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IMPROVED OMIT SET DISPLACEMENT
RECOVERIES IN DYNAMICS ANALYSIS

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ABSTRACT

Two related methods for improving the dependent (OMIT set) displacements after performing a Guyan reduction are presented. The theoretical bases for the methods are derived. The NASTRAN DMAP ALTERs used to implement the methods in a NASTRAN execution are described. Data are presented that verify the methods and the NASTRAN DMAP ALTERs.

1.0 INTRODUCTION

A NASTRAN user is faced with two major challenges when solving a dynamic eigenvalue problem. First, an eigenvalue solution is expensive to perform for most structural problems encountered in engineering analysis, and second, many more degrees of freedom (DOF) are required to define a structure's elastic properties than are required to define its inertial properties.

A popular method for meeting these challenges is to reduce the problem size using Guyan reduction (Reference 1). Guyan reduction allows the user to preserve the elastic properties of the problem set while reducing the problem size to one that is more manageable for a dynamic eigenvalue analysis. At the same time, the mass properties are also condensed with some penalty associated with the redistribution of mass from the coordinates eliminated during the Guyan reduction. The present paper describes two approaches that correct the inaccuracies caused by the condensation of the mass matrix without unduly affecting the solution time.

The theoretical development of the improvement methods is provided in Section 2. Section 3 describes the NASTRAN DMAP ALTERs used to implement the algorithms used for both methods. Verification of the two methods, the second of which is a refinement of the first, is presented in Section 4. Conclusions and recommendations are provided in Section 5.

2.0 THE IMPROVEMENT METHOD

We begin by deriving the Guyan reduction scheme.

The dynamic eigenvalue problem is given by the equation

$$([K] - \lambda[M])\{\phi\} = 0 \quad (1)$$

where

K = the structural stiffness matrix
M = the structural mass matrix
 λ = the system eigenvalue
 ϕ = the eigenvector or modal displacements.

We can partition Equation 1 into independent DOF, designated in NASTRAN as the analysis set, or A-set, and dependent DOF, designated as the OMIT set, or O-set. After performing this operation Equation 1 becomes

$$\begin{bmatrix} \bar{K}_{aa} & K_{ao} \\ K_{ao}^T & K_{oo} \end{bmatrix} - \lambda \begin{bmatrix} \bar{M}_{aa} & M_{ao} \\ M_{ao}^T & M_{oo} \end{bmatrix} \begin{Bmatrix} \phi_a \\ \phi_o \end{Bmatrix} = 0 \quad (2)$$

where the subscript "a" denotes A-set DOF and the subscript "o" denotes O-set DOF.

Looking at the lower partition of Equation 2 we can say

$$K_{ao}^T \phi_a + K_{oo} \phi_o - \lambda M_{ao}^T \phi_a - \lambda M_{oo} \phi_o = 0 \quad (3)$$

The Guyan reduction method (Reference 1) makes the assumption that the inertial forces on the O-set displacements are much less important than the elastic forces transmitted by the A-set displacements. A constraint equation for Guyan reduction can be derived by ignoring the mass terms in Equation 3. The resulting constraint equation is given by

$$\phi_o = G_o \phi_a \quad (4)$$

where

$$G_o = -K_{oo}^{-1} K_{ao}^T \quad (5)$$

This relationship constitutes a Ritz transformation of the eigenvalue problem. The transformation written in terms of the full displacement set is

$$\{\phi\} = \begin{Bmatrix} \phi_a \\ \phi_o \end{Bmatrix} = [G] \{\phi_a\} = \begin{bmatrix} I \\ G_o \end{bmatrix} \{\phi_a\} \quad (6)$$

Using this Ritz transformation, the reduced mass and stiffness matrices become

$$[M_{aa}] = [G]^T [M] [G] \quad (7)$$

and

$$[K_{aa}] = [G]^T [K] [G] \quad (8)$$

The mass of the system is redistributed based upon the elastic connections between the O-set DOF and the A-set DOF as shown in Equation 7.

The reduced mass and stiffness matrices shown in Equations 7 and 8, are then used to compute the eigenvalues and the A-set displacements of the reduced system. Once the A-set displacements have been computed, the Guyan reduction transformation of Equation 4 is used to recover the O-set displacements. This back transformation ignores the inertial terms of the O-set displacements.

An improved back transformation for ϕ_o can be found using Equation 3 (see Reference 2). For mode i , this back transformation is given by

$$\{\phi_o\}_i = -[K_{oo} - \lambda_i M_{oo}]^{-1} [K_{ao}^T + \phi_o - \lambda_i M_{ao}^T] \{\phi_a\}_i \quad (9)$$

Though Equation 9 will yield improved results, the first term on the right hand side must be inverted for each mode calculated during the eigenvalue analysis, a computationally inefficient process. Clearly, a more direct substitution would make the O-set displacement recovery more efficient.

Recasting Equation 3 for all the computed modes, we get

$$K_{ao}^T \phi_a + K_{oo} \phi_o - M_{ao}^T \phi_a \underline{\lambda} - M_{oo} \phi_o \underline{\lambda} = 0 \quad (10)$$

where $\underline{\lambda}$ is a square matrix with the system eigenvalues along the diagonal. Solving for the ϕ_o displacements that are not multiplied by $\underline{\lambda}$, we get

$$G_o \phi_a + K_{oo}^{-1} M_{ao}^T \phi_a \underline{\lambda} + K_{oo}^{-1} M_{oo} \phi_o \underline{\lambda} = \phi_o \quad (11)$$

From Equation 11 we can see that a closed form solution for ϕ_o does not exist. It is possible, however, to use Equation 11 to obtain an improved approximation to ϕ_o .

A first approximation to ϕ_o can be determined by using the O-set displacements recovered by Equation 4, or

$$\phi_o^{(1)} = G_o \phi_a \quad (12)$$

Substituting these O-set displacements into Equation 11 yields

$$G_o \phi_a + K_{oo}^{-1} M_{ao}^T \phi_a \underline{\lambda} + K_{oo}^{-1} M_{oo} \phi_o^{(1)} \underline{\lambda} = \phi_o^{(2)} \quad (13)$$

where $\phi_o^{(2)}$ are the corrected O-set displacements. These corrected displacements can be substituted back into Equation 13 for $\phi_o^{(1)}$ and a better approximation, $\phi_o^{(3)}$, can be computed. This process can be repeated until the displacements at the $(i+1)$ iteration are the same as the displacements at the i^{th} iteration. These "super" improved displacements will be identical to those computed using Equation 9, and can be determined without the computational penalty associated with inverting an O-set by O-set sized matrix for each mode.

To summarize, three methods for recovering the O-set displacements after performing the Guyan reduction and the reduced eigenvalue analysis have been presented. These three methods are:

- 1) Standard Guyan reduction recovery using Equation 4, henceforth designated as Guyan displacements.
- 2) Improved O-set displacement recovery using Equations 12 and 13, henceforth designated as improved displacements.
- 3) Successively iterated improved O-set displacements using Equation 13, henceforth designated as "super improved" displacements.

The reader will note that the A-set displacements are identical for all three methods described above. It is assumed that the eigenvalues and the A-set displacements computed during the eigenvalue analysis are "accurate". In other words, the accuracy (or inaccuracy) of the Guyan reduction itself is not in question.

Thus far, we have discussed improvements only in the O-set displacements. More importantly, any quantity computed using these O-set displacements, such as element forces or element stresses, will also be improved by methods 2 and 3.

The theory and methodology for improving the O-set displacements has been provided. The following section describes the implementation of the improved displacement recoveries in NASTRAN.

3.0 IMPLEMENTATION IN NASTRAN

With the methodology in hand, the implementation in NASTRAN becomes an exercise in defining the data blocks and the NASTRAN DMAP modules required to perform the desired operations. The DMAP ALTER sequences used to recover the improved displacements are provided in Figure 1. The first ALTER places the UPARTN module following the SMP1 module while the second ALTER places the DMAP modules used to recover the improved displacements after the SDR1 module. The user controls the recovery method with the parameters defined in the DMAP ALTERs. The allowable parameter values and the resulting action taken are provided in Table 1. Note that if no A-set is defined, the O-set recovery section is skipped.

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$
$ DMAP Alter to obtain required matrices for improvement. Place after the SMP2 Module.
ALTER ii $ where ii = DMAP statement number of Module SMP2
UPARTN      USET,MFF//,MAOT,,MOO/*F*/*A*/*O* $
$
$ DMAP Alter to perform O-set displacement improvement. Place after the SDR1 Module.
ALTER jj $ where jj = DMAP statement number of Module SDR1
COND        SKIPIM,OMIT $
$
$ This PARAM defines whether Guyan recovery or improvement
$ recovery is to be performed (NOIMP < 0, Guyan recovery)
PARAM       /*NOP*/NOIMP = -1 $
COND        SKIPIM,NOIMP $
$
$ This PARAM defines what recovery improvement will be performed
$ If NREPT = 0, improve once, NREPT > 0, iterate NREPT times
PARAM       /*NOP*/NREPT =10 $
$
$ MATGEN creates a square matrix from the LAMA table
MATGEN      LAMA,MLAMA/3/2 $
MPYAD       GO,PHIA,/PHIO/0/1/0/ $
FBS         LOO,,MAOT/C1/1/1 $
SMPYAD      C1,PHIA,MLAMA,,,PHIO/A/3///1 $
FBS         LOO,,MOO/B/1/1 $
LABEL      IMPRV $
SMPYAD      B,PHIO,MLAMA,,,/C/3///1 $
ADD         A,C/PHIO/(1.0,0.0)/(1.0,0.0) $
REPT        IMPRV,NREPT $
UMERGE      USET,PHIA,PHIO/PHIF/*F*/*A*/*O* $
UMERGE      USET,PHIF,/PHIN/*N*/*F*/*S* $
MPYAD       GM,PHIN,/PHIM/0/1/0/ $
UMERGE      USET,PHIN,PHIM/PHIG/*G*/*N*/*M* $
LABEL      SKIPM $

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Figure 1. O-set Displacement Improvement DMAP ALTERs

Table 1. DMAP Parameter Settings		
Execution Type	NOIMP	NREPT
No A-set	N/A	N/A
Guyan	-1	N/A
Improved	0	0
Super Improved	0	# repetitions

Once the O-set displacements have been recovered, the rest of the standard solution sequence is executed. This allows the user to define all data recoveries using the familiar NASTRAN Case Control Deck commands. Displacements, element forces, element stresses, or any other user requested data will be printed and handled in the normal fashion. No special provisions are required to view the improved data.

4.0

METHOD VERIFICATION

Two sample problems were created to verify the method and the DMAP described in Section 3. The first sample problem consists of a simple four story building. This problem was used to verify the methodology and the DMAP ALTERs shown in Figure 1. The second problem consists of a 3600 DOF substructured model. Element forces for this model were recovered from a transient response analysis using the three O-set displacement recovery methods and compared to the benchmark element forces obtained when no Guyan reduction was performed. These sample problems verify the improvement methods and the DMAP ALTERs.

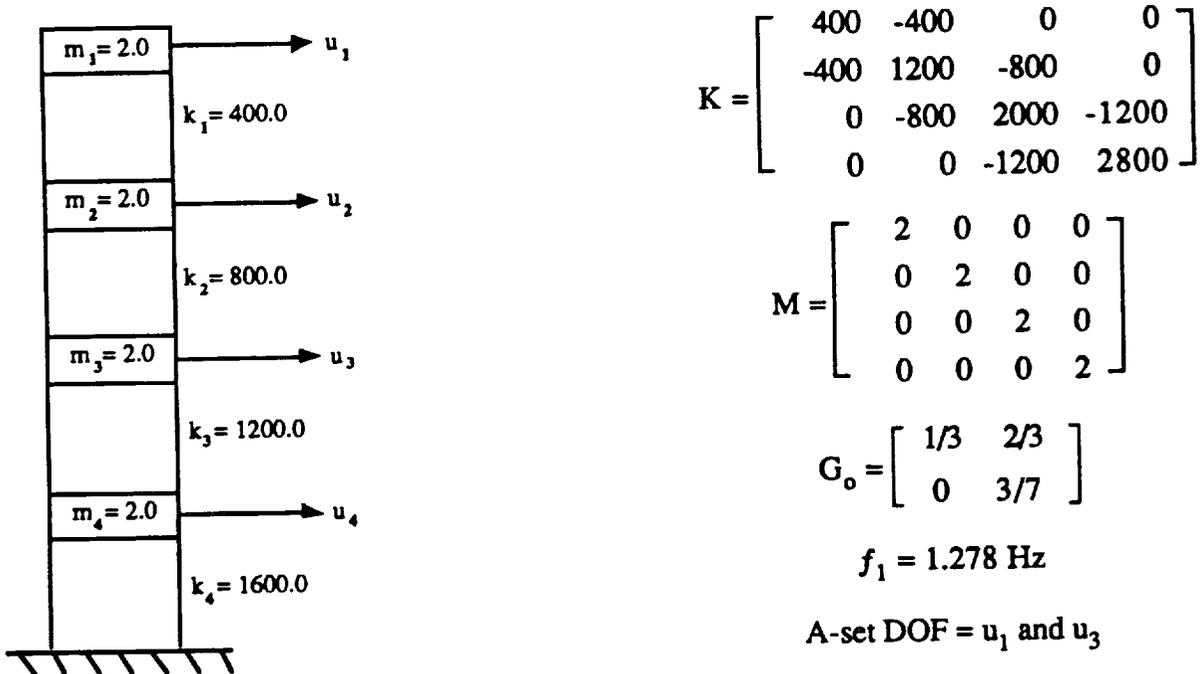


Figure 2. Simplified Four Story Building

The four story building used for sample problem 1 is shown in Figure 2. This problem was selected because it is easily represented with NASTRAN elements and may be solved using the NASTRAN program. It may also easily be solved by hand so that the data produced by the DMAP ALTERs can be verified. Data were recovered for the first mode only.

Table 2 presents the O-set displacements for the three methods as well as the unreduced benchmark displacements. The data in Table 2 were recovered from NASTRAN using the DMAP ALTERs described in Section 3. The reader can easily verify that the Guyan results are identical to those recovered by hand using Equation 4, the improved results are identical to those recovered by hand using Equation 11, and the super improved data are identical to those recovered by hand using Equation 9. These data verify the DMAP sequence described in Section 3.

Table 2. Displacement Comparison				
Disp.	Guyan	Improved	Super Improved*	Benchmark (No A-set)
u_1	1.0000	1.0000	1.0000	1.0000
u_2	0.6015	0.6681	0.6764	0.6775
u_3	0.4023	0.4023	0.4023	0.4069
u_4	0.1724	0.1806	0.1810	0.1828
MAC	0.99730	0.99995	0.99998	N/A

* These data were recovered using 10 iterations

The Modal Assurance Criterion (MAC) defined in Reference 4 is used to measure the accuracy of the eigenvectors provided in Table 2. MAC values will vary between zero, indicating no correlation between modes, to unity, indicating perfect correlation between modes. Based on the MAC values, it is clear that both improvement methods produce better O-set displacements than the standard Guyan recovery method produces alone.

The advantage of using the improved O-set recovery methods is clearer when element data, e.g. element forces or stresses, are compared. The modal spring forces for all three O-set displacement recovery methods are compared to the benchmark data in Table 3. From this it is clear that the improved displacements produce spring forces that are vastly superior to those of Guyan reduction.

Based on this simple problem, the displacements can be dramatically improved by using the methods described in Section 2. The next sample problem will show this more clearly.

The second sample problem uses the 3600 DOF Spacelab Pallet model shown in Figure 3. A transient response analysis was performed with this model in two configurations, an unreduced configuration and a Guyan reduced configuration. Transient element forces of all the bar elements were recovered using four distinct PHASE3 executions, i.e. no A-set, Guyan, improved, and super improved.

Table 3. Spring Force Comparison				
Spring Forces	Guyan	Improved	Super Improved	Benchmark (No A-set)
F_1	159	133	129	129
F_2	159	213	219	217
F_3	276	266	266	269
F_4	276	289	290	293
ΔF_1	30	4	0	N/A
ΔF_2	-58	-4	2	N/A
ΔF_3	7	-3	-3	N/A
ΔF_4	-17	4	-3	N/A

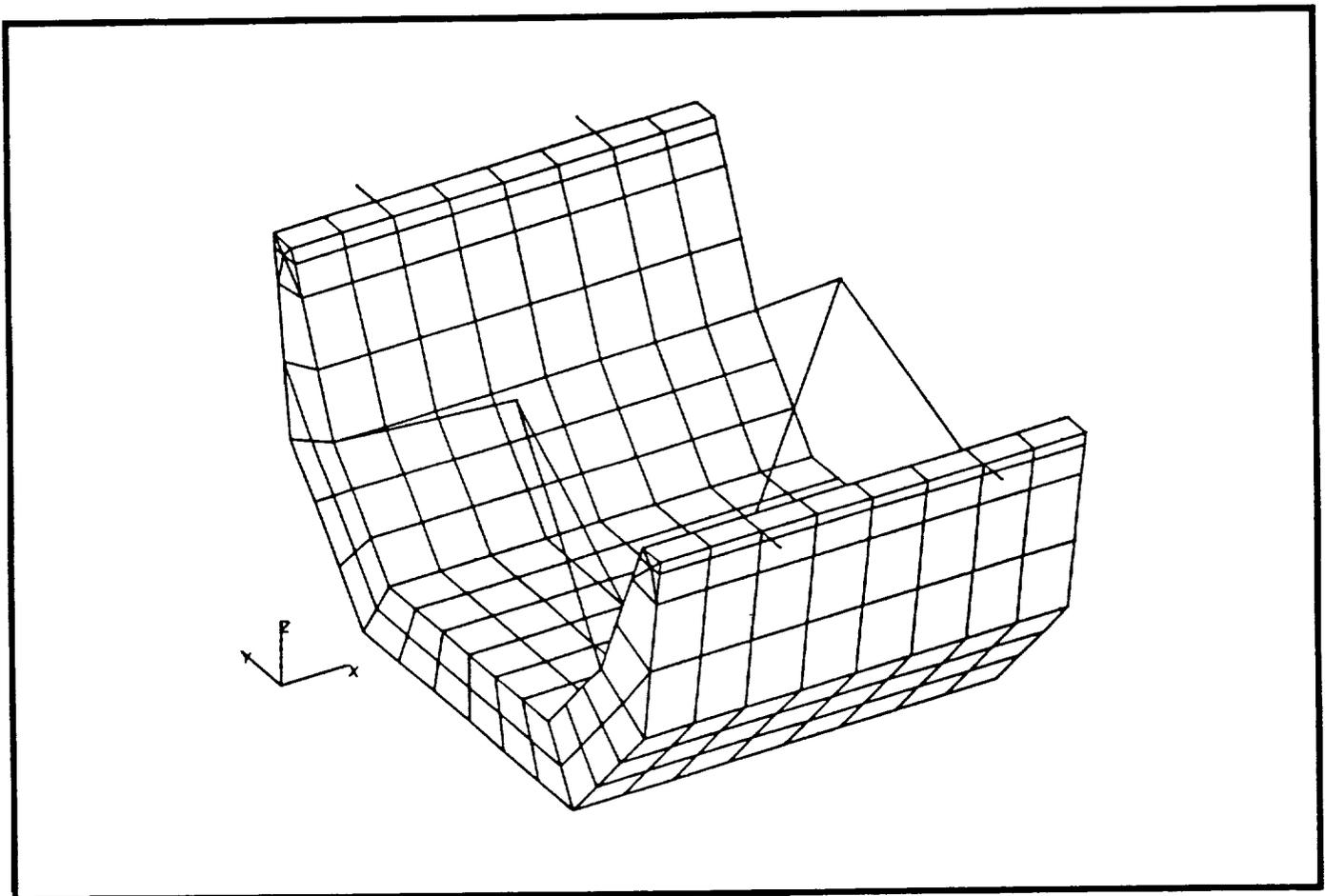


Figure 3. NASTRAN Model for Sample Problem 2

The maximum absolute values for all of the bar forces for the three recovery methods were compared to the benchmark case. These comparisons are shown in Table 4. The data are arranged according to a percentage difference range. For each of the recovery methods, the percentage of the forces falling within this range as well as the maximum difference between the benchmark data and the data produced by the three recovery methods within this delta percentage range are provided.

For example, in the range between two and five percent, 8.53 percent of the forces from the Guyan recovery method fell within this range with the maximum difference between the Guyan recovered data and the benchmark data being 397. For the improved recovery method, only 0.10 percent of the forces fell into this range with a maximum difference between the benchmark and the improved data being 5. The percentage of items falling in this range for the super improved method was 0.09, with a maximum delta of 7.

$\Delta\%$ Range	Guyan		Improved		Super Improved*	
	Percentage in Range	Maximum $ \Delta $	Percentage in Range	Maximum $ \Delta $	Percentage in Range	Maximum $ \Delta $
0-2	89.05	1045	99.90	102	99.76	114
2-5	8.53	397	0.10	5	0.09	7
5-10	1.48	48	0.00	0	0.03	4
10-25	0.60	82	0.00	0	0.00	0
25-50	0.03	0	0.00	0	0.00	0
>50	0.32	2281	0.00	0	0.13	36

* These data were recovered using 10 iterations

As was the case for the simplified model used for sample problem 1, the improved recovery methods produce data that are superior to those data computed using Guyan reduction. The data appear to be the most accurate for the simple improvement method. This is especially true when the computer CPU time required to produce the data is considered. The improved displacement recoveries required 30 percent more CPU time than the Guyan recovery, while the super improved displacement recoveries required 150 percent more CPU time than the Guyan recovery.

Because of the simplicity of this model, however, it would be premature to use these data to cast the super improved method aside without first considering more complex models with equally complex loading.

5.0

CONCLUSIONS AND RECOMMENDATIONS

Two methods for improving the O-set displacements were provided. It was demonstrated that both improvement methods produce O-set displacements that are vastly superior to those produced using the standard Guyan recovery alone. In addition, the NASTRAN DMAP ALTERs required to perform these operations were presented along with the supporting data used to verify them. It remains only to determine whether the additional accuracy that may be obtainable through the iterative procedure of Method

3 is justified by the extra computational effort. After all, a significant degree of approximation is already guaranteed by the initial use of Guyan reduction to determine the A-set displacements.

Because this study did not provide enough information to determine which of the two improved recovery methods was best suited for the problems encountered in most engineering applications, it is recommended that additional studies be performed to compare improved displacements from a set of models with varying complexity to the benchmark unreduced data. These additional cases can be used to definitively determine which improvement method is better in terms of accuracy and computational efficiency. Finally, it would be of great interest to compare the results of a multi-mode transient response analysis before and after modal improvement to assess its practical significance in terms of the end result.

6.0

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