3.

In laminar flow we have a conduction by the irregular molecular motion and a transmission by the orderly flow past the boundary layer. In turbulent flow the ensuing pulsating fluid motions cause a higher heat transmission, in which molecular components likewise participate as heat carriers. The molecular conduction and convection induced by the pulsating motion is commonly expressed as "turbulent heat conduction." It is impossible to calculate this turbulent conduction directly, because we do not know the mechanics of the pulsating motion. But we can draw inferences about the measure of heat transference from the friction by assuming the basic mechanics of the momentum interchange in the first case of heat transference to be identical in the second.

On this premise Reynolds* evolved the coefficient of heat transference from the resistance factor for smooth pipes. Prompted by the results of Nusselt, Professor Prandtl** showed that a direct, parallel conclusion (from flow resistance to heat transfer in turbulent flow) is permissible under stated conditions. Then Professor von Kármán extended the semi-empirical equations of the turbulent friction to the differential equations for turbulent heat conduction, so that it is now possible to interpret the heat transference in all cases where the velocity field of

*O. Reynolds, "An Experimental Investigation of the Circ. . . .," Phil. Transact. of the Roy. Soc., Vol. 174 III; and "Proceedings," Manchester Lit. and Phil. Soc., 1874, p. 9.

**L. Prandtl, "Relation between Heat Exchange and Resistance to Flow," Phys. Zeitschr., 1910, pp. 11 and 1672.

the turbulent flow is known. One of his students, H. Latzko,* then calculated the turbulent heat transfer of a fluid past a flat plate and through a circular pipe.

Various researches have been made on the emission of heat between a flat surface and moving air, but they are confined to summary measurements of the heat delivery.**

The object of the present study was to define experimentally the field of temperature and velocity in a heated flat plate when exposed to an air stream whose direction is parallel to it, then to calculate therefrom the heat transference and the friction past the flat plate, and lastly, compare the test data with the mathematical theory.

To ensure comparable results, we were to actually obtain or else approximate:

- a) two-dimensional flow;
- b) constant plate temperature in the direction of the stream.

To approximate the flow in two dimensions, we chose a relatively wide plate and measured the velocity and temperature in the median plane. Considerable difficulty was experienced in trying to maintain a constant plate temperature over its whole length. If the heat is evenly distributed and the conduction in the plate itself is not very intense the plate temperature would

*H. Latzko, "The Transmission of Heat on a Turbulent Fluid or Gas Flow," Abh. d. Aerodyn. Inst. d. Techn. Hochschule Aachen, No. 1, p. 36; see also Z. angew. Mathem. Mechn., Vol. I, 1921, p. 268.

**W. Jürges, "The Heat Transmission Past a Flat Wall," supplement to Gesundheits-Ingenieur, No. 19, R. Oldenbourg, München-Berlin, 1924.

rise in the flow direction. This rise had to be neutralized. This was accomplished, after various attempts, by having recourse to an electrically heated plate and a series of rheostats.

The first experiments were made with an ice-filled box, but were not satisfactory, so we substituted Glauber's salt (sodium sulphate). This method likewise proved unsuitable and we changed to a hollow copper plate with vapor heating (Fig. 1), with which we achieved better results. The advantage of using saturated vapor for heating lies in the freedom of the plate temperature from the velocity of the air stream and in the inappreciable effect of the barometric pressure. The vapors, produced from ether and alcohol heated in a glass vessel, passed through copper tubing into the hollow box-like plate which was equipped on the inside with fins to ensure even distribution of the vapor inflow.

The four narrow sides of the plate were housed in wood, first to minimize heat losses, and second, to facilitate mounting in the wind tunnel. The front of the plate was tapered to form an entrance section. The heat emitted by the plate was to be measured from the condensation at point II; a third inlet (III), served to check the regularity of the vapor inflow. But it proved impossible to determine the heat from the condensation - first, because the feed pipes, which had to be of metal (copper), dissipated part of the heat from the ether vapors and then, outlet opening II likewise allowed some vapor to escape. These dif-

difficulties precluded the rough check on the heat energy which we had deemed so desirable, even though the heat transmitted by the plate was to be primarily interpreted from the heat content of the air stream. Aside from this, the test set-up yielded the desired results. The plate temperature was assumed equal to the vapor temperature which was read on a mercury thermometer inserted in opening IV. The mild fluctuations which occurred were probably due to a slight overheating, since the ether was heated through a water bath which was difficult to regulate.

Unwilling to forego the rough check on the heat, we discarded this method for heating the plate direct by electricity. The plate, which really was a box (Fig. 2), was made of 3.5 mm thick copper, on the inside of which we mounted a variable rheostat with 18 separate heating coils. The axis of the coils was perpendicular to the direction of the air stream for regulating the heat by switching the individual coils on or off as required. The box was filled with oil to ensure satisfactory temperature balance. Moreover, it was to be expected that, due to natural convection in the oil bath, the temperature of the plate would not be constant vertically, which in a given case must be considered in the calculations. The side walls were made of heavy copper sheet, while the four narrow sides were covered with wood. The front was tapered to form a 10-centimeter entrance section (Fig. 2). To make possible a direct comparison with Jürges' test data, we made the plate 50 cm long, as in his tests.

Since it was impossible to base the determination of the plate temperature on that of the oil bath because of local temperature rises through the heating coils, we resorted to thermocouples. They were made on the principle* that the point of contact as well as a correspondingly long portion of the wires should assume the temperature of measurement, otherwise the contact point is unable to do so. The wires were 0.6 mm thick, of constantan and manganin metal and soldered in 60-millimeter long grooves. In order to ascertain the number and the arrangement of these thermocouples, we made two special ones which were pressed to the plate by suitable clamps, and with which we effected various temperature measurements. As a result, we mounted 17 surface thermocouples, all on one side of the plate so as not to interfere with the measurements in the boundary layer which were made on the other side. This of course, does not leave the flow conditions undisturbed, but we can nevertheless assume this disturbance to be without marked effect on the temperature distribution in the wall. The procedure was as follows:

We divided the plate into five sections of 5 cm each, with 3 thermocouples on each dividing line. The temperature values were graphed and their average determined, thus supplying the mean temperature coefficients for five sections. Since the thermocouples for the boundary layer had an 8-centimeter test length, parallel to the plate and perpendicular to the air stream, we

*Knoblauch-Hencky, "Introduction to Accurate Technical Temperature Measurements," p. 59. R. Oldenbourg, München-Berlin, 1926.

confined ourselves to the temperature readings of the two middle sections, which were 5 cm apart. The average, which yielded the mean plate temperature, was also determined graphically.

Now, whereas the mean temperature values revealed a linear increase, perpendicular to the direction of the stream, we had, in fact, measured the temperature in the central axis of the plate, because in this plane the temperature deviations were exactly neutralized.

The temperature in the plane of measurement, in which the contact point of the thermocouple was displaced, could be regulated to within 2 to 3 per cent accuracy in excess plate temperature over the air outside. As a result, the ratio of the mean plate temperature to the temperature measured in the plate center remained constant even under mild temperature fluctuations. In particular, it enabled us to predict the mean plate temperature from one single voltmeter reading. The cold junction of the thermocouples was placed in ice.

Owing to the slight discrepancy between the temperature measurements and the respective voltmeter deflections, the test points were not changed until one wire had first been connected in parallel to the subsequent one. Thus the pointer of the voltmeter moved only enough to indicate the discrepancy in temperature in the two test points, which made for rapid sequence in readings.

The limited extent of the boundary layer called for careful selection and construction of our measuring devices. The temper-

ature can be measured with resistance thermometers or with thermocouples. We preferred the latter.

But before deciding on any definite material, shape or wire thickness, we followed Dr. F. N. Scheubel's suggestion and made a systematic study of the sources of errors which are apt to occur. To ensure correct temperature readings, two factors must be taken into account:

First, the thermocouple, exposed to an air stream, must not set up any material turbulence. Second, the point of contact of the thermocouple must actually be able to assume the temperature in which it is used. The first exigency is taken care of by using fine gauge streamline wires.*

One contact point of the thermocouple is placed at the pertinent point of the boundary layer, where the temperature is to be measured. This, however, does not ensure a correct temperature record at this point, when the other parts of the wires pass through a zone whose temperature differs from that at the point of contact, for in this case an exchange of heat takes place. It is very expedient to use fine gauge wires stretched parallel to the plate but perpendicular to the direction of the air stream. If it were possible to realize an exact two-dimensional temperature field, there would be no temperature gradient at all in this direction. As a matter of fact, the temperature changes linearly with the height, and it may be assumed that the

*A method which includes the thermal lag of the test wires in measurements made by the hot-wire method, is given later.

same amount of heat flows in at one side of the contact point as flows out on the other, so that the point of contact still records the correct temperature.

The importance of heat compensation becomes readily apparent from the following comparative tests made with six different thermocouples of different shape and wire thickness but under otherwise identical conditions (Figs. 3 and 3a).

- Thermocouple I - made of 0.05 mm copper and constantan wire. The two wires were inserted, twisted, and soldered. The test length was 43 mm; the rest was of 0.6 mm copper and constantan wires.
- Thermocouple II - like I, but the 80 mm test length was of 0.3 mm copper and constantan wires. The contact was formed by butt-joining the two wire ends.
- Thermocouple III - same as II, but 0.6 mm wires.
- Thermocouple IV - 0.1 x 3 mm copper and constantan strip; test length 80 mm, where both strips were superposed for a space of 30 mm and then soldered.
- Thermocouple V - of 0.6 mm copper and constantan wires. The wires form a 90° angle at point of contact.
- Thermocouple VI - same as V, but with 130° angle.

From an examination of Figures 3 and 3a, it is clear that the angle-shaped thermocouples yield altogether fallacious results, and measurements made with such thermocouples in an intensely changing field of temperature must be regarded with caution, as Ludowici's*, for instance.

*W. Ludowici, "Measurements in the Boundary Layer of Flowing Gases," V.D.I., Vol. 70, 1926, p. 1122 ff.

The effect of the wire diameter is relatively small compared to that of its shape.

After heating the plate the measurements were made on three sections, namely, forward, middle ($x_{th} = 17.5$ cm) and rear ($x_{th} = 34$ cm), and confirmed by subsequent check tests with thermocouples Nos. 1, 2, 5 and 6. Curves 1 to 3 show that the diameter of the wire has some effect even for an identical set-up, that is, the temperature rises as the diameter decreases. The curves for 0.3 mm and 2×0.05 mm wire diameter (test No. 1) meet; it may be assumed that a limit has been reached at which the element indicates the exact temperature at the test point with any degree of accuracy. The working lengths of the thermocouples used in the main tests were of plain manganin and constantan wire 0.1 mm thick. The same kind of wire but 4×0.1 mm thick was used for the cold junctions (in ice) and for clamping to the holder. We substituted manganin for copper because the first has the same conductivity as constantan. We also made some experiments regarding the suitability of platinum-platinum-rhodium thermocouples which, however, turned out unsatisfactorily for reasons of mechanical strength, in addition to the fact that their thermal conductivity is only about one-fourth that of the constantan-manganin couples.

The thermocouples were clamped on a bakelite frame (Fig. 4), which was fastened to the steel holder on a workbench, so the element could be shifted in the direction of the stream (x) and perpendicular to the plate (y). The y coordinates were

read with a vernier to 1/50 mm accuracy; the x coordinates with a conventional scale. The whole set-up was mounted on a concrete block in front of the tunnel entrance cone.

At zero position there was an intimate contact between plate and thermocouple, as checked with an electric contact device and a small signal lamp.

The thermocouple denoted in this position, particularly at higher velocities, a lower temperature than the plate, on account of the cooling of the exposed parts of the test wire, which was not specially pressed on the plate. For the surface temperature measurements we used the previously mentioned surface thermocouples, which we calibrated with the Knoblauch-Hencky liquid boiling apparatus. Saturated vapors are produced in it (by alcohol or ether) so that the temperature can be kept constant for any desired period. The second point of contact was placed in a container with crushed ice. Subsequently, we used hot water and thermos bottles with ice, which answered the same purpose - in fact, were better, because the inside wall of the bottle takes the temperature of the liquid in it.

The calibration curves of the thermocouples made of copper-constantan and of manganin-constantan wire are represented in Figures 5 and 6. Within the temperature differences of our tests the relation between thermoelectric current, thermal voltage, and temperature, respectively, was linear. It also will be noted that the line connecting the three (two) test points does

not pass through the origin of the coordinates, wherefrom it was concluded that linear dependence is strictly valid only within the measured limits. By lower temperature differences (beginning at 5° C) the calibration curve becomes parabolic, and under certain conditions, very irregular.

We always placed the cold junction in ice, so that the room temperature became the lowest test temperature difference. For defining the temperature, we measured the thermal current because the readings had to be taken in rapid succession.

The voltage for a manganin-constantan thermocouple was

$$3.76 \times 10^{-2} \left(\frac{\text{volt}}{100^{\circ} \text{C}} \right),$$

and

$$3.64 \times 10^{-2} \left(\frac{\text{volt}}{100^{\circ} \text{C}} \right).$$

for that made of copper-constantan wire.

According to Knoblauch-Hencky, the voltage for the latter ranges between

$$3.6 \times 10^{-2} \quad \text{and} \quad 4 \times 10^{-2} \left(\frac{\text{volt}}{100^{\circ} \text{C}} \right),$$

depending on the kind of material used.

The current was measured with Siemens and Halske dial voltmeters and a rotary-coil recording device. The sensitivity for one scale division was 10^{-4} volt. The resistance of the recording devices is about 18 Ω, and 182 and 176 Ω, respectively, for the installed series resistance at this voltage sensitivity. In this manner it was possible to regulate the deflection so as

to fit the corresponding temperature and to make available a large portion of the scale, which in turn enhanced the accuracy of the readings.

The velocity of the free stream was varied between 3 and 35 m/s, the lower limit being contingent on the control possibilities of the wind tunnel, and the maximum on the vibrations of the thermocouples at higher velocities. The pulsating motion brings the point of constant in the boundary layer within the zone of different temperatures, causing the galvanometer needle to oscillate.

The temperature difference ϑ_0 between plate and undisturbed air stream, ranged between 16 and 20 ($^{\circ}\text{C}$), although we made some experiments at 36 to 37.5 ($^{\circ}\text{C}$) super-temperatures. We anticipated the temperature curves to be similar because of the fact that the best transmission within this temperature range is proportional. The various velocities for $\frac{t_n - t_L}{\vartheta_0}$ are shown in Figures 7, 8 and 9, plotted against y (t_n = temperature in boundary layer, t_L = temperature in free air stream).

The heat given off by the plate caused a slight rise in the temperature of the air stream, so the plate temperature itself arose, because the heat output remained constant. But since a measurement in the boundary layer never took longer than 5 to 10 minutes, we were justified in assuming that temperature as outside air temperature which we had measured after reaching the undisturbed air stream.

The temperature curves were taken at four test points designated with $x_h = 20, 30, 40$ and 50 , and with $x_{th} = 10, 20, 30$ and 40 , respectively. x_h here denotes the distance of the test points from the flow tip, the beginning of the hydrodynamical boundary layer; x_{th} , the distance preceding the entry of the thermal reaction. It forms the border line between the hot plate and the wood insulation of the flow tip.

The temperature curves exhibited in Figures 7-27, reveal - according to whether the flow is laminar or turbulent - two typically different shapes. The distribution curves for the turbulent flow show a much steeper gradient near to the wall than for the laminar flow. A notable feature of these two types of temperature distribution, induced by two different flow attitudes, is their contemporary existence as in the velocity distribution past the plate. Based on Figures 7-27, we append the types of flow for the individual temperature curves along with their respective Reynolds Numbers in the following table.

U _D (m/s)	x _h (cm)	U _D x _h (m ² /s)	Re = $\frac{U_D x_h}{\nu}$	Type of flow past plate	
				smooth	rough
3	10.8	0.324	20,220	laminar	laminar
3	20	0.6	37,500	"	"
3	30	0.9	56,300	"	"
3	40	1.2	75,000	"	"
3	50	1.5	93,700	"	"
5	20	1.0	62,500	"	laminar
5	30	1.5	93,700	"	"
5	40	2.0	125,000	"	turbulent
5	50	2.5	156,200	turbulent	"
7.5	20	1.5	93,700	laminar	laminar
7.5	30	2.25	140,500	"	turbulent
7.5	40	3.0	187,500	turbulent	"
7.5	50	3.75	234,200	"	"
10	20	2.0	125,000	laminar	laminar
10	30	3.0	187,500	turbulent	turbulent
10	40	4.0	250,000	"	"
10	50	5.0	312,500	"	"
15	20	3.0	187,500	"	"
15	30	4.5	282,000	"	"
15	40	6.0	375,000	"	"
15	50	7.5	469,000	"	"
19.3	20	3.86	241,000	"	"
19.3	30	5.79	362,000	"	-
19.3	40	7.72	482,000	"	-
19.3	50	9.65	603,000	"	-
20	20	4.0	250,000	"	"
20	30	6.0	375,000	"	"
20	40	8.0	500,000	"	"
20	50	10.0	625,000	"	"
25	20	5.0	312,500	"	"
25	30	7.5	469,000	"	"
25	40	10.0	625,000	"	"
25	50	12.5	781,000	"	"
35	20	7.0	438,000	"	"
35	30	10.5	656,000	"	"
35	40	14.0	875,000	"	"
35	50	17.5	1092,000	"	"

At $U_D = 5$ and $= 7.5$ m/s, consistent with $Re = 125,000$ and $= 140,500$ Reynolds Number, the flow past the smooth plate is still laminar, although it has already turned turbulent along the rough plate. At $Re = 100,000$ to $250,000$, a laminar flow prevails at the forward part of the plate, which gradually becomes turbulent as it passes the plate.

The pertinent curves of the temperature distribution plotted for higher x_h , that is, rear part of the plate, hence of turbulent character - intersect, by virtue of their gradient, the curves for lower x_h (forward part of plate), where the flow is still laminar. As a result the temperature along a straight line decreases parallel to the plate in the direction of the stream at the point of transition from laminar to turbulent flow. The rough plate had the roughness of a coarse file.

At high velocities (beginning at about 15 m/s) and up to 1 to 2 mm away from the wall, the rough plate showed a higher temperature gradient than the smooth plate. At greater distances (from about 1-2 mm on), the conditions become reversed; the smooth flow has the higher gradient, hence its curve is lower than for the rough plate.

At velocities below 15 m/s, no distinct differences can be noticed. Of course, it should be remembered that the zero position of the thermocouple was assumedly that setting at which the thermo-element was in intimate contact with the uppermost ridges of the roughened plate. We also measured the velocity distribu-

tion in these planes denoted by x_h and x_{th} .

The velocity was computed from the dynamic pressure, with the formula $u = \sqrt{\frac{2 \gamma_a h_a}{\rho_L}}$ (ρ_L expresses the air density conforming to the prevailing temperature). (Landolt-Börnstein, Phys. Chem. tables.)

Here, u = air speed,

γ_a = specific gravity of test liquid,

h_a = pressure head in Pitot tube,

ρ_L = air density with respect to temperature.

The dynamic pressure was defined with a dynamic pressure recorder, the Pitot tubes being thin, rectangular glass tubes 0.28 and 0.34 mm on the outside, and 0.15 mm on the inside. The diameter had to be small to prevent any appreciable disturbance on the test point which, however, delayed the pressure gauge from 5 to 10 minutes. At lower than 10 m/s speed in undisturbed stream, the measurement was not accurate. The zero position of the Pitot tube at the wall (checked by magnifying glass) was determined by contact between test tube and plate, although we disregarded the reading in this position.

As y coordinate of the test point, we used the distance of the tube axis away from the wall. The curves for the velocity and the temperature distribution are drawn for equal U_D and the same super-temperature ϑ_0 . In order to arrive at the effect of the heat on the velocity field, we defined the distribution of the latter with a nonheated plate, as shown in Figures

28 to 32. The difference between two such curves is not pronounced and does not exceed 2 to 3 per cent. This bears out Latzko's assumption, who stated that the effect of the temperature on the field of velocity could be disregarded at comparatively low super-temperatures. The figures further show the corresponding velocity curves for the rough plate. They are, as we anticipated, lower than for the smooth plate, because of the greater resistance and consequently, slower velocity of the rough plate.

The obtained test data now enable us to compute the heat transmission of the plate with respect to plate length x and velocity U_D .

The time rate of the heat passing through a flat section of width l , placed perpendicular to the plate is

$$Q(x) = \int_0^{\delta} \gamma c_p u(y) \vartheta(y) dy \left(\frac{WE}{m/s} \right) \quad (1)$$

where

- γ = specific weight of air
- c_p = " heat per unit weight (WE/kg °C),
- $C = c_p \gamma$ = " " " " volume (WE/m³ °C),
- $u(y)$ = velocity in boundary layer variable with $y/$ (m/s),
- $t_n - t_L = \vartheta(y)$ = super-temperature in boundary layer variable with y . (°C),
- t_n = temperature at test point,
- t_L = " of the air outside,
- y = vertical distance away from the wall (m),
- δ = boundary layer thickness (m).

So when we determine the heat volume $Q(x)$ of the air stream past the plate at point x_{n+1} and x_n in two such planes, the difference $Q(x)_{n+1} - Q(x)_n$ represents the heat transmitted by a plate strip of width l over $x_{n+1} - x_n$ length. The coefficient α , which denotes the heat per unit area in unit time and for 1° C temperature difference can now be determined from $Q(x)_{n+1} - Q(x)_n$ for the corresponding part of the plate.

Then the experimentally defined curves of the temperature and the velocity distribution are applied to formula (1) (Figs. 33-40) and the heat $Q(x)$ carried past the air stream is graphically integrated for $x_n = 20, 30, 40, 50$. The values of integrant $q = \gamma c_p u (t_n - t_L)$ are plotted against y and the entire heat $Q(x)$ carried off from the plate up to the corresponding test point x is computed (in watts) from the area enveloped by curve $q = f(y)$.

Now we compare our test results with the theoretical treatises of von Kármán and Latzko, E. Pohlhausen and Jürges measurements.

The heat transfer can be determined theoretically from formula (1) by using the distributing function for velocities $u(y)$ and for super-temperatures $\vartheta(y)$ conformal to the boundary layer theory. The theory postulates two physical characteristics: the Reynolds Number Re for $\frac{Ux}{\nu}$ and the Péclet factor Pe' , defined by $\frac{Ux c_p \gamma}{\lambda}$. It can be proved that the distributing functions of u and ϑ are precisely similar only when the two

factors are equal, i.e., when $\frac{c_p \gamma v}{\lambda} = 1$.

Pohlhausen* computed $u(y)$ and $\vartheta(y)$ with respect to these factors for the laminar zone, and arrived at

$$Q = \alpha(\sigma) \gamma \vartheta_0 \left(\frac{l}{Re}\right)^{1/2} \left(\frac{WE}{m/s}\right),$$

for a strip width l , where

$$\alpha(\sigma) = \frac{1}{\epsilon} = \frac{Pe'}{Re} = \frac{c_p \gamma v}{\lambda}.$$

For our case $\sigma = 0.744$ and $\alpha = 0.6$.

Von Kármán and Latzko made the calculation for the turbulent zone based upon Reynolds' and Prandtl's analogy between the transfer of friction and of heat. This analogy is perfectly valid as long as Reynolds' and Peclet's numbers are equal. On this premise the distributing functions for super-temperature and velocity are coincident and we can put

$$u = U \left(\frac{y}{\delta}\right)^{1/7} \quad \text{and} \quad \vartheta = \vartheta_0 \left(\frac{y}{\delta}\right)^{1/7}$$

with $\delta = 0.37 \left(\frac{\nu}{Ux}\right)^{1/5}$.

Written in formula (1), we arrive at

$$Q = 0.0356 \times C U \vartheta_0 \times \left(\frac{l}{Re}\right)^{1/5} \left(\frac{WE}{m/s}\right)$$

for a plate strip of width l . $\frac{Re}{Pe} = \epsilon$ being different from 1 for real gases, there is a certain arbitrariness in introducing the

Reynolds Number in the formula. But we can, however, estimate

*E. Pohlhausen, "Heat Exchange between Solid Bodies and Fluids by Slight Friction and Low Heat Conduction," Z. angew. Mathem. Mechan., Vol. I, 1921, p. 120.

the anticipated discrepancy by applying Peclet's number instead, in which case the value of the heat transfer then changes in the ratio of $\sqrt[5]{\epsilon} = \sqrt[5]{1.345} = 1.061$, which approximates to the theory by 6 per cent.

In Figures 41 and 42 we plotted the heat transmitted per 1 m^2 of plate area in one second, the velocity of the undisturbed air stream and 1° C temperature difference against $U x_h$ and $U x_{th}$ and Re and Pe' ; x_h denotes the distance of the entrance section and x_{th} the distance from the entrant thermal reaction. This dual representation seemed fitting because of the analogy between friction and heat conduction in accordance with the theory which postulates the same boundary conditions for both cases, i.e., the coincidence of incipient thermal and hydrodynamical reaction. The condition was not complied with because the section (tapered, 10 cm long) was not heated. However, a comparison of the respective temperature and velocity distribution curves revealed no difference except in the laminar flow at around $U_D = 10 \text{ m/s}$; but as soon as the flow becomes turbulent, these curves - converted to the same scale - become congruent, and only a very slight disparity prevails at the first test point $x_h = 20$.

From this it is concluded that the neutralizing effect of the turbulence gradually compensates the difference caused by the delayed thermal reaction along the plate, and that thereby the thermal processes are the same as if the beginning of its

own and that of the hydrodynamical reaction had been simultaneous. Obviously this compensation along the plate is complete only when the entrance section is relatively small as compared to the length of the heated portion of the plate.

When we compare the test data with the just cited theoretical formulas, we note first, the close agreement in the heat transfer coefficient up to $Re = 2 \times 10^5$ with Pohlhausen's function. Between $Re = 2 \times 10^5$ and 5×10^5 , the conditions have no well-defined character. The flow past the plate is neither wholly laminar nor turbulent throughout.

The test points for velocity U_D yield a series of heat transference curves which are analogous to Blasius and Gebers' * curves for the resistance to flow.

For $Re = 5 \times 10^5$ to $Re = 11 \times 10^5$, the test data are in satisfactory accord with the von Kármán-Latzko curve, particularly when the calculation of the plate length starts with the beginning of the thermal reaction. It must be borne in mind that our method for defining the heat volume makes great demands on the measuring accuracy, so that the scattering is naturally more pronounced than if the heat volume had been defined by one direct measurement.

One curve in Figure 42, taken from Jürges' ** measurements, is included for comparison. He attested to a marked discrepancy.

*Results of Aerodynamic Test Laboratory, Issue III, p. 5, 1927, Ergebnisse der Aerodyn. Versuchsanstalt zu Göttingen.

**Zeitschr. angew. Mathem. Mechan., Vol. 9, 1929.

between his measurements and von Kármán-Latzko's computed heat volumes, which is ascribed to the following:

Jürges uses, exactly as we did, a plate 50 cm long, in front of which he placed a 31-centimeter long entrance section. It is evident that in this case the premise of the theory (contemporary start of hydrodynamical and thermal reaction) does not hold. Whereas in our tests the ratio of the length to the hydrodynamical entrance section of the thermal measuring length was 1 : 5, it amounted to around 3 : 5 in Jürges' case. We suspect Jürges*, as well as ten Bosch,** wrote the total length, 31 + 50 cm in Latzko's formula for the Reynolds Number, so naturally the figures for the heat transmission are lower than the experimental values.

It becomes apparent from Figure 42 that the discrepancies of Jürges' figures are much lower. Within $Re = 3 \times 10^5$ and 7×10^5 our test points agree satisfactorily with his; at higher or lower figures than these, Jürges' figures are slightly higher, our test points approaching the theoretical curves somewhat closer.

In Figure 43 is exhibited the heat transfer plotted against the velocity per 1 m^2 area and 1° C temperature difference. The plotted test points are in accordance with the measured values graphically obtained at point $x_{th} = 50 \text{ cm}$ from equation

*Zeitschr. angew. Mathem. Mechan.; Vol. 9, 1929, p. 45.

**ten Bosch, "The Transmission of Heat," 2d edition, Berlin. Julius Springer, 1927, p. 125.

$Q(x) = \int_0^{\delta} \gamma c_p u (t_n - t_L) dy$. For comparison, we further include the von Kármán-Latzko and the Jürges curves. The application of the Péclet figure revealed $\frac{\lambda}{c_p \gamma v} = 1.345$ instead of ± 1 .

Our test points and the theoretical curve $f(P_e')$ check very closely. Greater scattering, about 14 per cent, shows but 2 points for $U_D = 20$ and 35 m/s.

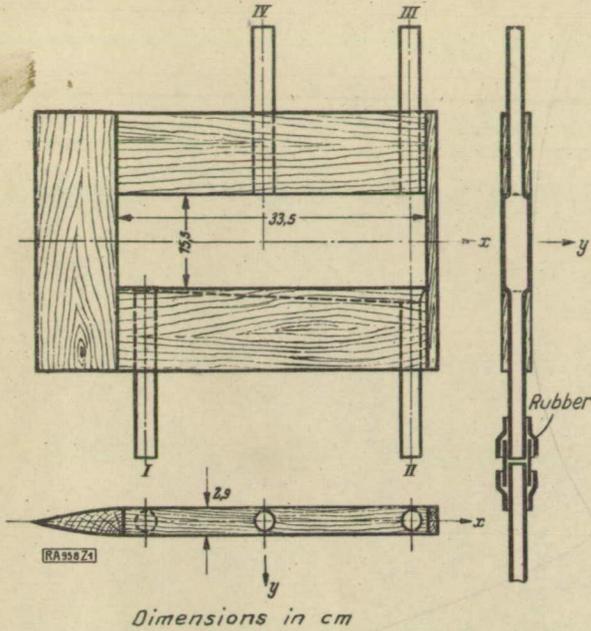
To demonstrate the effect of plate length on the ^{heat} ~~last~~ transfer, Figure 44 shows $Q(x)/\delta_0 U$ plotted against x_{th} , for four points: $x_{th} = 20, 30, 40$ and 50 cm; the average of the eight points was formed for the eight velocities between $U_D = 10$ m/s and $U_D = 35$ m/s. The points thus obtained are shown connected by a curve on Figure 44, and at a logarithmic scale on Figure 45. A straight line drawn through these points yields $Q(x)/\delta_0 U$ as a power function of plate length x_{th} having an exponent which is indicated by the slope of the straight line. Its value is $n = 0.89$.

The limits of analogy between flow resistance and transference of heat as confirmed by our test data are of vital importance for the theory. For we are now able to compare the temperature fields with the corresponding velocity fields, and the flow resistance with the heat transfer.

For comparing the temperature with the velocity field, the following is appropriate: The plate temperature reaches its maximum at $y = 0$, while the velocity is zero. At the boundary

layer for $y = \delta$, the super-temperature is zero while the velocity attains its maximum value - that of undisturbed flow. To ensure the same character and the same boundary conditions for the temperature (distribution curves) as for the velocity distribution curves, the difference "plate temperature minus boundary layer temperature" was plotted as variable. This curve actually has the same character as the curve of the velocity distribution. For representing both curves at the same scale, the speed values $100 u/U$ were expressed in per cent of the speed of undisturbed air flow and the temperature values $100 \theta/\theta_0$ in per cent of super-temperature of the plate, and the results plotted on Figures 46 to 55. It is seen that the conformal temperature and velocity distribution curves are in agreement with Prandtl's theory of similarity for speeds above 15 m/s (where the flow past the plate is already turbulent). The discrepancies become more pronounced at $U_D = 10$ m/s (where the flow is still laminar). The satisfactory accord between the fields of temperature and velocity, notwithstanding the delayed entry of the thermal reaction, is explained by the compensating effect of the prevailing turbulence.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.



Dimensions in cm
Fig.1

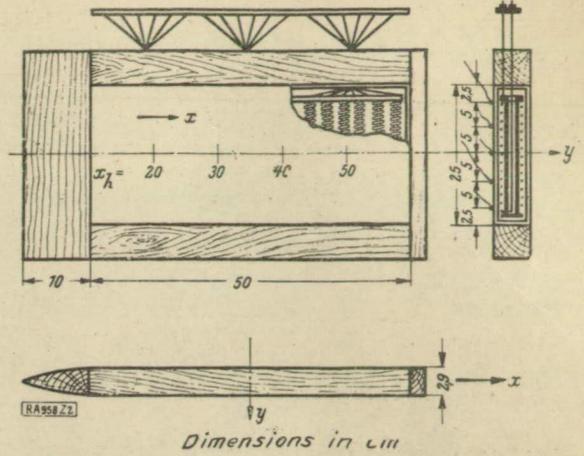


Fig.2

No.	Symbol	Date	Shape	Dimensions mm
1	*	25.2.27	2-005twisted	
2	o	27.2.27		0,3 φ
3	□	7.2.27		0,6 φ
4	◇	9.12.26		0,1x4 mm
5	△	12.2.27		0,6 φ
6	◊	8.2.27		0,6 φ

No.	Symbol	Date	Shape	Dimensions mm
1	*	25.2.27	2-005twisted	
2	o	27.2.27		0,3 φ
3	□	7.2.27		0,6 φ
4	◇	9.12.26		0,1x4 mm
5	△	12.2.27		0,6 φ
6	◊	8.2.27		0,6 φ

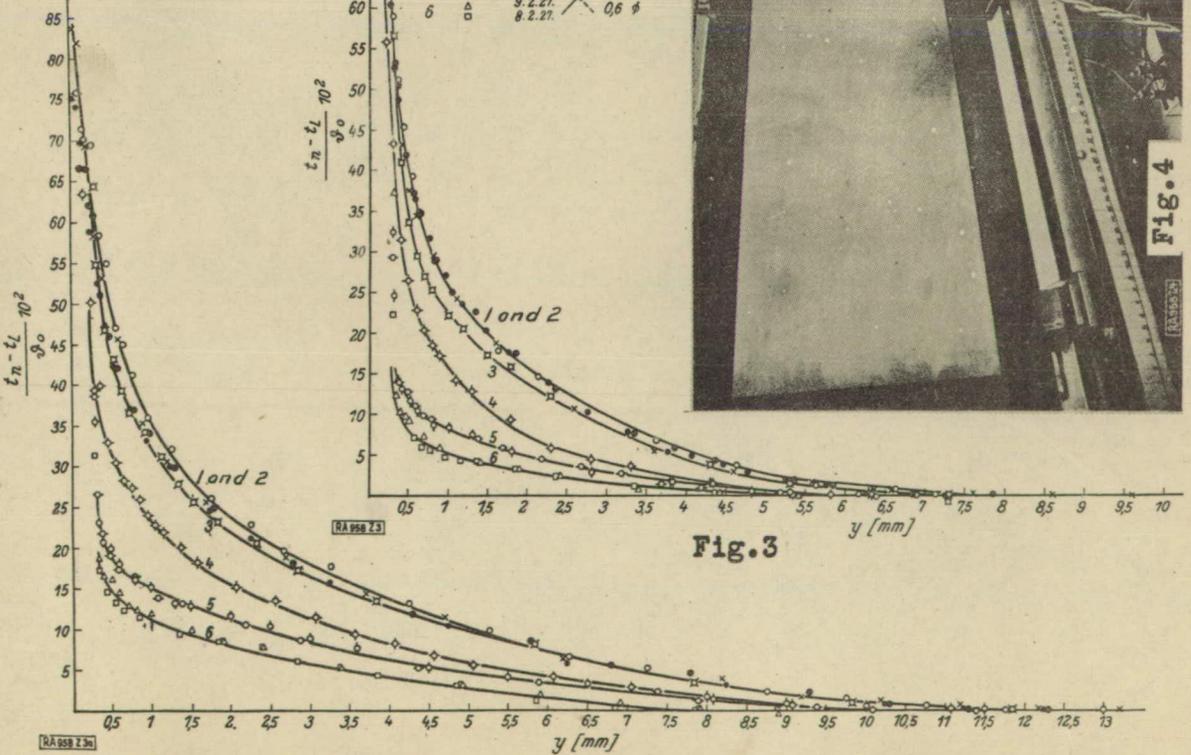


Fig.3

Fig.3a

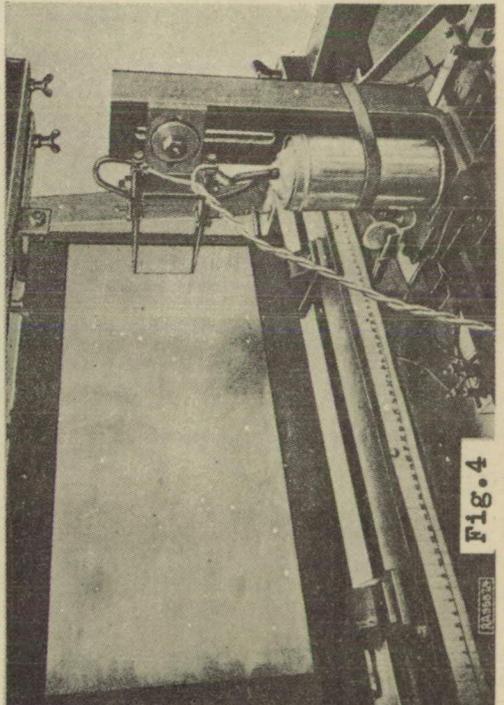


Fig.4

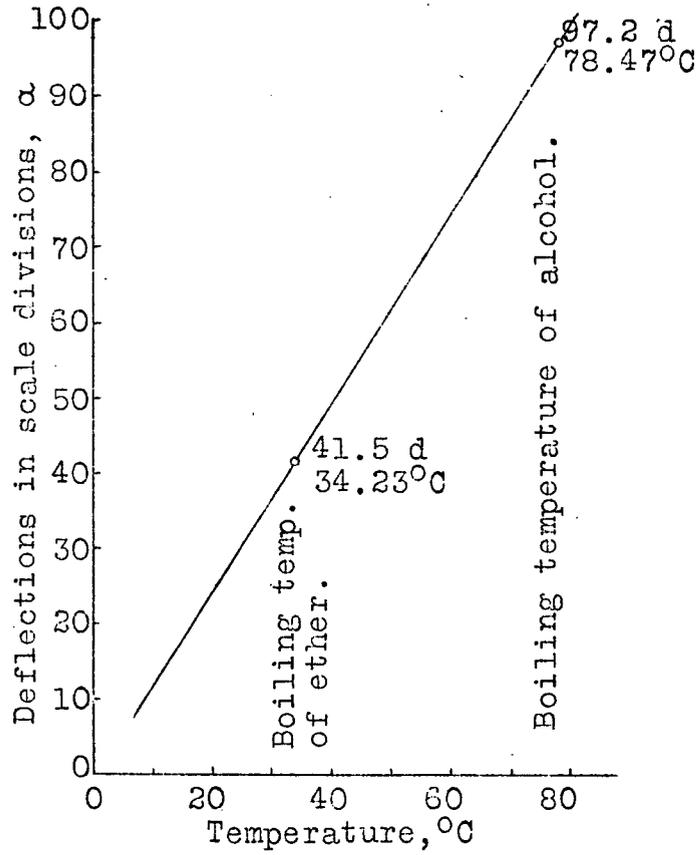
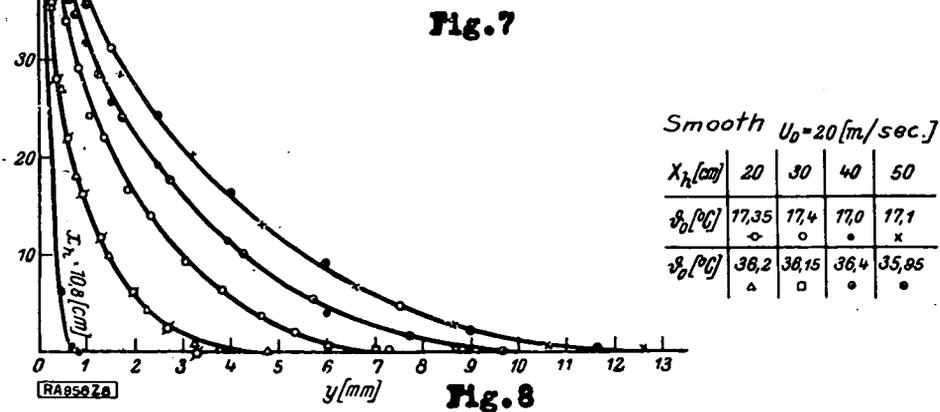
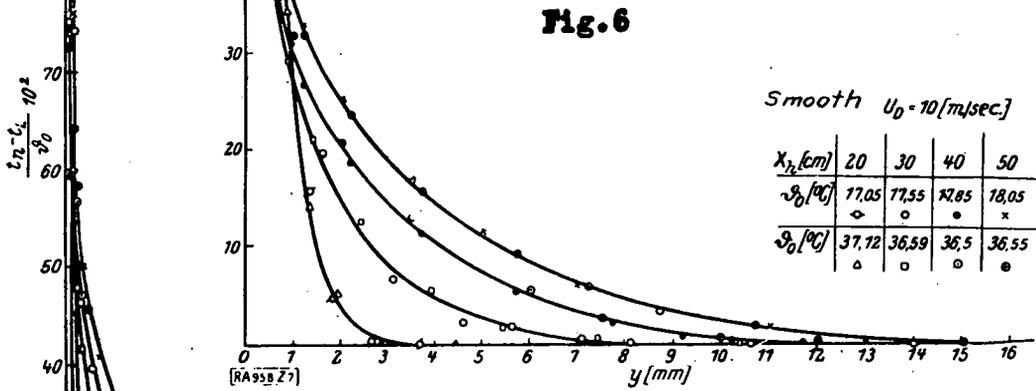
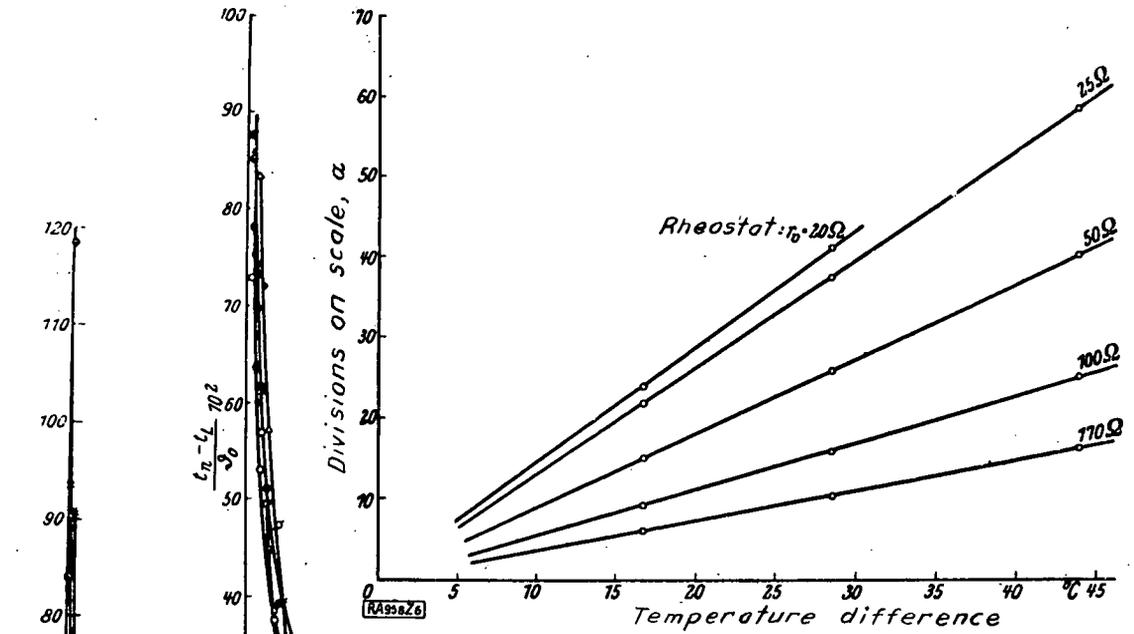


Fig.5



Smooth $U_0 = 5$ [m/sec.]

X_h [cm]	20	30	40	50	20
ψ_0 [deg]	77.65	77.27	77.5	77.55	77.4

Smooth $U_0 = 7.5$ [m/sec.]

X_h [cm]	20	30	40	50
ψ_0 [deg]	77.55	78.35	78.7	78.9

Table for Fig.12

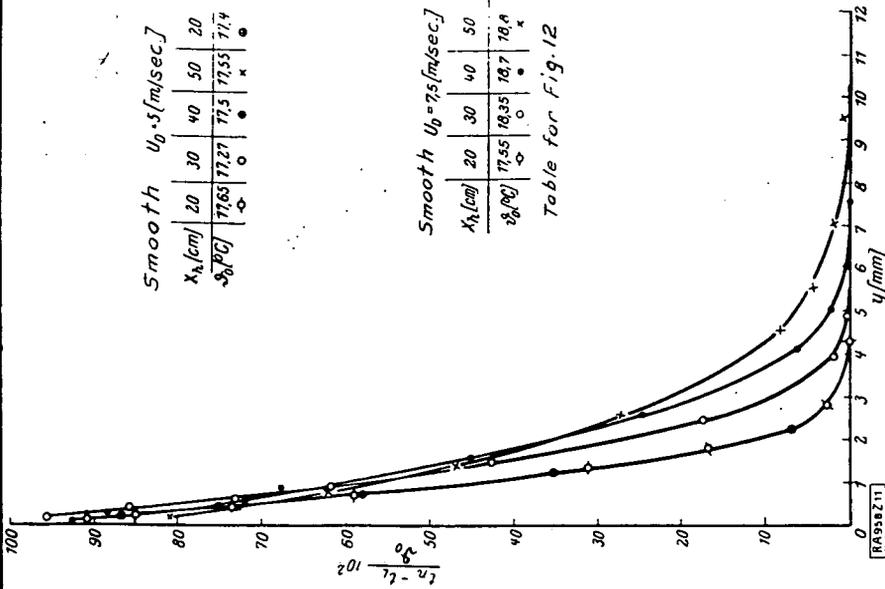


FIG.11

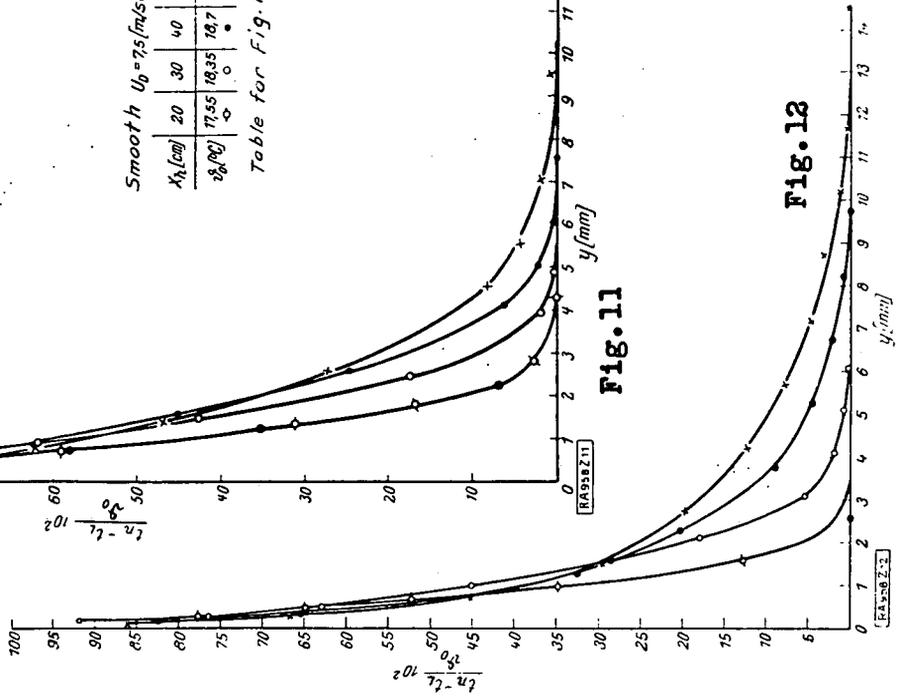


FIG.12

Smooth $U_0 = 25$ [m/sec.]

X_h [cm]	20	30	40	50
ψ_0 [deg]	77.8	78.05	77.9	78.4
ψ_0 [deg]	36.75	36.9	37.4	37.3

Smooth $U_0 = 3$ [m/sec.]

X_h [cm]	10.0	20	30	40	50
ψ_0 [deg]	76.5	76.3	76.2	76.25	76.4

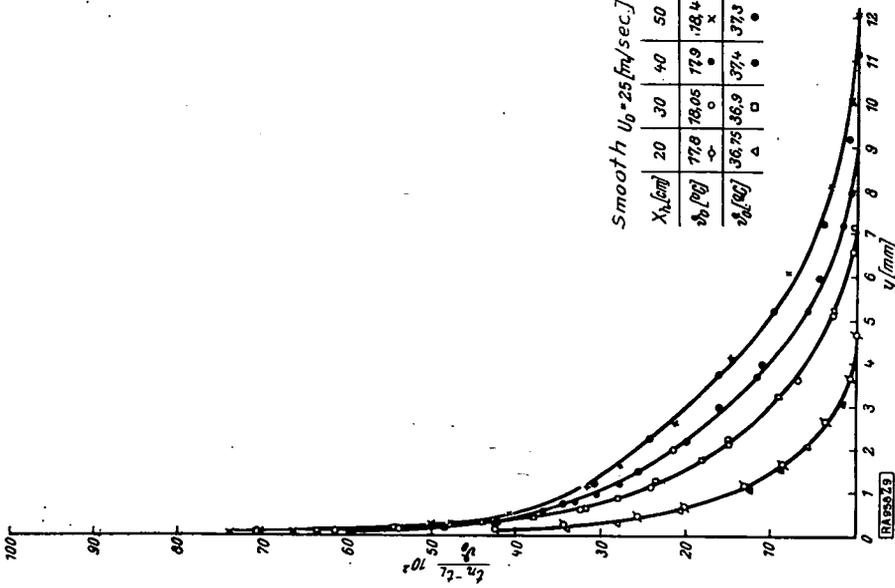


FIG.9

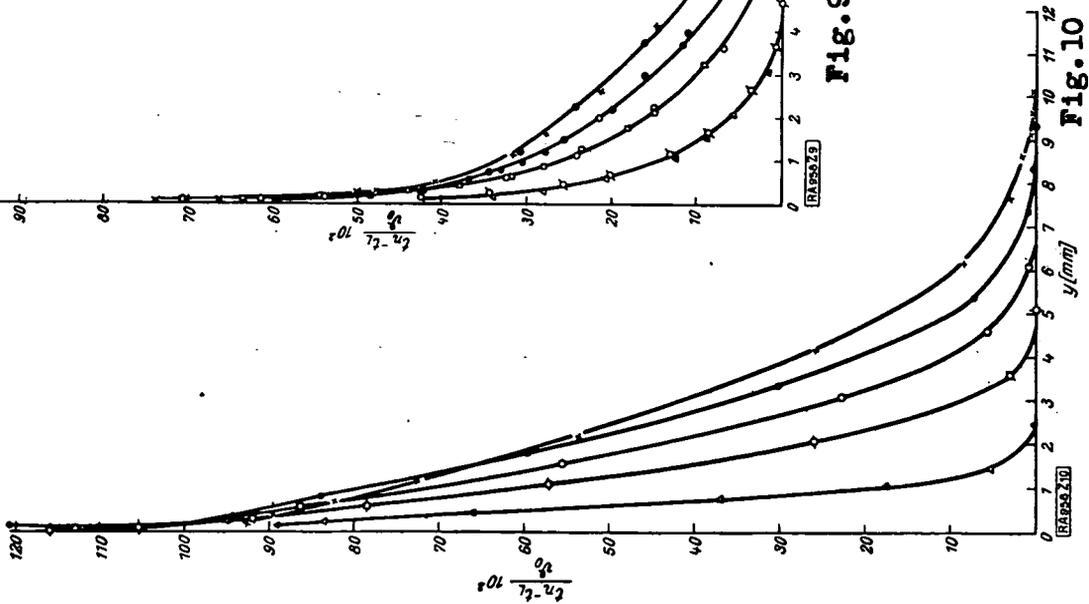
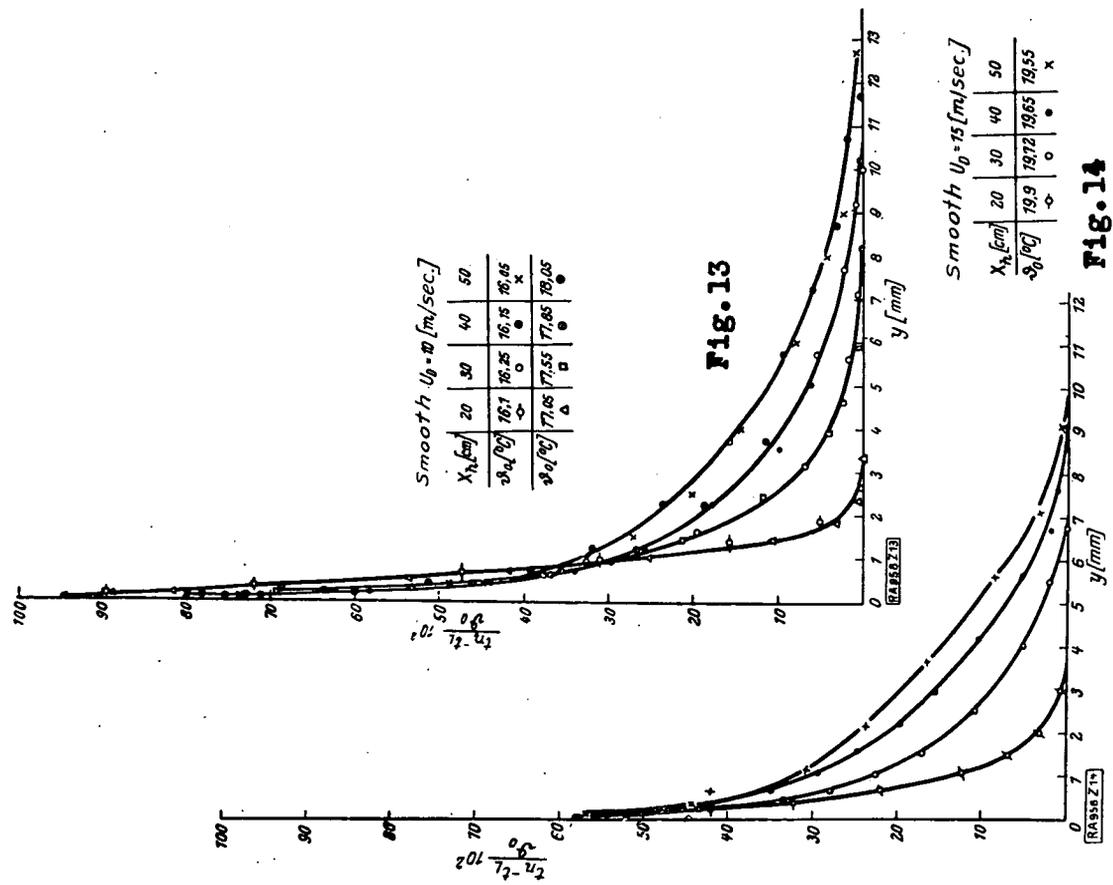
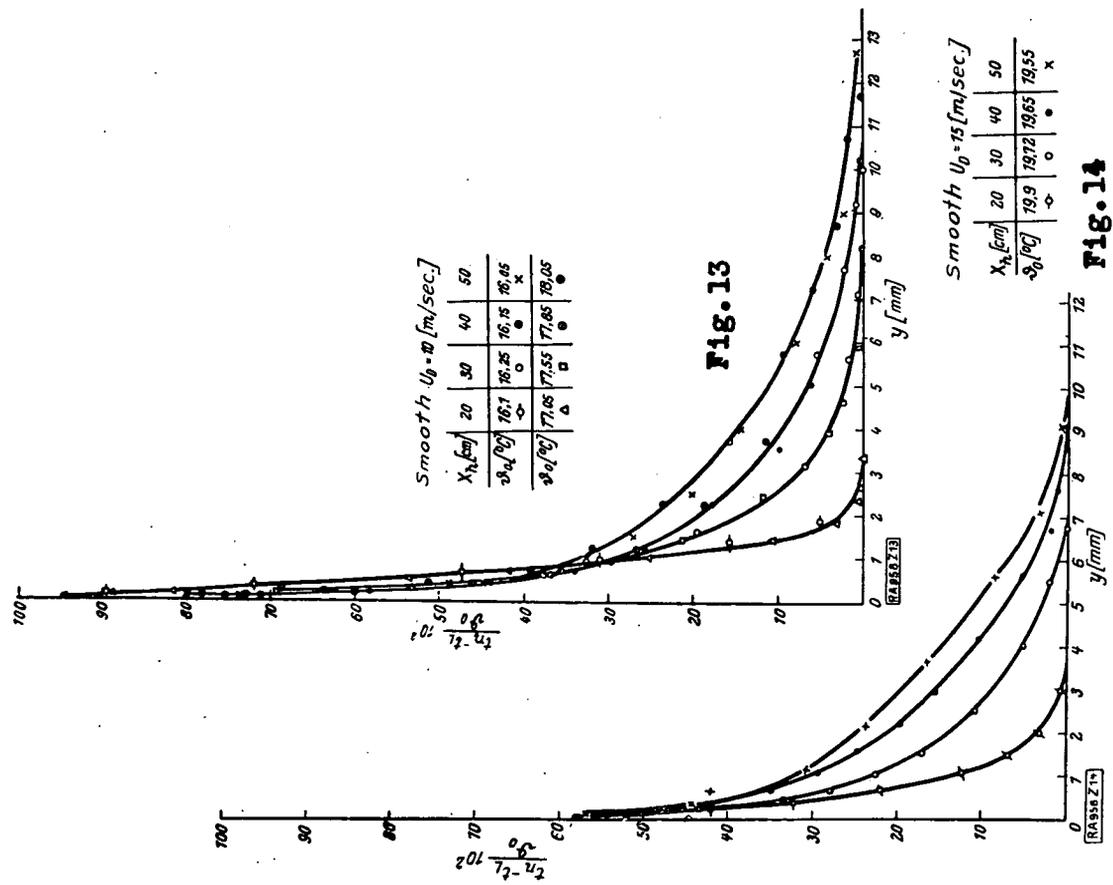
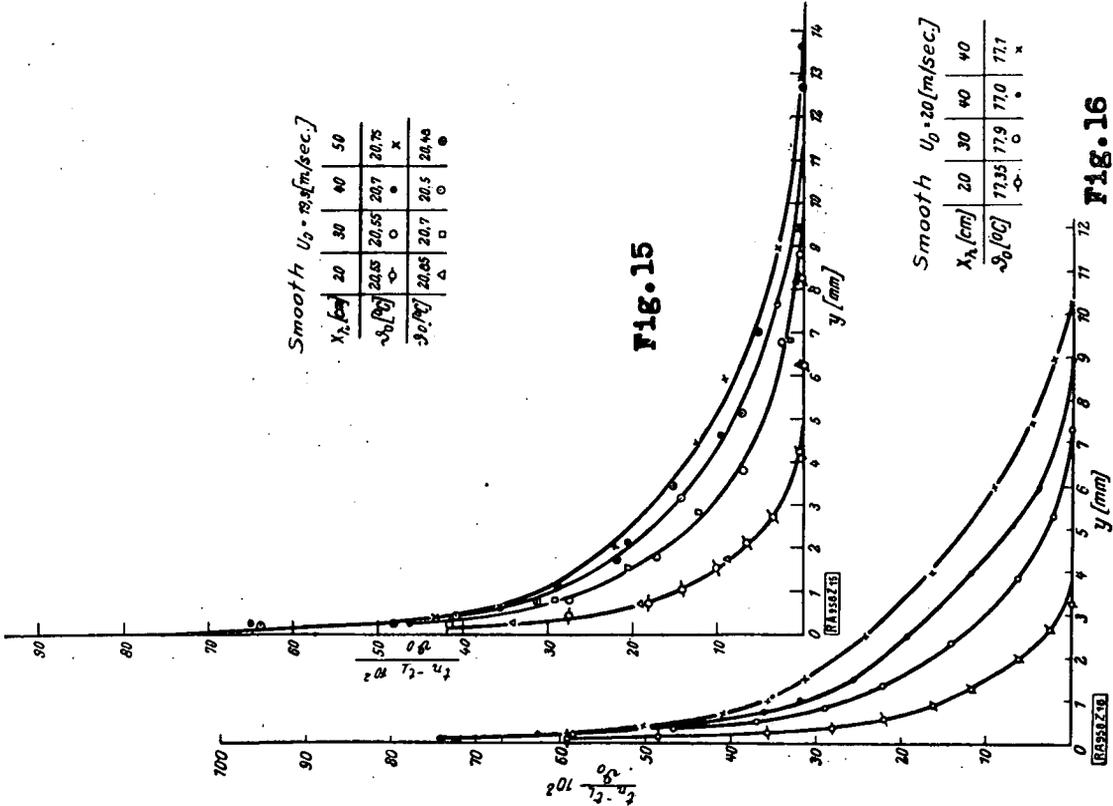
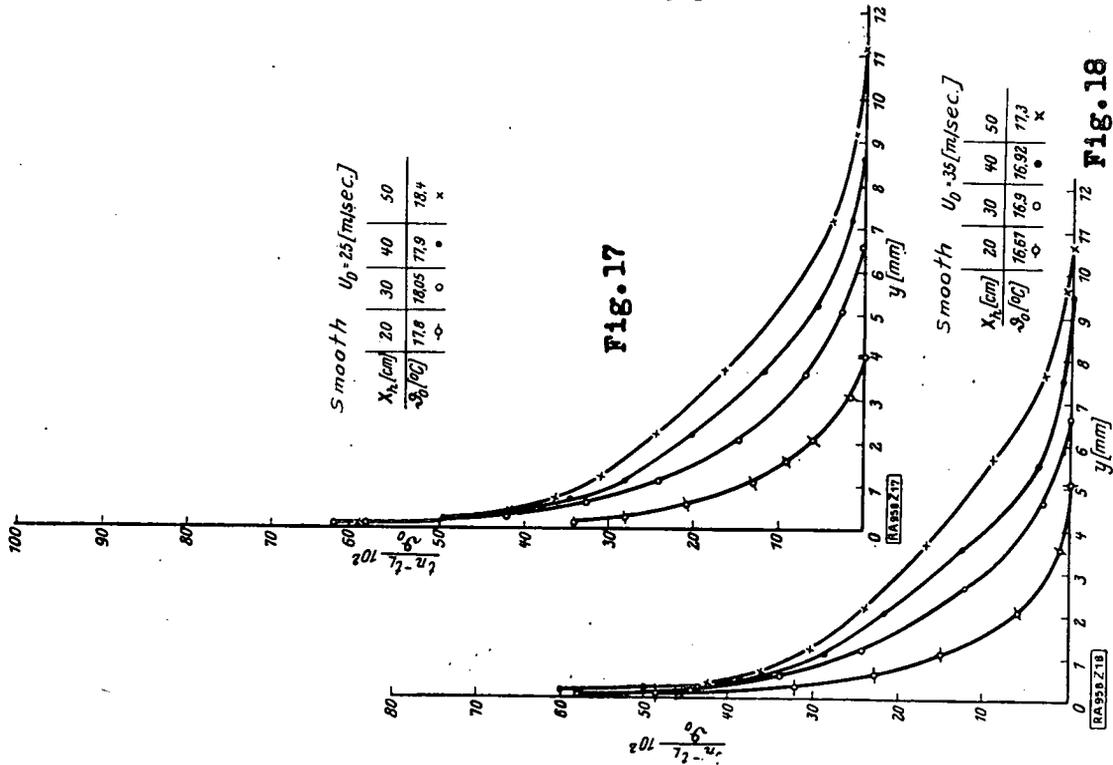
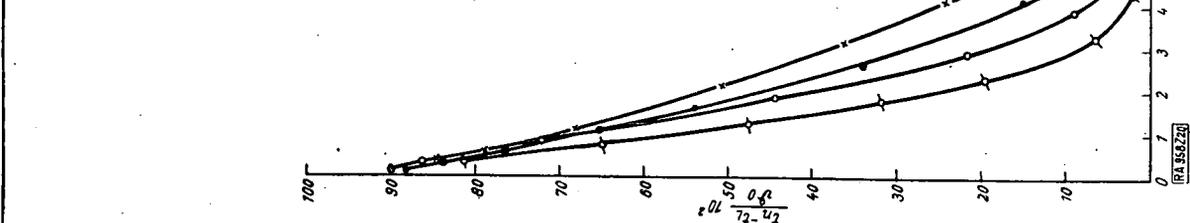
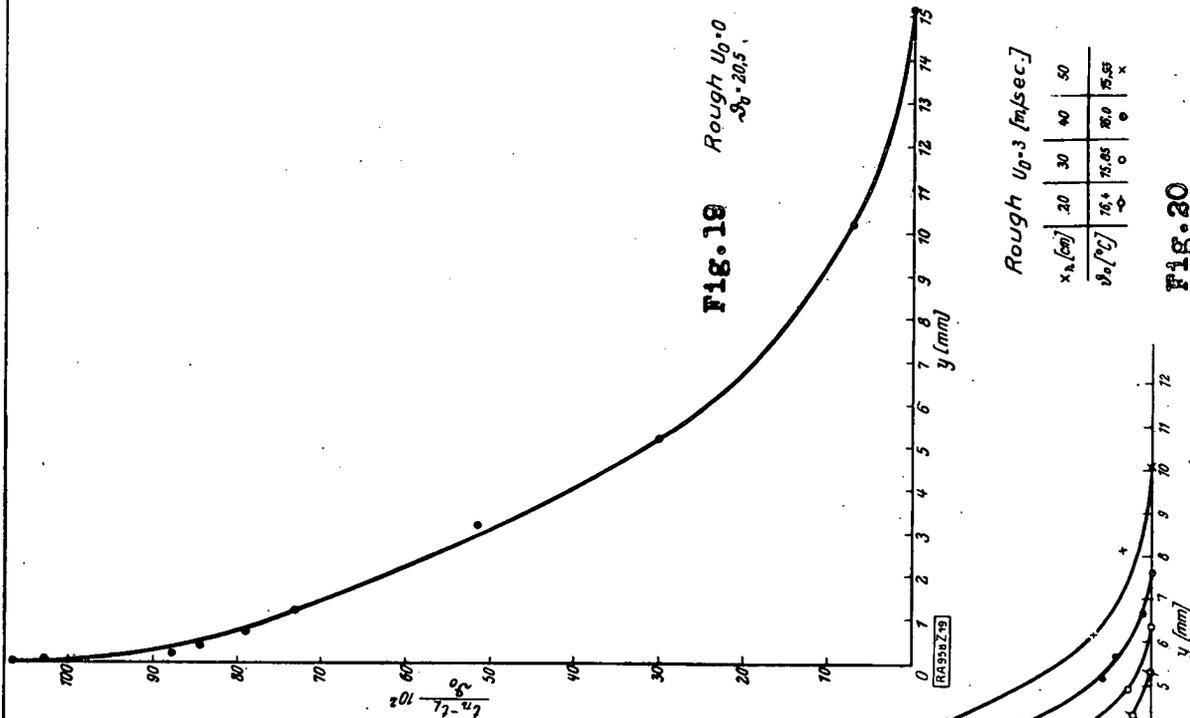


FIG.10



Smooth $U_0 = 20$ [m/sec.]

X_h [cm]	20	30	40	40
$T_n - T_l$ [°C]	17.35	17.9	17.0	17.7



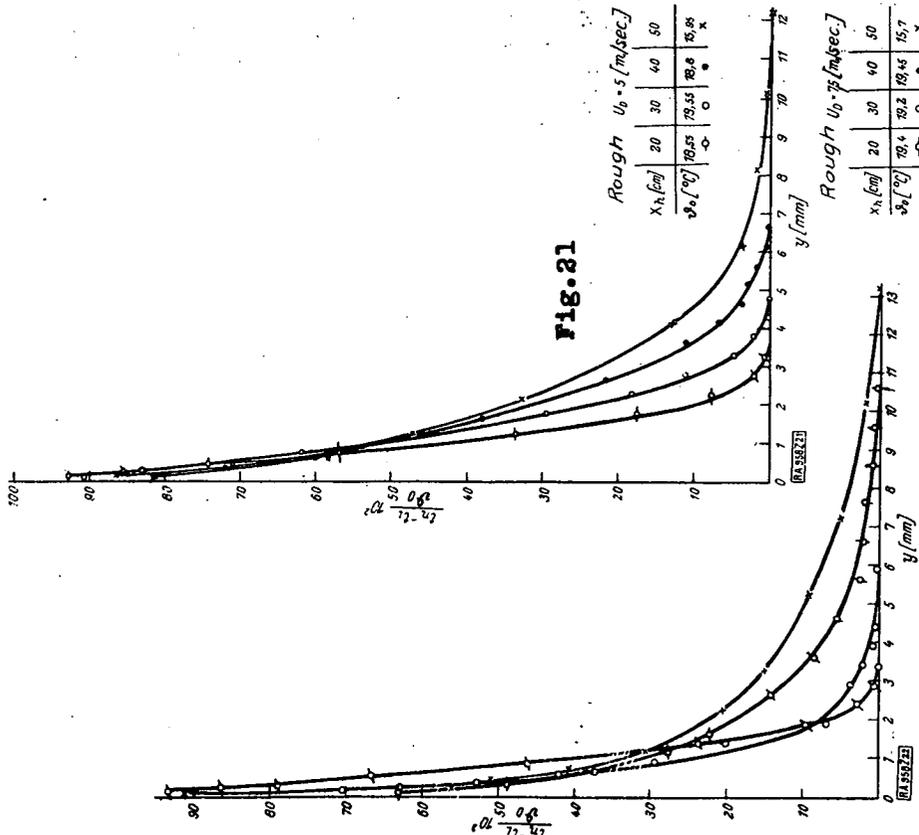


Fig. 21

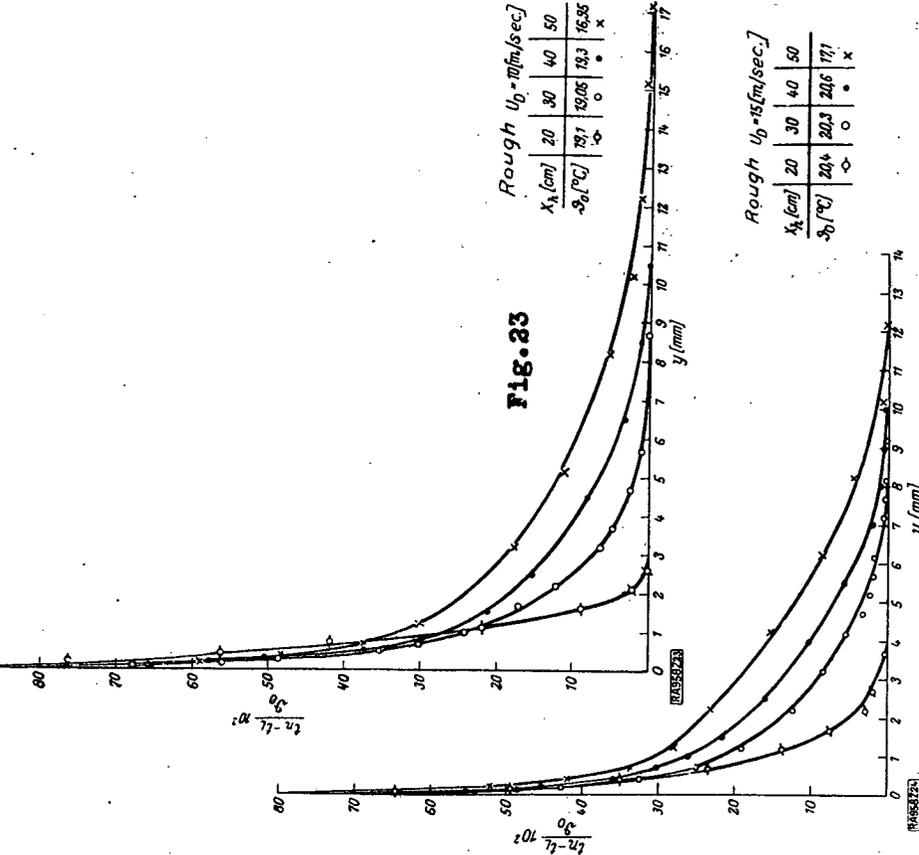


Fig. 23

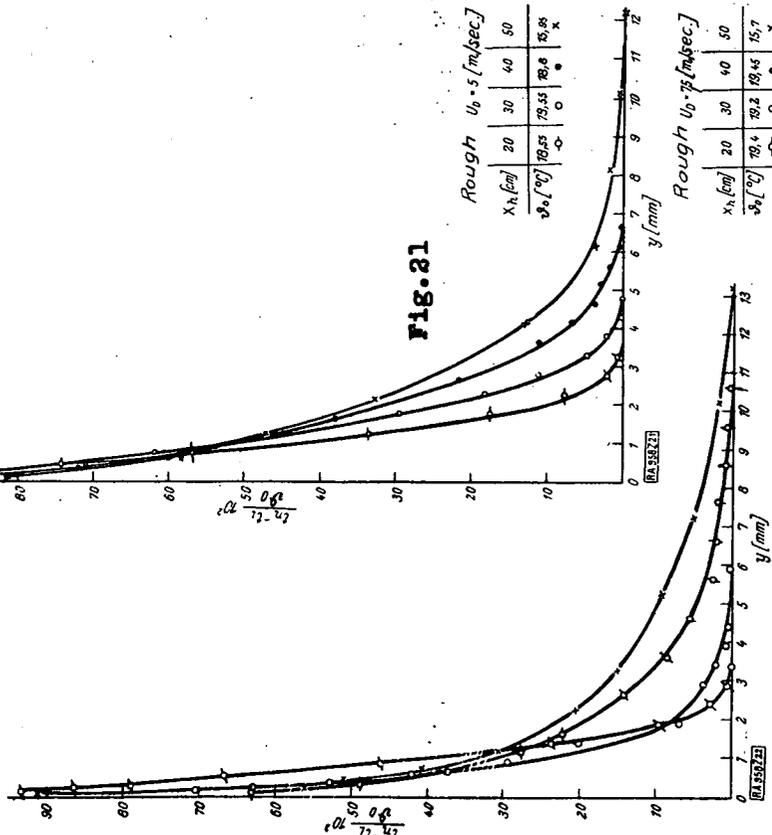


Fig. 22

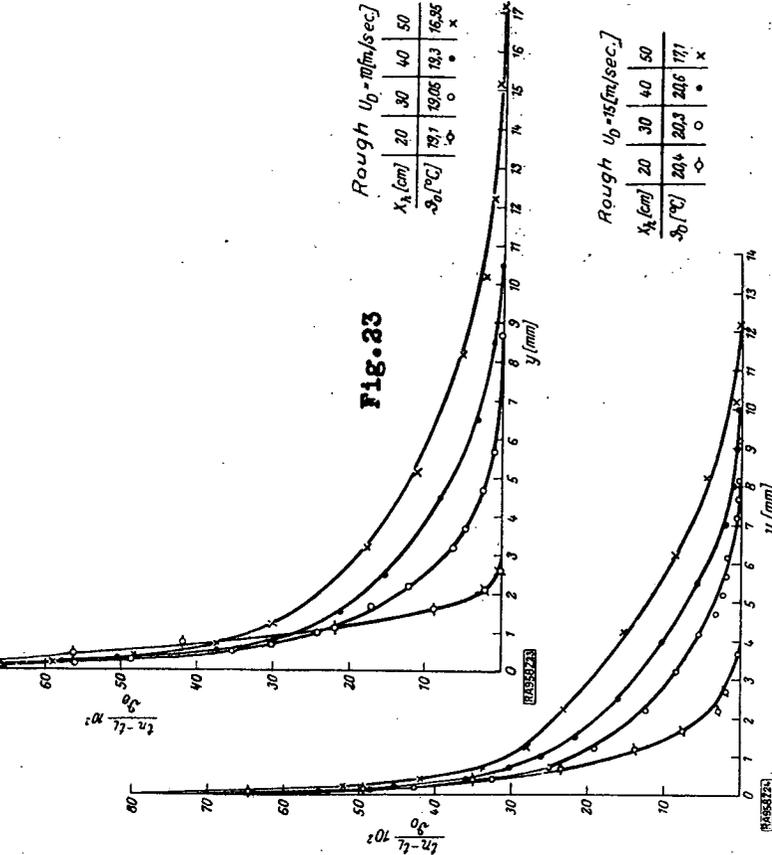
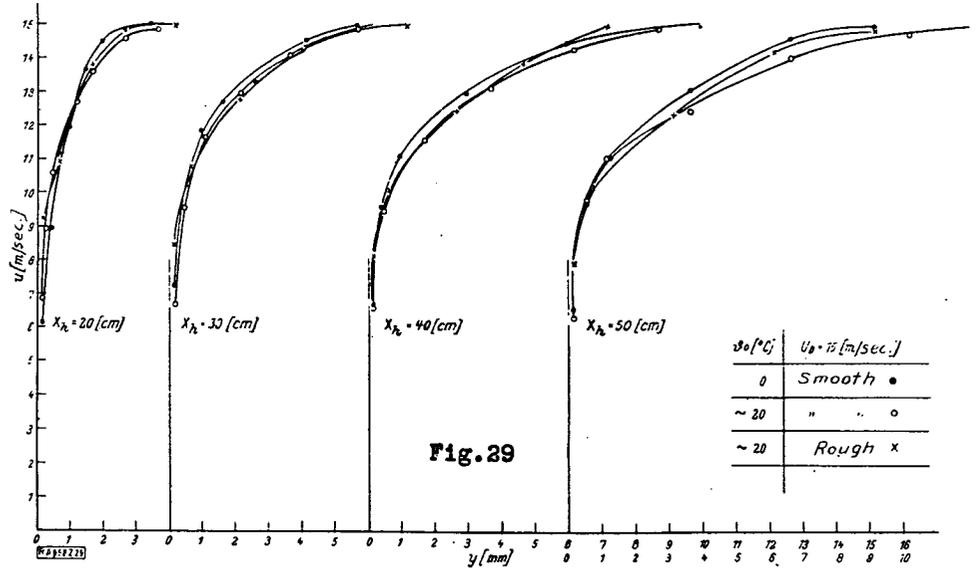
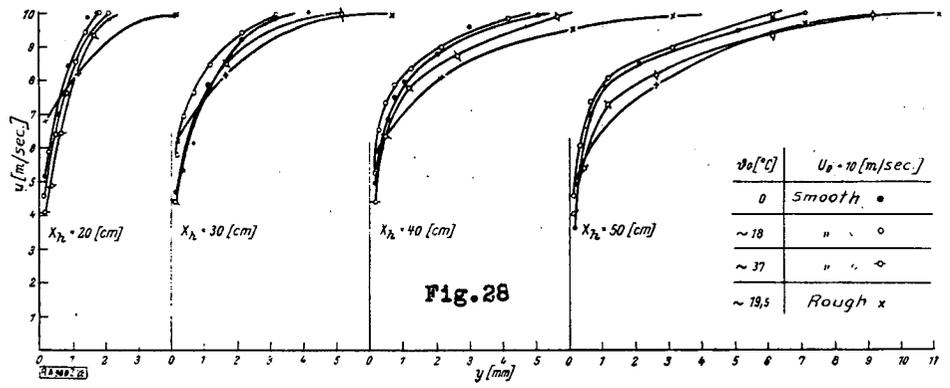
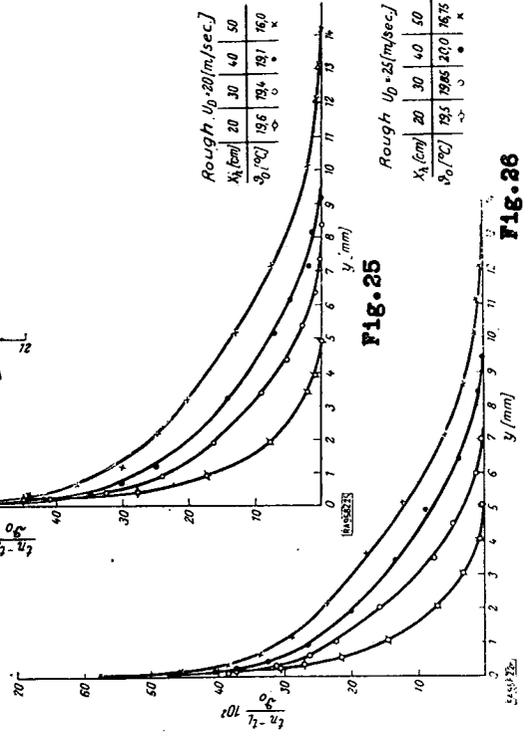
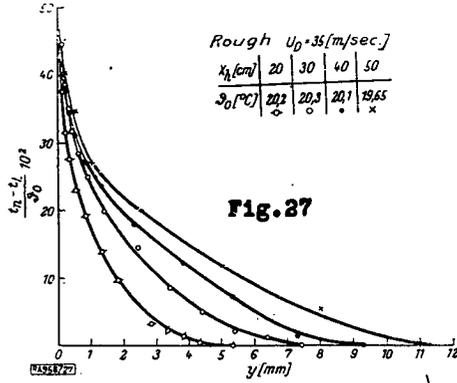
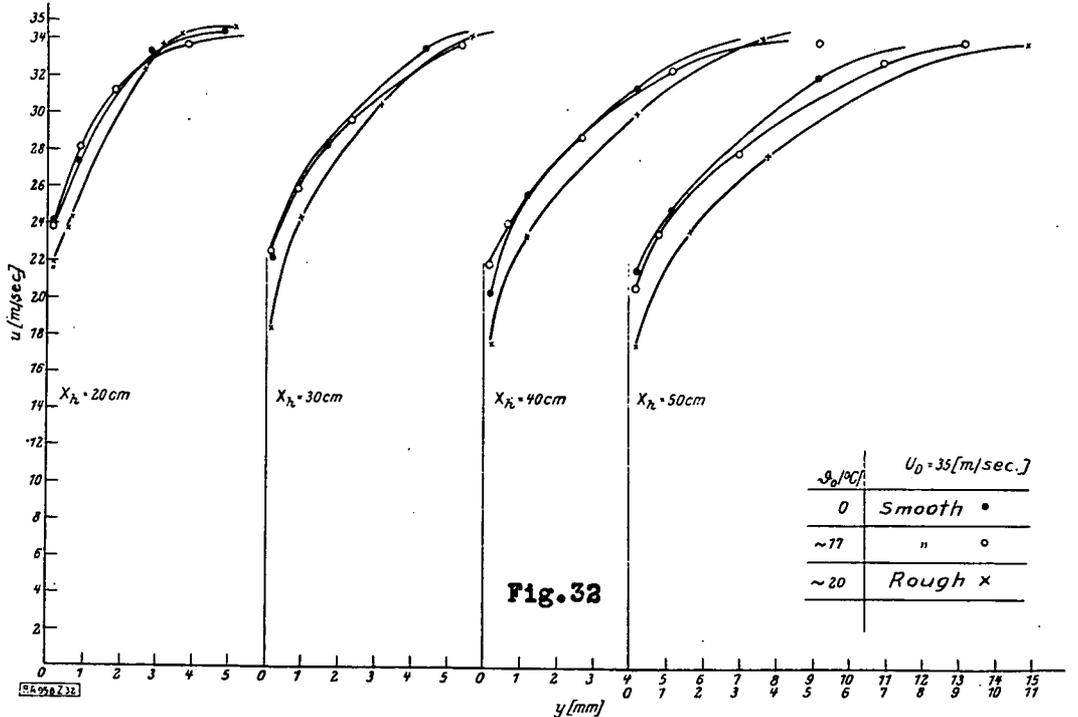
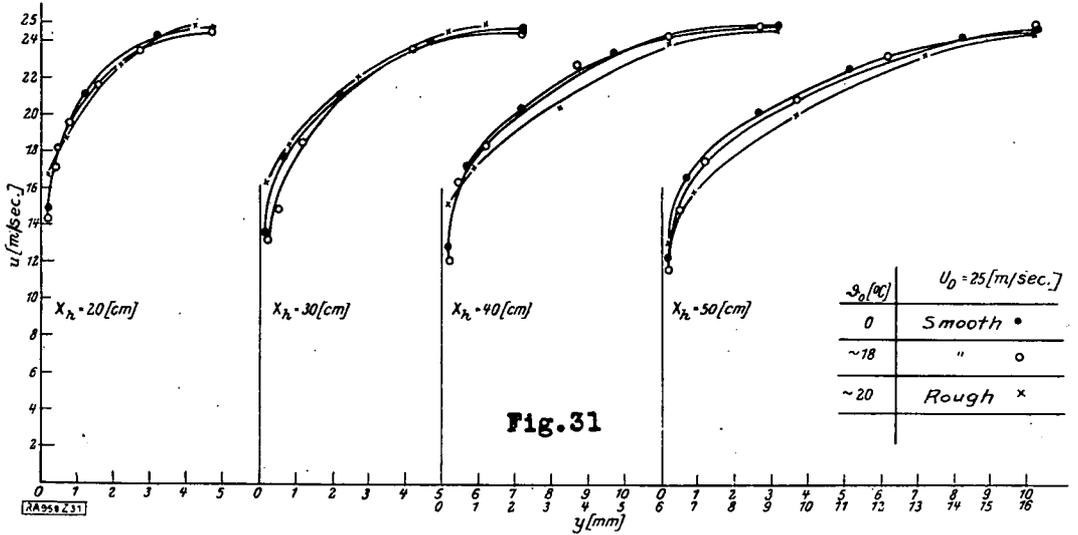
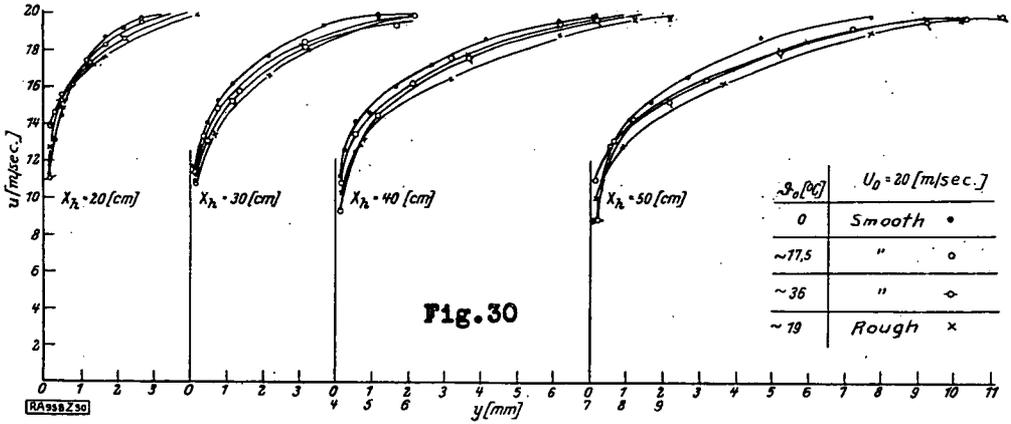


Fig. 24





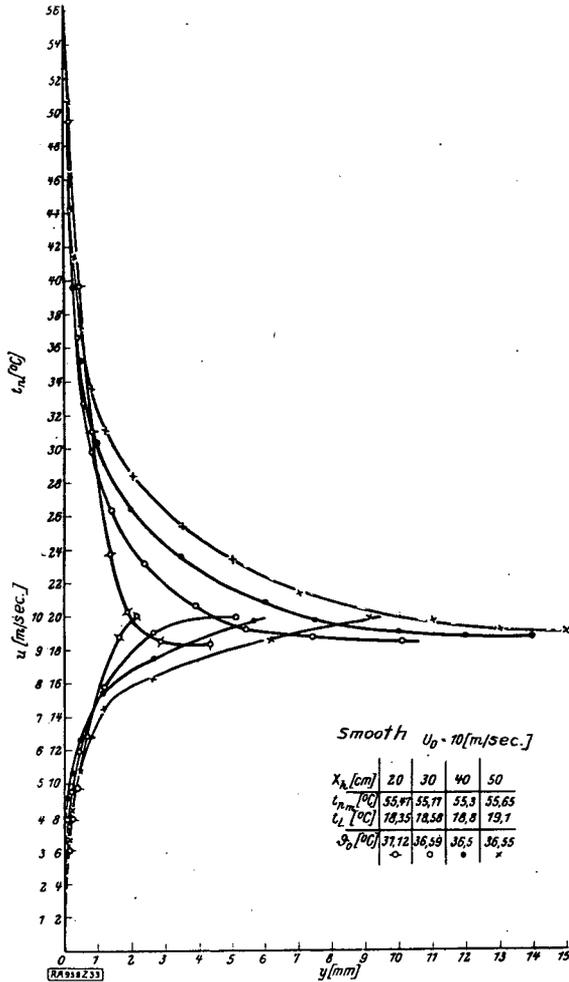


Fig.33

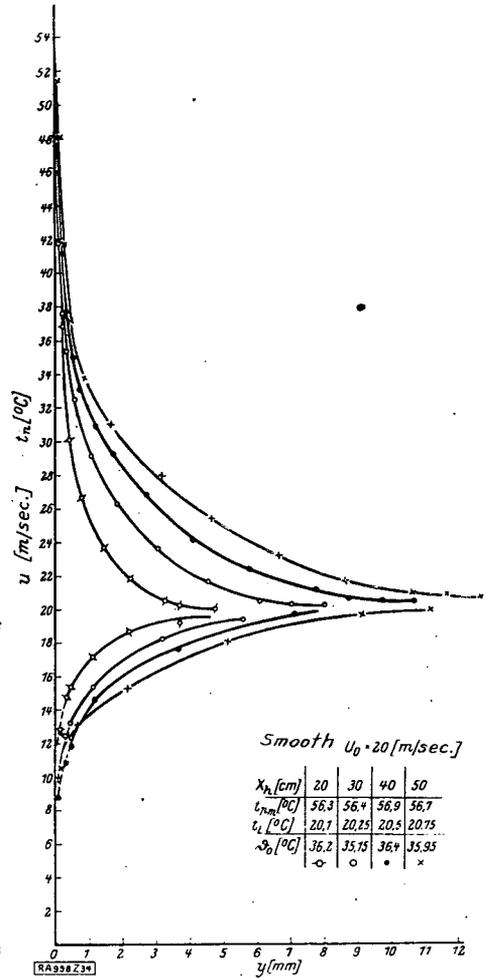


Fig.34

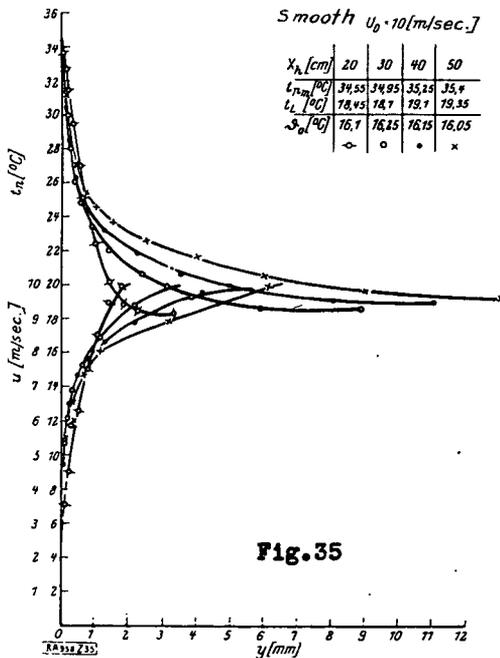


Fig.35

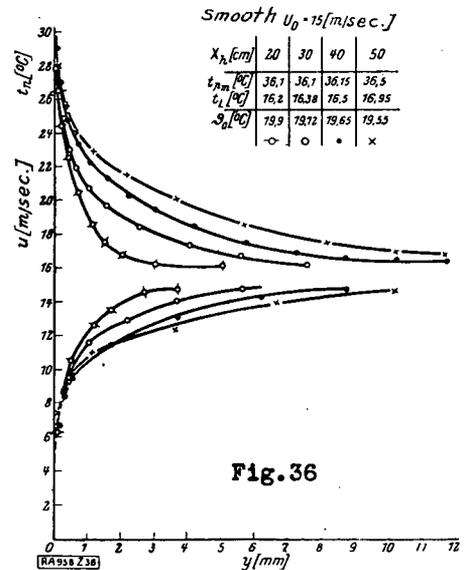
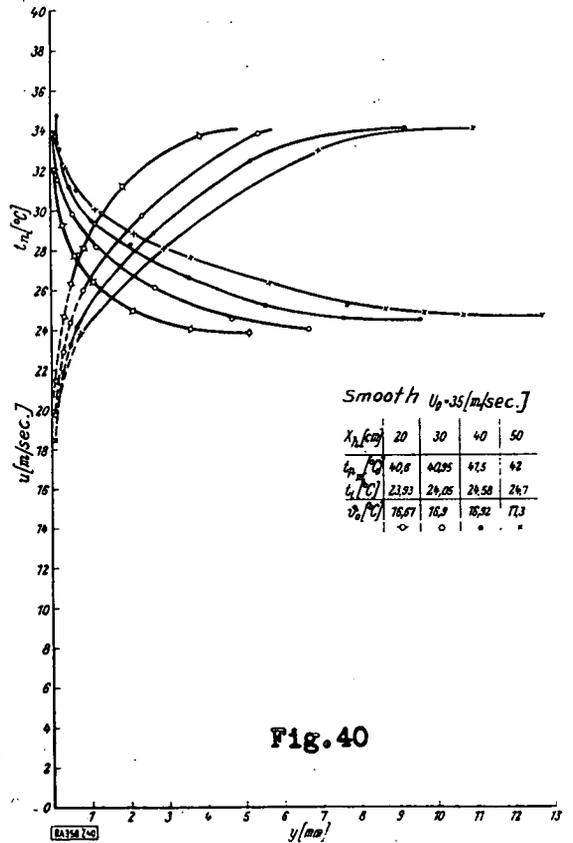
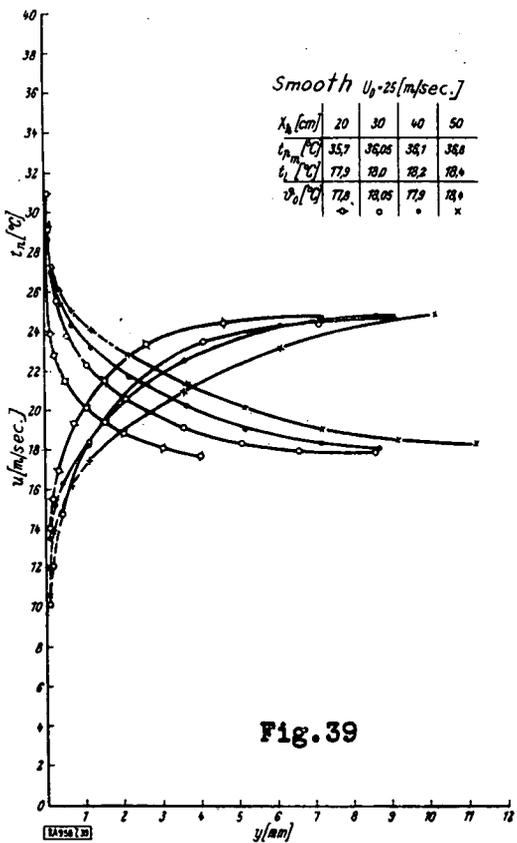
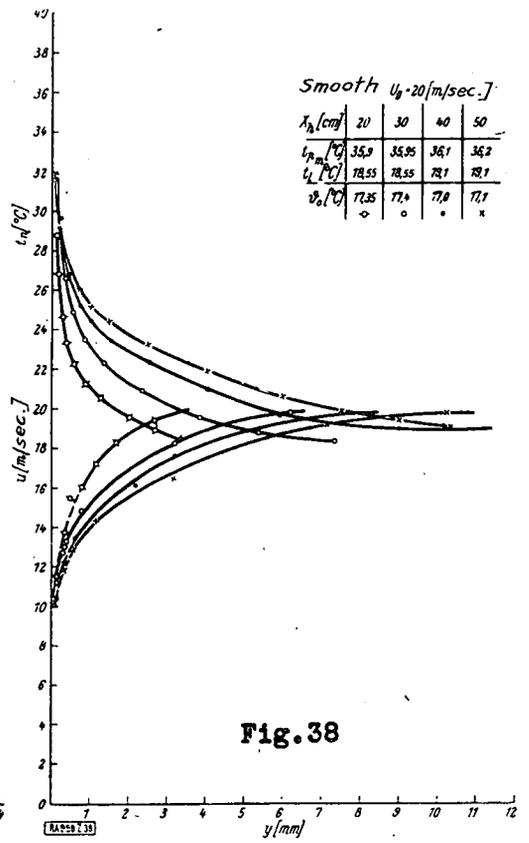
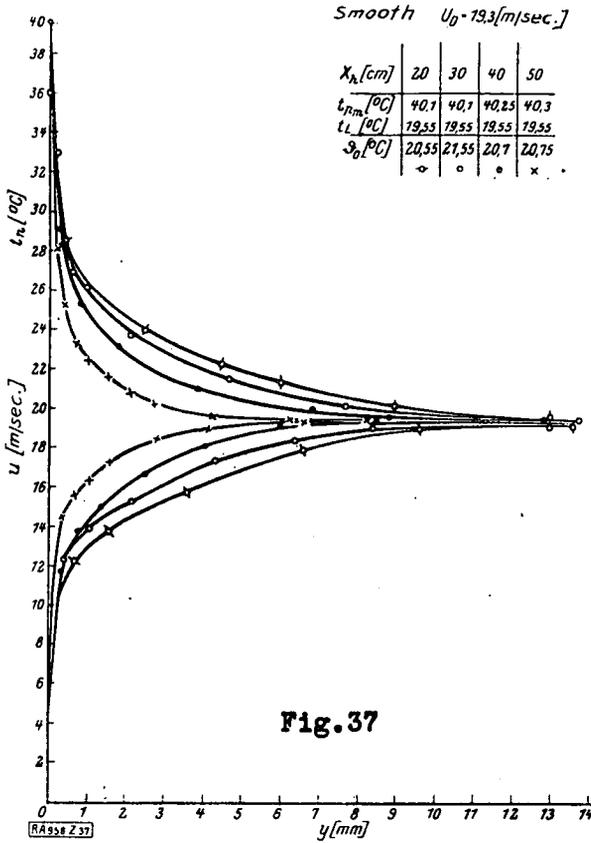


Fig.36



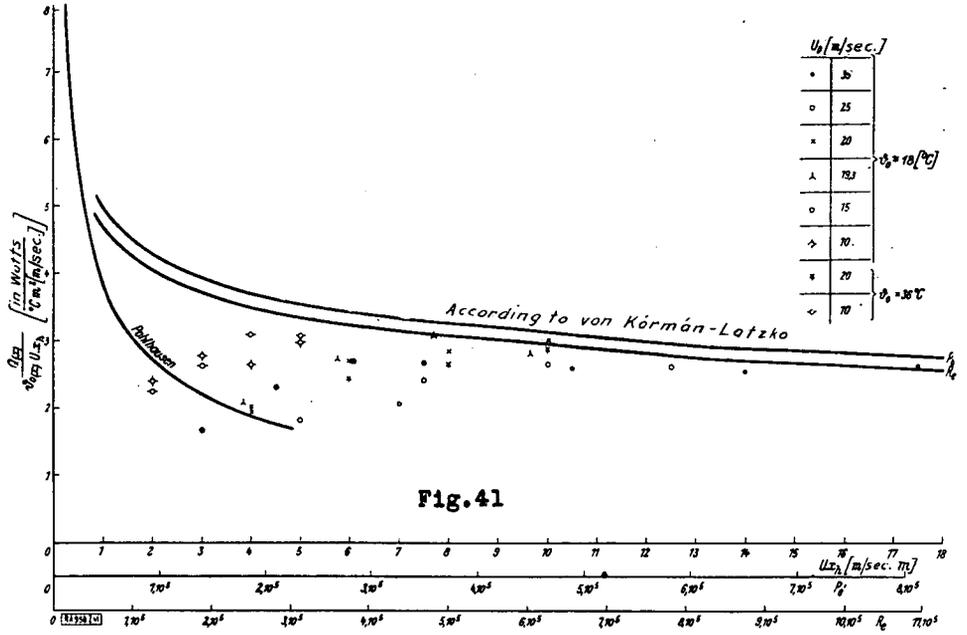


Fig.41

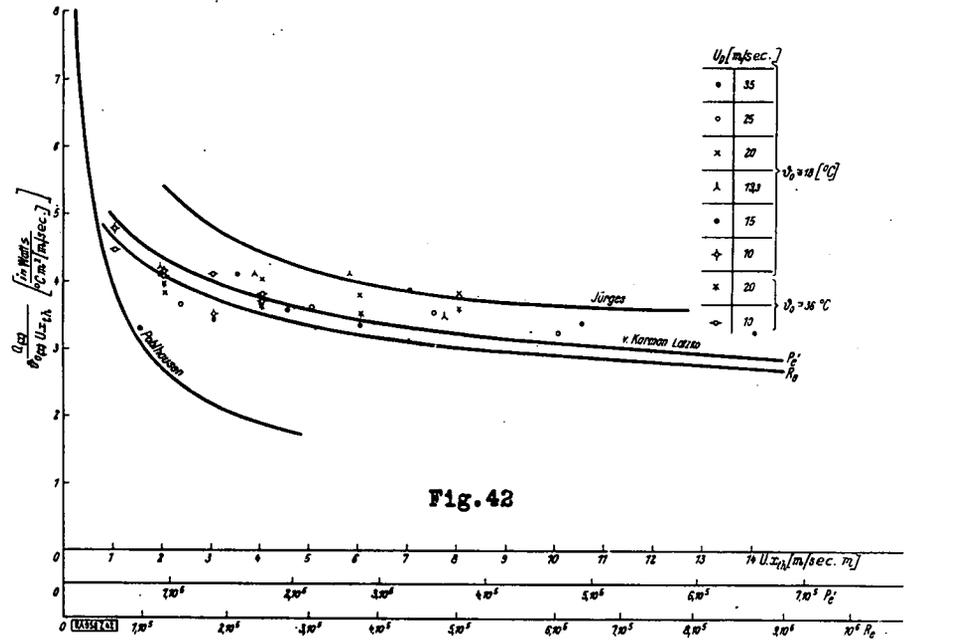


Fig.42

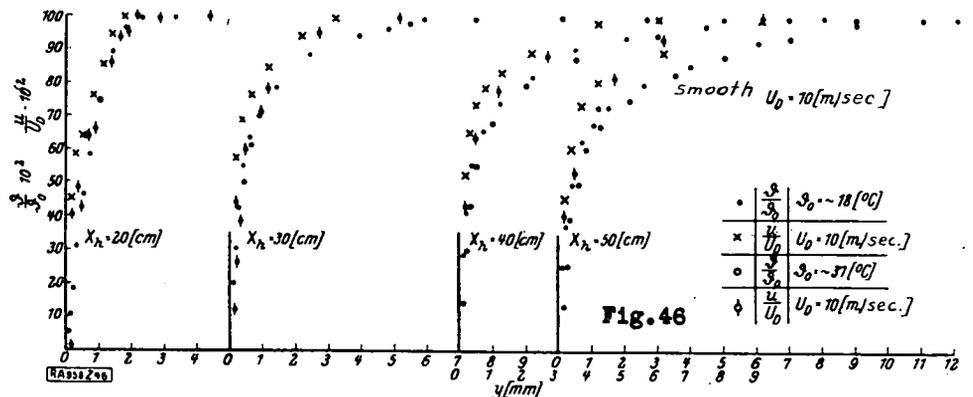
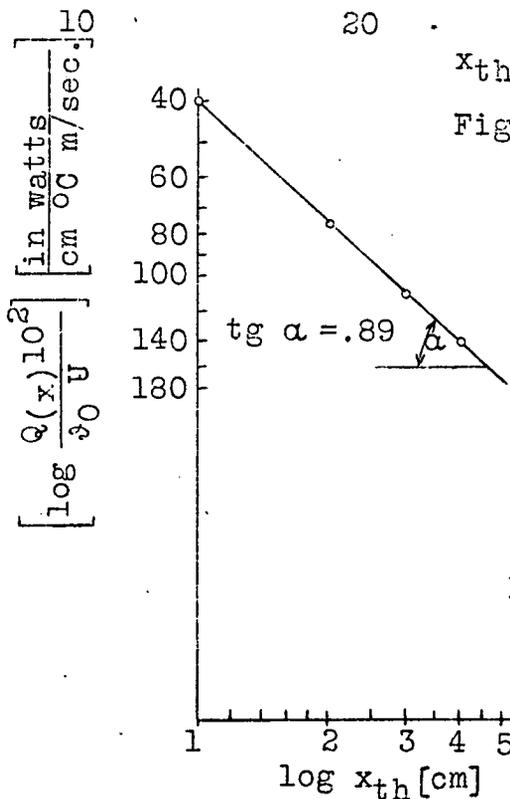
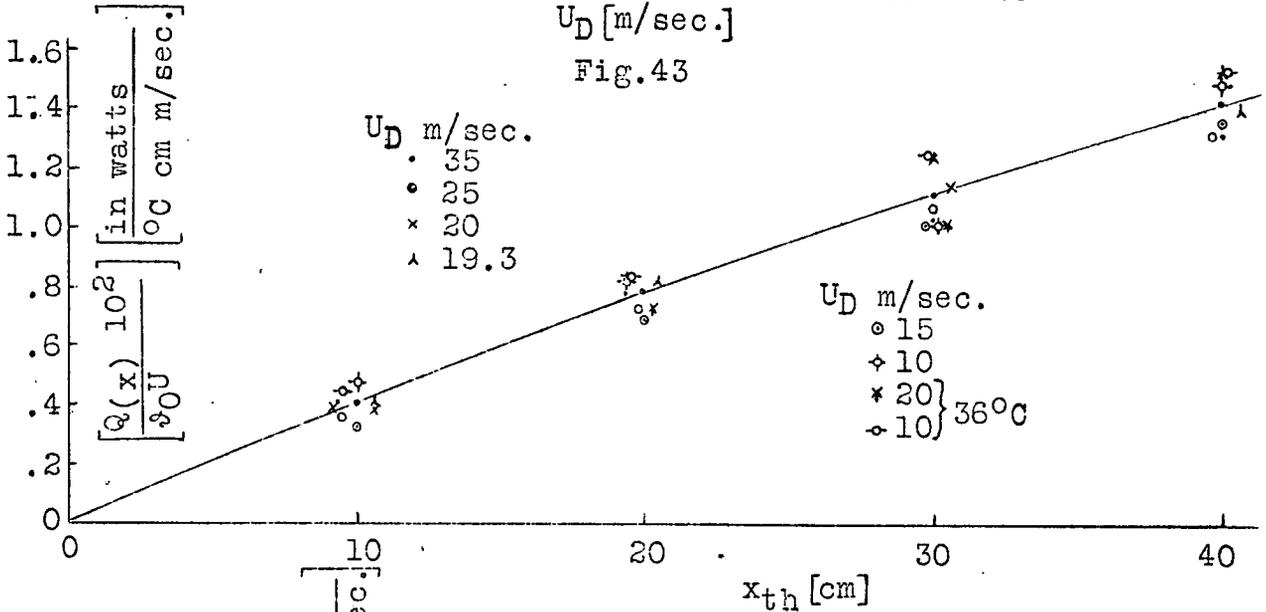
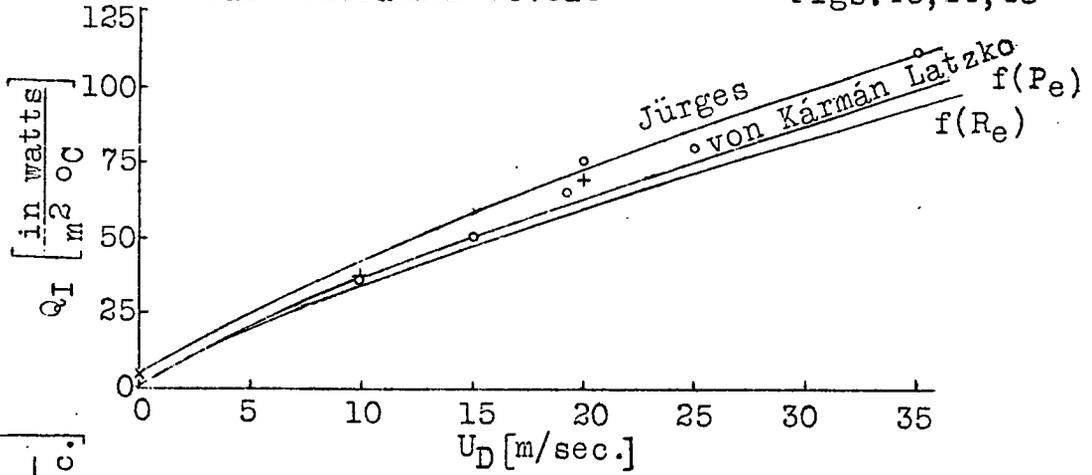


Fig.46



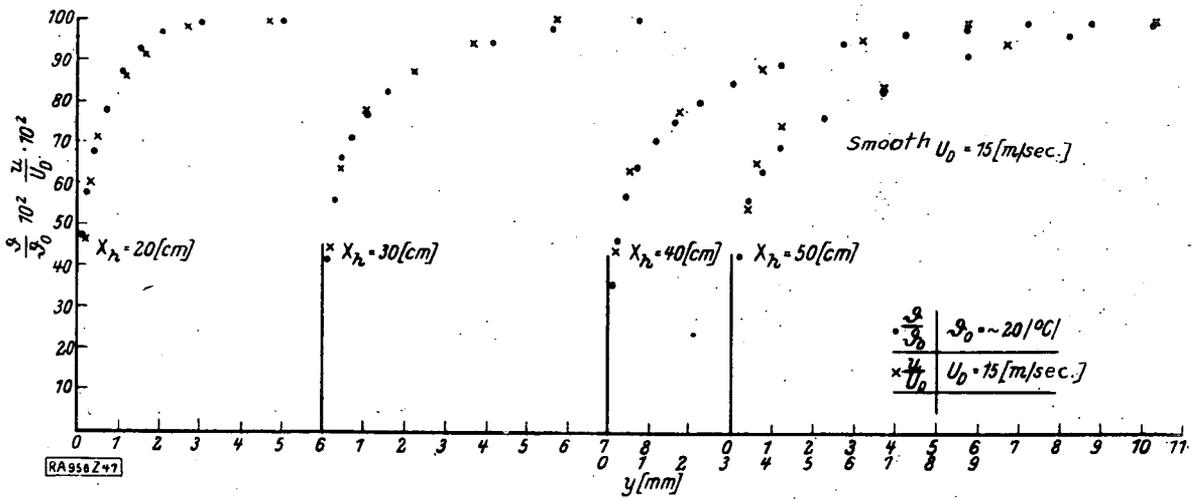


Fig.47

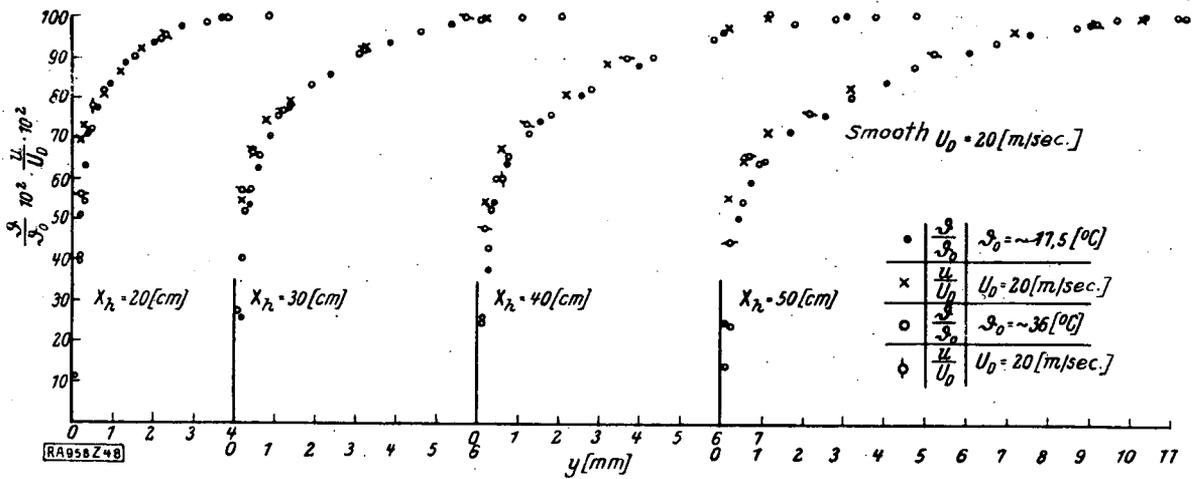


Fig.48

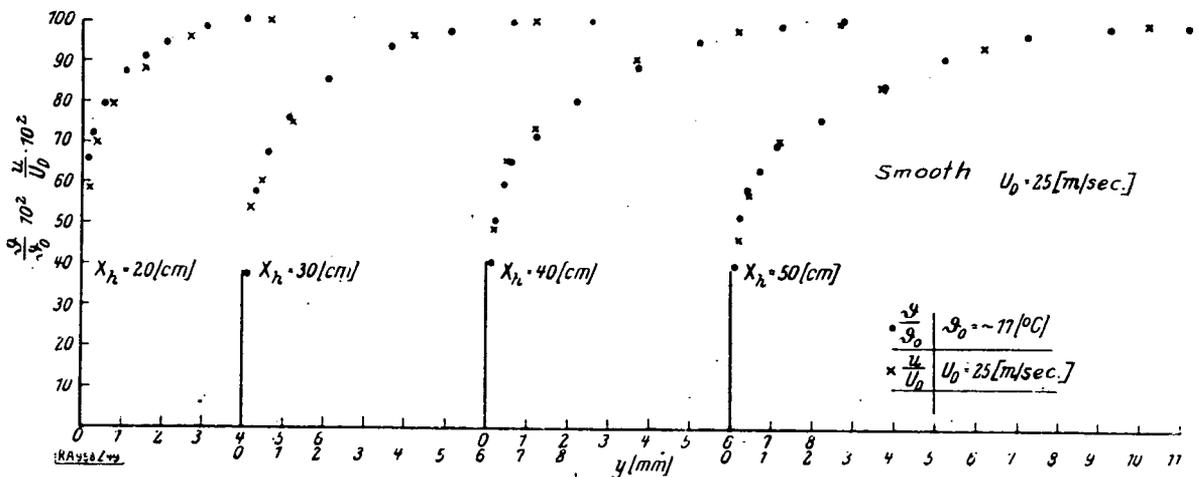


Fig.49

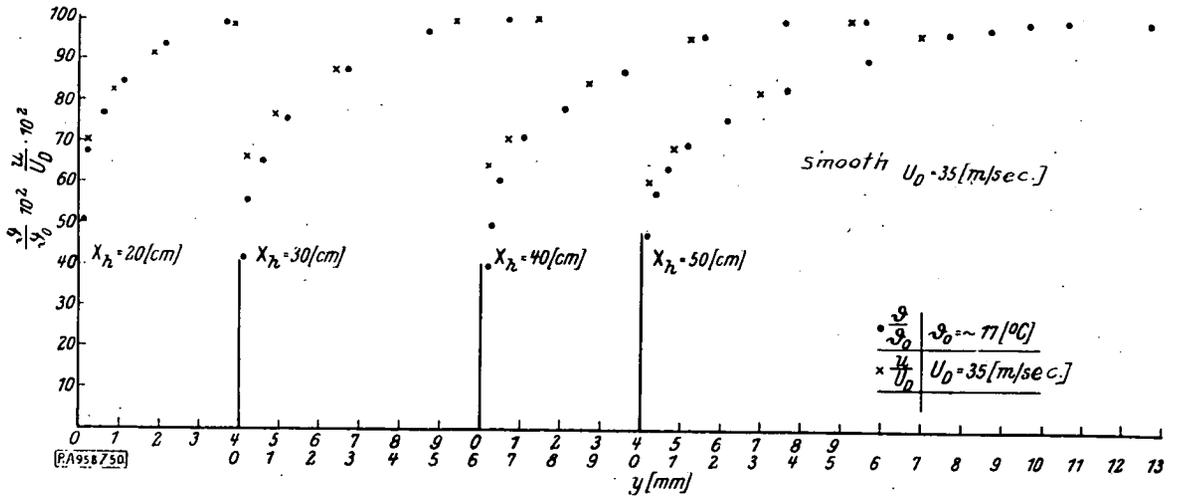


Fig.50

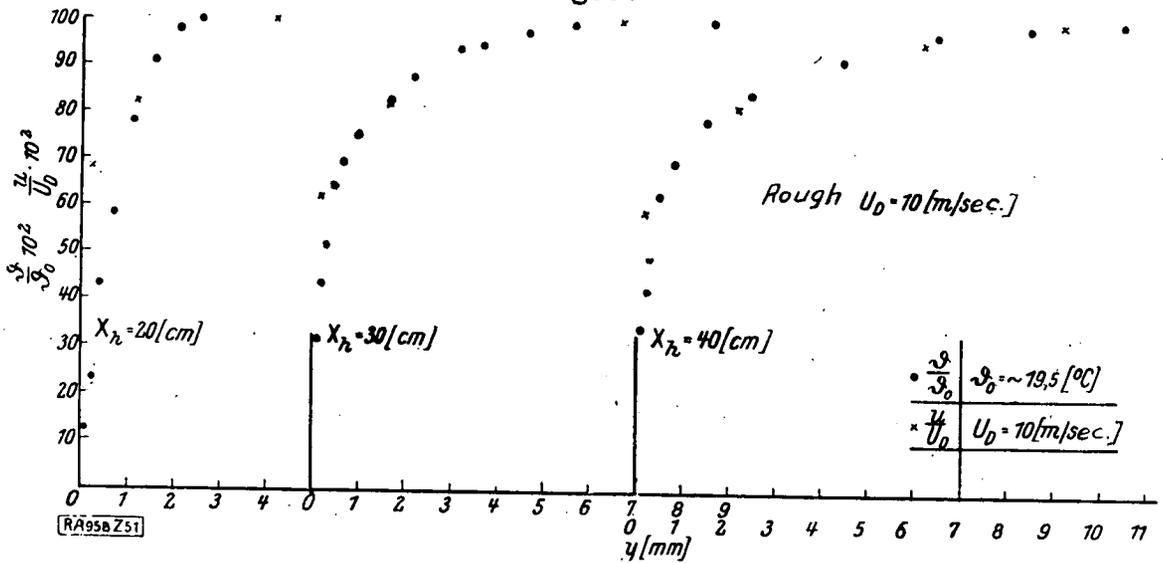


Fig.51

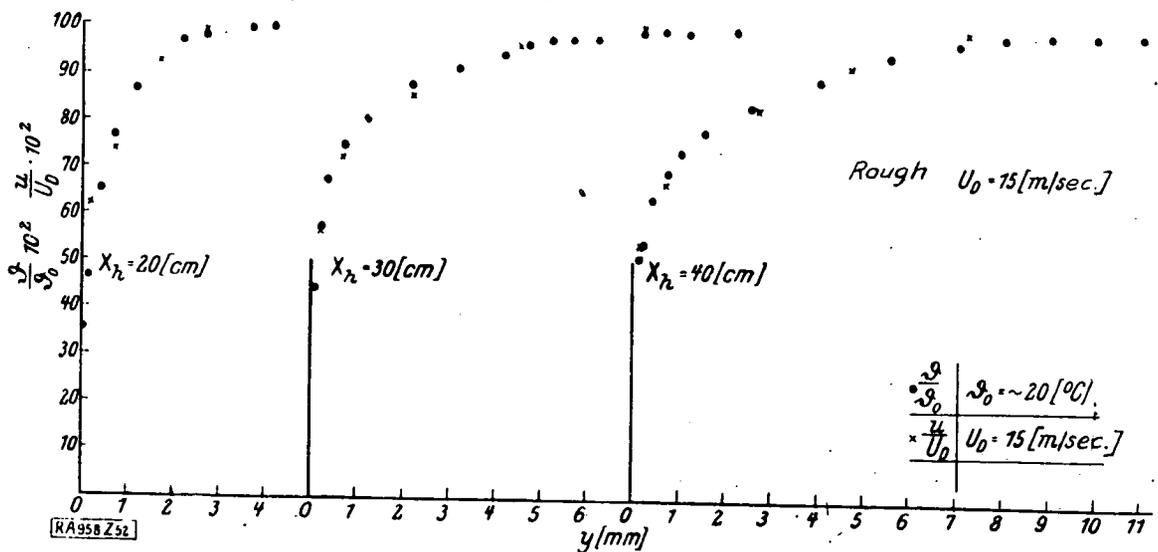


Fig.52

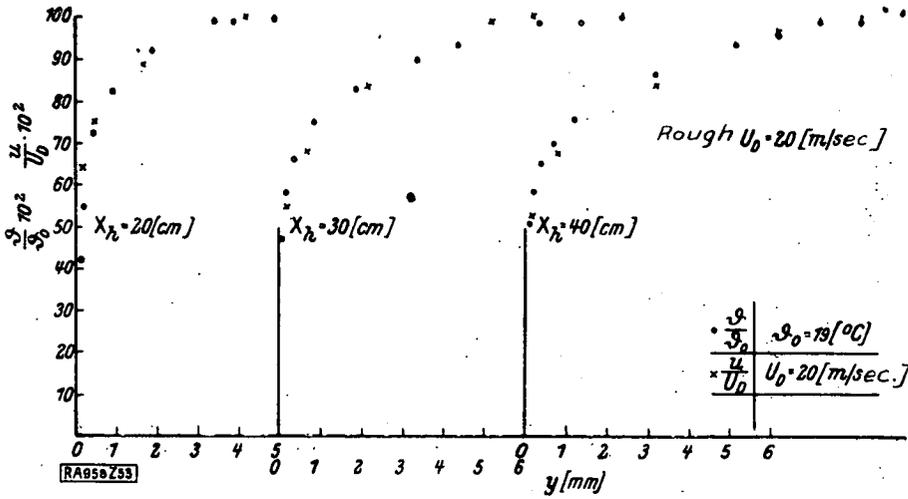


Fig.53

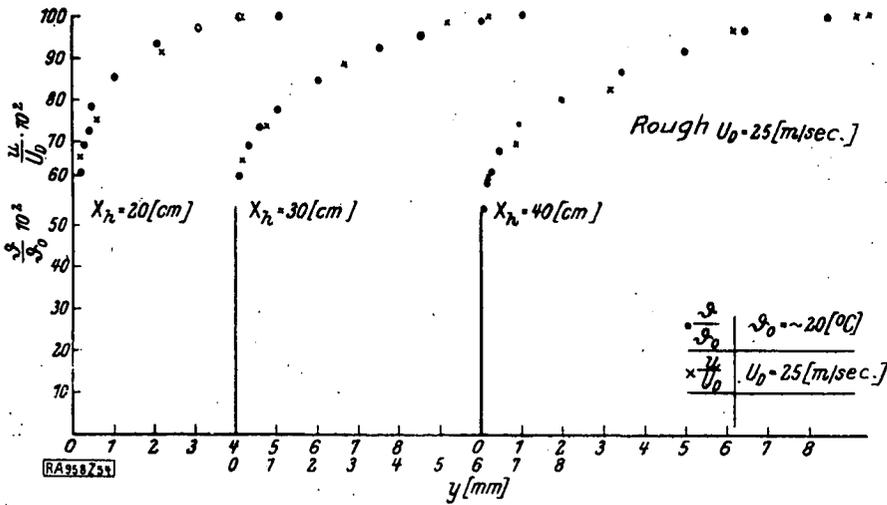


Fig.54

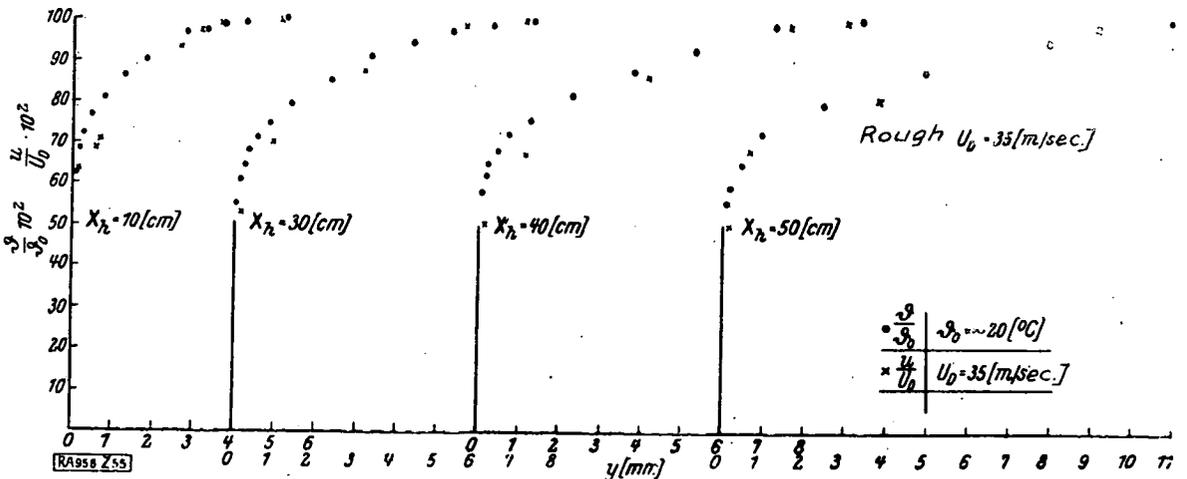


Fig.55