An Analytical Investigation of Transient Effects on Rewetting of Heated Thin Flat Plates

J.A. Platt
Lewis Research Center
Cleveland, Ohio

Prepared for the
Fundamentals of Heat Transfer in a Microgravity Environment, ASME 1993 Winter Annual Meeting
sponsored by the American Society of Mechanical Engineers
New Orleans, Louisiana, November 28–December 3, 1993
AN ANALYTICAL INVESTIGATION OF TRANSIENT EFFECTS ON REWETTING OF HEATED THIN FLAT PLATES

J.A. Platt
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

ABSTRACT
The rewetting of a hot surface is a problem of prime importance in the microgravity application of heat pipe technology, where rewetting controls the time before operations can be re-established following depriming of a heat pipe. Rewetting is also important in the nuclear industry (in predicting behavior during loss-of-coolant accidents), as well as in the chemical and petrochemical industries.

Recently Chan and Zhang (1992) have presented a closed-form solution for the determination of the rewetting speed of a liquid film flowing over a finite (but long) hot plate subject to uniform heating. Unfortunately their physically unreasonable initial conditions preclude a meaningful analysis of start-up transient behavior. The current work presents a new nondimensionalization and closed-form solution for an infinitely-long, uniformly-heated plate. Realistic initial conditions (step change in temperature across the wetting front) and boundary conditions (no spatial temperature gradients infinitely far from the wetting front) are employed. The effects of parametric variation on the resulting simpler closed-form solution are presented and compared with the predictions of a "quasi-steady" model. The time to reach steady-state rewetting is found to be a strong function of the initial dry-region plate temperature. For heated plates it is found that in most cases the effect of the transient response terms cannot be neglected, even for large times.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>b,c</td>
<td>variables of integration</td>
</tr>
<tr>
<td>c_p</td>
<td>specific heat</td>
</tr>
<tr>
<td>h</td>
<td>boiling heat transfer coefficient</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity</td>
</tr>
<tr>
<td>m</td>
<td>function defined in equation (22)</td>
</tr>
<tr>
<td>Q</td>
<td>uniform heat flux to the plate</td>
</tr>
<tr>
<td>q</td>
<td>dimensionless heat flux</td>
</tr>
<tr>
<td>S</td>
<td>plate thickness</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
</tr>
<tr>
<td>T_s</td>
<td>saturation temperature</td>
</tr>
<tr>
<td>T_0</td>
<td>Leidenfrost temperature</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>U</td>
<td>rewetting velocity</td>
</tr>
<tr>
<td>u</td>
<td>dimensionless rewetting velocity</td>
</tr>
<tr>
<td>X,Y</td>
<td>length coordinates in Eulerian system</td>
</tr>
<tr>
<td>X'</td>
<td>length coordinate in Lagrangian system</td>
</tr>
<tr>
<td>\alpha, \beta</td>
<td>functions used in temperature transformations (eqs. (11) and (18))</td>
</tr>
<tr>
<td>\eta</td>
<td>dimensionless length coordinate</td>
</tr>
<tr>
<td>\theta</td>
<td>dimensionless temperature</td>
</tr>
<tr>
<td>\nu</td>
<td>transformed dimensionless temperature (eqs. (11) and (18))</td>
</tr>
<tr>
<td>\rho</td>
<td>density</td>
</tr>
<tr>
<td>\tau</td>
<td>dimensionless time</td>
</tr>
</tbody>
</table>
**INTRODUCTION**

Space Station Freedom (SSF), when built, will require more power than any previous space facility. The heat generated as a result of this power must be transported to radiators where it will be discharged to space. One of the designs for the SSF thermal management system involves use of heat pipes in the radiators. Successful prediction of rewetting characteristics of heat pipes in a spacecraft environment requires an understanding of the physical mechanisms that control the wetting of a hot surface.

The problem of rewetting a surface which is significantly above the boiling temperature also has numerous industrial applications, most notably in the quenching of metals and in cooling nuclear fuel rod bundles during loss-of-coolant accidents in water reactors. Nuclear applications have been behind many of the previous analytical efforts at understanding rewetting, such as those of Yamanouchi (1968); Thompson (1972); Duffey and Porthouse (1973); Coney (1974); Sun et al. (1974, 1975); Blair (1975); Tien and Yao (1975); and Dua and Tien (1976), which all found analytical expressions for the steady-state rewetting velocities by employing a variety of boiling models to various unheated surface geometries.

Analysis of the unsteady start-up of rewetting and of the effect of uniform heating has been sparse. Bukur and Isbin (1972) performed a numerical analysis of start-up transients for a few cases corresponding to rewetting in nuclear loss-of-coolant accidents. They found that the effect of start-up transients for these cases was negligible. However, parametric ranges appropriate to heat pipe applications were not studied.

A work has recently been published (Chan and Zhang, 1992) which presents, among other things, a closed-form analytical solution of the unsteady rewetting of a heated thin flat plate. However, the initial conditions considered preclude any meaningful examination of start-up transients. Furthermore, no parametric analysis of plate heating was presented.

In the present work a simpler closed-form solution for rewetting velocity is derived for a thin heated flat plate of infinite length. Results are presented for a wide range of initial plate temperatures and heating rates, thus making the results applicable to microgravity heat pipe rewetting problems. The time-dependent start-up and heating effects of these solutions are discussed and full-solution heated-plate results are compared with those of a simplified quasi-steady solution.

**PROBLEM FORMULATION**

Consider the problem of rewetting, by a liquid film, an infinitely-long thin plate initially hotter than the Leidenfrost temperature (fig. 1). The plate is uniformly heated on its underside while the top dry region is considered adiabatic and the top wet region has a heat loss proportional to the difference between the plate temperature and liquid saturation temperature. The one-dimensional unsteady formulation of the problem, in dimensionless form, with appropriate boundary and initial conditions, can be represented as (see Appendix A):

\[
\rho c_p \left( \frac{1}{\tau^*} \frac{\partial \theta_w}{\partial \tau} - \frac{U^*}{X^*} u \frac{\partial \theta_w}{\partial \eta} \right) = \frac{k}{X^*} \frac{\partial^2 \theta_w}{\partial \eta^2} + \frac{Q}{(T_0 - T_\infty)S} - \frac{h}{S} \theta_w \eta^* \quad (\eta < 0)
\]

\[
\left. \frac{\partial \theta_w}{\partial \eta} \right|_{(-\infty, \eta)} = 0
\]

\[
\theta_w(0,\tau) = 1
\]

\[
\theta_w(\eta,0) = 0
\]
\[
\rho c_p \left( \frac{1}{t^*} \frac{\partial \theta_d}{\partial \tau} - \frac{U^*}{X^*} u \frac{\partial \theta_d}{\partial \eta} \right) = k \frac{\partial^2 \theta_d}{X^* \frac{\partial \eta^2}{\partial \eta}} \]
\[ + \frac{Q}{(T_0 - T_s)S} \quad (\eta > 0) \]
\[
\frac{\partial \theta_d}{\partial \eta} \bigg|_{(\eta, \tau)} = 0
\]
\[
\theta_d(0, \tau) = 1
\]
\[
\theta_d(\eta, 0) = \theta_1
\]

where \( \tau \equiv \frac{t}{t^*} \), \( \eta \equiv \frac{X'}{X^*} \), \( u \equiv \frac{U}{U^*} \), \( \theta \equiv \frac{T - T_s}{T_0 - T_s} \).

\( (\eta = 0 \) is the location of the wetting front.) The reference time, length, and velocity are yet to be determined. In selecting dimensionless parameters the approach taken is to obtain unit-order dimensionless variables. This results in different parameters than those of previous investigators (such as Chan and Zhang (1992); Sun et al. (1974); and Duffey and Porshouse (1973)). One relationship between these reference values can be found by recognizing that the two terms on the left-hand side of the equation should be of the same order of magnitude, since they both arise from the unsteady term of the Eulerian formulation (eqs. (A4) and (A5)). Thus we have the relationship:

\[ U^* = \frac{X^*}{t^*} \quad (3) \]

Another relation is obtained by assuming that boiling is the dominant term on the right-hand side of the equation. (The assumption is amply justified for a problem involving sputtering.)

\[ \frac{\rho c_p}{t^*} = \frac{h}{S} \quad (4) \]

Finally a reference length is chosen by balancing conduction with the unsteady and boiling terms:

\[ \frac{\rho c_p}{t^*} = \frac{k}{X^* \frac{\partial \eta^2}{\partial \eta}} \quad (5) \]

Equations (3) to (5) can be combined to yield the appropriate reference length, time, and velocity:

\[ X^* = \sqrt{\frac{kS}{h}} \quad t^* = \frac{\rho c_p S}{h} \quad U^* = \frac{1}{\rho c_p} \sqrt{\frac{kS}{h}} \quad (6) \]

Thus the dimensionless systems of equations for the wet and dry regions are:

\[ \frac{\partial \theta_w}{\partial \tau} = \frac{\partial^2 \theta_w}{\partial \eta^2} + u \frac{\partial \theta_w}{\partial \eta} + q - \theta_w \quad (\eta < 0) \]

\[ \frac{\partial \theta_d}{\partial \eta} \bigg|_{(\eta, \tau)} = 0 \]

\[ \theta_w(0, \tau) = 1 \]

\[ \theta_w(0, \tau) = 0 \]

\[ \theta_d(0, \tau) = 1 \]

\[ \theta_d(\eta, 0) = \theta_1 \]

where \( q \equiv \frac{Q}{S(T_0 - T_s)} \).

Here \( \theta \), the dimensionless temperature, is a function of position and time, and \( u \), the dimensionless rewetting velocity, is a function of time. \( \theta_1 \) and \( q \) are constants, with \( \theta_1 \) greater than unity.

In order to effect a closed-form solution to the problem, \( u \) will be treated as a constant for any given time. This approximation makes the equations linear and thus more tractable. Matching the \( \eta = 0 \) temperature gradients for any given \( \tau \) will yield \( u \) for that time.

**SOLUTION FOR THE WET REGION**

First we solve the boundary condition as \( \eta \rightarrow -\infty \) to obtain an expression for \( \theta \) as a function of \( \tau \). The relevant equation and initial condition are:

\[ \frac{\partial \theta_w}{\partial \tau} = q - \theta_w \quad \theta_w(0) = 0 \quad (9) \]
The solution is easily found to be:

$$\theta_w(-\infty, 0) = q(1 - e^{-r})$$

(10)

Note that for \( q > 1 \) complete dryout of the plate is expected for a finite time, \( \ln(q/(q - 1)) \). These high heat flux conditions are excluded from further consideration.

It is interesting to contrast the initial and boundary conditions of the present work with those of Chan and Zhang, who developed a closed-form solution for rewetting speed on a heated plate of finite (but very large) length. Chan and Zhang's boundary condition far from the wetting front was \( \theta(-\eta_{L1}, r) = 0 \) (where \( \eta_{L1} \gg 1 \)), which models the end of the plate in contact with a liquid reservoir at saturation temperature and results in spatial temperature gradients far from the wetting front when \( q \neq 0 \).

Chan and Zhang's initial condition for both the wet and dry regions was \( \theta(\eta, 0) = \theta_1 \). This is a physically unreasonable condition for the wet region and as a result the rewetting velocity could not be determined for small times.

By employing a transformation to get the problem in the form of a conduction equation:

$$\nu_w \equiv \left[ \theta_w - q(1 - e^{-r}) \right] e^{\eta \theta_w + \eta \beta w \tau}$$

(11)

where \( \alpha_w = \frac{u}{2} \) and \( \beta_w = 1 + \frac{u^2}{4} \)

the following system of equations is obtained:

$$\begin{align*}
\frac{\partial \nu_w}{\partial \tau} &= \frac{\partial^2 \nu_w}{\partial \eta^2} \\
\nu_w(\eta, 0) &= 0 \\
\nu_w(0, \tau) &= 1 - q(1 - e^{-r}) e^{\eta \theta_w} \\
\nu_w(-\infty, \tau) &= 0
\end{align*}$$

(12)

This system is very close to the elementary conduction problem of a semi-infinite body, initially at constant temperature, that is subject to a step change in boundary temperature at time 0. By employing Duhamel's theorem (Myers (1987)), we can use the solution to that problem to obtain a solution for the present problem as:

$$\nu_w = -\frac{\eta}{2 \sqrt{\pi}} \int_{-\infty}^{r} \frac{1 - q(1 - e^{-c}) e^{\beta_w c}}{(r - c)^{3/2}} \exp \left[ -\frac{\eta^2}{4(r - c)} \right] dc$$

(13)

By reversing the transformation of equation (11) an expression for the dimensionless temperature in the wet region is obtained:

$$\begin{align*}
\theta_w &= q(1 - e^{-r}) - \frac{\eta e^{-\alpha_w \eta}}{2 \sqrt{\pi}} \\
&\quad \times \int_{-\infty}^{r} \frac{(1 - q)e^{-\beta_w(r-c)}}{(r - c)^{3/2}} \exp \left[ -\frac{\eta^2}{4(r - c)} \right] dc
\end{align*}$$

(14)

This can be rewritten as:

$$\begin{align*}
\theta_w &= q(1 - e^{-r}) + (1 - q)e^{-(\alpha_w + \beta_w \tau)\eta} \\
&\quad + qe^{-r} e^{-(\alpha_w + \beta_w \tau)\eta} \\
&\quad + \frac{\eta e^{-\alpha_w \eta}}{\sqrt{\pi} r} \int_{0}^{1} \left( 1 - q \right) \exp \left[ -\frac{\beta_w \tau}{b^2} - \frac{\eta^2 b^2}{4 r} \right] db
\end{align*}$$

(15)

where \( b \equiv \frac{\sqrt{r}}{\sqrt{r - c}} \)

Substituting the expressions for \( \alpha_w \) and \( \beta_w \), the wet-side temperature gradient at \( \eta = 0 \) is:

$$\begin{align*}
2 \frac{\partial \theta_w}{\partial \eta} \bigg|_{\eta=0} &= \frac{\sqrt{u^2 + 4 - u}(1 - q)}{\sqrt{u^2 + 8 - u} + qe^{-r}} \\
&\quad + \frac{2(1 - q)}{\sqrt{\pi} r} \int_{0}^{1} \exp \left[ -\frac{(u^2 + 4)r}{4b^2} \right] db \\
&\quad + \frac{2qe^{-r}}{\sqrt{\pi} r} \int_{0}^{1} \exp \left[ -\frac{(u^2 + 8)r}{4b^2} \right] db
\end{align*}$$

(16)
SOLUTION FOR THE DRY REGION

The solution for the dry region follows a similar procedure to that for the wetted region. In this case the boundary condition far from the wetting front is:

$$\theta_d(\infty, r) = \theta_1 + qr$$  \hspace{1cm} (17)

Upon applying the transformation:

$$\nu_d = (\theta_d - qr - \theta_1)e^{\alpha_d \eta + \beta_d r}$$  \hspace{1cm} (18)

where \(\alpha_d = \frac{u}{2}\) and \(\beta_d = \alpha_d^2 = \frac{u^2}{4}\)

the following system of equations is obtained:

$$\frac{\partial \nu_d}{\partial r} = \frac{\partial^2 \nu_d}{\partial \eta^2}$$

$$\nu_d(\eta, 0) = 0$$

$$\nu_d(0, r) = (1 - \theta_1 - qr)e^{\beta_d r}$$

$$\nu_d(\infty, r) = 0$$  \hspace{1cm} (19)

By application of Duhamel’s theorem the following results are achieved:

$$\theta_d = (\theta_1 + qr) - (\theta_1 + qr - 1)e^{-\alpha_d \eta}$$

$$+ (\theta_1 + qr - 1)e^{-\alpha_d \eta} \frac{\eta}{\sqrt{\pi r}}$$

$$\times \int_{b=0}^{1} \exp\left(-\frac{\beta_d r}{b^2} - \frac{\eta^2 b^2}{r^2}\right) db$$  \hspace{1cm} (20)

$$2 \frac{\partial \theta_d}{\partial \eta} \bigg|_{\eta=0} = 2u(\theta_1 + qr - 1)$$

$$+ \frac{2(\theta_1 + qr - 1)}{\sqrt{\pi r}} \int_{b=0}^{1} \exp\left(-\frac{u^2 r}{4b^2}\right) db$$  \hspace{1cm} (21)

Matching expressions in equations (16) and (21) for the temperature gradient at \(\eta = 0\) yields an expression that, given \(\theta_1\), q, and r, can be solved iteratively for u:

$$2u(\theta_1 + qr - 1) - \left(\frac{u^2 + 4 - u}{1 - q}\right)$$

$$- \left(\frac{u^2 + 8 - u}{2}qe^{-r} + \frac{2}{\sqrt{\pi r}}\right)$$

$$\times \left[(\theta_1 + qr - 1)m(u^2) - (1 - q)m(u^2 + 4) - qe^{-r}m(u^2 + 8)\right] = 0$$  \hspace{1cm} (22)

where \(m(x) \equiv \int_{b=0}^{1} \exp\left(-\frac{x^2}{4b^2}\right) db\)

This expression is significantly simpler than the only other comparable solution in the literature, equation (52) of Chan and Zhang (1992).

For q = 0 and \(r \to \infty\), equation (22) reduces to a solution equivalent to the one first derived by Yamanouchi (1968):

$$u = \sqrt{\frac{4}{2(\theta_1 - 1)^2 - 1}}$$  \hspace{1cm} (23)

RESULTS AND DISCUSSION

The dependence of velocity on initial plate temperature, for the case of unheated plates (q = 0), is shown in figure 2. As expected, the rewetting velocity is reduced as the initial plate temperature is increased.

An unusual characteristic of the present solution is the initial infinite rewetting velocities for \(\theta_1 > 2\). This is caused by the temperature step-function initial condition, which results in infinite wetting-front temperature gradients in both the wet and dry regions as \(r \to 0\). The sign of these

![Figure 2](image-url)
infinite velocities depends on the relative temperature differences between the wet and dry initial temperatures (0 and \( \theta_1 \), respectively) and the wetting-front (Leidenfrost) temperature, \( \theta = 1 \).

Note that this step function initial condition, while producing infinite positive or negative initial velocities, does not produce an infinite initial movement of the wetting front. This can be shown by examining \( u_\tau \) for small \( \tau \). Unless \( u_\tau \rightarrow 0 \) as \( \tau \rightarrow 0 \) the physically impossible result occurs of a nonzero movement of the wetting front, either finite or infinite, for vanishingly small time. Such a physically impossible result does not occur for the present solution (fig. 3).

A significant result of the present work is that the time to reach steady-state velocity increases sharply as the temperature of the plate increases (fig. 4, solid line). For \( \theta_1 = 2 \) about 0.1 s is required to reach steady state while for \( \theta_1 = 5 \) about 100 s is required.

The implications of this result for nuclear applications are minimal because the high boiling heat transfer coefficients for such cases result in small reference times. Using the values of density, specific heat, and plate thickness given by Bukur and Isbin (1972) for the experiment of Yamanouchi (1968), and using a heat flux of 10^6 kcal/m^2 hr °C (Yamanouchi, 1968), \( t^*_0 = 0.035 \) sec. (The numerical solutions of Bukur and Isbin (1972) confirm that the effects of initial conditions are minimal for nuclear applications.)

The time to reach steady state can be significant for other applications, for which the boiling heat transfer coefficient is several orders of magnitude lower. For the experiment of Grimley et al. (1988), using values derived by Chan and Zhang (1992), \( t^*_0 = 7.4 \) sec and the time to reach steady state is approximately 150 s. Thus for nonnuclear situations such as heat pipe rewetting, the amount of time to reach steady state could be significant.

Results for heated plates (figs. 5 to 7) establish that, given enough time, for all cases velocities will first become negative and then will approach zero. (The dry region plate temperatures for which these transitions occur are so large that in most real situations they would be physically unreasonable.) The fact that \( u \rightarrow 0 \) as \( \tau \rightarrow \infty \) can be accounted for simply by the effect of increased dry region temperature; as the temperature of the plate gets very large rewetting occurs very slowly.

An explanation of the negative velocities is more difficult and can be aided by a comparison of the results of the complete equation (eq. (22)) with those of a "quasi-steady" solution based on just the first two terms of equation (22):

\[
\begin{align*}
\frac{u}{4(1 - q)^2} & = \left( \frac{2\theta_1 + 2qr - 1 - q^2 - (1 - q)^2}{(2\theta_1 + 2qr - 1 - q^2 - (1 - q)^2)^{1/2}} \right) \\
& = \left( \frac{4(1 - q)^2}{(2\theta_1 + 2qr - 1 - q^2 - (1 - q)^2)^{1/2}} \right)
\end{align*}
\]

(24)

The velocity in this quasi-steady solution is unconditionally positive (the same would also be true for a solution based on the first three terms of eq. (36)), indicating that the source of the negative velocities is in the last three terms, system.

A comparison of the full solution and the quasi-steady solution for \( \theta_1 = 2 \) (fig. 8) confirms the importance of the transient terms. Agreement between the two solutions occurs only for \( q = 0.01 \). The reason for the lack of agreement in the other cases is evident from an examination of figure 4, where the broken lines map the plate temperatures for \( \theta_1 = 2 \) and various levels of heating. Only the \( q = 0.01 \) curve ever reaches what can be termed a "quasi-steady" state, where the velocity can be accurately approximated.
by an infinite time solution at the same plate temperature; and even in that case the solution becomes unsteady again for larger values of $\tau$. Therefore it is concluded that, for $q > 0$, the effect of the transient terms is significant in almost all cases. The negative velocities are evidently the result of localized heating in the wetting front region, as opposed to the effect of heating on the dry plate temperature far from the wetting front.

CONCLUSIONS
An analytical investigation has been undertaken of the transient rewetting of an infinitely-long, heated flat plate initially above the Leidenfrost temperature. An approximate closed-form solution for the rewetting velocity has been obtained by applying some simplifying assumptions to the equations for heat conduction in the plate.

With a step function initial temperature condition, the initial rewetting velocity is infinite (either positive or negative), although this result is explainable and is not physically unrealistic. The time required to reach steady-state wetting velocity is found to increase sharply with increasing plate temperature.

The effect of heating the plate is to decrease rewetting velocity over time, with dryout (negative rewetting velocity) eventually occurring in all cases (although the
plate temperatures for which this dryout would occur would generally be too large to be achieved in an actual system). Quasi-steady states (heated conditions for which transient terms do not affect the results) are found to occur for only a narrow range of time and heat flux.

APPENDIX A: DERIVATION OF EQUATIONS

For the problem of rewetting an infinitely-long plate initially hotter than the Leidenfrost temperature (fig. 1) the two-dimensional, unsteady conduction equation for temperature in the plate is:

\[ \rho c_p \frac{\partial T}{\partial t} = \kappa \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \]  

(A1)

The plate is uniformly heated on its underside while the top dry region is considered adiabatic and the top wet region has a heat loss proportional to the difference between the plate temperature and liquid saturation temperature. Thus for a thin plate the conduction term in the Y-direction can be simplified as:

\[ k \frac{\partial^2 T}{\partial y^2} \approx \begin{cases} \frac{Q}{S} - \frac{h}{S} (T - T_s) & \text{(wet)} \\ \frac{Q}{S} & \text{(dry)} \end{cases} \]  

(A2)

Far from the wetting front, conditions will be unaffected by movement of the wetting front and temperatures will be independent of X, leading to boundary conditions of:

\[ \frac{\partial T}{\partial x} \bigg|_{x' = 0} = 0 \]  

(A3)

Therefore we have two one-dimensional, unsteady conduction problems to solve (one each for the wet and dry regions). An additional boundary condition for each of the problems is the temperature at the wetting front \( (X = X_{wf}) \), which is assumed to be the Leidenfrost temperature. The systems of equations (without initial conditions, which are not yet specified) are:

\[ \rho c_p \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} + \frac{Q}{S} - \frac{h}{S} (T - T_s) \quad (X < X_{wf}) \]  

(A4)

\[ \frac{\partial T}{\partial x} \bigg|_{x' = 0} = 0 \]

Upon transformation to a Lagrangian coordinate system \( (X', t) \) with its origin at the wetting front equations (A4) and (A5) become:

\[ \rho c_p \left( \frac{\partial T}{\partial t} - U \frac{\partial T}{\partial X'} \right) = \kappa \frac{\partial^2 T}{\partial X'^2} + \frac{Q}{S} - h(T - T_s) \quad (X' < 0) \]  

(A6)

\[ \frac{\partial T}{\partial X'} \bigg|_{X' = 0} = 0 \]

\[ T(0,t) = T_0 \]

\[ \rho c_p \left( \frac{\partial T}{\partial t} - U \frac{\partial T}{\partial X'} \right) = \kappa \frac{\partial^2 T}{\partial X'^2} + \frac{Q}{S} \quad (X' > 0) \]  

(A7)

\[ \frac{\partial T}{\partial X'} \bigg|_{X' = 0} = 0 \]

\[ T(0,t) = T_0 \]

where \( U \) is the rewetting velocity, which in general is a function of time.

Initial conditions are sought which satisfy time-independent, \( U = 0 \) forms of equations (A6) and (A7). For the dry region an isothermal initial condition is the only possibility:

\[ T(X' > 0, 0) = T_1 \]  

(A8)

where \( T_1 \) is a temperature greater than the Leidenfrost temperature. For the wet region, although another solution is possible, an isothermal initial condition, with the plate at
the saturation temperature of the liquid, is adopted for simplicity:

\[ T(X' < 0,0) = T_s \]  \hspace{1cm} (A9)

The initial conditions therefore are nothing more than a step function across the wetting front.

REFERENCES


An Analytical Investigation of Transient Effects on Rewetting of Heated Thin Flat Plates

J.A. Platt

National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135-3191


The rewetting of a hot surface is a problem of prime importance in the microgravity application of heat pipe technology, where rewetting controls the time before operations can be re-established following depriming of a heat pipe. Rewetting is also important in the nuclear industry (in predicting behavior during loss-of-coolant accidents), as well as in the chemical and petrochemical industries. Recently Chan and Zhang (1992) have presented a closed-form solution for the determination of the rewetting speed of a liquid film flowing over a finite (but long) hot plate subject to uniform heating. Unfortunately their physically unreasonable initial conditions preclude a meaningful analysis of start-up transient behavior. The current work presents a new nondimensionalization and closed-form solution for an infinitely-long, uniformly-heated plate. Realistic initial conditions (step change in temperature across the wetting front) and boundary conditions (no spatial temperature gradients infinitely far from the wetting front) are employed. The effects of parametric variation on the resulting simpler closed-form solution are presented and compared with the predictions of a "quasi-steady" model. The time to reach steady-state rewetting is found to be a strong function of the initial dry-region plate temperature. For heated plates it is found that in most cases the effect of the transient response terms cannot be neglected, even for large times.