Approximate Green’s Function Methods for HZE Transport in Multilayered Materials

John W. Wilson
Langley Research Center
Hampton, Virginia

Francis F. Badavi
Christopher Newport University
Newport News, Virginia

Judy L. Shinn and Robert C. Costen
Langley Research Center
Hampton, Virginia
Abstract

A nonperturbative analytic solution of the high charge and energy (HZE) Green's function is used to implement a computer code for laboratory ion beam transport in multilayered materials. The code is established to operate on the Langley nuclear fragmentation model used in engineering applications. Computational procedures are established to generate linear energy transfer (LET) distributions for a specified ion beam and target for comparison with experimental measurements. The code was found to be highly efficient and compared well with the perturbation approximation.

Introduction

Green's function was identified as the likely means of generating efficient high charge and energy (HZE) shielding codes for space engineering that are capable of being validated in laboratory experiments (ref. 1). A derivation of Green's function as a perturbation series gave promise for the development of a laboratory-validated engineering code (ref. 2), but computational inefficiency provided a major obstacle to the code development (ref. 3). More recently, nonperturbative approximations to the HZE Green's function have shown promise in providing an efficient validated engineering code (refs. 4 and 5). Described in the present report is a laboratory code using a nonperturbative Green's function to derive linear energy transfer (LET) spectra behind multilayered targets for ion beams with $Z$ (charges) $\leq 28$ corresponding to the major components of the galactic cosmic ray spectrum.

Green's Function for a Single Medium

We restrict our attention to the multicharged ions (atomic number $Z_j$ and atomic mass $A_j$) for which the Boltzmann equation may be reduced (ref. 6) to

$$\left[ \frac{\partial}{\partial x} - \frac{\partial}{\partial E} \tilde{S}_j(E) + \sigma_j \right] \phi_j(x, E) = \sum_k \sigma_{jk} \phi_k(x, E)$$

(1)

where $\phi_j(x, E)$ is the ion flux at $x$ with energy $E$ (in MeV/amu), $\tilde{S}_j(E)$ is the change in $E$ per unit distance, $\sigma_j$ is the total macroscopic reaction cross section, and $\sigma_{jk}$ is the macroscopic cross section for the collision of an ion of type $k$ to produce an ion of type $j$. The solution to equation (1) is to be found subject to the boundary condition

$$\phi_j(0, E) = f_j(E)$$

(2)

which, for laboratory beams, has only one value of $j$ for which $f_j(E)$ is not zero and where $f_j(E)$ is described by a mean energy $E_o$ and energy spread $\sigma$ such that

$$f_j(E) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left[ -\frac{(E - E_o)^2}{2\sigma^2} \right]$$

(3)

The usual method of solution is to proceed toward solving equation (1) as a perturbation series (refs. 1 and 6). In practice, the computational requirements limit the usefulness of the technique for deep penetration (ref. 3).
The Green’s function \( G_{jm} \) is introduced as a solution of

\[
\left[ \frac{\partial}{\partial x} - \frac{\partial}{\partial E} \tilde{S}_j(E) + \sigma_j \right] G_{jm}(x, E, E_o) = \sum_k \sigma_{jk} G_{km}(x, E, E_o) \tag{4}
\]

subject to the boundary condition

\[
G_{jm}(0, E, E_o) = \delta_{jm} \delta(E - E_o) \tag{5}
\]

where \( \delta_{jm} \) is Kronecker’s \( \delta \) and \( \delta(E - E_o) \) is Dirac’s \( \delta \) function. The solution to equation (1) is given by superposition as

\[
\phi_j(x, E) = \sum_k \int G_{jk}(x, E, E') f_k(E') \, dE' \tag{6}
\]

If \( G_{jk}(x, E, E') \) is known as an algebraic quantity, the evaluation of equation (6) may be accomplished by simple integration techniques, and then the associated errors in numerically solving equation (1) are avoided (ref. 7).

The above equations can be simplified by transforming the energy into the residual range \( (r_j) \) as

\[
r_j = \int_0^E \frac{dE'}{\tilde{S}_j(E')}
\]

and defining new field functions as

\[
\psi_j(x, r_j) = \tilde{S}_j(E) \phi_j(x, E)
\]

\[
G_{jm}(x, r_j, r_m') = \tilde{S}_j(E) \sum_k \sigma_{jk} G_{km}(x, r_k, r_m')
\]

Thus, equation (4) becomes

\[
\left( \frac{\partial}{\partial x} - \frac{\partial}{\partial r_j} + \sigma_j \right) G_{jm}(x, r_j, r_m') = \sum_k \nu_{jk} \sigma_{jk} G_{km}(x, r_k, r_m') \tag{11}
\]

with the boundary condition

\[
G_{jm}(0, r_j, r_m') = \delta_{jm} (r_j - r_m') \tag{12}
\]

and with the solution to the ion fields given by

\[
\psi_j(x, r_j) = \sum_m \int_0^\infty G_{jm}(x, r_j, r_m') \tilde{f}_m(r_m') \, dr_m' \tag{13}
\]

Note that \( \nu_j \), which is the range scale factor as \( \nu_j r_j = \nu_m r_m \), is taken as \( \nu_j = Z_j^2/A_j \). The solution to equation (11) is written as a perturbation series

\[
G_{jm}(x, r_j, r_m') = \sum_i G_{jm}^{(i)}(x, r_j, r_m') \tag{14}
\]
in which
\[
G_{jm}^{(0)}(x, r_j, r'_m) = g(j) \delta_{jm} \delta(x + r_j - r'_m)
\]
and
\[
G_{jm}^{(1)}(x, r_j, r'_m) \approx \frac{\nu_j \sigma_{jm} g(j, m)}{x (\nu_m - \nu_j)}
\]
where \(G_{jm}^{(1)}(x, r_j, r'_m)\) is zero unless
\[
\frac{\nu_j}{\nu_m} (r_j + x) \leq r'_m \leq \frac{\nu_j}{\nu_m} r_j + x
\]
for \(\nu_m > \nu_j\). If \(\nu_j > \nu_m\), as can happen in neutron removal, the negative of equation (16) is used and the upper and lower limits of equation (17) are switched. The higher terms are approximated as
\[
G_{jm}^{(i)}(x, r_j, r'_m) \approx \sum_{k_1, k_2, \ldots, k_{i-1}} \frac{\nu_j \sigma_{jk_1 k_2 \ldots k_{i-1} m} g(j, k_1, k_2, \ldots, k_{i-1}, m)}{x (\nu_m - \nu_j)}
\]
In the above equations,
\[
g(j) = e^{-\sigma_j x}
\]
and
\[
g(j_1, j_2, \ldots, j_{n-1}, j_n, j_{n+1}) = \frac{g(j_1, j_2, \ldots, j_{n-1}, j_n) - g(j_1, j_2, \ldots, j_{n-1}, j_{n+1})}{\sigma_{j_{n+1}} - \sigma_{j_n}}
\]
Note that \(G_{jm}^{(i)}(x, r_j, r'_m)\) is purely dependent on \(x\) for \(i > 0\), which we represent as \(G_{jm}^{(i)}(x)\). (See ref. 3.) In terms of the above, the solution to equation (1) becomes (from ref. 3)
\[
\psi_j(x, r_j) = e^{-\sigma_j x} \hat{\psi}_j (r_j + x)
+ \sum_{m, i} G_{jm}^{(i)}(x) \left[ \hat{F}_m (r'_m) - \hat{F}_m (r'_{m\ell}) \right]
\]
In equation (21), \(r'_{m\ell}\) and \(r'_{m\ell}\) are given by the upper and lower limits of the inequality of equation (17). The symbol \(\hat{F}_m(r'_m)\) refers to the integral spectrum
\[
\hat{F}_m(r'_m) = \int_{r'_m}^{\infty} \hat{f}_m(r) \, dr
\]
We note that
\[
\hat{F}_m(r'_m) \equiv F_m(E')
\]
with
\[
F_m(E') = \int_{E'}^{\infty} f_m(E) \, dE
\]
We now introduce nonperturbative terms for the summation in equation (21).

First, we recall that the $g$ function of $n$ arguments was generated by the perturbation solution of the transport equation neglecting ionization energy loss (ref. 1) given by

$$\left[ \frac{\partial}{\partial x} + \sigma_j \right] g_{jm}(x) = \sum_k \sigma_{jk} g_{km}(x)$$

subject to the boundary condition

$$g_{jm}(0) = \delta_{jm}$$

for which the solution is

$$g_{jm}(x) = \delta_{jm} g(m) + \sigma_{jm} g(j,m) + \cdots$$

It is also true that

$$g_{jm}(x) = \sum_k g_{jk}(x - y) g_{km}(y)$$

for any positive values of $x$ and $y$. Equation (29) may be used to propagate the function $g_{jm}(x)$ over the solution space after which

$$\mathcal{G}_{jm}(x, r_j, r_m') \approx e^{-\sigma_j x} \delta_{jm} \delta(x + r_j - r_m') + \frac{\nu_j \left[ g_{jm}(x) - e^{-\sigma_j x} \delta_{jm} \right]}{x (\nu_m - \nu_j)}$$

The approximate solution of equation (1) is then given by

$$\psi_j(x, r_j) \approx e^{-\sigma_j x} \tilde{f}(r_j + x) + \sum_m \frac{\nu_j \left[ g_{jm}(x) - e^{-\sigma_j x} \delta_{jm} \right]}{x (\nu_m - \nu_j)} \left[ \tilde{F}_m(r_m') - \tilde{F}_m(t_m') \right]$$

which is a relatively simple quantity (ref. 4).

**Green's Function in a Shielded Medium**

The major simplification in the Green's function method results from the fact that the scaled spectral distribution of secondary ions to a first approximation depends only on the depth of penetration as seen in equations (16), (18), and (30). Our first approach to a multilayered Green's function will rely on this observation and assume its validity for multilayered shields.

If we consider a domain labeled as "1" that is shielded by a second domain labeled as "2," the number of type $j$ ions at depth $x$ in 1 due to type $m$ ions incident on domain 2 of thickness $y$ is

$$g_{12jm}(x, y) = \sum_k g_{1jk}(x) g_{2km}(y)$$
The leading term in equation (32) is the penetrating primaries as

\[ g_{12jm}(x, y) = e^{-\sigma_1 x - \sigma_2 y} \delta_{jm} + \left[ g_{12jm}(x, y) - e^{-\sigma_1 x - \sigma_2 y} \delta_{jm} \right] \]  

(33)

in which all higher order terms are within the brackets of equation (33).

The first term of the scaled Green's function is then

\[ G^{(0)}_{12jm}(x, y, r_j, r_m') = e^{-\sigma_1 x - \sigma_2 y} \delta_{jm} \delta \left[ x + r_j - (r_m' - \rho y) \right] \]  

(34)

in which \( \rho \) is the range scale factor for the two media

\[ \rho = \frac{R_{1j}(E)}{R_{2j}(E)} \]  

(35)

The ratio of range in water to range in aluminum for proton beams (eq. (35)) is shown in sketch A. We take a single value for \( \rho \) corresponding to 600 MeV/amu. The secondary contribution is similarly found by noting that equation (17) becomes

\[ \frac{\nu_j}{\nu_m}(x + \rho y) \leq r_m' \leq \frac{\nu_j}{\nu_m} r_j + x + \rho y \]  

(36)

from which the average spectrum is evaluated. The full approximate Green's function is then

\[ G_{12jm}(x, y, r_j, r_m') \approx e^{-\sigma_1 x - \sigma_2 y} \delta_{jm} \delta \left( x + \rho y + r_j - r_m' \right) \]

\[ + \frac{\nu_j \left[ g_{12jm}(x, y) - e^{-\sigma_1 x - \sigma_2 y} \delta_{jm} \right]}{(x + \rho y) (\nu_m - \nu_j)} \]  

(37)

Equation (37) is our first approximation to the Green's function in a shielded medium (two layers) and is easily modified to multilayers. We now consider the first spectral modification.
Showing that the first collision term has the properties

$$
G^{(1)}_{12jm} (x, y, r_j, r'_m) = \begin{cases} 
\frac{\nu_j \sigma_{1jm} e^{-\sigma_{1m} x - \sigma_{2m} y}}{\nu_m - \nu_j} & (r'_m = r'_{mu}) \\
\frac{\nu_j \sigma_{2jm} e^{-\sigma_{1j} x - \sigma_{2j} y}}{\nu_m - \nu_j} & (r'_m = r'_{m\ell})
\end{cases}
$$

(38)

is easy; we use these properties to derive a simple correction for the average spectrum as

$$
G^{(1)}_{12jm} (x, y, r_j, r'_m) = \frac{\nu_j g^{(1)}_{12jm} (x, y)}{(x + \rho y) (\nu_m - \nu_j)} + b_{jm} (x, y) (r'_m - \bar{r}_m)
$$

(39)

where $g^{(1)}_{12jm} (x, y)$ is the first collision term of equation (39) and

$$
\bar{r}_m = \frac{r'_{mu} + r'_{m\ell}}{2}
$$

(40)

is the midpoint of $r'_m$ between its limits given by equation (36). The $b_{jm}$ term of equation (39) has the property that

$$
\int_{r'_{m\ell}}^{r'_{mu}} b_{jm} (x, y) (r' - \bar{r}_m) \, dr' = 0
$$

(41)

thus ensuring that the first term of equation (39) is, indeed, the average spectrum as required. The spectral slope parameter is found to be

$$
b_{jm} (x, y) = \frac{\nu_j \nu_m \left( \sigma_{1jm} e^{-\sigma_{1m} x - \sigma_{2m} y} - \sigma_{2jm} e^{-\sigma_{1j} x - \sigma_{2j} y} \right)}{(x + \rho y) (\nu_m - \nu_j) |\nu_m - \nu_j|}
$$

(42)

A similarly simple spectral correction can be made to the higher order terms. The spectral correction given in equation (42) will be included in the present Green’s function code.

**LET Spectra for Laboratory Beams**

We use the boundary condition appropriate for laboratory beams given by equation (3). The cumulative spectrum is given by

$$
F_j (E) = \frac{1}{2} \left[ 1 - \text{erf} \left( \frac{E - E_0}{\sqrt{2\sigma}} \right) \right]
$$

(43)

The cumulative energy moment needed to evaluate the spectral correction is

$$
\bar{E}_j (E) = \frac{1}{2} E_0 \left[ 1 - \text{erf} \left( \frac{E - E_0}{\sqrt{2\sigma}} \right) \right] + \frac{\sigma}{\sqrt{2\pi}} \exp \left[ -\frac{(E - E_0)^2}{2\sigma^2} \right]
$$

(44)

The average energy on any subinterval $(E_1, E_2)$ is then

$$
\overline{E} = \frac{\bar{E}_j (E_1) - \bar{E}_j (E_2)}{F_j (E_1) - F_j (E_2)}
$$

(45)
The beam-generated flux is

\[
\psi_j (x, y, r_j) = e^{-\sigma_1 x - \sigma_2 y} \tilde{f}_j (r_j + x + \rho y) \\
+ \sum_{m,i} g_{jm}^{(i)} (x, y) \left[ \tilde{F}_m (r'_{mu}) - \tilde{F}_m (r'_{ml}) \right] \\
+ \sum_{m} h_{jm}^{(1)} (x, y) \left[ r'_m (\tilde{E}) - r'_m \right] \left[ \tilde{F}_m (r'_{mu}) - \tilde{F}_m (r'_{ml}) \right]
\]

(46)

where \(\tilde{E}\) is evaluated using equation (45) with \(E_1\) and \(E_2\) as the lower and upper limits associated with \(r'_{mu}\) and \(r'_{ml}\), respectively.

The differential fluence spectra for a 600-MeV/amu \(^{56}\text{Fe}\) beam with a 2.5-MeV/amu standard deviation incident on a water slab are shown in figure 1. A single layer of 5 cm of water is shown in figure 1(a) and a double layer of 2.5 cm of water followed by 2.5 cm of water is shown in figure 1(b). A consistency check is performed on the multilayered code by comparing it with the single-layered computation when the two layers are of the same size and composition. The ratio of multilayered results to single-layered results differs by less than \(\pm 1\) percent. The LET distribution is found by using the methods of reference 8. The corresponding LET spectra are shown in figure 2. The highest LET peak is due to the primary beam and the iron fragments. The successive peaks below iron are due to lower atomic weight fragments. The lowest LET peak consists of relativistic charge fragments of \(p, d,\) and \(t\) particles that are produced in abundance in HZE collisions (ref. 9). The peak near 10 MeV/cm consists of \(he\)- and \(\alpha\)-particles that are also produced abundantly. Many of the HZE fragments are produced with a charge near the projectile charge, as Shinn, Townsend, and Wilson found earlier for hydrogenic targets (ref. 10). Note that no distinguishable difference exists between the LET spectra of the single-layered code (fig. 2(a)) and the multilayered code (fig. 2(b)) at the same penetration depth in water, which further demonstrates code consistency.

A series of evaluations for a (2.24-g/cm\(^2\)) lead-scattering foil is shown in figure 3. The lead-scattering foil is usually part of the accelerator beam line at the Lawrence Berkeley Laboratory Bevalac accelerator with the result that the fragments from the lead target are seen as contamination. Clearly, these fragments must be modeled to properly interpret the attenuation of the beam in the water target in actual experiments. The corresponding LET spectra at various depths of a water target are shown in figure 4. The importance of the fragmentation in the scattering foil is seen in comparing figure 4(a) with figure 4(b). Note that fewer of the fragmentations result in fragments near the beam charge for the lead foil in comparison with the water target, as seen by comparing the relative magnitudes of the three highest LET peaks in figures 2(a) and 4(a). These differences are part of the reason why hydrogenic targets are important to space radiation protection. At greater depths the LET distributions begin to overlap, and distinguishing the different charge groups becomes more difficult. Such LET spectra will be compared with experimental measurements in the near future.

Concluding Remarks

A formalism for the evaluation of Green's function in multilayered target configurations has been derived and a computer code generated. The code satisfies a consistency check for multilayered-material calculations when the layers are of uniform composition by comparing the results with the single-layered code. The importance of the multilayered code in the analysis of
experimental ion beams has been shown by demonstrating the effects of a lead-scattering foil in the Bevalac beam line. An analysis of such experiments is in progress.

References


Figure 1. Differential fluence spectra for 600-MeV/amu $^{56}$Fe beam with 2.5-MeV/amu standard deviation incident on water slab.

(a) 0 cm of H$_2$O followed by 5 cm of H$_2$O.

(b) 2.5 cm of H$_2$O followed by 2.5 cm of H$_2$O.
Figure 2. LET distribution for 600-MeV/amu $^{56}$Fe beam with 2.5-MeV/amu standard deviation incident on water slab.

(a) 0 cm of H$_2$O followed by 5 cm of H$_2$O.

(b) 2.5 cm of H$_2$O followed by 2.5 cm of H$_2$O.
Figure 3. Differential fluence for 525-MeV/amu $^{56}$Fe beam with 2.5-MeV/amu standard deviation after passing through 2.24-g/cm$^2$ lead-scattering foil and water target.
Figure 3. Concluded.

(c) 10 cm of H$_2$O.
Figure 4. LET distribution for 525-MeV/amu ^{56}\text{Fe} beam with 2.5-MeV/amu standard deviation after passing through 2.24-g/cm^2 lead-scattering foil and water target.
Figure 4. Concluded.

(c) 10 cm of H\textsubscript{2}O.
Approximate Green's Function Methods for HZE Transport in Multilayered Materials

John W. Wilson, Francis F. Badavi, Judy L. Shinn, and Robert C. Costen

NASA Langley Research Center
Hampton, VA 23681-0001

L-17302

Unclassified Unlimited

Subject Category 93

A nonperturbative analytic solution of the high charge and energy (HZE) Green's function is used to implement a computer code for laboratory ion beam transport in multilayered materials. The code is established to operate on the Langley nuclear fragmentation model used in engineering applications. Computational procedures are established to generate linear energy transfer (LET) distributions for a specified ion beam and target for comparison with experimental measurements. The code was found to be highly efficient and compared well with the perturbation approximation.