TECHNICAL MEMORANDUMS

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No. 577

DETERMINATION OF THE BEST CROSS SECTION FOR A BOX BEAM SUBJECTED TO BENDING STRESSES

By A. Von Baranoff

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DETERMINATION OF THE BEST CROSS SECTION FOR A BOX BEAM SUBJECTED TO BENDING STRESSES.*

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The investigation of the strength characteristics of wood has entered a new stage, since the former haphazard manner of experimentation is being replaced by a more systematic and scientific method. What can be accomplished in this direction is shown by the reports of J. A. Newlin and G. W. Trayer, on the work done by them in the Forest Products Laboratory of the Department of Agriculture in 1923, for publication by the National Advisory Committee for Aeronautics.** The writers proceed from an obvious although much simplified conception of the structure of wood and arrive experimentally at an accurate determination of the still unknown functions of their preliminary hypothesis (Ansatz). This method is less recommended by the partially arbitrary assumptions than by the agreement of its results with experience. I have been requested by P. Brenner to analyze the results contained in the above-mentioned reports and to work out a

*"Die Ermittlung des günstigsten Querschnitts eines auf Biegung beanspruchten Kastenholms." From the 1927 Yearbook of the Deutsche Versuchsanstalt für Luftfahrt, pp. 83-85. This report was also published in Zeitschrift für Flugtechnik und Motorluftschiffahrt, February, 1927, pp. 81-83.

**J. A. Newlin and G. W. Trayer, "The Influence of the Form of a Wooden Beam on Its Stiffness and Strength," N.A.C.A. Technical Reports Nos. 180, 181 and 188 (Parts I, II, and III).

method of representation suitable for the use of designers.

Notation

a) Strength coefficients

K_z	kg/cm^2	tensile strength.
K_D	"	compressive strength
K_B	"	bending strength.
K_{B_0}	"	bending strength of a solid rectangular section.
F	-	form factor.

b) Mechanical quantities

M_b	cm kg	bending moment at failure.
W_D	cm^3	moment of resistance on compression face.
J	cm^4	inertia moment of symmetrical section
θ		ratio of resistance moments of unsymmetrical and symmetrical sections.

c) Beam dimensions

H	cm	total depth.
B	"	total width.
h	"	depth of flange of symmetrical beam section.
b	"	width of web.
h + x	cm	depth of compression flange of unsymmetrical beam section.
x	cm	layer transferred from tension flange to compression flange.

y cm distance from extreme compression fiber.

y_o " distance of neutral axis from extreme compression fiber.

n depth ratio of the two flanges.

σ function of supporting effect.

$\eta = \frac{h}{H}$

$\xi = \frac{x}{H}$

$\lambda = \frac{y}{H}$

$\lambda_o = \frac{y_o}{H}$

Explanation of Form Factor

The bending strength K_B of wood is no simple characteristic of the material, but depends largely on the form of the section. A box beam or I beam has a smaller bending strength than a solid beam of the same outside dimensions. By using a factor which is always less than unity, we can therefore write

$$K_B = F\ K_{B_o},$$

F being the form factor of the cross section.* It depends both on the dimensions of the section and on the strength coefficients.

*Our definition of the form factor differs somewhat from that of Newlin and Trayer. Note especially the formal explanation in the next paragraph.

Physical Significance of the Form Factor

The credit of first calling attention to the physical significance of the form factor F belongs to Newlin and Trayer.* They proceed from a simplified conception of wood structure by regarding it as a bundle of wood fibers. When the bundle is subjected to pressure parallel to the direction of the fibers, the result obviously depends on the strength of the individual fiber. The latter tends to buckle, however, so that the compressive strength of the wood is less than its tensile strength. The process may be conceived as follows. The individual fibers are variously stressed according to their location, so that the less stressed ones (nearer the neutral fiber) have a supporting effect on the more stressed ones (farther from the neutral fiber). The Euler buckling load does not therefore cause the buckling of the outermost fibers, which buckle only at a greater load, though not so great as the tensile strength. This explains why the bending strength is greater than the compression strength, while the form of the section determines the supporting effect and hence the bending strength. It is naturally very difficult to take the step from this obvious illustration to the exact mathematical expression of the supporting effect. Help is obtained by the introduction of new assumptions. The effect of an inner layer of fibers on the outermost layer on the compression face depends on two circumstances. The more the inner layer is stressed, the less its supporting effect will be. On the

*"Form Factors of Beams Subjected to Transverse Loading Only," N.A.C.A. Technical Report No. 181, Part II (1924).

other hand, the shorter the distance between the two layers, the greater will be the supporting effect. The magnitude of the stress depends on the distance from the compression face, so that the supporting effect between the inner and outer layers may be regarded as a function of this distance. We designate this function by $\sigma(\lambda)$ and assume that

$$\int_0^1 \sigma(\lambda)\, d\lambda = 1$$

for a solid rectangular cross section and that therefore the total supporting effect of a solid cross section is unity. On the contrary, the total supporting effect for a box section is

$$\frac{B - 2b}{B}\int_0^{\eta+\xi} \sigma(\lambda)\, d\lambda + \frac{2b}{B}\int_0^1 \sigma(\lambda)\, d\lambda.$$

Assuming the second integral to be unity, we may express the form factor as follows

$$F = c_1 + c_2 \left[\frac{B - 2b}{B}\int_0^{\eta+\xi} \sigma(\lambda)\, d\lambda + \frac{2b}{B}\right]$$

The two constants serve to satisfy two limiting conditions. For a solid section $(\eta + \xi = 1)$ it will be

$$F = 1 = c_1 + c_2.$$

In the absence of the supporting effect, due to disappearing depth of flange, it will be

$$F = \frac{K_D}{K_{B_0}} = c_1 .$$

From these conditions we obtain

$$F = \frac{K_D}{K_{B_o}} + \left(1 - \frac{K_D}{K_{B_o}}\right)\left(\frac{B - 2b}{B}\int_o^{\eta+\xi} \sigma(\lambda)\ d\lambda + \frac{2b}{B}\right).$$

If $2b$, in comparison with B, is disregarded,

$$F = \frac{K_D}{K_{B_o}} + \left(1 - \frac{K_D}{K_{B_o}}\right)\int_o^{\eta+\xi} \sigma(\lambda)\ d\lambda .$$

The integral

$$\int_o^{\eta+\xi} \sigma(\lambda)\ d\lambda = \frac{K_B - K_D}{K_{B_o} - K_D}$$

can be obtained as a function of $\eta + \xi$ by bending tests with beams with flanges of different depths. Figure 2 shows the results of these experiments which were made with Sitka spruce (See N.A.C.A. Technical Report No. 181). Similar tests with pine wood verified these results surprisingly well within a restricted range of flange depths (up to 20% of the depth of the beam).* The differentiation of this curve yields the function $\sigma(\lambda)$. As may be surmised from the general considerations, it has a maximum of about $\lambda \cong 0.35$.

*Made by the Deutsche Versuchsanstalt für Luftfahrt, with beams furnished by the Albatros Airplane Works of Berlin-Johannisthal.

Effect of the Depth of the Compression Flange on the Moment of Resistance

It is assumed that the bending moment at failure M_b is determined by the strength of the compression flange. Then

$$M_b = W_D\ F\ K_{B_0}.$$

We will next consider a symmetrical box section. The thickness of the web will be disregarded in the following discussion. A layer ξ will now be transferred from the tension flange to the compression flange. Both W_D and F change with ξ. W_D has a maximum, while F continues to increase. Their product W_D F likewise has a maximum. For this purpose θ and F θ are plotted against ξ in Figure 3 with η as parameter, θ being the ratio of the two resistance moments, namely,

$$\theta = \frac{W_D\ H}{J\ 2} = \frac{1 - \frac{H^4}{J}\left(\frac{1 - 2\,\eta}{2\,\eta}\right)\xi^2}{1 - \left(\frac{1 - 2\eta}{2}\right)\xi}.$$

For every value of η the function Fθ has a maximum, which in turn has the smallest value for a certain value of η. The lowest maximum lies at about $\eta = 0.1$. This is chiefly affected by the small bending strength of the wood. For small values of η, the small strength is offset by the favorable ratio of the resistance moments. For large values of η, on the contrary, the value of F increases considerably.

The Best Cross Section

The form factor enables us to find, for any given kind of wood, the box section which causes both flanges to be equally stressed and which has such an asymmetry that the product of its resistance moment times its form factor (θF) is a maximum.

We have just discussed the latter condition, finding a relation between η and ξ for which the maximum condition is fulfilled. The first condition (equal stressing of both flanges) requires the ratio of the resistance moments for the compression and tension faces to equal the inverse ratio of the strengths for the compression and tension faces. The inertia moment being eliminated, we obtain

$$F \frac{H - y_o}{y_o} = \frac{K_z}{K_{B_o}} ,$$

or, since

$$\frac{y_o}{H} = \lambda_o,$$

the equation

$$\lambda_o = \frac{F}{\frac{K_z}{K_{B_o}} + F}$$

Introducing the value of λ_o

$$\lambda_o = 0.5 - \frac{1 - 2\eta}{2\eta} \xi$$

which follows from the equation

$$\int (y - y_o) \, d\, y = 0$$

for the neutral axis, we then have, with the graphic maximum condition, three equations for the three unknown quantities λ_o, ξ and η, Such is the case for $K_D/K_{B_o} = 0.5$ in Figure 4. The intersection point of the maximum curve with the curves λ_o of the uniformly stressed flanges yields the solution of our problem. For given strength values we obtain the unknowns η and ξ and can now determine the cross section in an indeterminate scale. In Figure 5 the ratio of the flange depths

$$n = \frac{\eta + \xi}{\eta - \xi}$$

as also the sum of both flange depths 2η, is plotted against K_z/K_{B_o}, with K_D/K_{B_o} as parameter.

The absolute dimensions for the cross section can be determined from the condition of the moment to be absorbed. There are still two variables, namely, the height H, and the width B. Hence a new condition can be introduced at this point, namely, for the given moment the cross-sectional area must be a minimum. This problem has been discussed by R. Sonntag ("Wirtschaftlichste ⊏ - und I-förmige Holmquerschnitte," Zeitschrift für Flugtechnik und Motorluftschiffahrt, May 15, 1922, pp. 126-127).

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

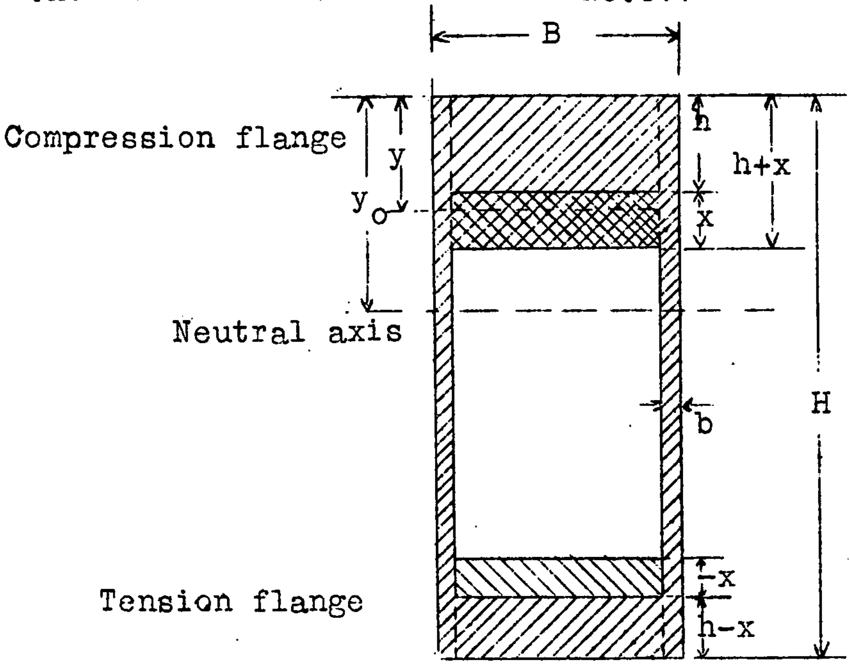


Fig.1 Cross section of box beam.

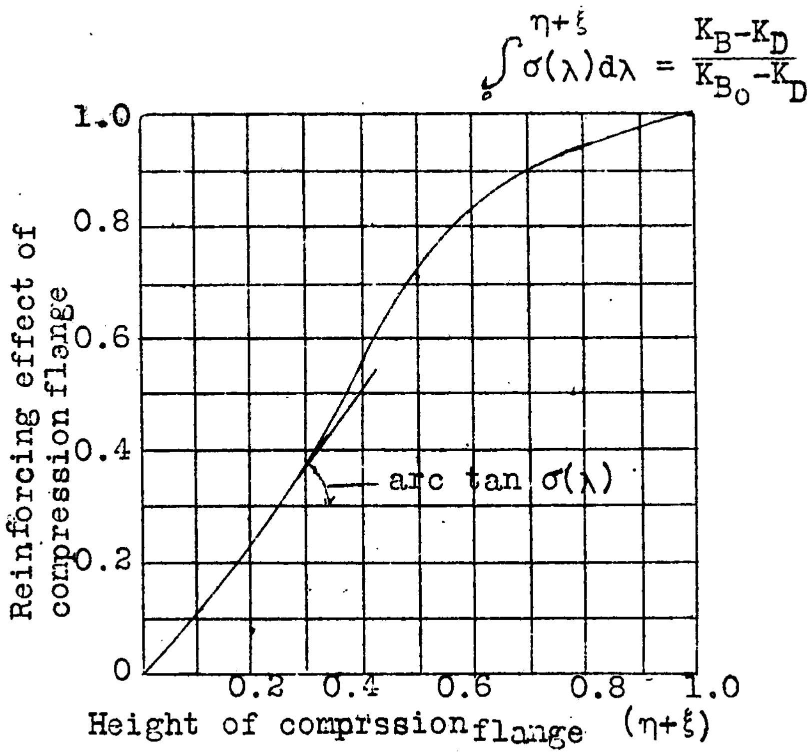
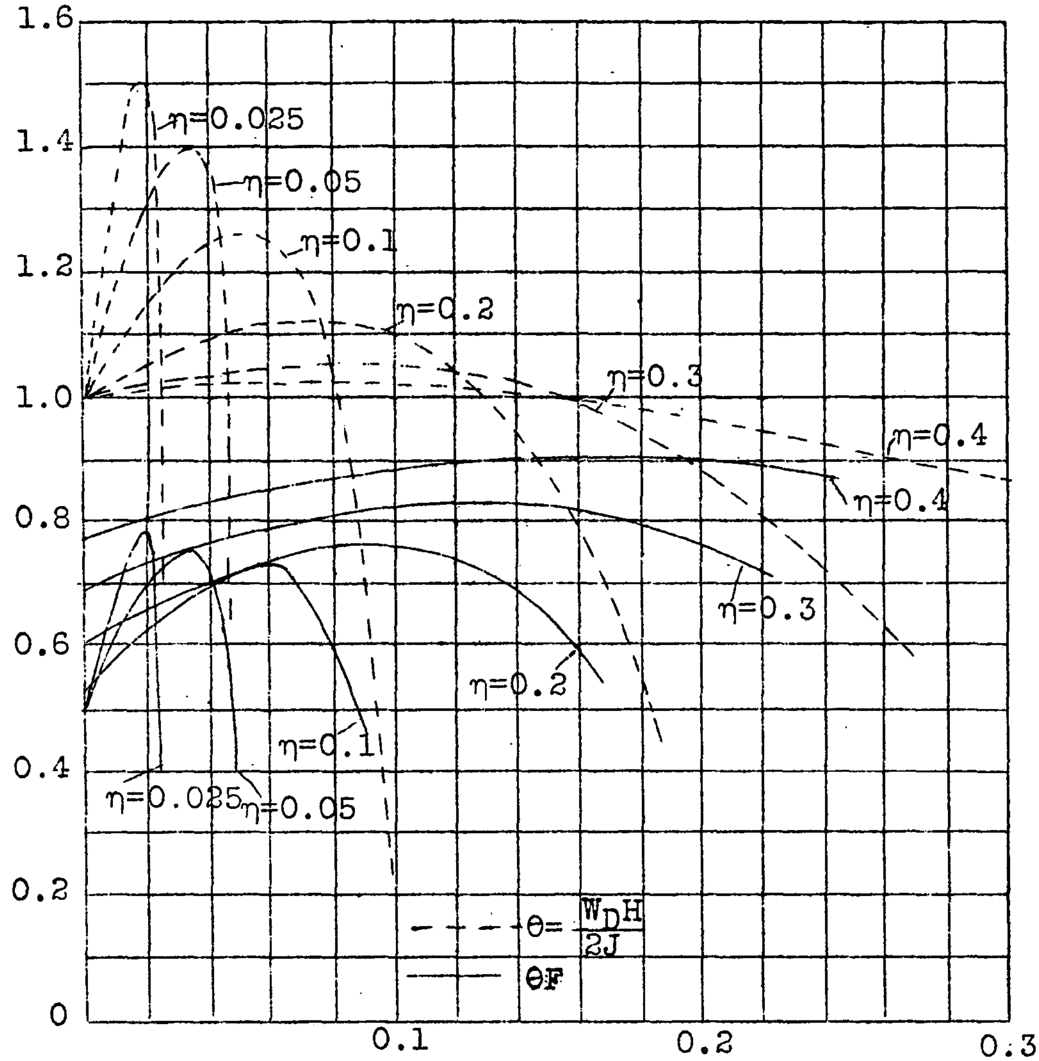
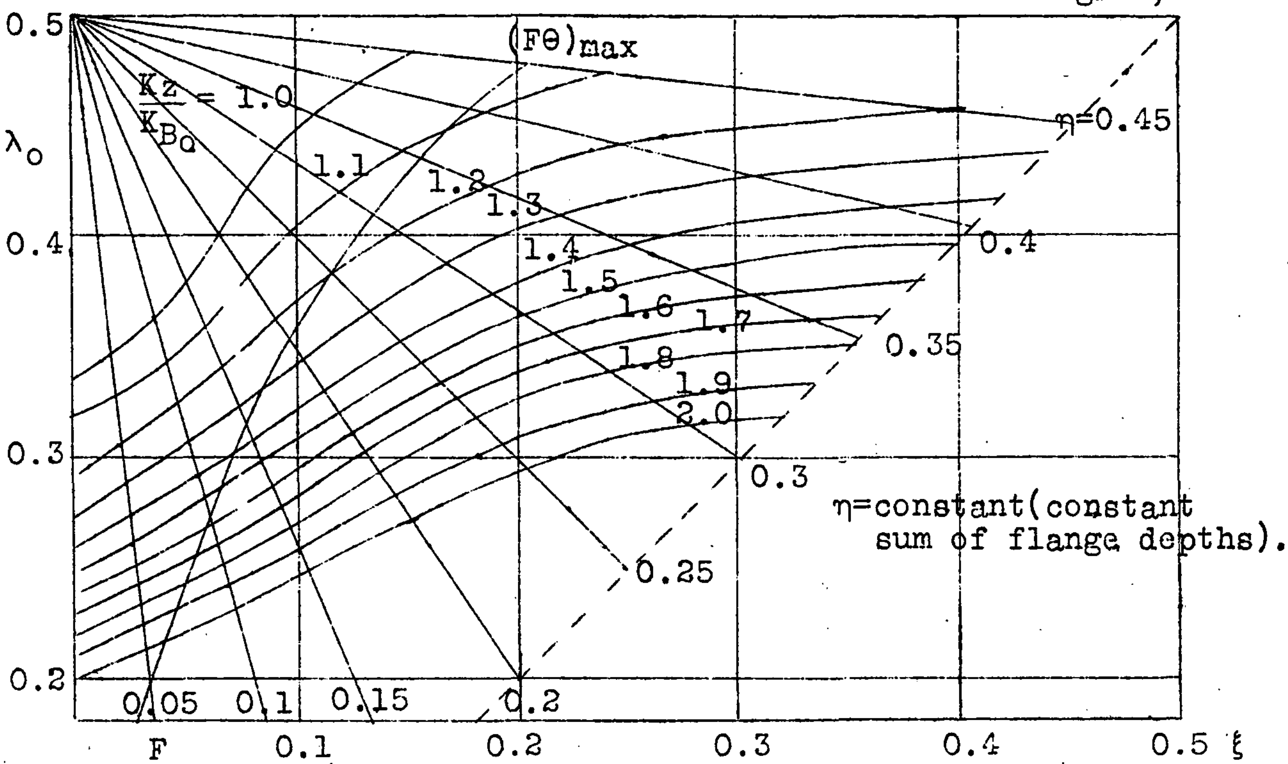


Fig.2 Bending strength plotted against depth of compression flange (according to experiments of Newlin and Trayer).



Layer transferred from tension flange to compression flange.

Fig.3 Resistance moment and product of form factor times resistance moment plotted against layer transferred from tension flange to compression flange.



$\lambda_o = \frac{F}{K_z/K_{B_o}+F}$ (simultaneous failure of tension and compression flanges) and the curves $(F\Theta)_{max}$.

Fig.4 For determination of best cross section from intersection point of the group of curves.

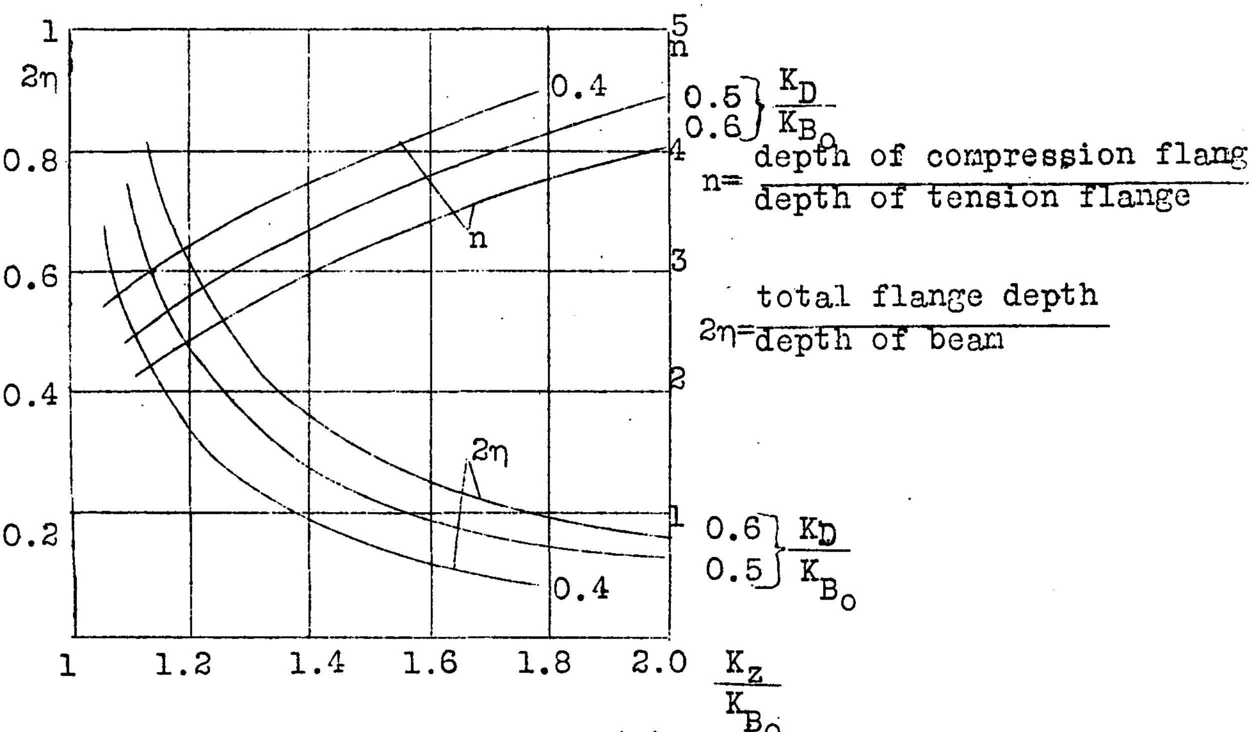


Fig.5 Ratio of flange depths (n) and sum of flange depths (2η) of best cross section plotted against the strength coefficients.