THEORY OF DISPERSIVE MICROLENS

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ABSTRACT

A dispersive microlens is a miniature optical element which simultaneously focuses and disperses light. Arrays of dispersive microlenses have potential applications in multi-color focal planes. They have a 100 percent optical fill factor and can focus light down to detectors of diffraction spot size, freeing up areas on the focal plane for on-chip analog signal processing. Use of dispersive microlenses allows inband color separation within a pixel and perfect scene registration. A dual-color separation has the potential for temperature discrimination. We discuss the design of dispersive microlenses and present sample results for efficient designs.

1.0 INTRODUCTION

For many applications it is desirable to determine the spectral characteristics of a light source. In certain applications it is necessary to do this over very small physical areas, for which miniature optical elements are required. The technology for manufacturing such devices has been available for a long time. Our interest is in describing a miniature optical element that simultaneously disperses and focuses light. We refer to such a device as a dispersive microlens.

Arrays of dispersive microlenses have potential applications in multicolor focal planes. They have a 100% optical fill factor and can focus light down to detectors of diffraction spot size, freeing up area on a focal plane for on-chip analog signal processing. Use of dispersive microlenses allows inband color separation within a pixel and perfect scene registration. A dual-color separation has the potential for use as a temperature discriminator.

We begin with our concept of the dispersive microlens and then give an analysis of its performance. This is followed by discussions of variations of the basic concept (Fresnel, multilevel analog, and multilevel Fresnel elements). We conclude with some examples of efficient designs.

2.0 DISPERSIVE MICROLENS CONCEPT

Dispersion of light can be caused either by material dispersion, where the material index of refraction is strongly wavelength dependent, or by coherent interference of light passing through (or reflecting from) a diffractive structure. The latter type of dispersion is seen in the familiar diffraction grating and is the type that we are interested in. For dispersive analysis, light is typically passed through (or reflected from) a diffraction grating and then focused by a lens. If an amplitude grating is used, half of the incident light energy does not get through the grating and leads to an inefficient setup. It is therefore desirable to use a phase grating to allow most of the incident light to be focused. Our objective is to combine the dispersion and focusing into a single optical element.

The efficiency of a diffraction grating depends upon its periodic structure. It is theoretically possible to obtain 100% efficiency (at a chosen wavelength) by using a periodic blazed structure. We therefore desire to construct an element which consists of a blazed diffraction grating combined with a focusing element. This concept is illustrated in Figure 1.

In our analysis we will derive the diffraction pattern due to a dispersive microlens and discuss how the different parameters affect the performance of the element.

3.0 ANALYSIS OF THE DISPERSIVE MICROLENS

For ease of exposition, we will assume the element to be illuminated by a normally incident plane wave. The analysis given below can be generalized to non-normal incidence or to light covering the full field of view of the lens. The analysis we give here can be applied to dispersive microlenses that are part of a telecentric system.

We will take the element to be optically thin so that the phase change of the plane wave travelling though it is proportional to the thickness of the element. The element sits upon a substrate (possibly air) through which the focusing and dispersion take place. We assume a scalar description of the field to be accurate enough for our purposes and employ the Fresnel approximation to the Rayleigh-Sommerfeld diffraction integral. This gives

\[
E(x, y, z) = \frac{n'}{i\lambda z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx' dy' \exp \left\{ \frac{i\pi}{\lambda z} \left[ (x - x')^2 + (y - y')^2 \right] \right\} E(x', y', 0)
\]

where \(E(x, y, z)\) is the electric field at point \((x, y, z)\). The field propagates in the \(z\) direction with the origin of the coordinate system lying on the substrate surface. The radiation has wavelength \(\lambda\) and the substrate has index of refraction \(n'\).

For the choice of a rectangular dispersive microlens of size \(w_x\) by \(w_y\), the evaluation of Eq. (1) at the focal plane of the lens yields

Here, $n$ is the refractive index of the element, $f$ is the focal length of the lens, $D$ is the modulation depth of the grating, $G$ is the grating period, $N = w_x/G$ is the number of grating periods across the element, and the sinc function is defined by $\text{sinc}(u) = \sin(\pi u) / (\pi u)$. The incident plane wave is taken to have an amplitude $E_0$.

The result Eq. (2) is essentially the Fraunhofer diffraction pattern of a blazed grating. The only difference is that we have replaced the usual frequency variables of Fourier space with explicit parameters involving the element and the radiation. It is important to do this since we will be interested in the element performance at different wavelengths and the wavelength is not a separate parameter in Fourier space.

Because of the nature of the result, we can use our knowledge of blazed gratings to interpret the diffraction pattern. The ratio of sine functions (due to interference effects from the grating) describes the size and position of the grating orders. Diffraction orders appear in the focal plane at positions where the argument of the sine function in the denominator is an integral multiple of $\pi$, i.e., at

$$x_j = \frac{\lambda f}{n'G}j$$

The product of sinc functions is the Fraunhofer diffraction pattern of a slit of dimensions $G$ by $w_x$, displaced in the focal plane because of the tilt of the blaze. The full diffraction pattern is proportional to the product of these two components, as shown in Figure 2.

The most efficient results for the dispersive microlens are obtained when the peak of the sinc functions is centered at a diffraction order. For a given material, this position is controlled by the radiation wavelength and the modulation depth of the grating. If we want the peak to fall at the $m^{th}$ order, then we must have $D = m\lambda/(n - 1)$. For the special case of the first diffraction order ($m = 1$), this defines a wavelength $\lambda_b = (n - 1)D$, which is called the blaze wavelength.

We now need to calculate efficiencies and dispersion to quantify the performance of the element. The efficiency of the $j^{th}$ order is the energy in the $j^{th}$ order divided by the incident energy on the element:

$$\eta_j = \frac{\int_{\text{order}} |E(x', y', f)|^2 dx' dy'}{\int_{\text{aperture}} |E(x', y', 0)|^2 dx' dy'}$$

(3)
For the case where there are a large number of periods across the element, this expression can be evaluated to give

$$\eta_j = \text{sinc}^2\left(\frac{\lambda_0}{\lambda - j}\right), \quad N > 1$$

Note that the maximum efficiency is obtained when $\lambda = \lambda_0/m$ and $j = m$, as expected. This maximum can be obtained only for a single selected wavelength. Since we are interested in a band of wavelengths, we cannot achieve unit efficiency for all wavelengths in the band. This equation tells us the wavelength spread, $\Delta\lambda/\lambda$, for which we can achieve a desired efficiency.

Perhaps a more useful performance measurement is the fraction of energy that gets into the main spot in the diffraction order. (This is important, for example, for determining the amount of energy falling on a detector placed under the element.) This is given by Eq. (3) except that the upper integral is evaluated over the main diffraction spot:

$$\theta_j = \frac{\int_{\text{main spot}} |E(x', y', f)|^2 dx' dy'}{\int_{\text{aperture}} |E(x', y', 0)|^2 dx' dy'}$$

Again, for a large number of grating periods, this can be evaluated to give

$$\theta_j = 0.903 \left\{ \frac{2}{N} \left[ 1 + 2 \sum_{k=1}^{N-1} \text{sinc} \left( \frac{2k}{N} \right) \right] \right\} \text{sinc}^2\left(\frac{\lambda_0}{\lambda - j}\right), \quad N > 1$$

In the limit $N \to \infty$, the factor in braces evaluates to 0.903 so that a maximum of 81.5% of the incident energy can be focused into the main diffraction spot.

We now examine the dispersion properties. Let us look at two wavelengths, $\lambda_0$ and $\lambda_1$, in the $j^\text{th}$ order. We have $x_j^{(\lambda_0)} = \lambda_0 f/n' G$ and $x_j^{(\lambda_1)} = \lambda_1 f/n' G$. Thus the wavelength separation in a particular order is given by

$$x_j^{(\lambda_1)} - x_j^{(\lambda_0)} = \frac{(\lambda_1 - \lambda_0)f}{n' G} j$$

Consequently, higher orders give increased dispersion.

Since the wavelength separation increases with order, eventually the orders for different wavelengths will overlap. This will first occur in adjacent orders. We can avoid overlap of orders by requiring $x_j^{(\lambda_{\text{max}})} + \lambda_{\text{max}} f/n' w_x < x_j^{(\lambda_{\text{min}})} - \lambda_{\text{min}} f/n' w_x$, which reduces to

$$j < \frac{\lambda_{\text{max}} - \lambda_{\text{min}}}{\lambda_{\text{max}} f/n' G} - \frac{\lambda_{\text{min}} f/n' W_x}{\lambda_{\text{max}} - \lambda_{\text{min}} w_x}$$

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Thus, we can get the maximum wavelength separation without spectral overlap of orders if we choose the optimal value of $j$ from Eq. (5). The characteristics of the diffraction patterns which depend on the order are shown in Figure 3. Spectral overlap is clearly seen as the order number increases.

4.0 DISPERSIVE MICROLENS DESIGN OPTIONS

So far, we have not addressed issues related to the fabrication of micro-optical elements. Although we have consistently referred to a dispersive microlens, there is, in fact, nothing in the theory that specifically addresses the size of the element (except, of course, the assumption of the validity of scalar wave theory). When one tries to fabricate miniature optical elements, a number of obstacles appear. We treat two of them here: the element thickness, which we address through the Fresnel element, and the surface smoothness, which we address through the multilevel analog element and the multilevel Fresnel element.

4.1 THE FRESNEL ELEMENT

A common methodology for fabricating micro-optical elements is through photolithographic techniques. The surface structure of the desired element is etched into a substrate to produce the desired result. Fast lenses or elements with deep surface reliefs present fabrication problems because of their large etching depth. One way to sidestep this problem is to convert the element into a Fresnel element whose maximum thickness corresponds to a $2\pi$ phase change at a given wavelength. This results in an element that has an overall thickness which is less than the type we have been discussing so far (the "analog" element). There is a tradeoff with using such a type of element, however. The $2\pi$ phase change can occur exactly only for a specific wavelength. Thus the element must be designed for a specific wavelength while all other wavelengths will suffer some degree of chromatic aberration. For the blazed grating, this wavelength is best taken to be an integral multiple of the blaze wavelength. We call this wavelength the design wavelength and denote it by $\lambda$. An example of the Fresnel element is shown in Figure 4.

In a first approximation, the Fresnel element introduces a focusing phase error (chromatic aberration) given by

$$\frac{\pi n'}{f}x^2\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

with a maximum error of

$$\frac{n'}{8f}w^2\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

waves, where $w$ is the larger of $w_x$ and $w_y$.

A more detailed and quantitative measure of the performance of Fresnel dispersive microlenses can be obtained by introducing a figure of merit that is the ratio of the energies reaching the main diffraction spot from the Fresnel and non-Fresnel elements. In other words, we define

\[ r_m^{\text{Fresnel}} = \frac{\theta_m^{\text{Fresnel}}}{\theta_m^{\text{analog}}} \]

where the element is designed to send light mainly to the \( m \)th order and the different \( \theta \)s refer to Eq. (4) evaluated for the different elements. Although other figures of merit are possible (e.g., a “chromatic” Strehl ratio), we choose this one since it provides a measure of the dispersive properties of the element along with how well the dispersed light is focused down to the theoretical spot size.

For a square element it can be shown that \( r_m^{\text{Fresnel}} \) depends on the following set of parameters: the order, \( m \), the number of grating periods across the dispersive microlens, \( N \), the ratio of the blaze and design wavelengths, \( \lambda_b / \lambda \), the ratio of the radiation and design wavelengths, \( \lambda / \lambda' \), and a parameter which we identify as a Fresnel number, \( \xi = n'w^2/\lambda f \). For the most common case of first-order diffraction where the blaze and design wavelengths are the same, this reduces to a set of only three parameters.

For most applications, only a few periods across the dispersive microlens are needed to give the desired dispersion; five is a typical number. In Figure 5 we plot \( r_m^{\text{Fresnel}} \) for dispersive microlenses with five grating periods in the case of first-order diffraction and \( \lambda_b = \lambda \). Note that the Fresnel dispersive microlens performs well only about a very narrow band about the design wavelength. This band gets smaller as the Fresnel number increases.

Similar plots for cases with up to nine grating periods show almost identical results. For cases of practical interest, then, we can ignore the effect of the number of grating periods and the results can be parameterized by using only the wavelength ratio and the Fresnel number.

Although not shown in Figure 5, it should also be noted that as the wavelength ratio approaches 1/2, the figure of merit begins to increase. This is because the Fresnel element operates efficiently at all the harmonics of the design wavelength, as does the dispersive microlens. The dispersive microlens is not used over a very large band, however, since the efficiencies decrease as the band gets large. Therefore, the behavior exhibited at the harmonics is usually out of the range of a given element design.

Although it may seem that the region of useful operation would be too small to be practical because of its narrowness, we can easily find cases where the Fresnel dispersive microlens is practicable. For example, an f/5, 100-\( \mu \)m square element operating in the infrared in silicon with a design wavelength of 10 \( \mu \)m has a Fresnel number ranging from 8.5 to 5.7 over a wavelength band from 8 to 12 \( \mu \)m. In this band, the figure of merit is at least 80%.

4.2 FABRICATION ISSUES

If the element is fabricated using standard photolithographic techniques, the surface relief of the element is approximated by etching a multilevel structure onto a substrate. The type of elements produced in this way have acquired the name binary optical elements. The etching of the
material is performed using a series of masks, each with an associated etching depth. The maximum number of levels that can be etched using k masks is $2^k$. The multilevel approximation changes the performance of the element. Both analog and Fresnel elements can be fabricated with multilevel approximations.

4.2.1 The Multilevel Analog Element

We will assume the multilevel approximation to the analog element to have $2^k$ levels. The number of levels determines the smoothness to which the depth of the element can be approximated. In Figure 6 we show the approximations to the element for two through eight levels.

We can get an estimate of the effect of the multistep structure on the diffraction pattern by using a multistep approximation to the grating (although not the focusing) part of the element. If we take $2^k$ levels and $2^k$ steps across a grating period, then a straightforward calculation shows that the diffraction pattern is given by Eq. (2) with the amplitude of the x-dependent sinc function modified by

$$\frac{\sin[\pi(\lambda_b/\lambda + j)]}{2^k \sin[\pi (\lambda_b/\lambda + j)]} \text{sinc} \left( \frac{j}{2^k} \right)$$

and a slight modification of the overall phase factor. If we optimize the grating for wavelength $\lambda$, Eq. (6) reduces to $\text{sinc}(j/2^k)$. The efficiencies and spot energies are affected in a similar manner.

Again, a more quantitative measure of performance can be constructed as was the case with the Fresnel element. We take the same figure of merit defined for the multilevel element:

$$r_m^{ML} = \frac{\theta_m^{ML}}{\theta_m^{\text{analog}}}$$

This figure of merit is parameterized in the same way as $r_m^{\text{Fresnel}}$ with the addition of another parameter, the number of levels, $2^k$. In most cases of practical interest, three binary masks are used in the fabrication process. We present in Figure 7 a plot of $r_m^{ML}$ for the same case as in Figure 5 with the additional parameter of $k = 3$ masks (8 levels).

This figure shows that the performance of the multilevel element is good at small Fresnel numbers but drops off rapidly at the Fresnel number increases. This behavior is expected because, in general, as the Fresnel number increases, either (a) the lens becomes faster so the stepped structure does not approximate the increased curvature of the surface as well, or (b) the wavelength gets smaller so the stepped structure looks less and less like a focusing element to the radiation.

If the same plot is made with an increasing number of levels, the same general result is found except that the fall-off behavior becomes more gradual and the oscillatory nature seen becomes shallower. For an infinite number of levels, one expects that there is no fall off and the oscillations disappear.

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4.2.2 The Multilevel Fresnel Element

The final type of element is that which combines the Fresnel element with the multilevel approximation. Because the Fresnel element is usually much thinner than its corresponding analog element, it is better approximated by a smaller number of levels than is the analog element. This means that in the case where a Fresnel element can be used, it is a better choice for fabrication since it will require fewer masks in the photolithographic process.

We can again define a figure of merit based upon the ratio of spot energies for the Fresnel and analog elements:

\[ \frac{r_{\text{ML Fresnel}}}{r_{\text{analog}}} = \frac{\theta_{\text{ML Fresnel}}}{\theta_{\text{analog}}} \]

A plot of the figure of merit for the case of five grating periods and eight levels is shown in Figure 8. Note that this figure is very similar to Figure 5, except that the amplitude is smaller. The peak value at \( \lambda = \bar{\lambda} \) is 0.95 = \( \text{sinc}^2(1/8) \), as might be expected from Eq. (6).

5.0 SAMPLE DESIGNS

To show more concrete examples of the performance of the dispersive microlens, we now present two sample designs, one operating in the infrared and one operating in the visible. We will examine how the elements perform under the different design options.

We first take the example mentioned at the end of section 4.1: an f/5, 100-\( \mu \text{m} \) square element operating in silicon over the wavelength band 8 \( \mu \text{m} \) < \( \lambda \) < 12 \( \mu \text{m} \). In this band, silicon has a near-constant index of refraction with a value \( n' = 3.4 \). We will take first-order diffraction and let \( \lambda_B = \bar{\lambda} = 10 \mu \text{m} \). We will take four grating periods across the element, so \( G = 25 \mu \text{m} \). Although the period is not much larger than twice the largest wavelength in the band (thus making the assumption of the validity of scalar theory debatable), it will be sufficient for our purposes. (We are currently investigating the effects of the vector nature of the electromagnetic field on this type of element, although we will not address that here.)

In Figure 9 we see a cross-cut through the center of the diffraction pattern at the element focus for three wavelengths: 8 \( \mu \text{m} \), 10 \( \mu \text{m} \), and 12 \( \mu \text{m} \). These three wavelengths are distinguished by different shades of gray. The different element types are distinguished by different line patterns.

There are several things to note in this figure. First, the analog and Fresnel element operate identically at the design wavelength, as expected. The multilevel element gives the worst performance among all the element types. This is because the approximation to the analog surface by only eight levels is still somewhat coarse. The Fresnel element operates very well at all wavelengths, mostly due to the relatively small Fresnel numbers (5.7 < \( g < 8.5 \)). Finally, the multilevel Fresnel element gives nearly as good performance as the Fresnel element. If the peak values of the multilevel Fresnel to the Fresnel values are compared, we find the ratios to be 0.95 at all the wavelengths, showing that using only three binary masks gives excellent performance for this design.

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We next consider the case of a dispersive microlens operating in the visible region, $0.4 \, \mu m < \lambda < 0.7 \, \mu m$. We take an f/10, 50-\mu m-square element, this time fabricated in a glass with $n' = 1.5$. The design and blaze wavelengths are taken as $\lambda_b = \lambda = 0.55 \, \mu m$. This gives a Fresnel number range of 18.8 at 0.4 \mu m to 10.7 at 12 \mu m. For increased dispersion, we now take $N = 5$ periods across the element. The results for this element are shown in Figure 10.

Because of the higher dispersion, we now see some spillover into the second order at 0.4 \mu m. The effect of this can be ameliorated either by a proper placement of detectors under the element or by reducing the wavelength band.

The analog and Fresnel elements again operate identically at the design wavelength; however, the performance at the edges of the band is very poor for the Fresnel element. The performance is worse at 0.4 \mu m because of the higher Fresnel number. This result shows that, even with a slow lens (f/10), performance of the Fresnel element will be unacceptable if the wavelength is too small and the waveband too wide. To use dispersive microlenses in the visible, therefore, it will be necessary to use analog elements.

We have not displayed the results for multilevel elements in Figure 10 because current photolithographic technology does not allow fabrication at the very small feature sizes required by the masks for these elements. The visible dispersive microlenses will therefore need to be fabricated using an analog technology (e.g., grayscale photolithography).

6.0 SUMMARY

We have presented an analytic theory for describing a micro-optical element which will simultaneously focus and disperse light, the dispersive microlens. Our theory describes how the diffraction pattern from the dispersive microlens is built up and the important parameters needed to quantify the results. The theory can be used to develop efficient designs for a given application.

There are four distinct design options available for the dispersive microlens (and, indeed, for any micro-optical element). These are the analog, multilevel analog, Fresnel, and multilevel Fresnel options. We have defined figures of merit for each of these options which can be used to determine when one type of design is acceptable as a replacement for another type of design. This is useful when fabrication issues become important. Our analysis has shown that Fresnel elements are only useful over a small waveband, except at very low Fresnel numbers. For the common case of eight levels used in photolithographic fabrication, multilevel elements are only useful for very small Fresnel numbers. We have been able to quantify how their performance improves as the number of masks used in the fabrication process increases. For cases where Fresnel elements can be used, the corresponding eight-level Fresnel element gives acceptable results.

In our analysis we have assumed a rectangular element. Other topologies, such as circular or hexagonal elements are possible. The analytical analysis of these elements is more difficult, although the principle remains the same.

For a discussion of the use of dispersive microlenses in systems applications of current interest, the reader is referred to Reference 6.

FIGURE 1. DISPERSIVE MICROLENS CONCEPT. The dispersive element is a combination of a blazed grating and a microlens. The grating has a width $w$, period $G$, and a modulation depth $D$.

FIGURE 2. DIFFRACTION PATTERN (IN THE $x$ DIRECTION) OF THE DISPERSIVE MICROLENS IN THE FOCAL PLANE. The full diffraction pattern is the product of the diffraction patterns due to the periodic structure of the grating and a single sawtooth of the grating. The normalized intensity $I/I_0 = |E/E_0|^2$ is shown. 

$\frac{n - 1D}{n'G}$  $\frac{2\lambda f}{n'GN}$
FIGURE 3. DIFFRACTION PATTERN (IN THE x-DIRECTION) OF THE DISPERSE
MICROLENS IN THE FOCAL PLANE FOR MULTIPLE WAVELENGTHS.
Dispersion of wavelengths is shown.

\[
\frac{(\lambda_{\text{max}} - \lambda_{\text{min}}) f}{n'G} \left[ \frac{}{n'G} \right] \frac{f}{n'G}
\]

ANALOG (NON-FRESNEL)  FRESNEL

FIGURE 4. AN ANALOG (NON-FRESNEL) ELEMENT AND ITS CORRESPONDING
FRESNEL COUNTERPART. The Fresnel element has a maximum thickness of
\( \frac{\lambda}{n - 1} \) which introduces a \( 2\pi \) phase change at wavelength \( \lambda \).
FIGURE 5. FIGURE OF MERIT FOR FRESNEL DISPERSIVE MICROLENS FOR THE CASE OF FIRST-ORDER DIFFRACTION, EQUAL BLAZE AND DESIGN WAVELENGTHS, AND FIVE GRATING PERIODS.

FIGURE 6. SUCCESSIVE MULTILEVEL APPROXIMATIONS TO A DISPERSIVE MICROLENS.
FIGURE 7. FIGURE OF MERIT FOR MULTILEVEL DISPERSIVE MICROLENSES FOR THE CASE OF FIRST-ORDER DIFFRACTION, EQUAL BLAZE AND DESIGN WAVELENGTHS, FIVE GRATING PERIODS, AND EIGHT LEVELS.

FIGURE 8. FIGURE OF MERIT FOR MULTILEVEL FRESNEL DISPERSIVE MICROLENSES FOR THE CASE OF FIRST-ORDER DIFFRACTION, EQUAL BLAZE AND DESIGN WAVELENGTHS, FIVE GRATING PERIODS, AND EIGHT LEVELS.
FIGURE 9. DIFFRACTION PATTERNS OBTAINED FROM DIFFERENT DISPERSIVE ELEMENT TYPES FOR A DESIGN OPERATING IN THE LWIR.

FIGURE 10. DIFFRACTION PATTERNS OBTAINED FROM DIFFERENT DISPERSIVE ELEMENT TYPES FOR A DESIGN OPERATING IN THE VISIBLE.