Research on Two Equation Models

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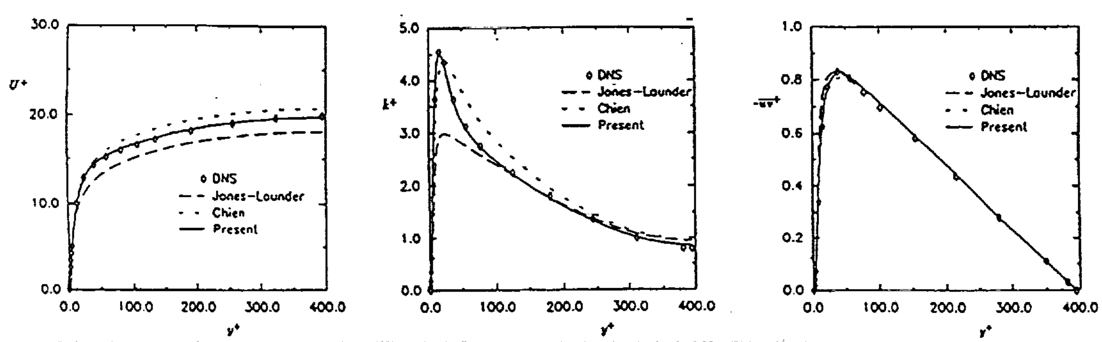
Motivations and Objectives:

- $k - \epsilon$ model is the most widely used turbulence model in engineering calculations.
- However, the following deficiencies need to be fixed:
  - Currently, most $k - \epsilon$ models for near wall turbulence contain geometry parameters.
  - The form of the ϵ equation lacks a solid theoretical foundation.
  - The $k - \epsilon$ model performs rather inadequately for flows with adverse pressure gradient.
  - The capability of $k - \epsilon$ models in predicting bypass transition due to the freestream turbulence needs improving.

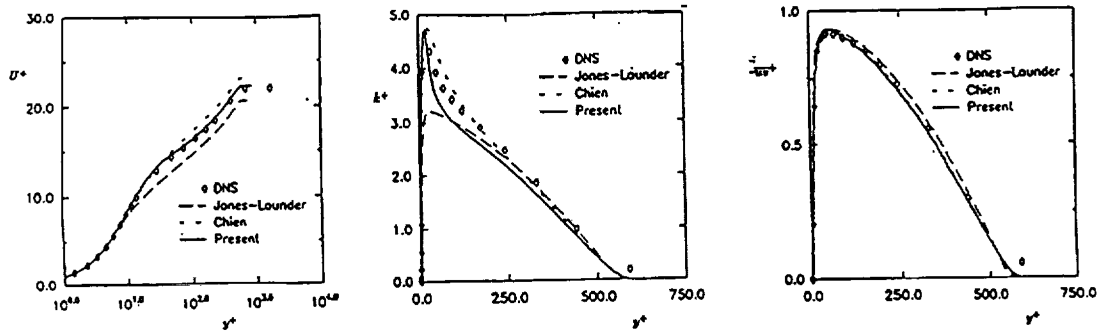
Modeling of Near Wall Turbulence

- Near wall $k-\epsilon$ model = Standard $k-\epsilon$ model + near wall effect.
- The near wall effect:
  - The time scale approaches the Kolmogorov time scale near the wall.
  - The damping function is parametrized by a new parameter which is independent of the coordinate system.
- The resulting model is Galilean and tensorial invariant.
- The resulting model is robust numerically.

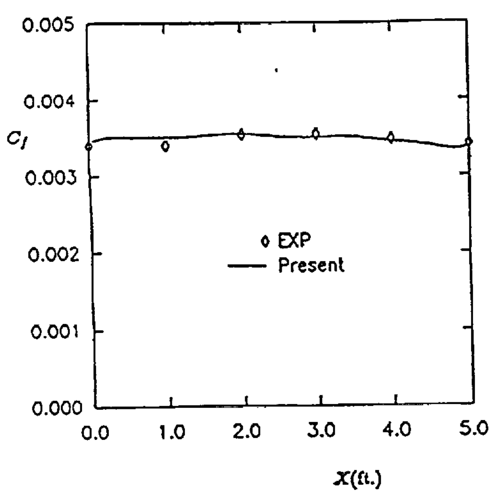
Turbulent Channel Flow at $Re_\tau = 395$



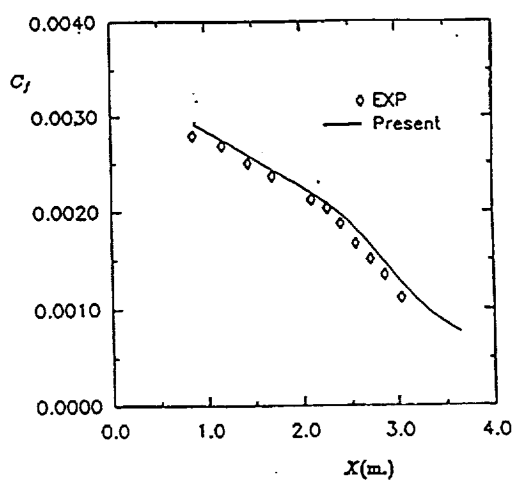
ZPG Turbulent Boundary Layert at $Re_\theta = 1410$



FPG Boundary Layer



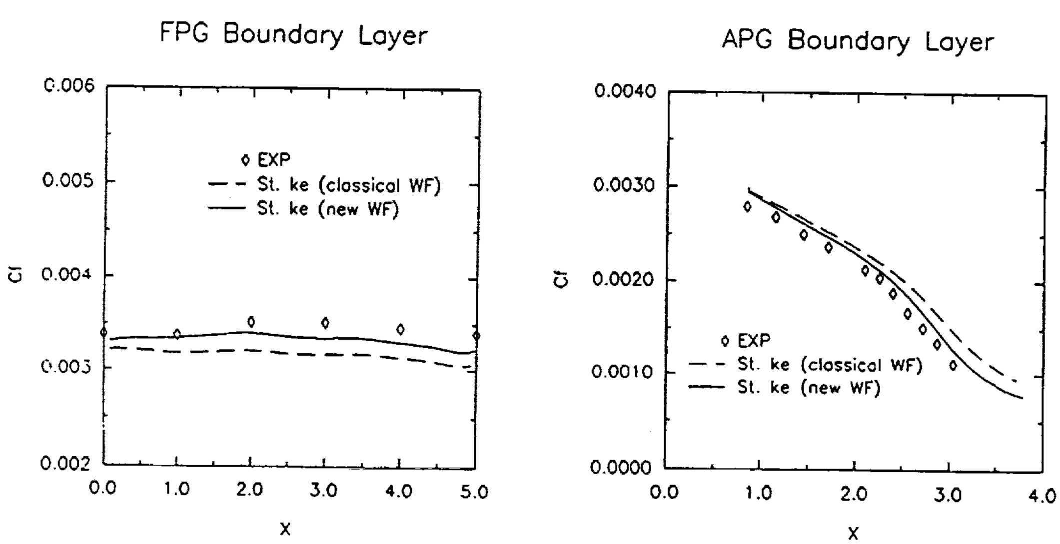
APG Boundary Layer



On the Wall Functions

- The advantages of the wall function approach:
  - Reduce the number of grid points by half, at least.
  - Reduce the numerical stiffness of the dissipation rate equation, by less grid stretching.
- The limit of wall function approach: the flow is assumed to be attached to the wall.
- Existing wall functions are based on the flat plate BL at zero pressure gradient.

  It is inadequate when the pressure gradient is not zero.
- A new set of wall functions are obtained:
  - They are based on the asymptotic behavior of the governing equations in the log layer.
  - They contain the effect of the pressure gradient.



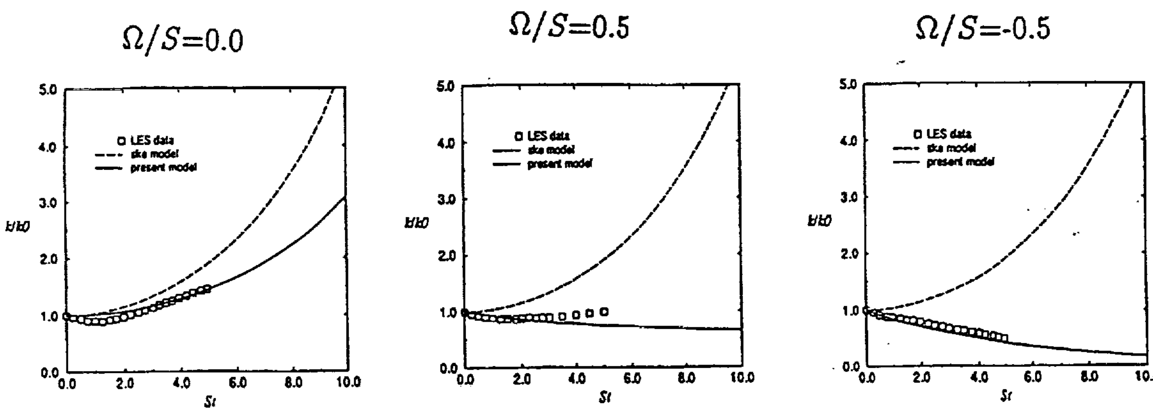
A Vorticity Dynamics Based Model for the Dissipation Rate Equation

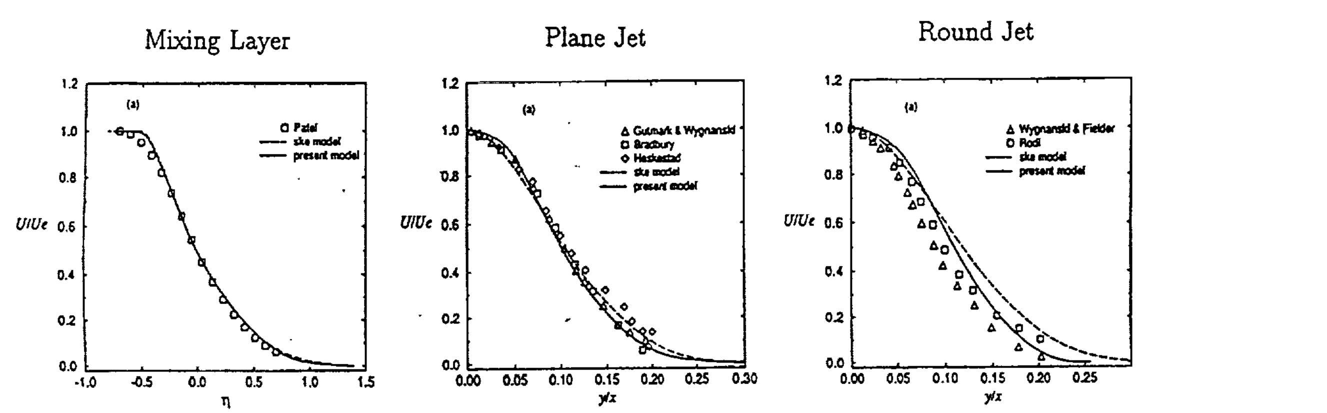
- The dynamic equation for the fluctuating vorticity is analyzed.
  (The terms in the fluctuating vorticity equation have clearer physical meanings than terms in the dissipation rate equation.)
- For large Reynolds numbers, $\epsilon = \nu\overline{\omega_i \omega_i}$.
- The resulting model equation has a better foundation than the standard ϵ equation.
- The resulting model equation always gives a positive production in dissipation rate.
  - The model calculation is expected to be more robust for complex flow calculations.

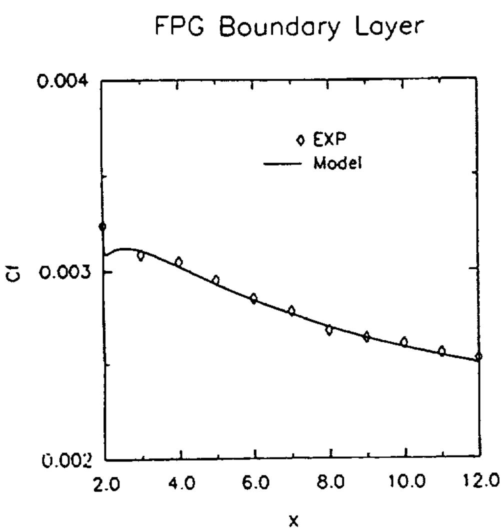
Flow Inhomogeneity and the Dissipation Rate Equation

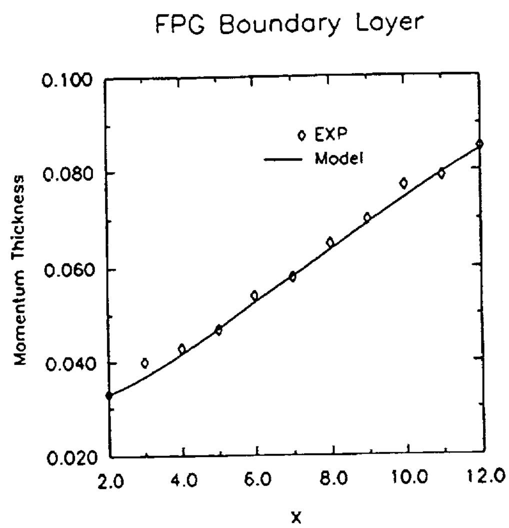
- The exact dissipation rate equation contains source terms due to the flow inhomogeneity.
- However, the existing ϵ equations are homogeneous.
  (The source terms are the same for both the homogeneous and inhomogeneous flows.)
- A new model equation for ϵ is proposed, which accounts for the inhomogeneity effect:
  - Flow inhomogeneity is represented by ∇S and ∇k.
  - Invariant theory is used to derive such a model equation.
- The resulting model equation accurately account for the effect of the pressure gradient.

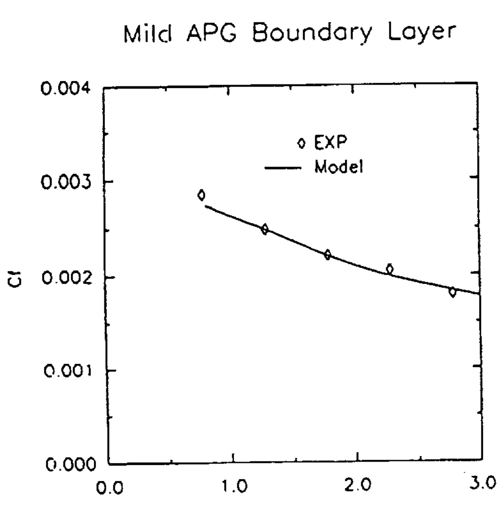
Rotating homogeneous shear flows

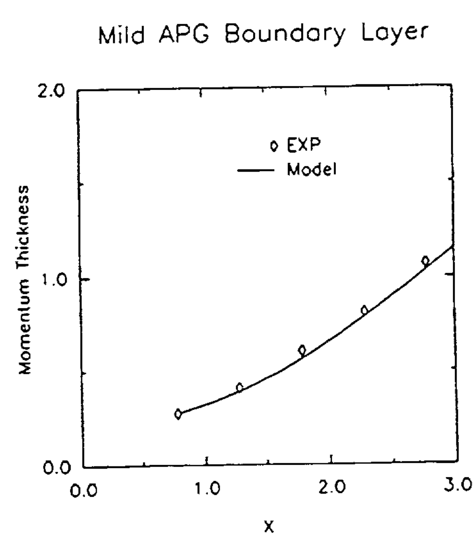




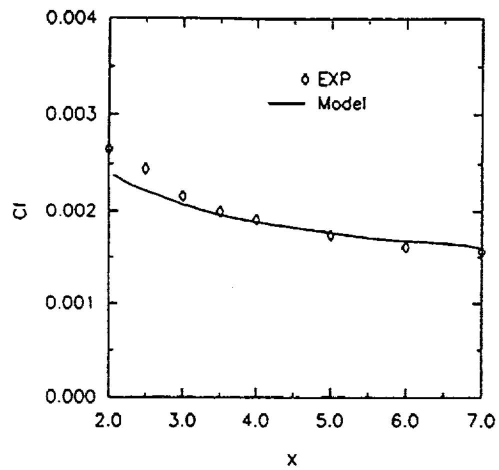

FPG Boundary Layer
0.004
0.003
0.002
Cf
2.0
4.0
6.0
8.0
10.0
12.0
x
EXP
Model


FPG Boundary Layer
0.100
0.080
0.060
0.040
0.020
Momentum Thickness
2.0
4.0
6.0
8.0
10.0
12.0
x
EXP
Model

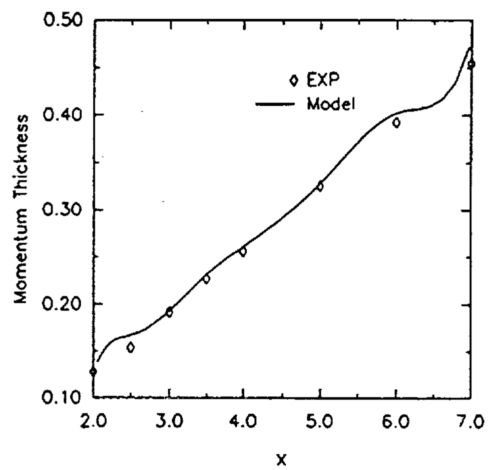

Mild APG Boundary Layer
0.004
0.003
0.002
0.001
0.000
Cf
0.0
1.0
2.0
3.0
x
EXP
Model


Mild APG Boundary Layer
2.0
1.0
0.0
Momentum Thickness
0.0
1.0
2.0
3.0
x
EXP
Model

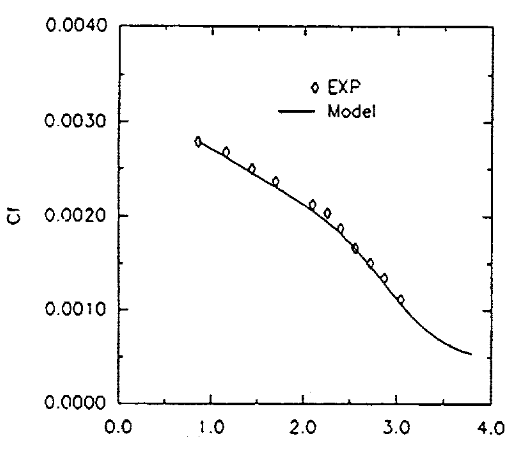
Moderate APG Boundary Layer



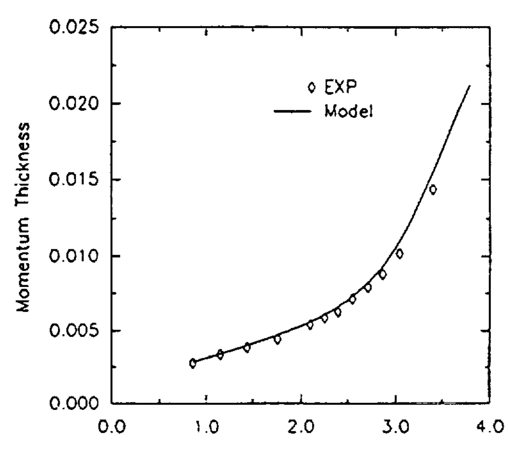
Moderate APG Boundary Layer



Strong APG Boundary Layer

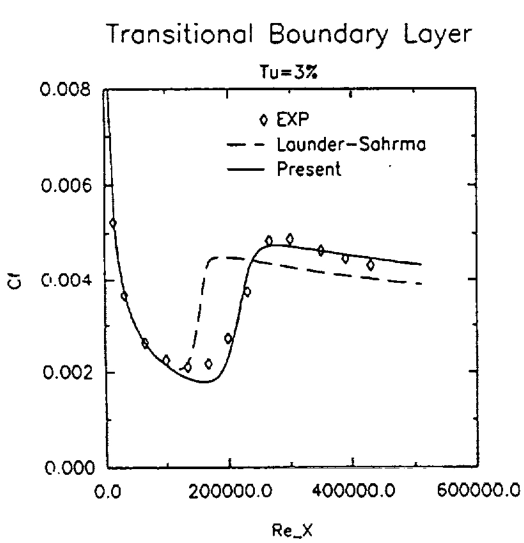


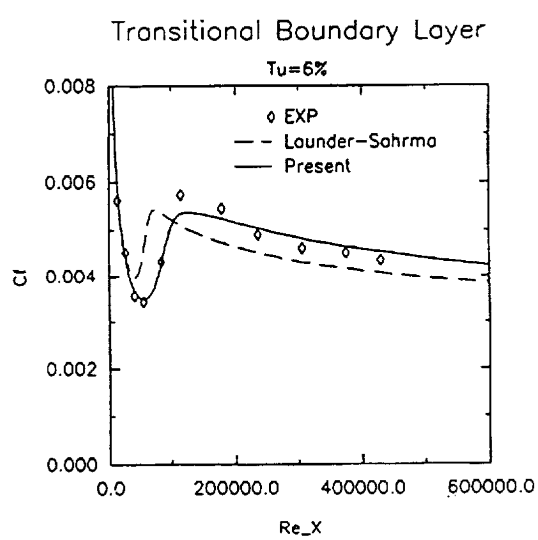
Strong APG Boundary Layer



Modeling of Bypass Transition

- Low Reynolds number $k - \epsilon$ models could mimic transition.
  - The predictions are not very good.
  - Among these models, the Launder-Sharma model gives the best prediction.
- New model for bypass transition is proposed by introducing the effect of the intermittency
- The transition model recovers to turbulence model at the end of the transition zone.
- Calculations of the benchmark flows show that the present model gives better predictions compared with the $k - \epsilon$ models without the intermittency effect.





Summaries:

The capabilities of $k-\epsilon$ model are enhanced in the following areas:

- A Galilean and tensorial invariant $k-\epsilon$ model for near wall turbulence.
- A new set of wall functions for attached flows.
- A new model equation for the dissipation rate:
  - It has a better theoretical basis.
  - It contains the contribution of flow inhomogeneity.
  - It captures the effect of the pressure gradient accurately.
- A better model for bypass transition due to freestream turbulence.