STUDY OF EINSTEIN-PODOLSKY-ROSEN STATE FOR SPACE-TIME VARIABLES IN A TWO PHOTON INTERFERENCE EXPERIMENT

Y. H. Shih, A. V. Sergienko and M. H. Rubin
Department of Physics, University of Maryland
Baltimore County, Baltimore, MD 21228

Abstract

A pair of correlated photons generated from parametric down conversion was sent to two independent Michelson interferometers. Second order interference were studied by means of a coincidence measurement between the outputs of two interferometers. The reported experiment and analysis studied this second order interference phenomena from the point of view of Einstein-Podolsky-Rosen paradox. The experiment was done in two steps. The first step of the experiment used 50 psec and 3 nsec coincidence time window simultaneously. The 50 psec window was able to distinguish a 1.5 cm optical path difference in the interferometers. The interference visibility was measured to be 38% and 21% for 50 psec time window and 22% and 7% for 3 nsec time window, when the optical path difference of the interferometers were 2 cm and 4 cm, respectively. By comparing the visibilities between these two windows, the experiment showed the non-classical effect which resulted from an E.P.R. state. The second step of the experiment used a 20 psec coincidence time window, which was able to distinguish a 6 mm optical path difference in the interferometers. The interference visibilities were measured to be 59% for an optical path difference of 7 mm. This is the first observation of visibility greater than 50% for a two interferometer E.P.R. experiment which demonstrates nonclassical correlation of space-time variables.

1 Introduction

Two photon interferometry has drawn a great deal of attention recently because it provides a tool to study the foundation of quantum mechanics and the fundamental properties of the electromagnetic field. A two photon interference experiment using two independent interferometers was proposed by J. D. Franson[1] which constituted a new type of E.P.R. experiment for space-time variables. Since then several experiments have reported the second order (second order in intensity, fourth order in field) interference effect.[2]-[5] These experiments have shown visibility less than 50% when the optical path difference of the interferometers are greater than the coherence length of the optical beam. The reason that the visibilities are less than 50% is due to the use of large coincidence time windows in these experiments. It has been pointed out that classical models predict a maximum of 50% visibility for these experiments.[2][3][6] Quantum theory predicts visibility greater than 50% for certain entangled states we called E.P.R. state. To make the
type of argument presented by E.P.R.[7] this state must be produced. For this experiment a short coincidence time window is needed to prepare an E.P.R. state.

Recently, a large set of measurements for a two photon interference experiment have been carried out in our laboratory. In this experiment parametric down conversion is used to produce the correlated two photons. The intensity of the down converted radiation used for the experiment is sufficiently low so that a two photon state is produced such that each beam contain at most one photon. Each photon is passed through an independent Michelson interferometer and is then detected by a coincidence counter. If the interferometers are set so that the optical path differences are longer than the coherence length of the fields, there is no first order interference (first order in intensity, second order in field). However, there is second order interference if the optical paths of the two interferometers are approximately equal. The interference arises from the frequency and wave number correlation in a given pair generated by the phase matching conditions, \( \omega_1 + \omega_2 = \omega_p \) and \( k_1 + k_2 = k_p \), where \( \omega_p \) and \( k_p \) are the pump frequency and wave number. The second order interference is measured by studying the visibility of the interference fringes that are generated by varying the optical path difference of the interferometers. The visibility of the interference can be estimated by classical and quantum models. The classical model never predicts visibility greater than 50%. However, for idealized condition, the quantum model predicts a 100% visibility when the coincidence time window is shorter than the optical path difference. In this case, the registration time of one photon traversing the long path and the other following the short path of the interferometers is outside the coincidence window and will not be registered by the coincidence counter.

As shall be explained below, the use of a short coincidence time window is equivalent to preparing a type of entangled state discussed in the original E.P.R. paper.[7]

We report in this paper an experiment which for the first time shows second order interference visibility greater than 50% for two independent interferometers. We also show in detail how the E.P.R. state is generated for the coincidence counting experiment.

## 2 E.P.R. Paradox and E.P.R. State

The E.P.R. paradox was based on the argument that non-commuting observables can have simultaneous reality.[7] E.P.R. first gave their criterion: if, without in any way disturbing the system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to this physical quantity. The gedanken experiment discussed by Einstein, Podolsky and Rosen was modified by Bohm in 1951.[8] In Bohm's version a singlet state \( | \psi \rangle \) of two spin \( \frac{1}{2} \) particles is produced by some source,

\[
| \psi \rangle = \frac{1}{\sqrt{2}} \left( | \hat{n}_1^+ \rangle \otimes | \hat{n}_2^- \rangle - | \hat{n}_1^- \rangle \otimes | \hat{n}_2^+ \rangle \right)
\]

where \( | \hat{n}_j^\pm \rangle \) quantum mechanically describe a state in which particle \( j \) has spin "up" or "down" along the direction \( \hat{n} \). For this state, if the spin of particle 1 is measured along the \( x \)-axis, particle 2 will be found to have its spin oppositely aligned along the \( x \)-axis with unit probability. Thus, the \( x \)-component of the spin of particle 2 can be measured without in any way disturbing it and so is an element of reality according to the E.P.R. criterion. It is similarly found that the other components of the spin of particle 2 can be determined as elements of physical reality and must exist without considering which component is being measured. Of course, this point of view is
different from that of quantum mechanics. Philosophical arguments aside, the predictability of the spin of particle 2 with 100% certainty after measuring the spin of particle 1 is a mathematical consequence of quantum theory applied to state of the form (1). States of the type (1) are a particular type of entangled state,[9][10] which will be called E.P.R. state. It is the E.P.R. state which leading to the nonclassical interference behavior of the two particle system. It is the E.P.R. state has no classical analog.

The existence of polarization E.P.R. states have been experimentally demonstrated.[11]-[14] The new type of E.P.R. experiment considers the measurement of position and time correlation in contrast to the historical measurement of polarization correlation. The key element is to seek an E.P.R. state for space and time variables. This is closer to the original E.P.R. gedanken experiment for the determination of position and momentum of a photon. In this case, see FIG. 1, the two-photon E.P.R. state sought is of the form,

\[ \Psi_{\text{EPR}} = \Psi(L_1, L_2) + \Psi(S_1, S_2) \]  

(2)

where the first amplitude corresponds to the photons both passing along the longer arms of the interferometers and the second amplitude corresponds to them both following the shorter arms. It is clear that this is an E.P.R. state of the type defined above, if photon 1 is determined in the long (short) arm, then, photon 2 follows the long (short) path. The photon path is then an element of physical reality according to the E.P.R. criterion. In practice state (2) is produced by parametric down conversion. If we assume perfect phase matching, then because \( k_1 + k_2 = \text{constant} \), a momentum measurement of one photon determines the momentum of the other. So the momentum of the photon is also an element of physical reality. If this state does exist, in idealized conditions, its signature is an interference visibility of 100% when the optical path difference of the two independent interferometers are equal.

However, the output of the interferometers is not state (2), but rather the state

\[ \Psi = \Psi(L_1, L_2) + \Psi(S_1, S_2) + \Psi(L_1, S_2) + \Psi(S_1, L_2) \]  

(3)

which differs because of the presence of the last two terms, which corresponding to one photon passing the long arm and another passing the shorter arm of the interferometers. State (3) can not give any determination of the paths of the photon. It gives a maximum of 50% visibility, which can not be distinguished from a classical model. However, it will be seen in the next section, that according to quantum mechanics, the last two terms of (3) can be suppressed by using a coincidence time window which is shorter than the optical path difference of the interferometers.

3 Theoretical Discussion

Our version of the new type of E.P.R. experiment is illustrated in FIG. 1. The photon pair generated from parametric down conversion is sent through two independent Michelson interferometers I and II. The optical path differences \( \Delta L_1 = L_1 - S_1 \) and \( \Delta L_2 = L_2 - S_2 \) can be arranged to be shorter or longer than the coherence length of each beam of the down conversion field. The coincidence measurement is between the two output of the interferometers.

The two photon state of the parametric down conversion can be considered as,

\[ \Psi = \int dk_1 \int dk_2 \delta(k_1 + k_2 - k_p) A(k_1) | k_1 \rangle \otimes | k_2 \rangle \]  

(4)
FIG. 1: Schematic diagram of the experiment
where \( k_1 \) is the signal, \( k_2 \) is the idler and \( k_p \) is the pump wave number, the \( \delta \) function comes from the perfect phase matching condition of the parametric down conversion, \( A(k) \) is the wave packet distribution function and its width determines the coherence length of the wave packet. After leaving the interferometers, the wave function becomes,

\[
\Psi = \frac{1}{2} \int dk_1 \int dk_2 \delta(k_1 + k_2 - k_p) \cdot A(k_1)
\]

\[
\cdot \left[ |k_1L \rangle |k_2L \rangle + |k_1S \rangle |k_2S \rangle + |k_1L \rangle |k_2S \rangle + |k_1S \rangle |k_2L \rangle \right]
\]

where \( |k_i \rangle = |k_i \rangle e^{i\varphi(M_i)} \), \( \varphi \) is the phase shift caused by passage of the wave through the system. The four terms of state (5) corresponding to the photons which have followed the long-long, short-short, long-short and short-long paths of the interferometers. State (5) is not an E.P.R. state, the coincidence rate can be estimated as, \( R_c = R_{\infty} |\Psi|^2 \)

\[
R_c = R_{\infty} \int dk_1 F(k_1) \cdot \{1 + \cos k_1 \Delta L_1 + \cos(k_p - k_1)\Delta L_2
\]

\[
+ \frac{1}{2} \cos[k_1(\Delta L_1 - \Delta L_2) - k_p \Delta L_2] + \frac{1}{2} \cos[k_1(\Delta L_1 - \Delta L_2) + k_p \Delta L_2]\}
\]

where \( |A(k_1)|^2 \equiv F(k_1) \). Function \( F(k_1) \) will generally have about the same width as \( |A(k)|^2 \). If \( \Delta L_1 \) and \( \Delta L_2 \) are greater than the first order coherence length of the wave packets, the second, third, and fourth terms in (6) will vanish. The last term contains \( \cos[k_1(\Delta L_1 - \Delta L_2) + k_p \Delta L_2] \); consequently, so long as \( |\Delta L_1 - \Delta L_2| < \) is less than the first order coherence length of the wave packet, this term gives rise to the interference fringes. If \( |\Delta L_1 - \Delta L_2| << \) coherence length (equal optical path difference) then the visibility of these fringes attain their maximum value of 50%.

A similar result can be obtained from a classical model.[6][15] In the classical analog to the above experiment the electric field leaving the interferometer \( j \) will be

\[
E_i = \frac{1}{\sqrt{2}} \int dk_1 A(k_1) e^{i(k_1 r - \omega t_1)} \cdot (e^{i\varphi(L_i)} + e^{i\varphi(S_i)})
\]

where we neglect the polarization vector. The intensity is given by

\[
I_i = \frac{1}{2} \int dk_1 |A_i(k_1)|^2 \cdot (1 + \cos \delta_i)
\]

where \( \delta_i = k_i \Delta L_i = \varphi(L_i) - \varphi(S_i) \). The modulation as a function of the optical path difference \( \Delta L_i \) is determined by the width of the function \( |A_i(k_1)|^2 \) and gives the first order interference coherence length of the field.

Now suppose the second order interference is measured, the coincidence counting rate \( R_c \propto < I_1 I_2 > \), where the bracket denotes an ensemble average,

\[
< I_1 I_2 > = \int dk_1 \int dk_2 < |A_1(k_1)|^2 |A_2(k_2)|^2
\]

\[
\cdot \cos^2(\frac{\hbar \Delta L_1}{2}) \cos^2(\frac{\hbar \Delta L_2}{2})
\]

In order to model parametric down conversion it is necessary to account for the correlation in the two beams that is imposed by the phase matching condition. To do this assume perfect phase matching and take

\[
< |A_1(k_1)|^2 |A_2(k_2)|^2 > = \delta(k_1 + k_2 - k_p) \cdot G(k_1)
\]
so that
\[ R_c = R_{c0} \int dk_1 G(k_1) \left\{ 1 + \cos k_1 \Delta L_1 + \cos (k_p - k_1) \Delta L_2 \right\} \]
\[ + \frac{1}{2} \cos [k_1 (\Delta L_1 + \Delta L_2) - k_p \Delta L_2] + \frac{1}{2} \cos [k_1 (\Delta L_1 - \Delta L_2) + k_p \Delta L_2] \] 
(10)

It is the same as (6) which we have derived from the state (5).

It is not surprising that a classical model gives the same answer as that of quantum mechanics, because the above calculations have dealt with the wave nature of radiation for both the quantum and the classical models. However, if one can take advantage of the particle nature of the photon, the quantum prediction will be different. This idea has been demonstrated in the early polarization E.P.R. experiment using a coincidence measurement to produce an E.P.R. state.[13] For the two photon interference experiment a coincidence measurement is not enough to suppress the last two terms of (5) unless the coincidence time window is shorter than the optical path difference of the interferometers. Then the registration time difference in which the photons follow the long-short and short-long paths are outside the time window, i.e., the last two terms of (5) will not be registered by the coincidence counter.[16] This "cut off" effect will result in an E.P.R. state, which has no classical analog,

\[ \Psi = \frac{1}{4} \int dk_1 \int dk_2 \delta(k_1 + k_2 - k_p) A(k_1) \cdot \left( | k_{1L} \rangle | k_{2S} \rangle + | k_{1S} \rangle | k_{2L} \rangle \right) \] 
(11)

E.P.R. state (11) can provide 100% interference visibility,

\[ R_c = R_{c0} \int dk_1 F(k_1) \cdot \left\{ 1 + \cos [k_1 (\Delta L_1 - \Delta L_2) + k_p \Delta L_2] \right\} \] 
(12)

To realize 100% visibility, besides equal optical path difference in the interferometers, a pump field with zero band width is required along with perfect phase matching for the parametric down conversion. One can easily arrange a narrow enough spectral band width of the pump field by means of a single mode laser as was done in this experiment, but, in principle, it is impossible to achieve perfect phase matching. When the finite size of the crystal and the finite interaction time of the down conversion is taken into account, the \( \delta \) functions of \( (k_1 + k_2 - k_p) \) and \( (\omega_1 + \omega_2 - \omega_p) \) are replaced by functions with non-zero widths giving \( k_1 + k_2 = k_p \pm \Delta k \) and \( \omega_1 + \omega_2 = \omega_p \pm \Delta \omega \).[17] In this case (12) becomes,

\[ R_c = R_{c0} \int dk_1 F(k_1) \cdot \left\{ 1 + \cos [k_1 (\Delta L_1 - \Delta L_2) + k_p \Delta L_2 \pm \Delta k \Delta L_2] \right\} \] 
(13)

The uncertainty \( \Delta k \) will reduce the interference visibility.

A detailed and careful study of the influence of the coincidence time window and the non-perfect phase matching can be found in reference (6). For a quasi monochromatic wave model, which is reasonable for parametric down conversion, the general solution of \( R_c \) may be written as

\[ R_c = R_{c0} \left\{ f_0 + f_1 \cos [k_1 (\Delta L_1 - \Delta L_2) + k_p \Delta L_2] \right\} \] 
(14)

where we assume that the optical path difference is much longer then the coherence length of the down conversion beams and ignore the trivial terms. The \( f \)'s depend on the detail of the experiment, in particular the coincidence time window and the uncertainty \( \Delta k \). For a large coincidence window, \( f_1/f_0 \) attains a maximum value of 50%. When the time window becomes shorter and shorter especially shorter than the optical path difference of the interferometers, \( f_1/f_0 \) reaches 100% for zero \( \Delta k \).
4 Experiment

The experimental arrangement is shown in FIG.1. A 351.1 nm single mode CW Argon laser beam was used to pump a 50 mm long potassium dihydrogen phosphate (KDP) nonlinear crystal for optical parametric down conversion. The coherence length of the 351.1 nm pump beam was measured to be longer than 5 meters. The KDP crystal was cut at TYPE I phase matching angle for generation of $\omega_1$ and $\omega_2$ photons. Both degenerate and nondegenerate (in frequency) photon pairs have been used in the experiment. In the degenerate case, $\lambda_1 = \lambda_2 = 702.2$ nm. The emission angles were about 2° relative to the pump. In the nondegenerate case, 632.8 nm and 788.7 nm signal and idler pair were generated. The signal and idler photons were emitted at angles 1.8° and 2.3° relative to the pump beam, respectively. The signal and idler photons then were selected by pinholes and sent to two independent Michelson interferometers I and II. The interferometers are 5 m apart in order to have space-like separated detections. Two Geiger mode avalanche photodiodes D1 and D2 with 1 nm spectral filters (centered at 702.2 nm for degenerate case and 788.7 nm and 632.8 nm for nondegenerate case, respectively) were used for monitoring the first order and the second order interferences by means of direct counting and coincidence counting. The coincident circuit provides 20 psec, 50 psec and 3 nsec time window. $N_1$, $N_2$, $N_s$ which correspond to the number of counts from detector 1, detector 2 and from the coincidence time window were recorded simultaneously. The above measurements have taken advantage of the state-of-the-art millimeter lunar laser ranging high resolution timing diagnostic technique, which has been developed at the University of Maryland.

The optical path difference $\Delta L_1 = L_1 - L_2$ and $\Delta L_2 = L_2 - S_2$ of the two independent Michelson interferometers I and II can be changed by step motors continually from white light condition to about 7.2 mm which is longer than both the coherence length of the down converted fields and the 20 psec time window. It is also possible to move one of the mirrors discontinuously to a maximum $\Delta L = 12$ cm.

The experiment was performed in two steps. First, we used a 50 psec and a 3 nsec time window simultaneously for the coincidence measurement. By comparing the interference visibilities for $\Delta L > 1.5$ cm between the 50 psec and 3 nsec coincidence window, we expect to see the "cut off" effect. 702.2 nm, photon pairs were used for the first step measurement.

1: $\Delta L_1 < \text{coherence length}$

We have measured the first order and the second order interference visibilities when both $\Delta L_1$ and $\Delta L_2$ were shorter than the coherence length of the field. We have also measured the first and second order interference visibilities when the optical path difference of one interferometer was shorter than the coherence length and that of the other was much longer than the coherence length. Fig. 2 (a,b) shows the second order and the first order interference visibilities with $\Delta L_2 = 5$ mm and $\Delta L_1$ scanned starting from the white light condition. 97% second order and 82% first order interference visibilities were observed at the beginning of the scan. All reported values are directly measured without noise reduction and theoretical corrections.

2: $\Delta L_1 > \text{coherence length}$

Fig. 3(a,b) reports two typical second order interference visibility measurements in which $\Delta L_2$ was set to a value which was longer than the coherence length and $\Delta L_1$ was scanned from white light condition. For each data point, the visibility was calculated from measurements similar to those shown in fig. 2. It is clear that the interference disappeared at about $\Delta L_1 = 500 \mu m$ which
Fig. 2(a): Second order interference near white light condition, showing visibility near 100%.

Fig. 2(b): First order interference near white light condition, showing visibility 82% (noise was not subtracted).
Fig. 3(a): Second order interference visibility with 50-psec coincidence window ($\Delta L_2 = 1.8$ mm, $\Delta L_1$ scanned from white light condition).

Fig. 3(b): Second order interference visibility with 50-psec coincidence window ($\Delta L_2 = 4$ mm, $\Delta L_1$ scanned from white light condition).
corresponding to the first order coherence length of the field (determined by the band width of the spectral filter) and reappeared around $\Delta L_1 = \Delta L_2$. These measurements were repeated many times.

Fig. 4 and table 1 report the second order interference visibility measurement for $\Delta L_1 = \Delta L_2$ with 50 psec time window and 3 nsec time window. The interference visibilities were measured to be $(38 \pm 6)\%$ and $(21 \pm 7)\%$ for the 50 psec window and $(22 \pm 2)\%$ and $(7 \pm 3)\%$ for the 3 nsec window, when the optical path difference of the interferometers were 2 cm and 4 cm, respectively. The ratios are about $1.7 \pm 0.3$ for $\Delta L = 2$ cm and about $3.0 \pm 1.6$ for $\Delta L = 4$ cm, respectively. The "cut off" effect is clearly demonstrated. However, we still need a visibility more than 50% in order to have a unambiguous quantum result.

The second step of the experiment used a 20 psec coincidence time window. Higher interference visibility (>50%) was expected at $\Delta L > 6$ mm. In this experiment, 632.8 nm and 788.7 nm photon pairs were used for the measurement. The wavelength 632.8 nm was used for easy alignment. We used a CW He-Ne laser beam as input signal to match the 632.8 nm down conversion mode. Both 632.8 nm and 788.7 nm radiation have much longer coherence length due to the stimulated down conversion (or so called induced coherence). The parametric amplified signal and idler radiation were used for careful alignment. High visibility first order interference of the stimulated down conversion beams were observed before taking data.

Fig. 5, 6 and 7 report the experimental results. Fig. 5 (fig. 6) is a typical measurement in which $\Delta L_1(\Delta L_2)$ was fixed at 7 mm and $\Delta L_2(\Delta L_1)$ scanned around 7 mm. Fig. 7. reports the measurement in which both interferometers were scanned around 7 mm. The 7 mm optical path difference was much longer than the coherence length of the down conversion beam, no first order interference can be observed in $N_1$ or $N_2$, however, the coincidence measurement $N_c$ showed clear interference fringes in the above measurements. The fringe visibilities are 59% with a period of 632.8 nm and 59% with a period of 788.7 nm for the type of measurements in fig. 5 and fig. 6, respectively. When both $\Delta L_1$ and $\Delta L_2$ are changed together the visibility is 58% with a period of 351.1 nm. The solid curves in fig. 5, fig. 6 and fig. 7 are the fittings for 632.6 nm, 788.7 nm and 351 nm, respectively. The standard deviation for these measurements is about 2%.

In summary:

1. The existence of E.P.R state has been observed by means of:
   (1). the "cut off" effect, i.e., the interference visibility comparison between 50 psec and 3 nsec coincidence time window.
   (2). direct measurement of more than 50% interference visibility for a 20 psec coincidence time window. This is the first observation of visibility greater than 50% for the two independent interferometers experiment.

2. The second order interference coherence (second order in intensity fourth order in field) is not limited by the coherence length of the pump beam only, but also by the non-perfect phase matching of the parametric down conversion. The uncertainty of the correlation in frequency determines the second order coherence length. We believe it is the non-perfect phase matching of the down conversion that reduced the visibility of the second order interference fringes in our experiment.

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Fig. 4: Second order interference visibility for equal optical path differences with 50-psec and 3-nsec coincidence time window.

Fig. 5: Second order interference fringes for 632.8 nm ($\Delta L_1 = 7$ mm, $\Delta L_2$ scanned around 7 mm, 100 second for each point) with 20-psec coincidence time window.
Fig. 6: Second order interference fringes for 788.7 nm ($\Delta L_2 = 7$ mm, $\Delta L_1$ scanned around 7 mm, 100 second for each point) with 20-psec coincidence time window.

Fig. 7: Second order interference fringes for 351.1 nm ($\Delta L_1 = \Delta L_2$, $\Delta L_1$ and $\Delta L_2$ scanned together around 7 mm, 100 second for each point) with 20-psec coincidence time window.
References


