Effects of Turbulence on the Geodynamic Laser Ranging System

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1. INTRODUCTION

The Geodynamic Laser Ranging System (GLRS) is one of several instruments being developed by the National Aeronautics and Space Administration (NASA) for implementation as part of the Earth Observing System in the mid-1990s (Cohen et al., 1987; Bruno et al., 1988). It consists of a laser transmitter and receiver in space and an array of retroreflectors on the ground. The transmitter produces short (100 ps) pulses of light at two harmonics (0.532 and 0.355 μm) of the Nd:YAG laser. These propagate to a retroreflector on the ground and return. The receiver collects the reflected light and measures the round-trip transit time. Ranging from several angles accurately determines the position of the retroreflector, and changes in position caused by geophysical processes can be monitored.

The atmosphere will have several effects on the operation of the GLRS. The most obvious atmospheric factor is cloud cover. When there are clouds between the satellite and a particular retroreflector, no measurement from that reflector is possible. Fortunately, most of the geophysical processes of interest are slow enough that many cloud-free observations are expected before significant motion is observed. The next factor to consider is refraction because of the overall temperature gradient in the atmosphere. This can be corrected using the dispersion of the atmosphere, and is the reason for using two colors of light (Querzola, 1979; Abshire, 1980; Abshire and Gardner, 1985).

The final atmospheric factor to consider is refractive turbulence. This is a random phase perturbation of the optical field as it propagates through a random field of refractive index inhomogeneities in the atmosphere. Possible effects on the optical field include a random time delay, pulse spreading, beam wander, beam spreading, and irradiance fluctuations or scintillation. Gardner (1976) and Abshire and Gardner (1985) calculated the amount of random time delay and found it to be negligible for the GLRS configuration. The pulse spreading has also been calculated (Muchmore and Wheelan, 1951; Bramley, 1968; Brookner, 1969; Brookner, 1970) and is also negligible (<< 1 ps) for the GLRS case. The other effects are considered in this report.
2. TURBULENCE CHARACTERISTICS

At optical frequencies, the refractive index of air can be approximated by

\[ n = 1 + 7.76 \times 10^{-7}(1 + 7.52 \times 10^{-3} \lambda^{-2})P/T, \]  

(1)

where \( P \) is the atmospheric pressure in millibars, \( T \) is the temperature in Kelvins, and \( \lambda \) is the wavelength of light in micrometers. Thus, small changes in temperature cause small changes in the refractive index. Small changes in temperature exist in the atmosphere because of turbulent mixing of air parcels.

Refractive turbulence in the atmosphere can be characterized by three parameters. The outer scale, \( L_0 \), is the length of the largest scales of turbulent eddies. The inner scale, \( l_0 \), is the length of the smallest scales. For separations greater than the inner scale and less than the outer scale, the structure function of refractive index is given by

\[ D_n(\rho) = C_n^2 \rho^{2n}. \]  

(2)

where \( \rho \) is the separation of two observation points at positions \( p \) and \( p + \rho \), and the structure function is defined by

\[ D_n(\rho) = \langle [n(p) - n(p + \rho)]^2 \rangle. \]  

(3)

This implies that \( C_n^2 \) is a measure of the strength of refractive turbulence.

In the lowest few hundred meters of the atmosphere, turbulence is generated by radiative heating and cooling of the ground. During the day, solar heating of the ground drives convective plumes. Refractive turbulence is generated when these warm plumes mix with the cooler air surrounding them. At night, the ground is cooled by radiation and winds mix the cooler air near the ground with warmer air higher up. Periods of extremely low turbulence exist at dawn and dusk when no temperature gradient exists in the lower atmosphere. Turbulence levels are also very low when the sky is overcast and solar heating and radiative cooling rates are low.

Values of turbulence strength near the ground vary widely. Lawrence et al. (1970) measured values from less than \( 10^{-16} \) to greater than \( 10^{-12} \) \( m^{-2/3} \) at a height of about 2 m. These values are typical of what we see at this height. At 2.5 m, Kallistratova and Timanovskiy (1971) measured values from less than \( 10^{-17} \) to almost \( 10^{-13} \) \( m^{-2/3} \). Under certain conditions, the turbulence strength can be predicted from meteorological parameters and characteristics of the underlying surface (Holtslag and Van Ulden, 1983; Thiermann and Kohnle, 1988; Andreas, 1988).
Using a theory introduced by Monin and Obukhov (1954), Wyngaard et al. (1971) derived a theoretical dependence of turbulence strength on height above flat ground in the boundary layer. During periods of convection (generally clear days), $C_n^2$ decreases as the $-4/3$ power of height. At other times (night or overcast days), the power is nearly $-2/3$. No theory for the turbulence profile farther from the ground exists. Measurements show large variations in refractive turbulence strength. They all exhibit a sharply layered structure, where the turbulence appears in layers of the order of 100 m thick with relatively calm air in between. In some cases, these layers can be associated with orographic features; that is, the turbulence can be attributed to mountain lee waves. Generally, the turbulence decreases as height increases to a minimum value at a height of about 3-5 km. The level then increases to a maximum at about the tropopause (10 km) and decreases rapidly above the tropopause.

Based on these type of data, Hufnagel and Stanley (1964) and Hufnagel (1974) developed a model of an averaged profile of $C_n^2$ for altitudes of 3-20 km. It is probably the best available model for investigation of optical effects. To extend the model to local ground level, one should add the surface layer dependence (i.e., $h^{-4/3}$). To see the general dependence of $C_n^2$ on altitude, we plotted the average Hufnagel profile in Fig. 1. It has been extended to ground level using a $h^{-4/3}$ dependence with a value of $10^{-12} \text{ m}^{-2/3}$ at a height of 2 m. Note that this type of combination of models generally leaves a step in the profile at $h = 3$ km. Although this is not physical, it does not prevent the model from producing valid results in optical propagation problems.

Fig. 1. Typical height profile of the refractive turbulence structure parameter $C_n^2$. The solid line is the Hufnagel model with a $-4/3$ height dependence near the ground. The dashed line is the Hufnagel-Valley model with 5-cm coherence length and 7-µrad isoplanatic patch.
Another attempt to extend the model to ground level is the Hufnagel-Valley model (Sasiela, 1988), referred to as the HVs/7 model because it produces a coherence diameter (separation required for two receivers on the ground to observe incoherent fields from a source at zenith) $r_0$ of about 5 cm and an isoplanatic angle (angular separation required for two mutually coherent sources at zenith to produce mutually incoherent fields at a point on the ground) of about 7 μrad for a wavelength of 0.5 μm. It is plotted as a dashed line in Fig. 1. Although it is not as accurate at modeling turbulence near the ground, it has the advantage that the moments of turbulence profile important to propagation can be evaluated analytically (Sasiela, 1988).

Less is known about the vertical profiles of inner and outer scales. Over flat grassland in Colorado, we typically observe inner scales of 5-10 mm near the ground (1-2 m). Banakh and Mironov (1987) report calculated values of 0.5-9 mm at similar heights. Larger values (up to ∼10 cm) are expected higher in the atmosphere.

Near the ground, the outer scale can be estimated using Monin-Obukhov similarity theory (Monin and Obukhov, 1954). For typical daytime conditions, the outer scale is about one-half the height above the ground. Above the boundary layer, the situation is more complex. Weinstock (1978) calculated that $L_0$ should be about 330 m in moderate turbulence in the stratosphere. Barat and Bertin (1984) measured outer scale values of 10-100 m in a turbulent layer using a balloon-borne instrument.

3. BEAM WANDER

The first effect to consider is the wander of an optical beam caused by refractive inhomogeneities in the atmosphere. This wander is generally characterized statistically by the variance of the angular displacement. Both the magnitude of the displacement and the component along a single axis are used. For isotropic turbulence, the variance of the magnitude is simply twice the variance of the component.

For the downlink, the beam wander variance can be written as

$$s^2 = 2.92 \phi^{-1/3} H^{-7/3} \sec^2 \theta \int_0^\infty dh h^2 C_n^2(h), \quad (4)$$

where $\phi$ is the full-angle beam divergence, $\theta$ is the zenith angle, and $H$ is the orbital height. For the GLRS system, $\phi$ is about 100 μrad, $H$ is 824 km, and $\theta$ is between 0° and 70°. Using the $C_n^2$ profile of Fig. 1 with no inner or outer scale effects, the rms beam wander at 70° is 7.7 nrad. This is much less than the beam divergence and can be neglected.
For the uplink, the beam wander variance can be approximated by

\[ s^2 = 2.92D^{-1/3} \sec \theta \int dh' C_n^2 (h') , \]

where \( D \) is the diameter of the retroreflector and \( h \) is its height above the ground. If \( h \) is set to zero and a pure power law dependence of \( C_n^2 \) on height is used, the integral does not converge. The simplest solution to this mathematical problem is to assume that the retroreflector is at some height above zero.

For the GLRS system, we will assume a 10-cm-diameter retroreflector at a height of 1 m. For a \( C_n^2 \) value of 10\(^{-12}\) m\(^{-2/3}\) at a height of 2 m and a -4/3 dependence, the beam wander varies from 6.8 \( \mu \)rad at a zenith angle of 0° to 11.6 \( \mu \)rad at a zenith angle of 70°. The diffraction angle for this size reflector is about 13 \( \mu \)rad for the green wavelength and about 8.7 \( \mu \)rad for the ultraviolet wavelength, so the uplink beam wander can be a significant fraction of the beam size.

Since the wander from the uplink alone cannot be neglected under the strongest turbulence conditions, it is necessary to consider the effects of the correlation between the turbulence on the downlink with that on the uplink. Although the beam wander on the downlink can be neglected, there is also an angular deviation or tilt across the retroreflector induced by the downlink turbulence. This would result in a wander component at the receiver. However, the beam is reversed by the retroreflector and then propagates back through the atmosphere. If the propagation were through the exact same portion of the atmosphere, the tilt from the downlink would exactly cancel the wander induced on the uplink and there would be no wander. If the two propagation paths are not identical, only partial cancellation is obtained (Chumside, 1989). In the case of the GLRS, the two paths are slightly different because of the motion of the satellite during the propagation of the pulse. The retroreflector is not a true retroreflector, but has been designed to accommodate this path separation. This case has not been treated in the literature.

The derivation can be done using a geometric optics analysis following Chumside and Lateitis (1987, 1990) and Chumside (1989). For small values of \( \alpha \), the angle between the incident and reflected beams, the wander variance can be expanded in a Taylor series in \( \alpha \). For a circular orbit of 824 km, the orbital period is about 100 min and the orbital velocity is 7.44 km s\(^{-1}\). The round trip time of a light pulse is 5.49 ms at zenith and increases to 16.1 ms at a zenith angle of 70°. The beam separation angle varies from 49.6 \( \mu \)rad at zenith to 27.5 \( \mu \)rad at 70°. We calculated the rms beam wander for a 10-cm retroreflector using the high turbulence profile (solid line) of Fig. 1. The result was less than 1 \( \mu \)rad at any zenith angle and beam wander effects can be neglected.
4. BEAM SPREADING

The next effect to consider is the turbulence-induced spread of an optical beam as it propagates through the atmosphere. Here we are talking about the short-term beam spread, which does not include the effects of beam wander. The primary effect of beam spreading is to spread the average energy over a larger area. Thus, the average value of the on-axis irradiance is reduced and the average value of the irradiance at large angles is increased. Since beam spreading is a statistical quantity, the amount of the spreading fluctuates in time. This aspect has not been treated in depth in the literature.

We can consider beam wander to be caused by turbulent eddies that are larger than the beam. Beam spread is caused by turbulent eddies that are smaller than the beam. There are more small eddies in the beam at any time, which implies that the beam spread at any instant is averaged over more eddies. Thus, the fluctuations of beam spread are smaller than those of beam wander. Also, the smaller eddies are advected across the beam more quickly, and changes in beam spread are faster than changes in pointing angle. The long-term beam spread is defined as the turbulence-induced beam spread observed over a long time average. It includes the effects of the slow wander of the entire beam. The short-term beam spread is defined as the beam spread observed at an instant of time. It does not include the effects of beam wander, and is approximated by the long-term beam spread with the effects of wander removed, although the two are not identical.

Yura (1973) and Tavis and Yura (1976) used the extended Huygens Fresnel principle to calculate the short-term spread of a Gaussian beam. The results are collected and summarized by Fante (1975, 1980). For \( \rho_0 \) and \( l_0 \) much less than \( D \), the short-term beam spread is approximately given by

\[
P_s = \left\{ \frac{4}{k^2D^2} + \frac{D^2}{4L^2} \left( 1 - \frac{L}{F} \right)^2 + \frac{4}{k^2\rho_0^2} \left[ 1 - 0.62 \left( \frac{\rho_0}{D} \right)^{1/3} \right] ^{6/5} \right\} ^{1/2},
\]

where

\[
\rho_0 = \left[ 1.46 k^2 \int_0^L dz \left( \frac{z}{L} \right)^{5/3} C_n^2(\zeta) \right]^{-3/5}
\]

If \( \rho_0 \) is much greater than \( D \), the turbulence-induced component of beam spreading can be neglected.

Valley (1979) presents more complicated integral expressions that include inner-scale and outer-scale effects. Breaux (1978) performed numerical calculations for the case of a truncated Gaussian beam with a central obscuration. By curve fitting, he obtained the following approximation:
The high-turbulence values of coherence length may be less than the aperture diameter of the reflector. If so, the reflected beam will not be diffraction limited even before it propagates back through the atmosphere. Propagation back through the atmosphere will further spread the beam. If $\rho_0$ is greater than $D$, the effects of turbulence are small compared to diffraction effects. If $\rho_0$ is less than $D/2$, the beam reversal in the retroreflector will translate most points in the field by more than $\rho_0$. These points will then propagate back through a perturbation that is uncorrelated with the initial perturbation. Therefore, it is reasonable to consider the effects of the downlink and the uplink statistically independent. As turbulence effects get larger, this approximation gets better. We can include the effects of uplink and downlink turbulence by multiplying $C_n^2$ by 2 in Eq. (10).

The ratios of the round-trip, short-term beam spread to the diffraction beam spread are plotted in Fig. 2 for the high turbulence profile of Fig. 1 and a 10-cm-diameter reflector at a height of 1 m. The solid lines use the Gaussian aperture formula of Eq. (6) with $D = 7.07$ cm [an exp(-2) intensity diameter of 10 cm]. The dashed line is the uniform circular aperture formula of Eqs. (8) and (9) with a 10-cm aperture diameter. In the ultraviolet and at large zenith angles in the visible, $D/r_0$ is greater than 7.5 for this turbulence profile, and the uniform aperture formula does not apply. Where both are valid, the numbers are fairly similar after normalization by the diffraction limit.

Fig. 2. Ratio of short-term beam spread to diffraction-limited value as a function of zenith angle $\theta$ for Gaussian-aperture formula (solid line) and circular-aperture formula (dashed line).
\[ p_s = \left( 1 + 0.182 \frac{D^2}{r_0^2} \right) p_d, \quad (8) \]

where \( D \) is the effective aperture, \( r_0 = 2.099 \rho_0 \), and \( p_d \) is the diffraction limited value. This expression is valid for \( D/r_0 < 3 \). For \( 3 < D/r_0 < 7.5 \), the expression is

\[ p_s = \left[ 1 + \frac{D}{r_0} - 1.18 \left( \frac{D}{r_0} \right)^{-3/3} \right]^{1/2} p_d. \quad (9) \]

These expressions agree fairly well with available data (Dowling and Livingston, 1973; Cordray et al., 1981; Searles et al., 1991) and are similar to the previous calculations.

For the GLRS downlink, the beam spread is calculated using the point source phase coherence length for propagation from the ground to the satellite. If we use the turbulence profile of Fig. 1 and a zenith angle of 70°, we estimate that the phase coherence length is about 13 m for the 532-nm wavelength and about 8 m for the 355-nm wavelength. The corresponding long-term beam spreads are 13 nrad and 14 nrad. Thus, we conclude that beam spread on the downlink can be neglected.

For the uplink, we calculate \( \rho_0 \) for propagation from space to the earth. The formula is

\[ \rho_0 = \left[ 1.46 k^2 \sec \theta \int dh' C_n^2(h') \right]^{-3/5}. \quad (10) \]

For the turbulence profile of Fig. 1, the coherence length for propagation from a satellite at 70° zenith angle to a height of 1 m is 4.0 mm at 355 nm and 6.6 mm at 532 nm. The formula assumes that \( \rho_0 \) is much greater than the inner scale, which may not be valid under the conditions of this example. However, these values of \( \rho_0 \) are not much less than expected \( l_0 \) values and are not expected to be too far off.

For the 532-nm wavelength at zenith, the Fried coherence length \( r_0 \) is about 26 mm using the profile from Fig. 1. Fried and Mevers (1974) used astronomical data to infer \( r_0 \) values at two sites. They found a log-normal distribution of values ranging from about 30 mm to about 350 mm. Walters et al. (1979) observed values of about 20 mm to about 300 mm, also at an astronomical site. Walters (1981) made measurements at mountain and desert sites and found a similar range of values. Thus, the turbulence profile used seems to be a reasonable high-turbulence limit.
The big difference in the numbers is in the diffraction. Equation (6) implies a diffraction limited beam spread of \( \lambda/\pi D \), where \( D \) is the \( \exp(-1) \) intensity diameter of the transmitter. If we convert from \( \exp(-1) \) values to \( \exp(-2) \) values and convert to the full angle divergence, the corresponding beam spread is \( 4\lambda/\pi D \). The full angle to the first minimum for a uniform aperture is \( 2.44\lambda/D \), which is almost twice as high. The difference is partly due to the difference in definitions of beam divergence and partly due to the fact that a Gaussian beam will be diffracted less than a uniform one. In the visible, \( 2.44\lambda/D = 13 \) \( \mu \)rad; the turbulence-induced beam spread can be six or seven times this, even at zenith. In the ultraviolet, the diffraction is less, but turbulence has more of an effect. The net result for this example is that the beam spread will be on the order of 100 \( \mu \)rad for both wavelengths near zenith.

We note that the beam spread depends on the \( 1/3 \) power of the height of the retroreflector above the ground under conditions of high turbulence. This implies that the irradiance in the center depends on the \( 2/3 \) power of reflector height. Thus, doubling the height will increase the average power at the center of the beam by almost 60%. This may be worth considering at sites where daytime surface turbulence is expected to be severe.

5. SCINTILLATION

The refractive index perturbations that distort the optical phase front also produce amplitude scintillations at some distance. The first cases to be considered were plane and spherical wave propagation through weak path-integrated turbulence. The weak turbulence condition requires that fluctuations of irradiance be much less than the mean value. Tatarskii (1961) used a perturbation approach to the wave equation. Lee and Harp (1969) used a more physical approach to arrive at the same results. These results are summarized in a number of good reviews (Lawrence and Strohbehn, 1970; Fante, 1975, 1980; Clifford, 1978).

For propagation from the satellite to the ground, the plane wave formula is valid. The variance of irradiance fluctuations (normalized by the mean irradiance value) is given by

\[
\sigma_i^2 = \exp \left[ 2.24 k^{7/6} \sec^{11/6} \theta \int_0^\infty dh h^{5/6} C_n^2(h) \right] - 1. \tag{11}
\]

For the GLRS downlink, the rms fluctuations vary from 62% for the 0.532 \( \mu \)m link at zenith to 308% at 70° and from 83% for the 0.355 \( \mu \)m link at zenith to 650% at 70°. Near zenith, these values are small enough that the weak turbulence approximation is probably not too bad. Off zenith, the available theory is much more complex. Note that the visible values are similar to measured values of stellar scintillation (Jakeman et al., 1978; Parry and Walker, 1980), as one would expect.

For the uplink, the effects of the finite beam must be considered. Kon and Tatarskii (1965) calculated the amplitude fluctuations of a collimated beam using the perturbation
technique. Schmeltzer (1967) extended these results to include focused beams. Fried and Seidman (1967), Fried (1967), and Kinoshita et al. (1968) used these results to obtain numerical values for a variety of propagation conditions. Ishimaru (1969a, 1969b, 1978) used a spectral representation to obtain similar results.

The case of interest, however, is not a collimated beam transmitted from the ground. Turbulence on the downlink adds scintillation. It also adds phase distortion at the reflector that creates additional scintillation as the beam propagates back up to the satellite. The case of a retroreflector embedded in refractive turbulence can be treated in the same weak-turbulence approximation that has been used throughout. Most work in this area has been done in the Soviet Union. An excellent review of this work is given in Banakh and Mironov (1987).

One interesting feature of the results of retroreflector calculations is that the fluctuations in the reflected light are maximum at the optical axis and decrease as the observation point moves off the axis. This effect might tend to counteract the tendency of a beam wave to have minimum fluctuations on the axis. However, these calculations are all for uniform turbulence and do not account for the propagation to the far field. They have also only been done for reflectors that are very large or very small in comparison to the Fresnel zone size.

For observation points near the center of the returned beam,

\[
s_{l}^{2} = \exp \left\{ 8.70 \frac{k^{3/6}}{L^{11/6}} \int_{0}^{1} du \left( C_{n}^{2}(u) \Re \left[ 4g_{2}^{5/6} - 4g_{1}^{5/6} \right] \right) \right\} - 1 ,
\]

where

\[
g_{1} = \frac{1}{2} \frac{\delta u^{2}}{1 + \delta^{2}} ,
\]

\[
g_{2} = g_{1} + i \left[ u(1-u) + \frac{1}{2} \frac{\delta^{2}u^{2}}{1 + \delta^{2}} \right] ,
\]

\[
g_{3} = \frac{\delta^{2}u^{2}}{(1 + \delta^{2})^{2}} - \left( 1 - u + \frac{\delta^{2}u^{2}}{1 + \delta^{2}} \right)^{2} ,
\]

\[
g_{4} = \frac{1 + \delta^{2} - u}{(1 + \delta^{2})^{2}} \delta u .
\]
Representative values of $\sigma^2$ have been calculated using this expression. In Fig. 3, we have presented the variance as a function of the distance of the observation point from the beam axis for vertical propagation. From this figure, we see that the variance is reduced as the observation point moves off the optical axis, in agreement with previous reflected beam results. It does not increase as with the upward propagating beam case. Thus we conclude that the round-trip propagation effects must be included to properly account for turbulence in the GLRS.

Figure 3 includes values for one reflector at a height of 1 m above the ground and one at a height of 10 m. We see a significant difference at both wavelengths. In the center of the ultraviolet beam, the improvement obtained by raising the reflector is about a factor of 2 in the variance. Of course, the turbulence profile considered here is for strong daytime turbulence near the ground. At night, the improvement would be less.

In Fig. 4, we investigated the zenith angle dependence of the visible wavelength with a reflector at 1 m. At a zenith angle of about 30°, the variance begins to increase rapidly. At these scintillation levels, the weak turbulence approximation of this theory is invalid. Investigation of the scintillation at these levels must be done with a numerical simulation of the type done by Lightsey et al. (1991).

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![Graph](image.png)

**Fig. 3.** Irradiance variance $\sigma^2$ for the GLRS geometry as a function of the distance of the observation point from the optical axis $\rho$ divided by the Fresnel zone size $(L/k)^{1/2}$. Solid lines are for a retroreflector height of 1 m and dashed lines for a height of 10 m.
Fig. 4. Irradiance variance $\sigma_I^2$ for the GLRS geometry as a function of zenith angle $\theta$ for the visible wavelength $\lambda$ and a reflector height of 1 m.

6. CONCLUSIONS

The first conclusion is that the effects of beam wander can probably be neglected.

The next conclusion is that turbulence-induced beam spreading will probably be significant under conditions of high turbulence. The available theory can be used to make reasonable estimates of the magnitude of this effect.

The most significant conclusion of this report is that substantial scintillations can be expected. The round-trip propagation geometry must be taken into consideration when scintillation levels are evaluated; the uplink beam propagation calculation is qualitatively unable to predict the effect of moving the point of observation off of the optical axis.

Furthermore, the weak-turbulence theories that have been developed for scintillation are not valid under the strong-turbulence conditions that can be expected at times in the GLRS system. A numerical simulation will probably be necessary to calculate values for various cases.

We recommend that a numerical simulation be performed to evaluate the scintillation for round-trip propagation to a retroreflector in the case of strong turbulence near the reflector. Following this, an experiment should be performed to verify the results. A first experiment could be done in the laboratory with a layer of artificially generated turbulence in front of the reflector. This could be followed by an aircraft experiment using atmospheric boundary layer turbulence in a configuration similar to the actual GLRS geometry.
REFERENCES


