Efficient Dynamic Optimization of Logic Programs

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Abstract

A summary is given of the dynamic-optimization approach to speedup learning for logic programs. The problem is to restructure a recursive program into an equivalent program whose expected performance is optimal for an unknown but fixed population of problem instances. We define the term "optimal" relative to the source of input instances and sketch an algorithm that can come within a logarithmic factor of optimal with high probability. Finally we show that finding high-utility unfolding operations (such as EBG) can be reduced to clause reordering.

Purpose

This paper presents an outline of the motivation, problem, methods, and results of some recent work on dynamic optimization of programs. An earlier paper (Laird, 1992b) contains details, experimental results, and more complete references to related work. In addition, this paper discusses some new results, particularly the efficient handling of unfolding transformations.

Dynamic Optimization

"Speedup learning" refers generally to the problem of learning to perform more efficiently with practice (a form of skill learning). A particular case of speedup learning, called dynamic optimization, is the problem of improving the average-case performance of a program without affecting its the correctness. Unlike static optimization, dynamic optimization requires sample runs or other experience with the distribution of problem instances to be solved.

One approach—known variously as memoization or caching—is to retain results of solved problems for subsequent reuse in order to reduce the amount of redundant computation. Explanation-based generalization is a familiar example of this method. Another approach is to formulate and refine useful rules about a search process. SOAR and PRODIGY have successfully exploited this idea.

The approach taken here, however, is different: the structure of the program is modified in order to improve the average-case performance; and instead of making a priori assumptions about the "average case", we learn what we need to know about it by statistical sampling.

As a generic task, automated program speedup has great potential for commercial return. Data processing programs are very complex because they must handle correctly every contingency, but most of these contingencies occur rarely, if ever, during the lifetime of the program. Restructuring the program so that it runs fastest on the kinds of problems that it encounters most often is a sensible approach, one that is best accomplished by automation: not only is restructuring code a difficult and error-prone process, humans users can seldom supply more than qualitative understanding of the statistical properties of the data that the program will be processing.

Formal Problem Definition

The goal of this work is to find practical optimization methods, not just to prove abstract or asymptotic properties of the problem. Still, a formal statement of the dynamic-optimization problem is useful.

Fix a computational language \( \mathcal{L} \). Let \( \tau \) be a family of correctness-preserving program transformations for programs over \( \mathcal{L} \). We are given as input three things: (1) a program \( P \) in \( \mathcal{L} \); (2) an unknown stochastic process \( \mathcal{S} \) that can be invoked to generate problem instances for the program; and (3) a computable cost function \( \mathcal{C} \) that assigns a real-valued cost to any computation. In cases where there may be multiple

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1We also need some reasonability conditions on the cost
solutions, the cost of running a program on an input instance is taken to be the cost of finding the first solution. The task is to find a program \( P' \) such that (1) \( P' \) is equivalent to \( P \) in the sense that there is a finite sequence of transformations in \( \tau \) that maps \( P \) into \( P' \), and (2) \( P' \) is optimal with respect to \( C \) and \( S \), i.e., the expected cost of solving problems drawn from \( S \) is minimal (as measured by \( C \)) among all programs equivalent to \( P \).

For concreteness we apply our work to the Prolog language with three transformations (defined below): predicate unrolling, clause reordering, and unfolding of pairs of clauses. The cost functions I used in the experiments were CPU time and the number of atomic unifications. The problem instances have been independently selected with replacement from some fixed distribution, often with unbounded support (i.e., the number of possible problems is infinite).

The transformations we admit are as follows: (see Figure 1)

- (Unrolling) Copy all clauses of a predicate \( p \) and assign the predicate a new name, e.g., \( p_{\text{copy}} \). Some references to the old predicate may be changed to the new name. In practice, we shall change all references in the tails of the unrolled clauses to refer to the new name.
- (Reordering) Reorder the clauses of a particular predicate.
- (Unfolding) Unfold two clauses \( C_1 \) and \( C_2 \) by resolving a premise goal from \( C_2 \) with the head of \( C_1 \). The result is a new clause to be added to the program.

These are not the only possible semantics-preserving transformations for logic programs, but they are sufficient to obtain good results in practice and simple enough to understand mathematically.

Properly, the dynamic optimization problem defined above is ill-posed if one can construct a Prolog program and an input source such that there exists an infinite sequence of equivalent programs each of which has lower expected cost than its predecessor. In fact one can do just this. But in practice this technicality is removable by requiring only that we construct a program whose expected cost is, with high probability, within \( \epsilon \) of optimum, for arbitrary \( \epsilon > 0 \).

More than a technicality is the fact that the (corresponding decision) problem is NP-hard. Even if we restrict the transformations to clause reordering, the problem of determining whether or not there is a reordering such that the expected cost is no greater than \( C \) is NP-complete, as demonstrated by a reduction from the minimal set cover problem. We are, therefore, unlikely to find a polynomial time solution to the problem, but we are certainly free to look for a polynomial-time approximation algorithm with some performance guarantee. Indeed, this is our approach.

The Learn/Optimize Cycle

Our main point of departure from the caching approach is to separate the learning phase from the program transformation phase. A similar approach has been employed by Greiner and Orponen (1991) for non-recursive query languages. During the learning phase one collects statistics about the probabilities and costs of success and failure of specific clauses at specific points (contexts) in the proof. The transformation phase uses these statistics to select transformations that are likely to improve performance. Applying these transformations results in a revised program.

Note: In the programs below only the predicates are shown, without their arguments—e.g., \( p \) instead of \( p(...) \).

(a) Initial program:
\[
\begin{align*}
[C1]: & \quad p \leftarrow p. \\
[C2]: & \quad p.
\end{align*}
\]

(b) After an unrolling: (Clauses \( C_3 \) and \( C_4 \) are copies of \( C_1 \) and \( C_2 \), resp., but with the predicate renamed.
\[
\begin{align*}
[C^1]: & \quad p \leftarrow p_{\text{copy}}. \\
[C2]: & \quad p. \\
[C3]: & \quad p_{\text{copy}} \leftarrow p_{\text{copy}}. \\
[C4]: & \quad p_{\text{copy}}.
\end{align*}
\]

(c) After reordering the \( p_{\text{copy}} \) predicate:
\[
\begin{align*}
[C^1]: & \quad p \leftarrow p_{\text{copy}}. \\
[C2]: & \quad p. \\
[C4]: & \quad p_{\text{copy}}. \\
[C3]: & \quad p_{\text{copy}} \leftarrow p_{\text{copy}}.
\end{align*}
\]

(d) Let \( \theta \) be a unifier for the underscored literals above; after unfolding \( C_1 \) through \( C_4 \), we obtain:
\[
\begin{align*}
[C^1.1]: & \quad p\theta. \\
[C2]: & \quad p. \\
[C^1.2]: & \quad p \leftarrow p_{\text{copyrest}}. \\
[C4]: & \quad p_{\text{copy}}. \\
[C3]: & \quad p_{\text{copy}} \leftarrow p_{\text{copy}}. \\
[C3.1]: & \quad p_{\text{copyrest}} \leftarrow p_{\text{copy}}.
\end{align*}
\]

Here, \( C_{3,1} \) is a copy of \( C_3 \); note that the case \( C_4 \) is covered by the unfolded clause \( C_{1,1} \).

Figure 1: Basic Prolog transformations

In this paper we treat only "pure" Prolog, but the techniques extend to most impure constructs as well.

3The standard EBG algorithm unfolds the program through all the resolution steps used in solving a single problem instance (4).
To summarize, the learn/optimize cycle gradually un-
substantially greater improvement.

and optimizing the initial calls in the program pro-
done, I found that the additional steps of unrolling
later, since more of the search space will be avoided.

reason is that advice obtained early in a long search
folding some of them for some number of steps? The
program by reordering the existing clauses and un-

In general, the cycle of learning and optimizing is re-
peated, with transformations occurring at successively
deeper levels of the program, until some kind of conver-
gence is achieved. Unrolling is followed by reordering
or unfolding in order to effect optimizations at specific
points in the computation. As the program is unrolled,
the total number of clauses increases, but in practice
the physical size of the program is a negligible part
of the run time. Note that unrolling has the effect of
separating out a finite, non-recursive initial part of
the computation from the later, recursive parts. For
example:

• In the transformation from Figure 1(a) to 1(b),
the initial call (to p) is distinguished from the re-
cursive calls to pcopy. Thus our learning phase
can gather different statistics for the initial pro-
gram call to p and for the subsequent recursive
 calls to pcopy.

• If, in Figure 1(a) the subgoal p from clause C;
is solved more efficiently using a different clause
order (C$ before C`) from that of the main goal
p, then the program will be unrolled as in Figure
1(b) and reordered as in Figure 1(c).

An unfolding transformation may increase by one the
number of clauses in a procedure (e.g., p in Figure
1(c-d)). The “catchall” clause (C'1,2) invokes a differ-
ent version of the unfolded predicate, one that omits
the clause that was unfolded (here, C4 is omitted in
pcopyrest).

Unrolling the program in this way is another distin-
guishing feature of our approach. A reasonable ques-
tion is: instead of unrolling, why not just optimize the
program and optimize it for calls first at
depth 0, then at depth 1, and so on. If on some cy-
cle no optimizations can be found and depth d, the
cycle still continues by optimizing depth d + 1. The
procedure halts, not because it runs out of transfor-
mations, but because of finite limits on the accuracy
of the learning algorithm. At this point the final op-
timization is to reorder the clauses of the recursive
procedures (without unrolling).

Below, we discuss several specific issues concerning the
learn/optimize cycle.

Well-foundedness

A problem can arise when some of the clauses of a
procedure overlap in the cases that they cover, i.e.,
more than one clause can successfully resolve cer-
tain goals. Suppose, for example, that in Figure 1(b)
C3 and C4 each cover almost all provable instances of
the pcopy predicate, with equal expected cost. Then
with the clause order shown, we will charge C3 with
almost all the cost of solving pcopy goals. C3 thus
appears to be more expensive than C4, and the trans-
formation phase will therefore place C4 before C3 in
the revised program. But after reordering and repeat-

ing the learning phase, C4 is now charged for the cost
of pcopy, and the original order appears preferable.
By continuing to reverse the order of these two clauses
we could get stuck in a loop that never improves the
program. As noted above, finding an optimal ordering
of a set of clauses is NP-complete, so no exact algo-

rithm is likely to be significantly faster than testing
every possible ordering.

A simple solution to this problem is that once the
clause order of a predicate is chosen, we should never
change it again, even if subsequent learn/optimize cy-
cles make a different order seem better. It turns out
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What about unfolding transformations? For these
there is no inverse folding transformation in the admis-
sible set, so once carried out they cannot be undone.
However the order of the unfolded clauses (e.g., C'1,1,
C'1,2, and C2 in Figure 1(d)) may be changed on the
next cycle. Below we shall show that the question of
whether the statistics can mislead us into performing a
sub-optimal unfolding transformation can be finessed,
and the search for unfolding transformations reduces
to one of finding good clause orderings.

Convergence

As discussed above, it may happen that as a result
of a transformation the measured expected cost of the
program actually increases on the next cycle; even so,
we continue the cycle even after a temporary increase,
because transformations on subsequent cycles can re-
duce the mean cost of the program substantially. Consequently, this is not a hill-climbing procedure.

If the learning algorithm were to continue to unroll clauses and collect statistics on them to arbitrary accuracy, the learn/optimize cycle might never halt: ever deeper transformations might be found that continue reducing the expected cost of the program, ultimately by very small amounts. In practice, however, it is the learning algorithm, specifically its finite sample-size limits, that bounds the cycle. The deeper one goes in the search tree, the lower the probability that the nodes will be encountered on any given problem, and hence the larger will be the number of instances that must be solved in order to estimate the likelihoods and costs accurately.

Slightly more formally, one can characterize the program to which the process converges by defining a program to be $d$-optimized if all its calls at depths $d$ and below are optimal. Then if the learning algorithm provides correct statistics (in the PAC sense), the optimized program will be (probably approximately) $d$-optimized to some depth $d$.

**Order of Transformations**

Suppose that for a subgoal at some point in the program, an unfolding transformation and a reordering transformation would each be effective, according to the results of the most recent learning phase. Which transformation(s) should we perform? My intuition tells me that the clauses should be reordered before any of them should be unfolded with subsequent clauses, but initially I had difficulty justifying this fact. Below I'll argue why it is in fact true.

**The Learning Algorithm**

The statistics collected during the learning phase are driven by the need to predict the efficiency of program transformations during the subsequent phase. Consider the predicate $p$ in Figure 1(b) and the decision about the ordering of the clauses $C_1$ and $C_2$. The solution to this problem is well known: we need to estimate for each clause the a priori probability $p_1$ that the clause will succeed (independently of whether the other clause leads to a solution) and the expected cost $C_i$ of applying the clause (regardless of success or failure). Then if we change the program by placing these clauses in decreasing order of $p_1/C_i$, the ordering of the $p$ clauses is optimal—provided no two clauses cover (solve) the same problem instance. If multiple clauses cover some instances, we still adopt the same ordering: although this may not be an optimal ordering, it is (as noted above) a greedy approximation to optimal. On subsequent passes of the cycle, the ordering of the clauses $C_1$ and $C_2$ will not be altered, so statistics on them need not be collected. Instead, clause $C_3$ will be unrolled ($C_4$ has no subgoals to unroll) and statistics $p_1$ and $C_i$ will be collected for clauses $C_3$ and $C_4$.

Efficient statistical methods for estimating $p$ and $C$ for a clause to within any given accuracy and confidence are well known, and a sufficient sample size is easily computed. For some clauses, $p$ may be fall below the accuracy limit and thus be estimated as zero: such clauses are not optimized further. Typically the deeper one gets in the program (i.e., the more unrolling transformations have been performed), the smaller the likelihood that any particular problem instance will invoke that clause, and the larger the sample size required to collect the needed statistics. Consequently we impose a lower bound on the absolute likelihood that a clause will be invoked and refuse to optimize clauses whose likelihood falls below that threshold. It turns out (Laird, 1992b) that this lower bound is not enough to guarantee termination of the cycle: we must also impose an absolute upper bound on the depth of the unrolling. In practice, however, this absolute depth will probably not be reached.

Finally we are left with optimizing the recursive clauses (like pcopy in Figure 1(d)). Unlike the clauses that precede it in the computation, such clauses are invoked multiple times. A Markov-tree learning algorithm (called the TDAG algorithm) is used to compute the probabilities of success for these recursive procedures. Unfortunately there is no polynomial time bound on the sample size needed to estimate the probabilities for such clauses, since for Markov processes successive events are not statistically independent. The sample size needed to ensure the accuracy of the statistics is highly dependent upon the structure of the underlying call graph and the transition probabilities determined by the input source. As a practical approach, I have determined this sample size heuristically, with good results.

**Finding Unfolding Transformations**

We have seen that the necessary statistics and the corresponding procedure for selecting good clause-reordering transformations are straightforward, and that we can quantify the relationship between the optimal order and our approximation to it. What about unfolding transformations? In a previous paper (Laird, 1992a) I outlined a rather complicated algorithm for finding unfolding transformations, with special statistics collected for the purpose during the learning phase. But it turns out that virtually the same algorithm as for reordering transformations applies. In fact, unfolding can be viewed as a special case of reordering.

The simple program in Figure 1(c) represents the SLD search procedure for resolving goals with predicate $p$. In Figure 2 a portion of the depth-first search tree for this program is shown explicitly. The SLD-resolution procedure searches this tree in pre-order left-to-right.
Figure 2: Search tree for the program of Fig. 1(c).

Figure 3: Reordered Search tree.

Figure 4: The program of Fig. 1(b) reordered to depth two, according to Fig. 3. The substitution $\theta$ is that shown in Fig. 1(d), unifying the pcopy consequent of clause $C_1'$ with the head of $C_4$. The substitution $\phi$ unifies the pcopy term in clause $C_1'$ with the head of clause $C_3$.

In the figure each node is labeled with the sequence of clauses used to resolve subgoals. For example, to reach the node labeled $C_1' - C_3$, one resolves the p-goal using clause $C_1'$, tries to resolve the resulting pcopy-subgoal using clause $C_4$ but fails, backtracks, and tries again using clause $C_3$.

The order of the nodes $C_1'$ and $C_2$ at depth 1 is established, as described above, by placing them in order left-to-right with decreasing values of $p/C$. The order of the clauses $C_3$ and $C_4$ below $C_1'$ is determined similarly.

Note that the option to consider node $C_2$ after node $C_1' - C_4$ but before $C_1' - C_3$ is not available if we may only reorder clauses for a single predicate. It is, however, if we consider reordering all the nodes at depths $\leq 2$—i.e., the nodes $C_1' - C_4$, $C_1' - C_3$, and $C_2$. To do so we need to estimate the $p/C$ values for the pairs of steps that lead to the depth-two nodes—e.g., we need to find the likelihood that a successful solution will be found using clause $C_1'$ followed by $C_4$, in comparison to the other two possibilities of $C_1'$ followed by $C_3$ or $C_2$. Suppose we do this and we discover that the $p/C$ value for $C_2$ is between that of the two nodes at depth two; then we should reorder the tree as in Figure 3. The corresponding program is shown in Figure 4.

Note that there are now three clauses for the predicate $p$, since there are three nodes of depth $\leq 2$ in the tree; the first and third clauses have each been unfolded one level.

This example illustrates how appropriate unfolding transformations can be found using the same learning statistics as for reordering. Actually, there is a minor difference between reordering one-step nodes and two-step nodes: in the event of failure, the program of Figure 4 must unify the p goal with the head of a p-clause three times, instead of two in Figure 1(c); this small additional expense can be estimated and taken into account in the decision about whether to change the order of the tree in Figure 2 to that of Figure 3.

Although we have treated only a special case, the general procedure for determining unfolding transformations is the same: Determine the best ordering for pairs...
of resolution steps instead of single steps; if in the resulting order a clause at the lower level has only one immediate child node (e.g., the nodes labeled C1 in Figure 4), go ahead and unfold. By iterating this procedure during the learn/optimize cycle, one can end up unfolding multiple steps into a single clause, without the need to estimate statistics for more than two steps at a time.

Conclusion: Where do we go from here?

The work described here builds upon and extends the work of many researchers, including Smith and Genesereth (1985), Prieditis and Mostow (1987), Gooley and Wah (1989), and Greiner and Orponen (1991). The principal contributions are as follows:

- We have shown that recursive programs can be dynamically optimized efficiently and effectively. Previous work has reported difficulties speeding up recursive programs.
- The nature of that optimization can be quantified better than previous heuristic methods were able to.
- We have shown how search control can be embedded in the program instead of being added on in the form of a time-consuming meta-theory that must be evaluated outside the actual program.
- By averaging over several runs we have removed the dependency of the resulting optimized program on the order of the examples. This has been a consistent problem with caching methods.
- We have integrated the utility estimates into the learning procedure: no transformations are even generated unless their utility justifies it with high probability. The learning data and the utility evaluation data are one and the same.
- We have eliminated all vestiges of the "generalization-to-N" anomalies and tricky "operationality" decisions that occur with EBG-based methods. In fact, unfolding itself has been reduced to a minor transformation that occurs after consecutive pairs of clauses have been reordered.

This work is still in progress. Future plans call for redoing the previous dynamic optimizer for Prolog to incorporate the improvements described here and to handle many of the so-called impure constructs in Prolog, including negation-as-failure, call, and or. Mathematical analysis of this approach is still incomplete, and we cannot yet argue that a strategy different from unrolling and reordering will not provide superior optimization.

Finally, for dynamic optimization of programs to be of commercial value, we must be able to optimize programs written in commercial programming languages.

Prolog is not a commercial data-processing language and, in my estimation, is not likely to become one. The method described here does not apply to procedural languages like C and Fortran, where order of decisions cannot be changed. One could add nondeterministic search primitives to such languages, but such new constructs are unlikely to gain wide acceptance. Constraint-logic programming, on the other hand, is growing in popularity as a programming language, and it is quite likely that these optimization methods are applicable. Also, very similar dynamic-optimization problems occur in database query languages, where further opportunities for commercialization may be found.

References