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BENCHMARKING THE QUAD4/TRIA3 ELEMENT

BY

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INTRODUCTION

The QUAD4 and TRIA3 elements are the primary plate/shell elements in NASTRAN*. These elements enable the user to analyze thin plate/shell structures for membrane, bending and shear phenomena. They are also very new elements in the NASTRAN library. These elements are extremely versatile and constitute a substantially enhanced analysis capability in NASTRAN. However, with the versatility comes the burden of understanding a myriad of modeling implications and their effect on accuracy and analysis quality. The validity of many aspects of these elements were established through a series of benchmark problem results and comparison with those available in the literature and obtained from other programs like MSC/NASTRAN[2] and CSAR/NASTRAN[3]. Nevertheless such a comparison is never complete because of the new and creative use of these elements in complex modeling situations. One of the important features of QUAD4 and TRIA3 elements is the offset capability which allows the midsurface of the plate to be noncoincident with the surface of the grid points. None of the previous elements, with the exception of bar (beam), has this capability. The offset capability played a crucial role in the design of QUAD4 and TRIA3 elements. It allowed modeling layered composites, laminated plates and sandwich plates with the metal and composite face sheets. Even though the basic implementation of the offset capability is found to be sound in the previous applications, there is some uncertainty in relatively simple applications. The main purpose of this paper is to test the integrity of the offset capability and provide guidelines for its effective use. For the purpose of simplicity, references in this paper to the QUAD4 element will also include the TRIA3 element.

BACKGROUND

The QUAD4 element was added to the COSMIC/NASTRAN element library in 1987. Although similar in use to the MSC/NASTRAN QUAD4 element of 1980, there are differences in the theoretical formulation of the two. These differences are primarily in the hardening of shear deformation and numerical integration.

The formulation for the QUAD4 isoparametric quadrilateral element incorporates a bilinear variation of geometry and deformation within the element. The QUAD4 element has 5 degrees of freedom (dof) per node, i.e., the stiffness for rotation about the normal to the mid-surface at each node is not defined. Furthermore, it is assumed that plane sections remain plane and that the variation of strains through the thickness is linear. In addition, direct strain through the thickness is neglected (assumed to be zero).

The QUAD4 element may be used to model either membrane or bending behavior, or both. Transverse shear flexibility may be requested as well as the coupling of membrane and bending behaviors using nodal offsets or linear variation of material properties through the thickness. In addition, the QUAD4 element is capable of representing laminated composite materials, with an option to compute interlaminar shear stresses and layer failure indices.

*NASTRAN without qualification refers to COSMIC/NASTRAN[1].

The transverse shear stiffness is numerically conditioned to enhance the accuracy of the element for a wide range of modeling practices including very thick or thin elements, high aspect ratio elements, and skewed elements.[4]

FEATURES OF THE QUAD4

The QUAD4 element gives the NASTRAN user an accurate, all-purpose plate/shell/membrane element. It can be used in place of all QUAD and QDMEM elements. The QUAD4 element uses a linear, isoparametric formulation with bilinear variation of geometry and deformation. It can be used to model the following types of plates:

- Membrane plates
- Bending plates
  - Membrane/bending (without nonlinear coupling)
  - Membrane/bending (with offset coupling)
- Plates offset from the grid point plane
- Layered composite plates
- Laminated plates
- Sandwich plates (metal and composite face sheets)
- Thin and Thick plates

USE OF THE OFFSET CAPABILITY AND ITS IMPLICATIONS

There are several different ways to specify plate offsets in NASTRAN. They are as follows:

- Z0 field on CQUAD4 bulk data card
- Z0 field on PSHELL bulk data card
- Z0 field on PCOMP bulk data card
- Use of rigid element (RBAR) bulk data card
- Use of PCOMP card to model offset plate as unsymmetric laminate with very soft layer (value of E 2 to 3 orders of magnitude less than plate) serving as the offset space[5]

However, the use of the Z0 field is sufficient for most users to model plate offsets. The result of offsetting a plate depends on the loading condition. For out-of-plane loading (as in the examples), the offset has no effect on out-of-plane displacements, but in-plane displacements increase due to the rotational arc of the element. For in-plane loading, displacements are affected both in-plane and out-of-plane due to the combination of in-plane loading plus offset acting as a moment as well as rotational effects. Note that membrane/bending coupling will play an important part in the correct formulation of the problem, so material cards referenced by offset plates must be provided for both membrane and bending stiffness.

The user must be aware of the differences in the definition of the offset between the CQUAD4, PSHELL and PCOMP cards. The offset value that is used in the Z0 field on the CQUAD4 and PSHELL cards is the distance from the grid point surface to the element mid-plane of the CQUAD4 element. However, on the PCOMP card, the distance appearing on the Z0 field is measured from the grid point surface to the bottom surface of the CQUAD4 element. Also, the Z0 value may be positive or negative depending on the node numbering scheme (clockwise = negative Z0, counterclockwise = positive Z0) and the position of the CQUAD4 element relative to the grid point plane (element above grid point plane = positive Z0, element below grid point = negative Z0). Please note that this is different from what is documented in the User's Manual as of 3/3/90, which properly states offset definition for the PCOMP card only. See Figures 1 and 2 for further detail.

DIFFERENCES BETWEEN COSMIC/NASTRAN, CSAR/NASTRAN AND MSC/NASTRAN

As mentioned in the previous discussion of QUAD4 theory, the theoretical formulation of QUAD4 elements is different in different versions of NASTRAN. COSMIC/NASTRAN and ASTROS share the same QUAD4 element so results compare favorably between these two codes. The COSMIC/NASTRAN QUAD4 element tends to be slightly stiffer and exhibits a closer relationship to linear theory than CSAR and MSC QUAD4 elements. However, all codes give results that compare within 3% of empirical solutions.

EXAMPLE PROBLEMS

1. CANTILEVER PLATE

The cantilever plate problem consists of a semi-monocoque-like structure of plates (QUAD4 elements) attached to a bar (CBAR element) (see Figure 3). The structure is fixed at the wall and has a plane of symmetry on the left side. The cantilever plate can be modeled with the grid points running down the center of the CQUAD4 elements and the bar offset, with the grid points running down the center of the CBAR elements and the plates offset, or with the grid point plane separate and both the CQUAD4 and CBAR elements offset. The result of each of these three methods should compare to each other favorably. These results are located in Table 1.

Table 1

Maximum Displacements
z-displacements
x-displacements

CASE	COSMIC	CSAR	MSC
A. Cantilever Plate Offset on CQUAD	-7.741E-2 -1.963E-3	-7.69E-2 -1.961E-3	-7.76E-2 -1.961E-3
B. Cantilever Plate Offset on CBAR	-7.741E-2 +4.007E-4	-7.701E-2 +4.007E-4	-7.771E-2 +4.006E-4
C. Cantilever Plate CQUAD, CBAR Offset	-7.741E-4 -3.336E-2	-7.669E-2 -3.332E-2	-7.740E-2 -3.327E-2
D. Cantilever Plate Offset on PSHELL	-7.741E-2 -1.963E-3	N/A	N/A

Note: CSAR/NASTRAN and MSC/NASTRAN do no offer field on PSHELL card.

2. MODIFIED CANTILEVER PLATE

The cantilever plate problem was modified to examine some accuracy and user features of the offset capability. The first modification of the cantilever plate was to remove the offset entirely. This results in a cross-shaped cross section instead of a t-shaped cross section and as such is expected to give entirely different results (see Figure 4). The second modification to the cantilever plate is a modified load from a distributed load to a point load at the end of the bar. This gives us a configuration that can be easily compared to an empirical solution (see Figure 5). The third modification to the cantilever plate problem is to change the height of the bar so that a "stepped" cantilever plate results (see Figure 6). This is to display the interaction of the Z0 fields on the CQUAD4 and PSHELL cards. The results are located in Table 2.

Table 2

Maximum Displacements
z-displacements
x-displacements

CASE	COSMIC	CSAR	MSC
A. Cantilever Plate No offset	-2.794E-1 0.0	-2.789E-1 0.0	-2.794E-1 0.0
B. Cantilever Plate Theory=3.334E-2	-3.400E-2 (1.8% error)	-3.399E-2 (1.8% error)	-3.413E-2 (2.1% error)
C. Cantilever Plate Stepped config.	4.636E-2 -1.385E-3	N/A	N/A

The results from case A show that the cantilever plate run in example 1 with offsets removed show that the configuration is changed and the results are significantly different. This verifies that the offsets used in example 1 are indeed working and giving excellent results. The results from case B show that the cantilever plate with CQUAD4 offset and a point load on the tip of the structure give very close correlation with a theoretical solution of a T-shaped bar of the same dimensions. The results in case C show that placing a standard offset in the Z0 field on the PSHELL card is an efficient method to model a structure where many plates are offset by the same distance. The Z0 field on the PSHELL card can be overridden by an entry on the CQUAD4 card when a few have different offsets (the alternative method is to place an entry in EVERY CQUAD4 card, which can be quite laborious and unnecessary for a large model).

3. CLAMPED PLATE
Note: This problem derived from "Theory of Plates & Shells", by Timoshenko and Woinowsky-Krieger, P.206 (Reference 7)

The clamped plate model is a plate that is clamped on all four sides. Due to the symmetric nature of the structure, only 1/4 of the structure is modeled. There are no elements except CQUAD elements in this model. Three model densities are examined, a 3x3 grid, a 6x6 grid, and a 12x12 grid (see Figure 7). The model is tested with no offset and with a 1.0" offset. According to Reference 7, the empirical solution for this model is -8.806E-4 (no offsets are considered). The results are located in Table 3.

Table 3

Maximum Displacements
z-displacements

CASE	COSMIC	CSAR	MSC
A. Clamped Plate 3x3 grid, no offset	-8.499E-4	-8.95E-4	-8.776-4
B. Clamped Plate 6x6 grid, no offset	-8.743E-4	-9.00E-4	-8.923E-4
C. Clamped Plate 12x12 grid, no offset	-8.802E-4	-8.962E-4	-8.874E-4
D. Clamped Plate 3x3 grid, 1.0 offset	-8.499E-4	-1.478E-4	-8.961E-5
E. Clamped Plate 6x6 grid, 1.0 offset	-8.743E-4	-2.885E-4	-1.154E-4
F. Clamped Plate 12x12 grid, 1.0 offset	-8.802E-4	-5.515E-4	-2.639E-4

The results show that, in the no offset case, the COSMIC QUAD4 element is slightly stiffer and exhibits better correlation with linear theory as it asymptotically approaches the empirical solution. All cases, however, compare well with the empirical solution. In the offset cases, the reason for great differences in CSAR/NASTRAN and MSC/NASTRAN cannot be explained.

4. SANDWICH PLATE

The sandwich plate models 1/4 of a symmetric plate structure with all four edges constrained in the out-of-plane direction and a loading in the center of the symmetric section of the plate (see Figure 8). It is modeled using metal and composite sandwich plates. The metal sandwich plates are modeled using a separate material card to specify transverse shear properties. The composite sandwich plates are modeled in a two step process, first using a PCOMP card to input the properties of the composites, then from the output the equivalent properties as PSHELL/MAT2 cards is extracted, and rerun with modified PSHELL and MAT2 cards. This procedure is described at length in reference 4. Results are located in Table 4.

Table 4

Maximum Displacments
z-displacements
x-displacements

CASE	COSMIC	CSAR	MSC
A. Metal Sandwich No Offset	-3.747E-2 0.0	-3.770E-2 0.0	-3.792E-2 0.0
B. Metal Sandwich Offset on CQUAD	-3.72E-2 -5.00E-3	-2.960E-2 -2.406E-6	-3.690E-2 -1.520E-2
C. Composite Sandwich No Offset	-2.667E-2 0.0	-2.696E-2 0.0	-2.721E-2 0.0
D. Composite Sandwich Offset on CQUAD	-2.626E-2 +1.365E-4	-1.394E-2 +4.131E-7	-2.663E-2 +1.391E-4

The results show that both metal sandwich and composite sandwich plates can be modeled with and without offset. The reason for different results from CSAR/NASTRAN for offset cases cannot be explained.

5. LAMINATED PLATE

The laminated plate model is identical to the cantilever plate model (see Figure 9). The difference is that in this case both metal and composite laminated plates are used in place of the isotropic plate used in example 1. The problem is run with the CBAR offset from the CQUAD4 and with the CQUAD4 offset from the CBAR with the CQUAD4 offset on the PCOMP card. The reason that the Z0 field was used on the PCOMP card rather than the CQUAD4 card is that the Z0 field on the CQUAD card, when used in conjunction with a PCOMP card, appears to be inactive for both COSMIC/NASTRAN and CSAR/NASTRAN. It is operating in MSC/NASTRAN. Note that this is not the case for the CQUAD4/PSHELL combination, where the Z0 can be used on either card. Results are located in Table 5.

Table 5

Maximum Displacements
z-displacements
x-displacements

CASE	COSMIC	CSAR	MSC
A. Metal Laminate PCOMP Offset	-3.406E-2 -9.786E-4	-3.319E-2 -9.686E-4	-3.404E-2 -9.741E-4
B. Metal Laminate CBAR Offset	-3.406-2 +1.979E-4	-3.386E-2 +1.978E-4	-3.410E-2 +1.978E-4
C. Composite Laminate PCOMP Offset	-6.250E-2 -1.030E-3	-6.250E-2 -1.030E-3	-6.060E-2 -1.030E-3
D. Composite Laminate CBAR Offset	-6.250E-2 +1.150E-3	-6.250E-2 +1.150E-3	-6.230E-2 +1.150E-3

The results show that laminated plates, both metal and composite, can be accurately and easily modeled using offset capabilities.

CONCLUSION

The results of studies performed in this paper indicate that the offset feature provided in COSMIC/NASTRAN for the QUAD4/TRIA3 elements is performing as expected. The results are compared against empirical solutions and other NASTRAN variations (MSC and CSAR). These results generally show excellent agreement except in some comparisons with MSC and CSAR, where COSMIC results appear to be correct.

REFERENCES

1. COSMIC/NASTRAN User's Manual, Volume 1
2. MSC/NASTRAN User's Manual, Volume 1
3. CSAR/NASTRAN User's Manual, Volume 1
4. Venkayya, V.B., Tischler, V.A., QUAD4 Seminar, WRDC-TR-89-3046, April 1989
5. CSAR/NASTRAN User Newsletter, 4th Quarter 1991
6. MSC/NASTRAN Application Manual, Volume 2
7. Timoshenko & Woinowsky-Krieger, Theory of Plates and Shells, McGraw-Hill, 1952.

QUAD4 - OFFSET DEFINITION

(CQUAD4 PSHELL)

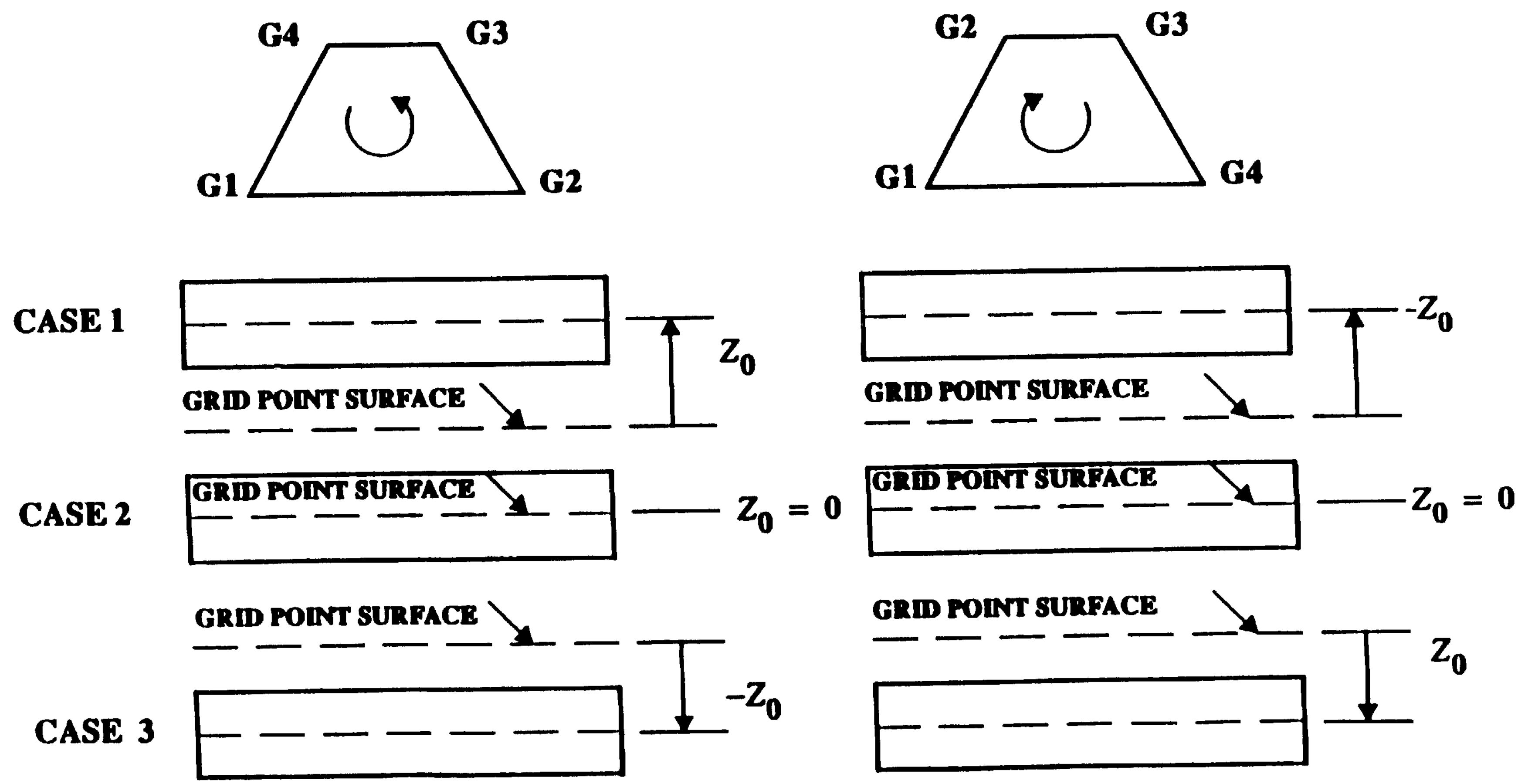


Figure 1

QUAD4 - OFFSET DEFINITION

(PCOMP PCOMP1 PCOMP2)

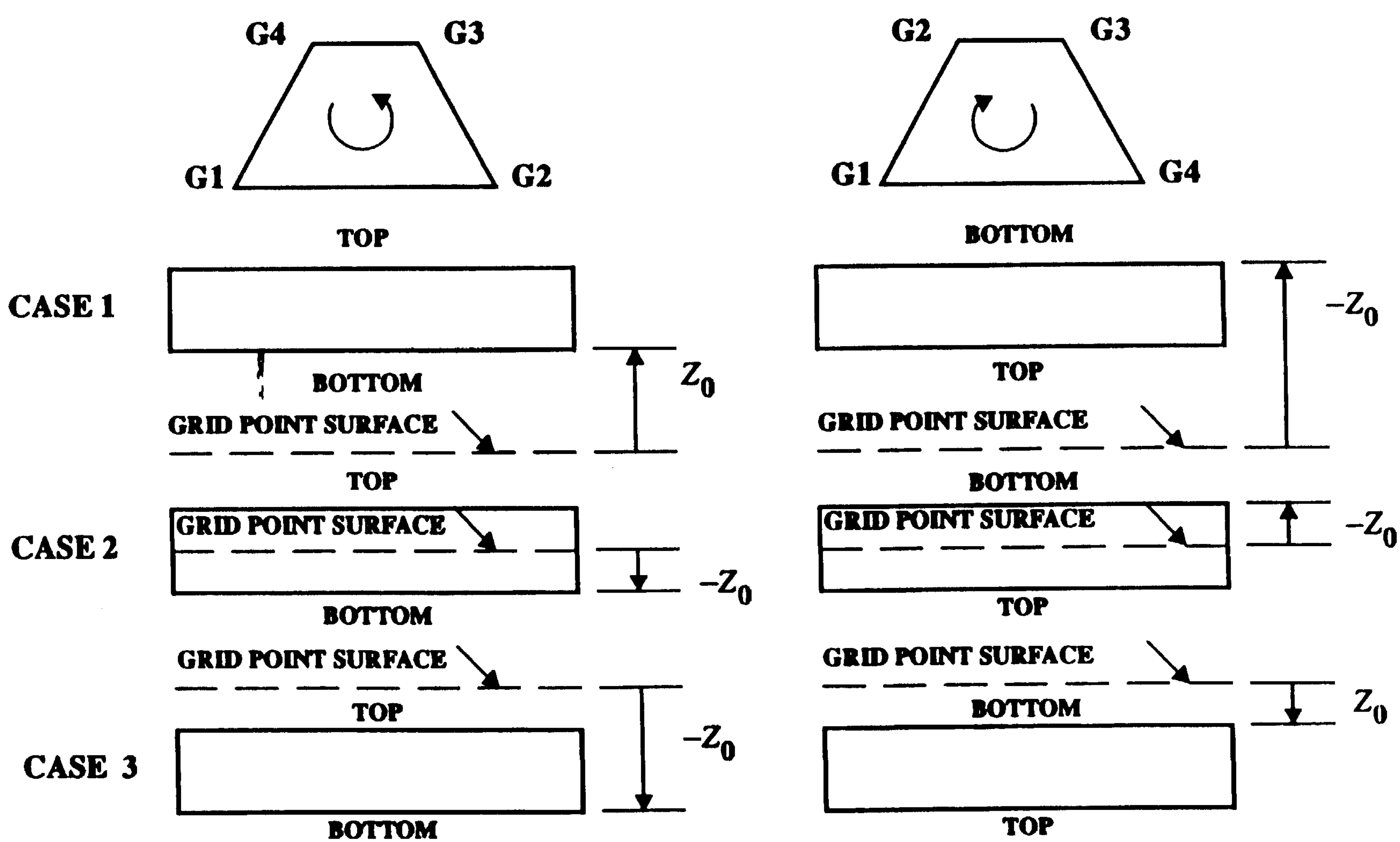
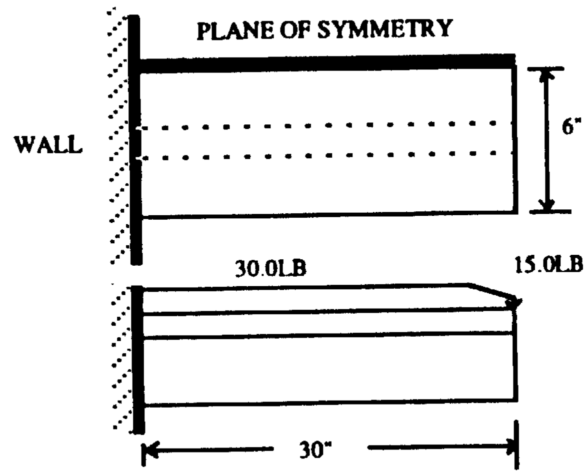


Figure 2

RESULTS

CANTILEVERED PLATE



MATERIAL PROPERTIES

E = 29.0E+6

NU = .3

I 1 (BAR) = .042

I 2 (BAR) = .0104

J(BAR) = .028625

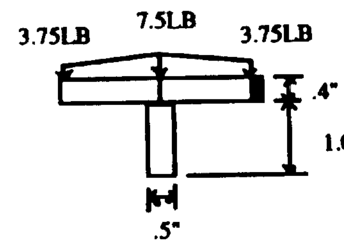
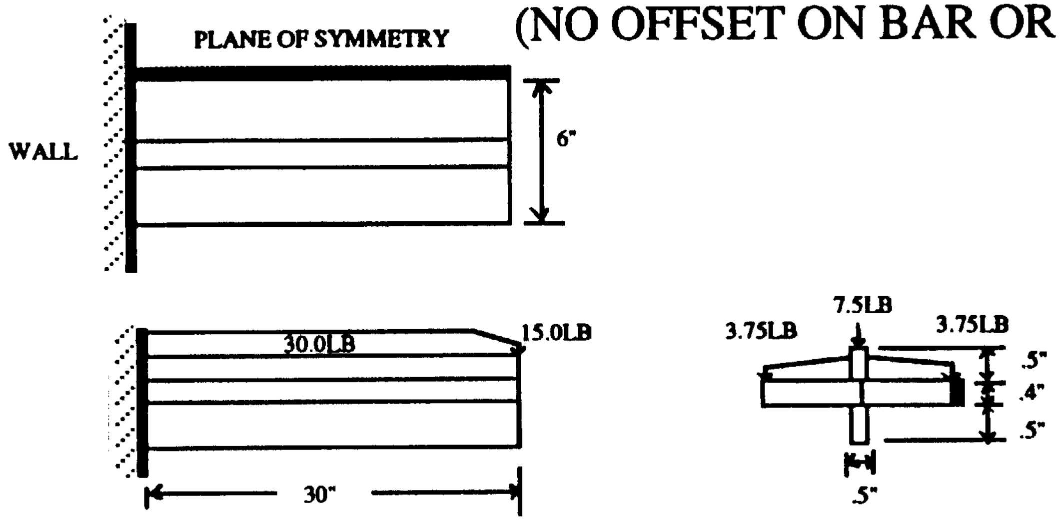


Figure 3

RESULTS

MODIFIED CANTILEVERED PLATE
(NO OFFSET ON BAR OR PLATE)



MATERIAL PROPERTIES

E = 29.0E+6

NU = .3

I 1 (BAR) = .042

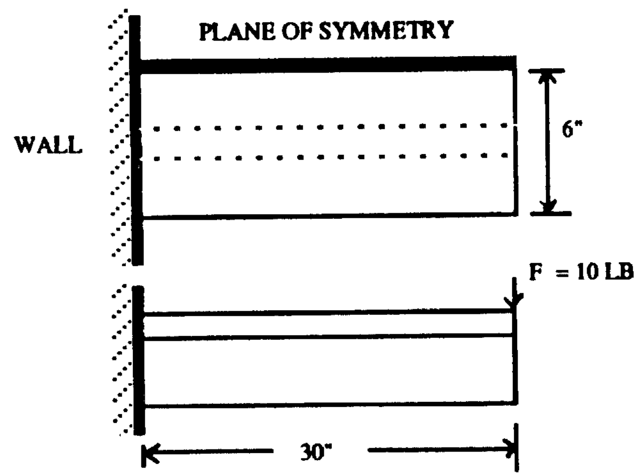
I 2 (BAR) = .0104

J(BAR) = .028625

Figure 4

RESULTS

MODIFIED CANTILEVERED PLATE
(NEW LOADING CONDITION)



MATERIAL PROPERTIES

E = 29.0E+6

NU = .3

I 1 (BAR) = .042

I 2 (BAR) = .0104

J(BAR) = .028625

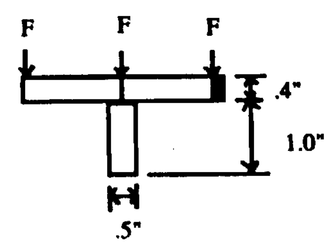
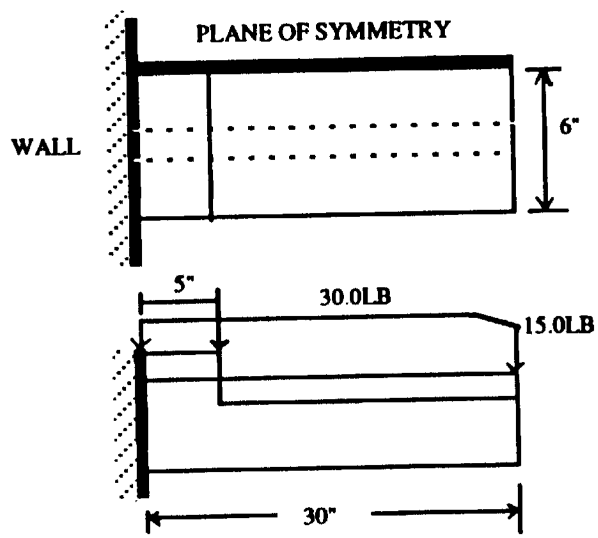


Figure 5

RESULTS

MODIFIED CANTILEVERED PLATE
(STEPPED PLATE)



MATERIAL PROPERTIES

E = 29.0E+6

NU = .3

I 1 (BAR) = .042

I 2 (BAR) = .0104

J(BAR) = .028625

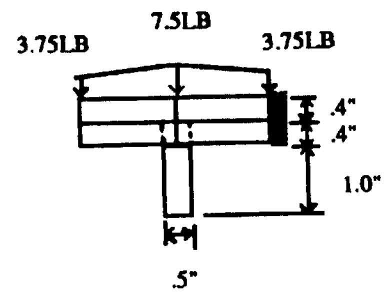
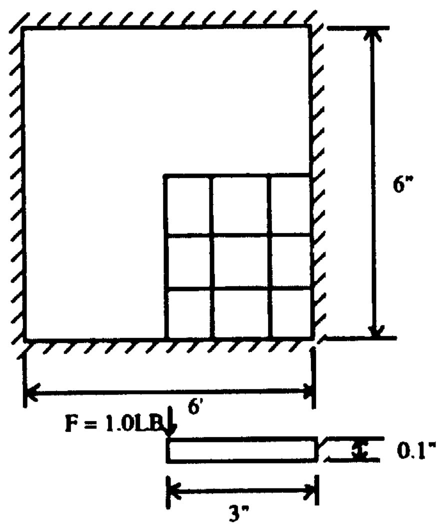


Figure 6

RESULTS

SIMPLE PLATE (CLAMPED)



MATERIAL PROPERTIES

E = 10.0 E +6

NU = .3

12I / T3 = 1.0

GRID MODELED AS 3X3, 6X6, AND 12X12

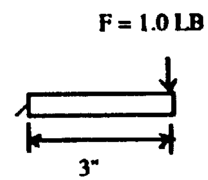
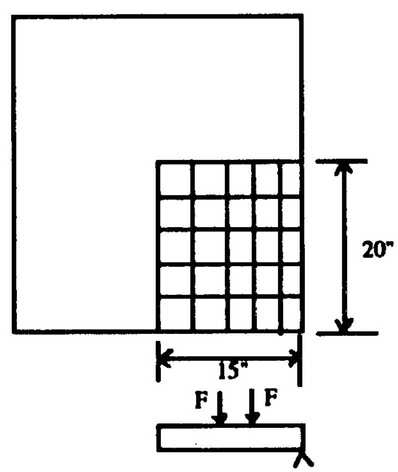


Figure 7

RESULTS

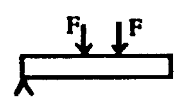
SANDWICH PLATE



SIMPLY SUPPORTED BOUNDARY CONDITION

MATERIAL PROPERTIES:

F = 1000 LB



METAL SANDWICH PROPERTIES

MAT1, 22, 10.0E+6, , .3

MAT1, 23, , 3.0E+4

PSHELL, 72, 22, .3776, 22, 31.2355, 23, 2.6483

COMPOSITE SANDWICH PROPERTIES

MAT8, 22, 18.5E+6, 1.6E+6, .25, .65E+6, 4.0E+5, .005

PCOMP, 72, .5, ... , +ABC

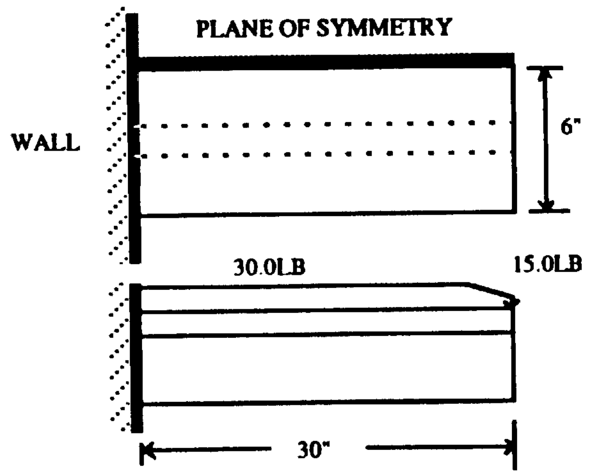
+ABC, 22, .0236, 45., YES, 22, .0236, -45., YES, +BCD

+BCD, 22, .0236, 90., YES, 22, .0236, 0., YES

Figure 8

RESULTS

LAMINATED PLATE



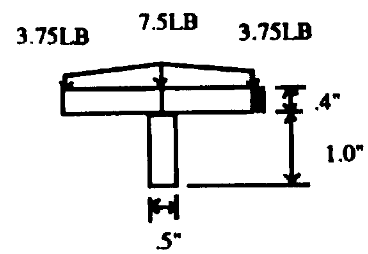
MATERIAL PROPERTIES:

METAL LAMINATE

E = 29.0E+6
NU = .3
RHO = 7.324E-4
SBOND = .65E+6
T = 4.1" PLIES

COMPOSITE LAMINATE

PCOMP SAME AS METAL LAMINATE
MAT8, 70, 1.85E+7, 1.6E+6, .25, .65E+6,,,.055,+MT5
+MT5, 0., 0., 100., 1.15E+5, 1.15E+5, 1.15E+5, 1.15E+5, 1.0E+15



NOTE: Z0 FIELD ON CQUAD4 WHEN USED WITH PCOMP IS INOPERABLE (CQUAD4/PSHELL OK)

Figure 9