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FLOQUET OPERATOR AS INTEGRAL OF MOTION

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Nonstationary quantum systems have no energy levels. However, for time-dependent periodical quantum systems, the notion of quasi–energy levels has been introduced in Ref. [1, 2]. The main point of the quasi–energy concept is to relate quasienergies to eigenvalues of the Floquet operator or monodromy operators which is equal to the evolution operator of a quantum system taken at the moment coinciding with the period of the system. The purpose of this article is to relate the Floquet operator to integrals of the motion and to introduce new operator which is the integral of motion and has the same quasienergy spectrum that the Floquet operator has. Implicitly, the result of the article was contained in Ref. [3], but we wish to have an explicit formula for this new integral of motion.

If one has the system with Hermitian Hamiltonian $H(t)$ such that $H(t+T) = H(t)$, the unitary evolution operator $U(t)$ is defined as

$$|\Psi, t\rangle = U(t)|\Psi, 0\rangle, \tag{1}$$

where $|\Psi, 0\rangle$ is the state vector of the system at the initial time. Then, by definition, the operator $U(T)$ is the Floquet operator and its eigenvalues have the form

$$f = \exp(-i\epsilon T), \tag{2}$$

with $\hbar = 1$, where ϵ is called the quasienergy state vector. The spectrum of quasienergy may be discrete or continuous for different quantum systems [3]. We wish to answer the following question. Is the quasienergy a conserved observable or not? This question is related to another question. Is the Floquet operator F(T) an integral of motion or not? The answer to the second question is negative. The operator U(T) does not satisfy the relation

$$\frac{\partial I(t)}{\partial t} + i[H(t), I(t)] = 0, \tag{3}$$

which defines the integral of motion $I(t)$. Thus, the Floquet operator $U(T)$ is not the integral of motion for the periodical nonstationary quantum systems. But as it was found in Ref. [3], the operator of the form

$$I(t) = U(t)I(0)U^{-1}(t) \tag{4}$$

satisfies equation (3) and this operator is the integral of motion of the quantum system. Thus, for periodical quantum systems, let us introduce the unitary operator M(t) which has the form

$$M(t) = U(t)U(T)U^{-1}(t). \tag{5}$$

This operator is the integral of motion due to the construction given by the formula (4) for any integral of motion. The spectrum of the new invariant operator $M(t)$ coincides with the spectrum of the Floquet operator $U(T)$. We have therefore answered the question about quasienergies. Since these numbers are defined as eigenvalues of the integral of motion $M(t)$, they are conserved quantities. Thus we generalize the concept of quasienergies connecting these quantum observables with the integral of motion of periodical quantum systems.

The construction given above allows us to introduce new invariant labels for nonperiodical systems, for example, with the time-dependence of the Hamiltonian corresponding to quasicrystal structure in time. For such systems, the analogue of the invariant Floquet operator (5) will be the operator

$$M_1(t) = U(t)U(t_1)U(t_2)U^{-1}(t). \tag{6}$$

This integral of motion is connected with the two characteristic times of the quasicrystal structure t_1 and t_2. For poly-dimensional structure, we can introduce the integral of motion

$$M_2(t) = U(t)\left[\prod_{i=1}^{i=n} U(t_i)\right] U^{-1}(t), \tag{7}$$

where $t_1, t_2, ..t_n$ are the characteristic times of the system. The eigenvalues of the operators $M_1(t)$ and $M_2(t)$ are conserved quantities, and they characterize the nonperiodical quantum systems with quasicrystal structure in time in the same manner as quasienergy describes the states of periodical quantum systems.

References

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