Tetravalent Chromium (Cr⁴⁺) as Laser-Active Ion for Tunable Solid-State Lasers

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ACCOMPLISHMENTS

During 10/31/91 - 3/31/92 the following summarizes our major accomplishments made under the NASA grant: NAG-1-1346

Numerical modeling of the four mirror astigmatically compensated, Z-fold cavity was performed. The simulation revealed several design parameters to be used for the construction of a femtosecond forsterite laser.

PUBLICATIONS AND PRESENTATIONS

During the period of this report the following paper was accepted for presentation:

1. V. Petričević, A. Seas, and R. R. Alfano, "Novel Cr$^{4+}$-Based Tunable Solid-State Lasers", *Recent Advances in the Uses Of Light in Physics, Chemistry, Engineering and Medicine*, at City College of New York
Modeling of a Four Mirror Astigmatically Compensated Cavity Design.

The major disadvantage of the three mirror cavity used for mode-locking of the forsterite laser\textsuperscript{1,2} is that when is used with long samples the beam area inside the gain medium is not uniform. This problem can be overcome by using a four mirror z-fold astigmatically compensated cavity. The z-fold astigmatically compensated cavity design takes advantage of symmetry to force the beam size in the xz and yz planes to be simultaneously minimum at the same spatial position in the laser medium\textsuperscript{3}.

The z-fold four mirror cavity design is shown in figure 1. The four mirror cavity can be thought of as a combination of two three mirror cavities as shown in figure 1. One three mirror cavity consists of mirrors R\textsubscript{1}, R\textsubscript{2}, and a flat mirror positioned at the image plane and the other consists of mirrors R\textsubscript{4}, R\textsubscript{3} and the flat mirror at the image plane. Both cavities use half of the gain medium. The folding angles of the mirrors R\textsubscript{2} and R\textsubscript{3} that compensate for astigmatism can be estimated using eq.

\[
2t \left( \frac{(n^2 - 1) \sqrt{n^2 + 1}}{n^4} \right) = 2f \sin \theta \tan \theta
\]

where: \(t\) - Half the thickness of the gain medium  
\(f\) - Focal length of the folding mirrors  
\(n\) - Index of refraction of the gain medium  
\(\theta\) - Angle of incidence of the folding mirrors

Numerical calculations were performed using eq. 1 to estimate the folding angle for mirrors R\textsubscript{2} and R\textsubscript{3} to achieve astigmatic compensation in the four mirror z-fold cavity. In the calculations it was assumed that R\textsubscript{2} = R\textsubscript{3}. Figure 2 shows the folding angle as a function of the laser crystal thickness in the case of the forsterite laser and assuming different values for the radius of curvature of the folding mirrors.
Fig. 1. Four mirror z-fold astigmatically compensated cavity design. \( \theta \) is the folding angle of mirrors \( R_2 \) and \( R_3 \).

Fig. 2. Folding angle, \( \theta \), vs. forsterite crystal thickness for astigmatic compensation in a four mirror cavity.

Other characteristics of the four mirror cavity can be determined by using the theory presented in references 4 and 5. A computer program was developed to simulate the beam size.
in the four mirror cavity used in the mode-locking experiments. For the program it was assumed that a 1 cm long chromium-doped forsterite crystal was positioned exactly at the center of the two folding mirrors $R_2$ and $R_3$. The round trip $ABCD$ matrix of the four mirror cavity was calculated by choosing a reference plane and following the beam for a complete round trip through the cavity (see figure 3). Note that this process was performed twice, once for the $xz$ and once for the $yz$ plane since the folding mirrors and the brewster-cut crystal behave differently in the two planes.

![Reference Plane](image)

**Fig. 3. Cavity configuration used in the numerical simulation.**

The $ABCD$ matrix for a complete round trip through the cavity is given by

$$M_t = M_2M_3M_4M_5M_6M_7M_8M_9M_{10}M_{11}M_1M_9M_8M_7M_6M_5M_4M_3M_2M_1$$

where:

- $M_2, M_4, M_8, M_{10}$ - $ABCD$ matrices for traveling a distance in free space
- $M_1, M_{11}$ - $ABCD$ matrices for reflection from a flat mirror
- $M_9, M_3$ - $ABCD$ matrices for reflection from a curved mirror at an arbitrary angle of incidence (Different for the $xz$ and $yz$ planes)
$M_5, M_7$ - ABCD matrices for interface between air and forsterite crystal

$M_6$ - ABCD matrix for traveling in the forsterite crystal (Different for the $xz$ and $yz$ planes)

The stability range of the four mirror z-fold cavity design was determined next. This was done by determining the separation between mirrors $R_2$ and $R_3$ for which the self consistent and perturbation stable requirements were satisfied. The four mirror cavity is stable for $\pm 1$ mm from optimum separation between mirrors $R_2$ and $R_3$ which is similar to the three mirror cavity design.

When the stability range was determined, the distance between the folding mirrors was chosen around the center of the stability range and another program was written to calculate the size of the beam waist at any point inside the resonator. This was done by moving the reference plane through out the cavity and calculating the total ABCD matrix for the new reference plane. Then the beam waist at the reference plane was estimated using eq. 2.

\[
\omega_2^2 = \frac{|B|\lambda}{\pi} \sqrt{\frac{1}{1-m^2}}
\]  

(2)

where

\[ m = \frac{A + D}{2} \]

The results of the calculations are shown in figures 4 and 5. Figure 4 shows the size of the beam waist as a function of the position in the cavity while figure 5 is a magnified version of figure 4 showing the behavior of the beam inside the forsterite crystal. Considering figure 4 the beam waist at zero or at mirror $R_1$ has a value of about 600 $\mu$m and is the same for the $xz$ and $yz$ planes. This fact can serve as an indication when aligning the four mirror cavity. A nice round output beam is indicative of astigmatic compensation while an elliptical beam at the output indicates that the folding angle is off the optimum value. The beam waist increases from 600 $\mu$m to 800 $\mu$m as it moves 85 cm from mirror $R_1$ to the folding mirror $R_2$. Still at mirror $R_2$ there is good agreement in
the size in the xz and yz planes. Mirror $R_2$ has a radius of curvature of 10 cm, forcing the beam to focus 5 cm away from the mirror inside the forsterite crystal. As it can be seen from figure 5 the beams inside the forsterite laser have their minimum waist at exactly the same spot. Once the beam comes out of the forsterite crystal expands rapidly up to mirror $R_3$ which collimates the beam. Finally the beam moves from mirror $R_3$ to $R_4$ where it gets reflected and the same transformations are repeated.

![Graph showing beam waist vs. position in the four mirror, z-fold, astigmatically cavity design.](image)

Fig. 4. Beam waist vs. position in the four mirror, z-fold, astigmatically cavity design.
Fig. 5.  Beam waist vs. position in the forsterite crystal for the four mirror, z-fold, astigmatically cavity design.

In order to be able to compare the results obtained from the analysis of the four mirror cavity design with the three mirror cavity the same computer program was modified to correspond to the three mirror cavity. For this program a 1 cm long brewster-cut forsterite crystal was placed in a three mirror cavity with $R_1 = 5$ cm, $R = 10$ cm, $R_2 = \text{flat}$ and the same procedure as with the four mirror cavity was repeated. The results of this simulation are shown in figures 6 and 7. Figure 6 shows the size of the beam waist as a function of the position in the cavity while figure 7 is a magnified version of figure 6 showing the behavior of the beam inside the forsterite crystal.

By comparing figures 5 with 7 it is clear that a four mirror z-fold cavity design has better mode characteristics as compared to the three mirror cavity. The four mirror cavity compensates for the astigmatic effects introduced by the brewster forsterite crystal and also offers a uniform mode area in the forsterite crystal. The cavity mode can be easily matched with the pump mode. In the three mirror cavity design the minimum beam waists for the two planes are formed at different positions in the forsterite crystal resulting in a non uniform mode area making hard to match pump
and cavity modes. Sub-100-μm beam size is achieved using either cavity configuration.

Fig. 6. Beam waist vs. position in the three mirror astigmatically compensated cavity design.

Fig. 7. Beam behavior inside the forsterite crystal in the three mirror astigmatically compensated cavity design.
REFERENCES


