TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 587

COUNTER-PROPELLER

By Ugo de Caria

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COUNTER-PROPELLERS.*

By Ugo de Caria.

Helicoidal propellers are very often inappropriately compared to a screw. We say inappropriately, because, while the female screw can furnish a reaction without moving and by undergoing only a slight elastic deformation, the fluid in which a propeller revolves reacts only by reason of a velocity increment imparted to it by the propeller itself. Hence, aside from the fact that a screw and a helicoidal propeller serve the common purpose of converting a motive couple into an axial thrust, they are not comparable because the physical phenomena produced by the two are substantially different. It follows that, while, in the case of the screw, the only losses of energy are due to friction, in helicoidal propellers the losses are due to the kinetic energy imparted to the fluid.

It is natural therefore that many scientists have sought to recover this lost energy in whole or in part. On analyzing the motion imparted to the fluid in the wake of a propeller, we find axial, tangential, and radial velocity increments in addition to the initial velocity. The radial increments are generally negligible in comparison with the dynamic effects, where the lost energy may be regarded as composed only of axial and

*"Le contro-eliche." From Aeronautica, June, 1930, pp. 417-420.

tangential energy.

The first part, which is intimately connected with the force of traction, is very difficult to recover. Nothing has yet been accomplished in this connection, despite the persistent endeavors of many scholars and investigators.

The tangential increments, namely, those which absorb the motive couple are, on the other hand, easily converted into axial increments, thus increasing the traction and hence the useful energy, without increasing the total energy expended. This result can be attained by using the counter-propeller which we are about to consider.

The counter-propeller is a fixed propeller smaller than the main propeller, mounted either fore or aft of the latter and performing the function of changing the direction of motion of the fluid filaments, which naturally tend to adopt a helicoidal form. Figures 1 and 2 are diagrammatic representations of counter-propellers mounted respectively, in front of and behind the main propeller.

They have been used, in accordance with the original designs of Dr. Rudolph Wagner and Professor Heustass, with 10 to 20% increase in efficiency on many ships. In 1927, designs for their use with aircraft propellers were made by A. Guglielmetti.*

*"Studio sull'applicazione delle contro-eliche agli aeri." Rendiconti tecnici della Direzione Generale del Genio Aeronautica, June, 1927, Vol. XV, No. 2.

They were never actually used, however.

We now propose to consider the real advantage of counter-propellers on aircraft and the best shape of the blades.

First of all, we wish to determine the possible energy absorption by the tangential increments. This process will be facilitated by the examination of the polygons of the relative velocities fore and aft of a generic section, of radius r, of one of the blades of a propeller (Fig. 3).

Let V represent the velocity of translation, $r\Omega$ the tangential velocity and W, equal to the sum of the two, the relative upstream velocity of the section itself, forming the angle α with the $r\Omega$. Moreover, let φ denote the angle of deflection of the current at that point, ψ the reduction coefficient of the velocity and W' the relative downstream velocity, so that $W' = \psi W$.

The energy absorbed by the blade element in the section under consideration, if the elementary mass of fluid actuated by the element itself per unit of time is denoted by dm, will be

$$dN = \frac{1}{2} (V'^2 - V^2) \, dm \tag{1}$$

while the energy absorbed by the tangential increments alone will be

$$dN'' = \frac{1}{2} (r\omega)^2 \, dm \tag{2}$$

Therefore

$$\mu = \frac{dN''}{d\ N} = \frac{(r\omega)^2}{V'^2 - V^2} \tag{3}$$

Since $\overline{AE} = \overline{AD} = V'$

$$V'^2 - V^2 = \overline{EB}^2$$

in which the fraction μ is equal to the square between the lengths of the segments $\overline{CD}$ and $\overline{EB}$.

Even without recourse to the analytical development, it is easy to see, by examining the figure, that the value of μ increases rapidly with increases in the ratio v/V and in the angle α.

If v were zero, $r\omega$ would depend only on ψ, but since the value of this coefficient generally differs only a few thousandths of unity, it follows that the tangential increment has very small values when the axial increment, and consequently the force of traction, is zero.

For normal values of the ratio v/V, the value of μ is also small when α is sufficiently small. Now α depends on the functioning ratio γ and on the ratio of the radii r/R. Hence the energy absorbed by the tangential increments can be large only for propellers on very swift aircraft or on aircraft equipped with engines using reduction gears. This may also be the case, if there is required of the propeller a relatively great force of traction on the portion of the blade near the hub, which is difficult in practice, however, due to constructional reasons.

The benefits obtained by propellers on boats, which have, in general and for various reasons, efficiencies much lower than aircraft propellers, are ascribable to the fact that, for the low translation speeds and the great forces of traction together with the necessarily small diameters, the values of the ratio v/V are large in the former. Moreover, for low rotational speeds, the values of γ obtained with the former are generally greater than those obtained with the latter. Lastly, for the relatively greater width of the blades of marine propellers, the profile drag also assumes important proportions.

Hence we believe that the advantage attainable with counter-propellers is much less for aircraft propellers than for boats. Nevertheless, their use may be desirable, if not in general, at least in cases where the ratio of the pitch to the diameter of the principal propeller exceeds the ordinary values.

Examination of Figure 3 shows that, if it is possible to deviate V' by the angle θ so as to make it coincide with the line of action of V, there will be an increase Δv in the axial increment and therefore a proportional increase in the force of traction. The deviation can be obtained by placing aft of the propeller fixed blades which will constitute the counter-propeller. We then have the arrangement shown in Figure 4, in which the same symbols are used as in Figure 3. On the other hand, with the counter-propeller placed in front of the main propeller, it will suffice to create an initial deviation equal and opposite to that generated by the propeller, as shown in Figure 5.

In both cases there will be developed on the fixed blades of the counter-propeller an aerodynamic reaction F_1 with a component T_1 in the direction of motion and a component C_1 at right angles to T_1. The component C_1 will always be such as to create a moment with respect to the axis of rotation of the propeller in the same direction as the propeller and will therefore reduce the reaction of the latter on the aircraft. This represents another appreciable advantage of the counter-propeller, which renders it particularly suitable for helicopters with a single lifting propeller.

On the other hand, the component T_1 will have the direction of the motion only when the counter-propeller is placed behind the main propeller, in which case it will increase the tractive force. On the contrary, if it is upstream, the fixed blades will exert a negative thrust. But, by reducing the pitch of the principal propeller (Fig. 5), it will be possible to obtain a thrust inclined more in the direction of the motion and consequently, for the same engine torque, a greater tractive force. In the final analysis, the total effect will perhaps be identical with the first case, to which particular reference is made in the following discussion.

The benefit obtainable with a counter-propeller obviously depends on the angle between F_1 and C_1. Assuming the complete elimination of the tangential increments, this angle will be equal, for every section of the fixed blades, to $\frac{1}{2}\theta$ (or to the angle of induction) diminished by ϵ = arc tan C_{rp}/C_p, due to the profile drag of the sections themselves (Fig. 4). In order to obtain a benefit, ϵ must therefore be less than $\frac{1}{2}\theta$. This necessitates a limitation in the diameter of the counter-propeller, because, as already stated, θ decreases rapidly as r increases. It is necessary, moreover, to adopt penetrating profiles and to arrange them suitably at appropriate angles of attack. Otherwise the use of the counter-propeller might easily prove disadvantageous.

It follows that the counter-propeller must be designed contemporaneously with the main propeller, since it is necessary to know exactly the value of the angle θ. This being known, the calculation is very simple and, after the number and aspect ratio of the blades has been fixed, it will only remain to determine the angle of setting of the various sections of the counter-propeller. It might be even better, after determining the best angle of setting for each section, to determine the best width for the blade. For the determination of these quantities, the reader is referred to another treatise of mine.*

*"Un nuovo metodo per il calcolo aerodynamico delle eliche," Aeronautica, March, 1930, pp. 175-180.

In order to determine the angle of deviation θ, we must have

$$\theta = \frac{4\ C_p}{\pi\ \lambda}$$

Hence the lift coefficient of the section must be

$$C_p = \frac{\pi}{4} \lambda\ \theta \qquad (4)$$

The aspect ratio of the given section will be furnished by the general formula

$$\lambda = \frac{dm}{\frac{\pi}{4} \frac{a}{g} W\ dS} \qquad (5)$$

in which the various symbols must be replaced by the values assumed by them in our case.

Let V_1 represent the component, in the direction of the translational motion, of the relative velocity V_e with which the section of the counter-propeller is struck by the fluid. In the downstream arrangement of the propeller (Figs. 3-4)

$$V_1 = V + v = V' \cos \theta \qquad (6)$$

If the counter-propeller is upstream, V_1 will coincide with V_c and with the velocity of translation (Fig. 5). Then let r_1 represent the radius of the given section of the counter-propeller, which may, with sufficient approximation, be considered equal to the radius of the corresponding section of the propeller blade. Let l_1 represent the corresponding width of the fixed blades and p_1 the number of such blades.

The elementary fluid mass affected per unit of time by the blade element in the vicinity of the given section will be

$$dm = \frac{1}{p_1} \frac{a}{g} 2 \pi V_1 r_1 dr \tag{7}$$

The area of the projection of the same element, on a plane tangent to the latter, will be

$$dS = l_1 dr \tag{8}$$

In place of W we must put the velocity at which the section is struck, which has already been designated by V_c. Substituting in equation (5), we have

$$\lambda = \frac{8}{p_1} \frac{r_1}{l_1} \frac{V_1}{V_c} \tag{9}$$

If the counter-propeller is mounted in front, V_1 coincides with V_c, as already stated. If, on the contrary, it is mounted behind, $V_c \equiv V'$ and hence, for equation (6), $V_1/V_c = \cos\theta$; but, since the angle θ cannot generally have very large values, it is possible even in this case, to have $V_1 = V_c$, and therefore

$$\lambda = \frac{8}{p_1} \frac{r_1}{l_1} \tag{10}$$

Substituting in equation (4)

$$C_p = \frac{\pi}{4} \frac{8}{p_1} \frac{r_1}{l_1} \theta = \frac{2 \pi r_1}{p_1 l_1} \theta \tag{11}$$

Equation (11) means that the lift coefficient to be assigned to the section having the radius r_1 for the purpose of recovering the tangential increments, must equal the ratio of the circumference having the radius r_1 to the total arc occupied by the blades multiplied by the angle of deviation θ.

The section must have the angle of incidence i_1 with respect to the relative velocity V_c. If i_o is the angle (generally negative) which the direction of zero lift makes with the tangent to the profile of the section and if $K = (d\ C_p/di)_\infty$ is the lift gradient of the same profile corresponding to the infinite aspect ratio (in general $K \cong 2.75$), we should have

$$i_1 = \frac{C_p}{K} + \frac{1}{2}\,\theta + i_o = \left(\frac{1}{K}\,\frac{2\,\pi\,r_1}{p_1\,l_1} + \frac{1}{2}\right)\theta + i_o \qquad (12)$$

This equation applies, on conserving the same value for K and expressing in degrees or in radians the values of the angles i_1, i_o and θ.

The angle of setting of the section, with respect to the direction of motion, will then coincide with i_1 in the case of the counter-propeller mounted upstream (Fig. 5), while it will equal i_1 diminished by the angle θ (Fig. 4) in the downstream arrangement.

We have already said that, in order to obtain the maximum advantage from the use of the counter-propeller, we must have the minimum value of the angle $\epsilon = \text{arc tan}\ C_{rp}/C_p$. In order to obtain the latter, it will be necessary to construct the polar of the profile corresponding to the infinite aspect ratio (Fig. 6), after deducting, from the polar plotted for any aspect ratio, the value of the induced drag

$$C_{ri} = \frac{2\ C_p^2}{\pi\ \lambda}$$

Thus the profile drag and the minimum value of λ for any point of the polar will be such that the junction of the latter with the origin of the axes will be tangent to the polar itself.

Knowing the best value of C_p, it is only necessary to substitute it in equation (11), to determine the width of the blade for stated values of p_1 and r_1. This will be

$$l_1 = \frac{2\ \pi\ r_1}{p_1} \frac{\theta}{C_p} \tag{13}$$

and the angle of coincidence will be

$$i_1 = \frac{C_p}{K} + \frac{1}{2}\ \theta + i_o \tag{14}$$

From equation (13) it follows that, with parity of profile, l_1 will decrease as r_1 increases, since θ decreases much more rapidly than r_1 increases. Therefore by adopting a single profile, or making all the sections similar, the blade will taper, thus assuming a form adapted to the effect of the temporary moments to which it is subjected. In this case the calculation is also facilitated, because in equation (13) it is necessary to vary only r_1 and θ, and in equation (14) only $\frac{1}{2}\ \theta$.

The diameter of the counter-propeller will be fixed, as already mentioned, by the condition $\epsilon \leq \frac{1}{2}\ \theta$. From this same inequality there follows the advantage evidently obtainable by the use of the counter-propeller.

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

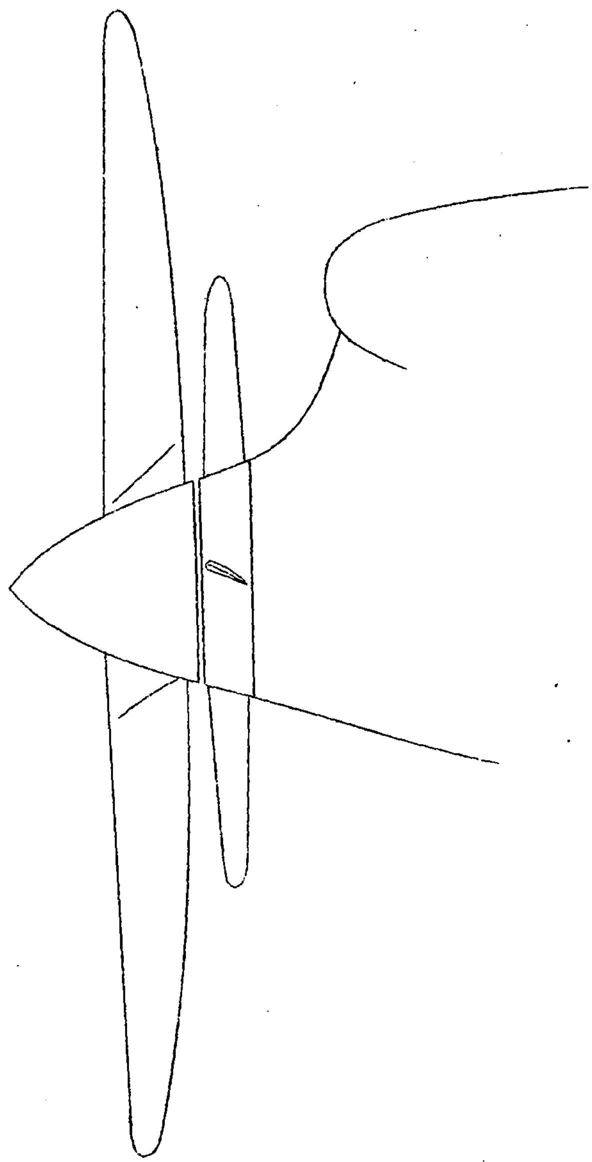


Fig.1

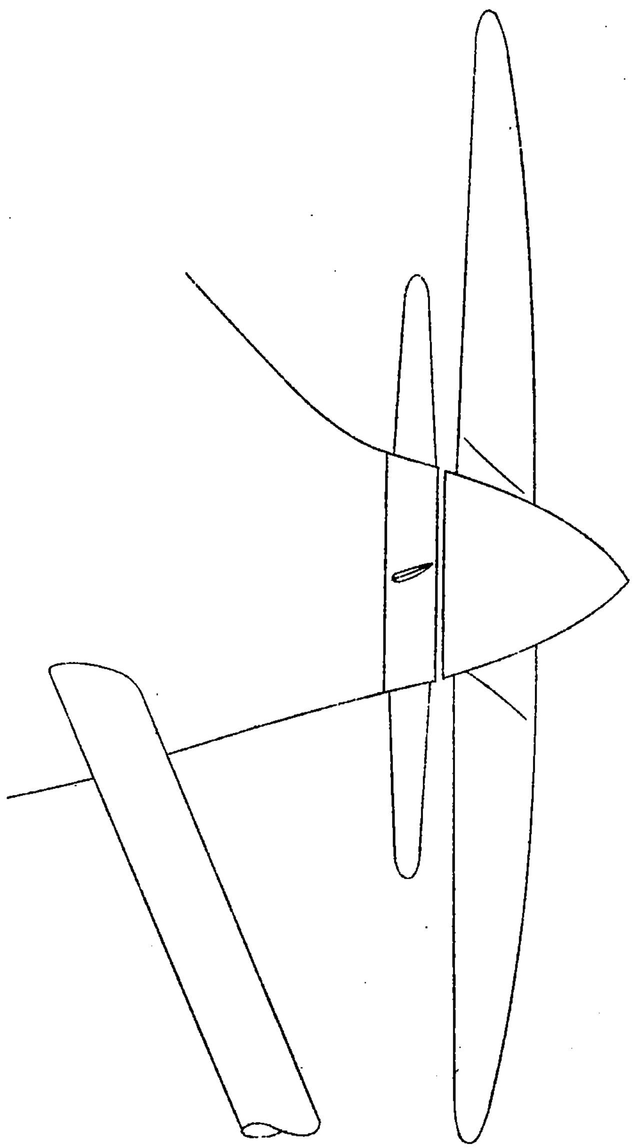


Fig.2

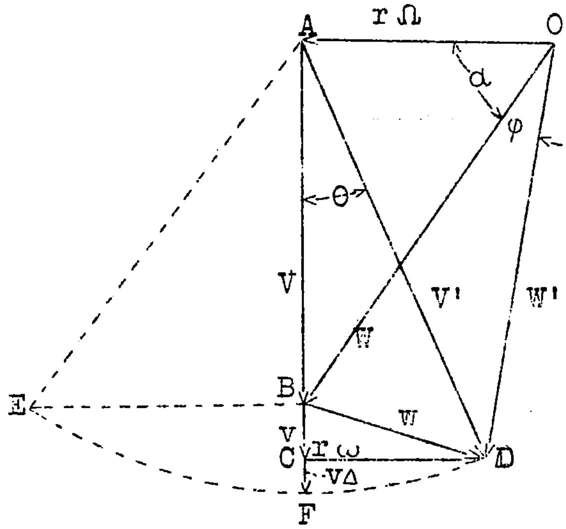

A
rΩ
O
α
φ
θ
V
V'
W'
W
B
E
w
v
rω
C
D
vΔ
F

Fig.3

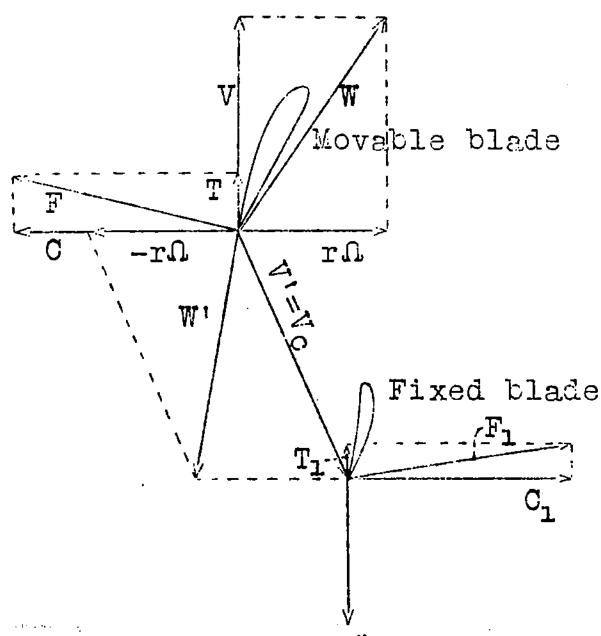
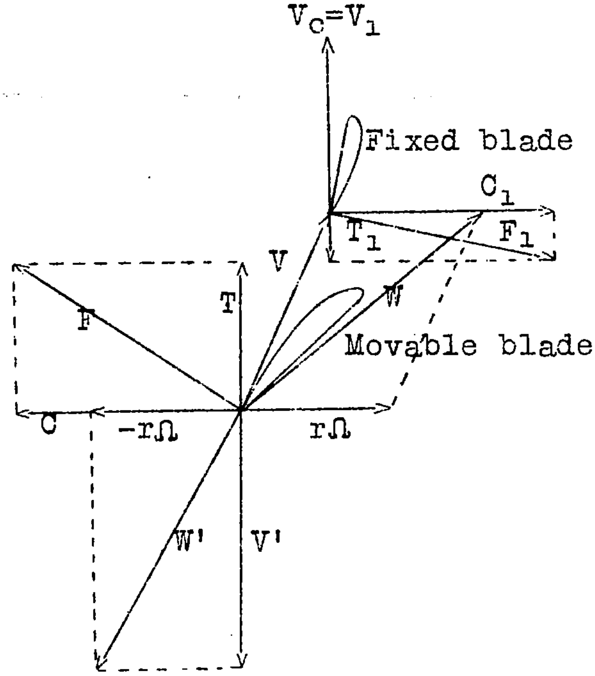
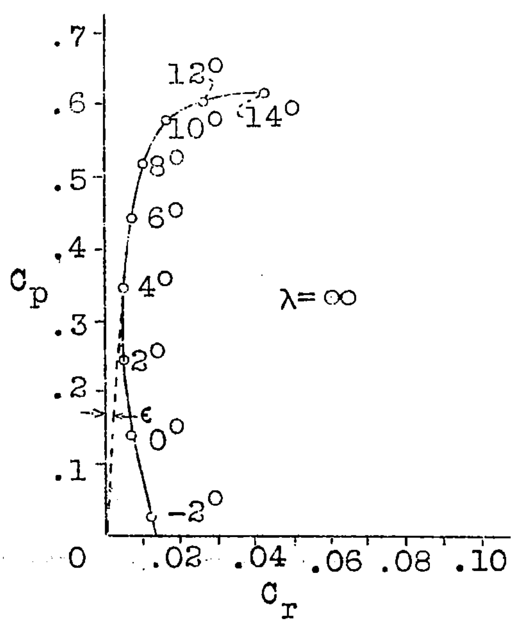

V
W
Movable blade
T
F
C
-rΩ
rΩ
W'
V'=Vc
Fixed blade
F1
T1
C1
V"

Fig.4



$V_c=V_1$
Fixed blade
C_1
T_1
F_1
V
T
F
W
Movable blade
C
$-r\Omega$
$r\Omega$
W'
V'

Fig.5



.7
.6
.5
.4
.3
.2
.1
0
C_p
12^o
14^o
10^o
8^o
6^o
4^o
2^o
0^o
-2^o
ϵ
$\lambda=\infty$
.02
.04
.06
.08
.10
C_r

Fig.6