Dielectric gratings have long been used for various applications at both optical and microwave frequencies; however, most of the optical applications were limited to performance regimes dictated by scalar theories and assumptions. Recently, with the development of finite difference time domain (FDTD) methods [1], both transient and steady-state solutions can be modeled to exploit the spectral filtering properties of chirped grating structures. In this paper, FDTD techniques are used to analyze various types of surface-relief structures including chirped dielectric gratings.

The FDTD method for solving Maxwell's time-dependent curl equations is accurate, computationally efficient, and straightforward to implement. Since both time and space derivatives are employed, the propagation of an electromagnetic wave can be treated as an initial-value problem. Second-order central-difference approximations are applied to the space and time derivatives of the electric and magnetic fields providing a discretization of the fields in a volume of space, for a period of time. The solution to this system of equations is stepped through time thus simulating the propagation of the incident wave. If the simulation is continued until a steady-state is reached, an appropriate far-field transformation can be applied to the time-domain scattered fields to obtain reflected and transmitted powers. From this information, diffraction efficiencies can also be determined.

In analyzing the chirped structure, a mesh is applied only to the area immediately around the grating. The size of the mesh is then proportional to the electric size of the grating. Doing this, however, imposes an artificial boundary around the area of interest. An absorbing boundary condition must be applied along the artificial boundary so that the outgoing waves are absorbed as if the boundary were absent. Many such boundary conditions have been developed that give near-perfect absorption. In this analysis, the Mör absorbing boundary conditions are employed [2].

Several grating structures were analyzed using the FDTD method. First, the method was validated by comparing its results to that of the coupled-wave approach [3]. Two periods of a binary periodic grating were modeled in the center of an extended surface. The period was two wavelengths and the depth was one wavelength as measured in free space. The refractive index of the dielectric was 1.5. Both TE and TM polarizations were calculated. A comparison between the two methods is illustrated in Figures 1 and 2. Figure 1 shows the amplitude and phase for TE polarization calculated at the interface, and Figure 2 shows the same for TM polarization. In the FDTD analysis, the simulation was run until a steady-state was reached. At this point, the amplitude and phase were extracted from the complex field
values. The agreement is quite good. The FDTD results appear to be noisier due to the imperfect absorbing boundary conditions applied at the edges of the computation region.

Figure 1. Comparison Between FDTD and Coupled-Wave, TE Case

Figure 2. Comparison Between FDTD and Coupled-Wave, TM Case
Next, the number of periods was increased to demonstrate the effects of finite structures. In addition to the case mentioned above with two periods, four more cases were run with the maximum number of periods being ten. The far-field diffraction patterns were calculated by doing a fast Fourier transform on the amplitude and phase data. These results are shown in Figure 3. As would be expected, the zero-order decreases and the higher orders increase in intensity as the number of periods is increased. Figure 4 shows similar results from the same exercise performed on gratings with four phase levels. Just as a side note, the computation time required to analyze the four level gratings was no more than that for the binary gratings. This is a definite advantage of using this technique over the coupled-wave approach or the method of moments.

![Figure 3. Far-Field Diffraction Pattern for Binary Grating](image)

The final type of grating modeled was chirped gratings. The Fresnel zone equation

\[
    r_m = \sqrt{\frac{2mf\lambda}{n} + \left[\frac{m\lambda}{n}\right]^2}
\]

where
- \(r_m\) = m-th zone radius
- \(f\) = focal length
- \(\lambda\) = free-space wavelength
- \(n\) = dielectric refractive index

was used to determine the radii of the zones. If a large enough region is simulated, the focussing effect of the lens can be seen. Of equal interest is the behavior of the fields near the grating. A video of the time frames from this simulation shows this behavior as well as the focussing effect.
The finite difference time domain technique is well suited to analyzing finite grating structures. The results were seen to be very close to those achieved with the coupled-wave method. Also, since this method incorporates time, transient pulses can also be modeled.

References

