The Faraday Ring Ammeter was the subject of this grant for a new innovative instrument for space plasma instrumentation. This report summarizes our progress in this work. Briefly, we have conducted an intensive series of experiments and trials over three years, testing some 5 configurations of the instrument to measure currents, resulting in 2 Ph.D theses, supported by this grant, and two flight configurations of the instrument. The first flight would have been on a NASA-Air Force collaborative sounding rocket, but was not flown because of instrumental difficulties described below. The second has been successfully integrated on the NASA Auroral Turbulence payload which is to be launched in February, 1994.

The experimental laboratory program to develop the FRA is based on attempts to construct an instrument to detect Faraday rotation in optical fibers and thereby determine the curl of the magnetic field, which is proportional to the current. Systems labelled FRA I, FRA I.5, FRA II, and FRA II.5 in increasing stages of sophistication, were developed as described in Appendix I, the first part of the first Ph.D thesis in this work. The thresholds that were considered possible in this work were as low as 1-3 uAmp per square meter.

The flight instrumentation was based on this work and is the subject of Appendix II, the second Ph.D thesis, which describes all the instrumental problems in making a spaceworthy instrument. That spaceworthy FRA prototype uses spun elliptically birefringent (SEB) fiber from an elliptical core preform. It presents the derivation of the retardance and Faraday rotation sensitivity for the SEB fiber. It also reviews the implications of these ex-
pressions for a low-birefringent precursor fiber and presents the implications for a high-birefringent precursor SEB fiber of sufficient length for a Faraday current sensor. Sensitivity data for a space plasma SEB fiber current sensor prototype using 650 meters and 320 meters of SEB fiber, with spin pitch equal to the linear beat length of the unspun precursor fiber, is included along with prototype mechanical and electrical descriptions.

The final prototype appears to be significantly poorer than we first thought possible with SEB fibers: 1000 uAmp per square meter. The basic difficulty lies with coherent interference of Rayleigh backscattered light within the fiber itself. A possible solution to this difficulty lies in Faraday Version III, which is presently far too complex to include in a sounding rocket payload, but which we believe can be simplified. The optical alignment stability needed for that version appears to be the biggest hurdle.

Work continues on internal funds at UNH on this method. It is hoped that these continuing efforts will lead to a successful spaceworthy instrument capable of measuring down to 1 uAmp per square meter and making significant breakthroughs in auroral physics.
APPENDIX I

Polarization Noise in Optical Fiber Faraday Ring Ammeters

by

Indu Saxena

A dissertation

Submitted in partial fulfillment of the requirements for

the degree of Doctor of Philosophy in

The Department of Physics

of

The School of Graduate Studies

of

The University of Alabama in Huntsville

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This necessarily incomplete list would be very wanting without the inclusion of the support from my family, without which this Ph.D. would not have been possible.

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Abstract

This dissertation presents the development of a new technique named the Faraday Ring Ammeter (FRA) for measuring the curl of magnetic fields, and thereby electric currents, in space plasmas. The technique relies on the magneto-optic (Faraday) effect in diamagnetic materials, for e.g. optical fibers. The FRA is a highly non-invasive, yet direct measurement technique for space plasma current densities, and is impervious to space-craft electromagnetic interferences.

The FRA uses a single mode optical fiber loop as the magneto-optic interaction medium. The nature of transmission through single-mode fibers and its limitations on the measurement of the induced optical activity in the fiber is studied. The Jones representation for a material possessing both linear and circular birefringence is described and applied to a fiber which is twisted and bent into a loop. The effect of the spectral wavelength of the source on the FRA measurement is analyzed within the constraints of space flightworthy systems.

A description of the design of the first spaceflightworthy Faraday Ring Ammeter, FRA-I is presented. Passive polarization control was implemented in FRA-I by reflecting the light at the fiber end such that it emerges at the front end of the fiber having its polarization rotated by an angle given only by Faraday rotation. The minimum current measured with FRA-I is smaller than any other published as a result of an optical fiber current measurement. The results of the FRA-I measurement are discussed. The factors limiting the resolution of FRA-I have
been determined.

FRA-II was designed to overcome the limitations of FRA-I, and was designed for insensitivity to low frequency vibrations. Experimental results from FRA-II showed that although the acousto-optic device it employed could switch light intensity at high frequencies, the polarization of the transmitted light was very unstable. The third developmental design, the FRA-II.v was suggested by the author, and the instabilities observed in FRA-I were, consequently, removed. The latter two versions (II, and II.v) are under further development by the group.

An exhaustive approach to solve the interfering noise problems of FRA-I and FRA-II was then adopted. The approach chosen was different from any of the previous FRA's and it was required to study the complete state of polarization transmitted through the fiber. For that purpose, a high speed digital ellipsometer (DRME) which gives the Stokes vector of the light was designed by the author. The Stokes vectors are sampled at a frequency of six kilosamples per second. The DRME implementation has been described. The DRME would enable the removal of noise at known frequencies, for e.g. those due to mechanical vibrations. The responses of two types of fiber samples to an electrical current (or magnetic stress) as a function of thermal ambient has been studied with the DRME. The results enable fiber selection from a variety of fibers, and compensation of thermal effects by operating at preferred ambient temperatures.

Keywords: fiber optic sensors, current sensor, polarimetric techniques.
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<td>AOM</td>
<td>Acousto-Optic Modulator</td>
</tr>
<tr>
<td>DOP</td>
<td>Degree of Polarization</td>
</tr>
<tr>
<td>SOP</td>
<td>State of Polarization</td>
</tr>
<tr>
<td>EOM</td>
<td>Electro-Optic Modulator</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Linear Retardance (in radians unless specified)</td>
</tr>
<tr>
<td>$\beta^L$</td>
<td>Linear Birefringence (or Retardance) in radians per unit length</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Circular Birefringence (in radians unless specified)</td>
</tr>
<tr>
<td>$\alpha^L$</td>
<td>Circular Birefringence in radians per unit length</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Total birefringence</td>
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<tr>
<td>$\chi$</td>
<td>Extinction Ratio</td>
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<td>$\chi_f$</td>
<td>Extinction Ratio for a fiber</td>
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<td>$\chi_p$</td>
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<tr>
<td>$\eta_c$</td>
<td>Light Coupling Efficiency into the fiber</td>
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<tr>
<td>$\eta_r$</td>
<td>Responsivity of photodetector in amperes/watt</td>
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<tr>
<td>$V_\pi$</td>
<td>Characteristic half wave voltage of EOM</td>
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$n_o, n_e$  Ordinary and extraordinary refractive indices of a crystal
Chapter 1

Introduction

1.1 Determination of Plasma Current Densities

Recently, in space plasma physics, the need for a direct, model-independent and non-intrusive measurement of space plasma current densities has increased tremendously. It is necessary to determine critical current densities at which the ionospheric plasmas become unstable for a better understanding of the phenomenon. Estimations (R.B.Torbert, et al.[5]) show that the current densities at which the instabilities occur are around 10-50 $\mu$A/m$^2$. There has been no method capable of performing this measurement however, and these minute space plasma currents have so far never been directly measured.

A direct method with the promised precision is the Faraday Ring Ammeter (FRA) technique. It measures the entire current independent of the characteristics of the distribution function of the $B$ field. It is the one technique which...
promises to satisfy the sensitivity requirement of a direct measurement of current in space plasmas. It provides the motivation for this dissertation. Nobody, to the knowledge of the author has attempted to perform the direct measurement of space plasma currents with the FRA or any other technique.

Existing laboratory techniques for direct measurements of currents have not been attempted in space plasmas as they are expected to cause perturbations larger than the measurand, and cannot achieve the required sensitivity. Therefore, so far, these currents are indirectly derived from intrusive magnetometer measurements. In this method for derivation of currents [10] (as outlined below), comprehensive assumptions about the spatial distributions of the magnetic fields have to be made. These assumptions could lead to large errors, but the knowledge about magnetospheric current flow is so essential to the understanding of magnetospheric physics that these risks have been taken.

The significance of the assumptions made for derivation of currents may be understood from (Primdahl[10]):

"Polar cap horizontal ionospheric currents have long been inferred from magnetic perturbations of polar magnetograms [Friis-Christensen and Wilhelm, 1975 and references]. These analyses can only yield equivalent current systems, and additional information is necessary for determining the real currents. Recent polar cap electric field observations ...indicate current systems that are not in agreement with the equivalent horizontal current systems deduced from local magnetic observations...."
It must be stressed that (1) all current densities determined from the rocket measurements are local in some sense, (2) that their validity rests upon relatively unrestricted assumptions (no neutral wind and homogeneity over relatively small dimensions \(\approx 50 \text{ km}\)), and (3) that any disagreement between these local measurements and the ground-based observations undoubtedly reflects a breakdown in assumption over large dimensions \(\approx 500 \text{ km}\)."

Typical plasma current density derivations currently follow one of the following two techniques as outlined by Primdahl [10]:

- **From Ohm's Law:**

  The electrical conductivity tensor \(\sigma_{ij}\) is calculated from atmospheric and magnetic field models, electron density and temperature profiles. The ionospheric models and conductivity tensors are combined with dc electric field \((E)\) measurements made along the rocket trajectory to calculate the current density \((J)\),

  \[
  J = \sigma E, \quad \text{or} \\
  J_i = \sigma_{ij} E_j 
  \]

- **From magnetometer measurements:**

  By measuring total magnetic field \(B\) with either precession or flux gate magnetometers, the ionospheric current density is obtained from

  \[
  J = \nabla \times \frac{B}{\mu_0} 
  \]

  To find \(\nabla \times B\) or \(J\), some additional assumptions about the geometry and spatial distribution of the currents have to be made. These assumptions, however, may
lead to large errors. The only way to eliminate assumptions and obtain more accurate space plasma current densities is by using direct current measurement techniques, as opposed to the indirect methods described above. Direct measurement of the plasma currents will open up completely new areas of study which have been impossible so far [6]. For example, the relative drift of ions and electrons, (where $v_i$ is the ion velocity and $v_e$ is the electron velocity), given by

$$<v_i>-<v_e>=\frac{j}{en_p}$$

(1.3)

can then be computed. This is the single most important parameter characterizing current driven plasma instabilities, which are thought to cause aurora and solar flares.

The principle of the FRA technique is the magneto-optic effect in optical (glass) fibers. It is a non-intrusive method for the measurement of currents in space plasmas. Since the FRA series of sensors are unaffected by EMI (electromagnetic interference) they are ideal for plasma current measurements from spacecrafts. Currents from several kiloamperes to megaamperes have been routinely measured in some high-voltage transmission lines and monitoring applications [11], [33] with the traditional FRA technique. The realm of measurements aimed for with the FRA corresponds to the expected plasma current density in the earth's magnetosphere: one to tens of microamperes per square meter. Therein lies the challenge accurate, direct measurement of space plasma currents a few microamperes per square meter, by improving a technique traditionally
used to measure currents of several megaamperes per square meter.

1.2 The Faraday Effect

Michael Faraday, in 1845, discovered that the plane of polarization of light traversing a dielectric material in the presence of a magnetic field undergoes a rotation. This phenomenon is called the Faraday effect. The angle of rotation caused by the Faraday effect is given by $\alpha_H$, where

$$\alpha_H = V \int_L H \cdot dl$$

$d\ell$ is along the propagation direction of the beam, $L$ corresponds to the length of the interaction medium (optical fiber), and $H$ is the strength of the magnetic field. $V$ is the Verdet constant, which is a measure of the magnitude of the magneto-optic interaction. $V$ is a characteristic of the material and depends on the frequency $\nu$ (or angular frequency $\omega = 2\pi\nu$) of the incident radiation. The Faraday effect is different from other causes of optical activity (e.g. that due to a twisted fiber). All other causes of optical activity are reciprocal, that is, the net rotation of the SOP is zero when light travels forward and back through the same material. This is not true for the Faraday effect and can be explained by its dependence on $H \cdot d\ell$. The rotation on the reverse propagation path is additive to that of the forward propagation, as the field direction is opposite to the reverse propagation vector. This is known as the non-reciprocal behavior of
Faraday rotation. It is therefore possible to obtain a rotation of $N\alpha$ by reflecting $N$ times back and forth in the same length of the interaction medium.

Classically, the relationship between the Verdet constant and angular frequency of the incident radiation, $w$, is given by

$$V(w) = \frac{\pi \nu^2 \omega_e}{\lambda n_0 B (w_0^2 - w^2)^2}$$

(1.5)

for an electronic resonance frequency of the medium $w_0$, $B$ the applied magnetic field, $w_p^2 = \frac{N\nu^2}{mc^2}$, $\omega_e = \frac{eB}{mc}$, and $e$ and $m$ the electronic charge and mass respectively. The units for $V$ are units of angle (minutes, degrees or radians) per unit magnetic field, per unit length, (typically minutes per Oe.cm). $V$ is related to the dispersion of refractive index, $n(\nu)$, in a medium (Becquerel, 1897) by:

$$V(\nu) = \frac{e}{2mc^2 \nu} \frac{dn}{d\nu}$$

(1.6)

or

$$V(\lambda) = -\frac{e}{2mc} \lambda \frac{dn}{d\lambda}$$

(1.7)

c is the speed of light in vacuum, and is $c = \nu \lambda$ for light of frequency $\nu$ and wavelength $\lambda$. $n(\nu)$ may be given by

$$n(\nu) = \sqrt{1 + \frac{w_p^2}{w_0^2 - w^2}}$$

(1.8)

where a predominant electron absorption frequency $w_0$ appears. However, for a more general quantum mechanical expression, inclusion of transitions between all the different possible energy levels is necessary:

$$n(\nu) = \sqrt{1 + \sum_j \frac{w_p^2 f_{ij}}{w_{ij}^2 - w^2}}$$

(1.9)
where $f_{ij}$ are the oscillator strengths between level $'i'$ and $'j'$, and determine the probabilities of the transitions. The general expression for the Verdet constant as a function of $w$ is then obtained

$$V(w) = \frac{\pi}{\lambda n(w) B} \sum_j \frac{w w_j w_{ij}}{(w_{ij}^2 - w^2)^2}$$  \hspace{1cm} (1.10)$$

For a closed path $L$ in Eqn.1.4, Ampere's Law allows the rotation $\alpha_H$ to be given by

$$\alpha_H = V'I$$  \hspace{1cm} (1.11)$$

where $V'$ is $\mu_0 V$, $\mu_0$ being the vacuum permeability.

### 1.3 Previous Fiber-Optic Ammeters

Optical fiber was first employed for measurement of current in 1978 (Smith [26] in 1978) followed by others (Rashleigh and Ulrich [30] in 1979). The lengths of fiber employed in these measurements were, typically, a few meters. This length was sufficient for the magnitudes of current required to be measured, typically $\sim$ kA or larger. As Sec. 4.1 shows, the traditional measurement scheme measured the current via a response function proportional to $\sin(IN_f)$, where $I$ is the current in the conductor and $N_f$ are the number of fiber turns around the conductor carrying the current.

The radii of the fiber coils in earlier optical fiber ammeters were also small (as compared to those that are required for space plasma current measurements).
The smallness was of the order of a few cm, large enough to allow a current carrying conductor to pass through. Smaller radii meant smaller lengths of optical fiber were required for the same $N_f$ than that for larger radii.

All these factors enabled reduced sensitivity of the fiber to environmental effects as smaller lengths of fiber were involved. In order to achieve higher current sensitivity, more turns of fiber, and consequently longer lengths of fiber have been employed in the FRA-I and subsequent FRA's being designed for space plasma current measurements. New problems have therefore been encountered in these designs, the most important being thermal response and depolarization, as discussed in the following chapters.

1.4 Structure of the dissertation

An understanding of the fiber, source and detector behaviour in the measurement system of the FRA is necessary to determine the required operational conditions under which the maximum precision can be achieved. The first space flight worthy instrument, the FRA-I, that was designed and built is described in Chapter 4. It has been shown to be capable of detecting as small as a tenth of an ampere, that is, two orders of magnitude better than present day laboratory plasma instruments [9]. This has been doubled to fifty milliamperes by a novel laboratory FRA technique that has been implemented (Sec. 4.6). An instrument (the DRME) has been designed and developed to study noise processes caused by
unwanted mechanical and thermal stresses superposed on the interesting magnetic stresses in the fiber. The relationship between the minimum detectable plasma current that the FRA can measure and the constraints of the measurement environment has been determined. For a set of given system parameters, the minimum current for an FRA can be determined from this relationship as outlined in Chapter 3.

This dissertation follows a chronological order in which the experiments were performed to a large degree. The factors governing the choice of optical fiber and wavelength of operation have been outlined in Chapter 3. The first space flight worthy instrument, which was based on the existing optical fiber current measurement technique, is described in Chapter 4. This includes the description of the FRA-I, most of which forms the article "Compact Optical Fiber Faraday Ring Ammeter" [1]. The series of FRA's which incorporated measurement techniques different from the traditional one, have been described in Sec. 4.6. The motivation in these techniques was to remove slowly varying linear birefringence effects which interfere with current measurement in the traditional FRA-I technique. To determine the time variation of the linear birefringence "noise" effects, a time domain ellipsometric technique was developed to sample ellipsometric parameters very fast (6 kilosamples per second). An EOM (electro-optic modulator) formed the polarization modulation element in the ellipsometer. The accurate characterization of the EOM is described in Chapter 5. The article "Characterization of an ADP electro-optic modulator in the near infra-red" [2] was a
result of this work. The design and operation of the high speed ellipsometer or DRME is Chapter 6. The experiments performed with this instrument are also discussed in this chapter. Some of Chapter 6 forms the article "A High Speed Digital Ellipsometer for the study of Fiber Optic Sensor Systems" [3]. The final chapter (Chapter 7) concludes with a summary of the research.
APPENDIX II

I. FIBERS

Single mode optical fibers have provided the motivation for the development of many polarimetric fiber-optic sensors. These fibers have the special property of transmitting a spatially coherent $LP_{01}$ mode and are ideally non-depolarizing, making coherent detection schemes possible. The emergence of the single mode laser and the superradiant diode as sources has fuelled the development of single mode fibers and their applications. Even though a single polarization state can be excited in multimode fibers by choosing the launching conditions appropriately, multimode fibers divide the optical electric field among many modes with different spatial distributions and propagation speeds. This phenomenon, together with internally and externally induced mode conversions, depolarizes the field for fiber lengths larger than 1 meter. In order to understand the single mode fiber as the choice for our application, some concepts on optical fibers are defined.

An optical fiber is a cylinder of dielectric materials capable of guiding light. It consists of a central core, with one or more cladding layers concentric to it, and is protected by a jacket, usually of some plastic, which allows the brittle glass to be bent without breaking. In general, the refractive index of the core is circularly symmetric, the simplest case being that of a step index profile along the radial direction:

$$n(r) = \begin{cases} n_1 & \text{for } r < a \\ n_0 & \text{for } r > a \end{cases}$$

of core radius $a$, and $n_1 > n_0$. The relative index difference between the value $n_1$ on axis and the value of the cladding index $n_0$ is

$$\Delta = \frac{n_1^2 - n_0^2}{2n_1^2} \approx \frac{n_1 - n_0}{n_1}$$

A wave travelling in the positive $z$ direction with a phase velocity

$$v = \omega/k$$

has time and axial distance dependence given by

$$e^{i(\omega t - kz)}$$

where $\omega$ is the angular frequency of the monochromatic radiation and $k$ is the propagation constant. A finite number of propagation constants are obtained from
solutions of Maxwell's equations under the boundary conditions that the fields must satisfy. For guided modes, the propagation constants must lie in the range

\[ n_0 k_0 < k < n_1 k_0 \]

where \( k_0 = 2\pi/\lambda \) is the propagation constant in vacuum for a wavelength \( \lambda \).

An approximate solution for guided modes of weakly guiding step index fibers (\( \Delta << 1 \)) is given by a set of very nearly linearly polarized LP modes. The simplified \( \text{LP}_{\mu\nu} \) mode actually corresponds to a superposition of \( \text{HE}_{\nu+1,\mu} \) and \( \text{EH}_{\nu-1,\mu} \) exact modes. So, as the \( \text{LP}_{01} \) mode propagates, its constituent two modes travel with slightly different velocities such that their relative phase relationship keeps changing as a function of \( z \). Only after a distance corresponding to the beat length of the fiber does the original phase relationship recur. The beat length \( L_\beta \) between modes of propagation constants \( k_1^L \) and \( k_2^L \) is

\[ L_\beta = \frac{2\pi}{(k_1^L - k_2^L)} \]

The corresponding relative retardation in phase of one linear polarization eigenstate with respect to its orthogonal state over unit length is called the linear birefringence of the single mode fiber, and is just the difference of the propagation constants

\[ \beta^L = k_1^L - k_2^L = \frac{2\pi}{L_\beta} \]

Correspondingly, if the propagating fields are decomposed into circular eigenstates, instead of linear ones, the relative difference in phase of the orthogonal circular states over a unit length is the circular birefringence of the fiber, \( \alpha^L \). For propagation constants \( k_1^c \) and \( k_2^c \) of the two circular eigenstates of the fiber

\[ \alpha^L = k_1^c - k_2^c \]

The normalized frequency parameter of the V-number of the fiber is

\[ V = (n_1^2 - n_0^2)^{1/2} k a \]

which determines the number of modes, \( N \), that the fiber can support. This number of modes is approximately \( N = V^2/2 \), which includes both HE and EH possible polarizations. If only the \( \text{LP}_{01} \) mode, corresponding to the dominant \( \text{HE}_{11} \) exact mode, can exist, it is called a single mode fiber. A fiber operates in a single mode if the V-number satisfies the condition

\[ V < 2.405 \]

The numerical aperture of a waveguide is defined as

\[ NA = \sin \theta_c = \frac{n\sqrt{2\Delta}}{n_e} \]

For a waveguide in air, \( n_e = 1 \), \( n = 1.46 \), \( \Delta = 0.01 \), and the maximum angle ray that the waveguide will support is \( \theta_c \approx 12 \) degrees.
A. Birefringence

The nature of the transmitted polarization of a single mode fiber can be obtained if its birefringences can be determined. The intrinsic birefringence of a fiber is due to built-in stresses caused in the manufacturing process, is constant and cannot be changed. The extrinsic birefringence of a fiber is due to environmental effects.

There are three basic birefringence phenomena which may occur within a dielectric material. In optical fibers, the dominant effect is invariably linear birefringence. Two other birefringences may also occur: circular birefringence and fast-axis rotation. These birefringences may be present in any fiber-optical system. Some of them are more important than others in specific applications.

Linear birefringence- Linear birefringence is the phenomenon whereby the phase velocity of linearly polarized light within a medium depends on the direction of polarization relative to two fixed orthogonal axes within the medium. The direction for which the phase velocity is a maximum is known as the fast axis and the direction for which the phase velocity is a minimum is known as the slow axis. In general, if linearly polarized light enters a medium with linear birefringence, it will emerge elliptically polarized.

Circular birefringence- Circular birefringence is the phenomenon whereby the phase velocities for right- and left-handed circularly polarized light within a medium are different. If linearly polarized light enters a medium with circular birefringence, it will emerge linearly polarized, but rotated about the propagation direction.

Fast-axis rotation- Fast-axis rotation may occur only in materials which have linear birefringence. The fast- and slow-axis directions in such materials may change continuously with longitudinal position. This effect could be caused by a longitudinal twist in a linearly birefringent material. If linearly polarized light enters a medium with fast-axis rotation along the fast- or slow-axis, the linear polarization will rotate with that axis and emerge linearly polarized along that axis. In general, however, if linearly polarized light enters this medium at an arbitrary angle to the fast- or slow-axis, it will emerge elliptically polarized.

B. Extrinsic Birefringences:

To easily detect a circular birefringence effect like the Faraday effect, it is desirable to have intrinsic and extrinsic circular birefringence dominate any linear birefringence on a global system scale (see Section D.). The following are the types of external birefringence effects occurring in an optical fiber system.

- Linear birefringences
  1. Bend induced:

    Bend induced linear birefringence, \( \beta_b \), is essentially a stress effect. It results from the lateral, compressive stress, \(-\sigma_x\), that builds up in a bent
fiber under the conditions of "large" deformations. This stress modifies the refractive index of the fiber material: The two resulting orthogonal propagation directions are in the plane of the bend and normal to it. The birefringence is given by:

$$\beta_b = k_x - k_y \Rightarrow K_L(\omega) \frac{r^2}{R^2}$$

(1)

where

- $K_L(\omega)$ is a wavelength and material dependent constant equal to:

$$0.25kn^3(p_{11} - p_{12})(1 + \nu) = 1.03 \times 10^6 \text{ rad/m}$$

for bare silica fiber, where $\nu$ is the Poisson number of the glass, and $p_{11}$ and $p_{12}$ are the strain-optical coefficients, with $(p_{11} - p_{12}) = -0.15$,

- $r$ is the radius of the fiber core,

- $R$ is the bend radius of the fiber,

- and $\beta_b$ is the difference in propagation constants caused by bending stresses of the fiber, i.e. the bend induced linear birefringence.

2. Lateral force induced:

Any laterally anisotropic forces elasto-optically induce birefringence in the fiber 64, 118. In a coiled fiber system, these forces may appear any place the fiber is attached by cement to a flat surface or they may result from vibrations of the ring on which the fiber is spooled. The birefringence is given by:

$$\beta_l = \frac{2}{r^2}kn^3(1 + \nu)(p_{12} - p_{11}) \frac{F}{E}$$

(2)

where

- $k$ is the wavenumber of light;

- $n$ is the index of refraction,

- $E$ is the uniform Young's elastic modulus of the fiber or fiber with cladding and jacket, as appropriate,

- $r$ is the outer radius of the fiber and its cladding,

- $F$ is the force on the fiber in N/m,

- and $\beta_l$ is the linear birefringence due to lateral forces on the fiber.

Clamping the fiber is avoided in the FRA and does not induce any linear birefringence, but vibrational forces continue to be a problem.

3. Electro-optically induced, Kerr effect:

The electro-optically induced birefringence is given by:

$$\beta_K = knK(E_K)^2$$

(3)

where
- $K$ is the normalized Kerr effect constant of the fiber which, for silica, is $\approx 2 \times 10^{-22} m^2/V^2$.
- $E_K$ is a transverse electric field,
- and $\beta_K$ is the linear birefringence induced by $E_K$.

There is no modification of polarization in an optical fiber for transverse electric fields up to $2.5 \times 10^4 V/m$.

- **Circular birefringences**

1. **Twist induced**:

   In a twisted or spun fiber, the strain-induced optical activity, $\alpha_\tau$, is proportional to the twist and is given by:
   
   $$\alpha_\tau = \frac{-\eta_0^2(p_{11} - p_{12})}{2} 2\pi \tau$$
   (4)
   
   where
   - $\tau$ is the number of $360^\circ$ rotations per unit length,
   - $\eta_0^2(p_{11} - p_{12})$ is material and construction dependent and is $\approx 0.16$ for silica,
   - and $\alpha_\tau$ is the twist induced circular birefringence.

2. **Magneto-optically induced, The Faraday Effect**:

   The Faraday effect is the rotation of the plane of polarization of light traversing a dielectric material in the presence of a magnetic field. The angle of rotation is given by:

   $$\alpha_H = V \int_L H \cdot dl$$
   (5)

   where
   - $dl$ is along the propagation direction of the light,
   - $L$ corresponds to the length of the interaction medium, in our case the optical fiber,
   - $H$ is the magnetic field strength,
   - and $V$ is the Verdet constant, a measure of the strength of the interaction, characteristic of the material and a function of the frequency $\nu$ and the index of refraction $n$:

   $$V = \frac{e}{2mc^2} \nu \frac{dn}{d\nu}$$
   or
   $$V = \frac{e}{2mc} \lambda \frac{dn}{d\lambda}$$
   (6)
The interaction of light with a magnetic field causes a circular birefringence per unit length. For a closed loop, $\alpha_H$ becomes:

$$\alpha_H = \mu_0 VNI = V'NI$$  \hspace{1cm} (7)

where
- $I$ is the current through the plane of the loop,
- $N$ is the number of fiber turns,
- $V'$ is the Verdet constant in the convenient units of rad/Amp,
- and $\mu_0$ is the vacuum permeability.

If the direction of the light beam is reversed, the value of the difference in propagation constants for right- and left-handed light is negated; however, the sense of right and left is now reversed and the angle of rotation adds if the observer looks in the same direction, i.e. the Faraday effect is non-reciprocal. The only difference between the rotation for the two directions is the Faraday rotation, since all other birefringence effects, both linear and circular, remain the same; therefore, if one can traverse both directions in the fiber simultaneously, the Faraday effect will be doubled and other birefringence noise effects will be cancelled.
C. Measuring the Faraday Effect

Jones Calculus gives the modification of the SOP of light transmitted through a non-depolarizing medium. If the input is a linearly polarized electric field, the output is given by:

\[
E = \begin{pmatrix} 0 & E_0 e^{i\omega t} \\ 1 & 0 \end{pmatrix}
\]

If the medium is an ideal rotator, \( E' = S E \):

\[
E' = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 0 & E_0 e^{i\omega t} \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -\sin \theta & \cos \theta \end{pmatrix} E_0 e^{i\omega t}
\]

An analyzer aligned along the x axis at the fiber's end divides the two orthogonal polarizations, \( E'_x \) and \( E'_y \), into different photo detectors. The Faraday effect is measurable as the rotation \( \theta \), measured clockwise from the horizontal x axis, which can be indirectly measured as \( T' \) given by:

\[
T' = \frac{E_x'^2 - E_y'^2}{E_x'^2 + E_y'^2} = -\cos 2\theta
\]

With the analyzer in this configuration, large changes in \( T' \) result from large changes in \( \theta \) and the system is sensitive to large currents.

If the analyzer is rotated by 45° or a \( \lambda/2 \) wave plate in front of the analyzer is rotated by 22.5°, then:

\[
E'' = \begin{pmatrix} \cos \frac{\pi}{4} & -\sin \frac{\pi}{4} \\ \sin \frac{\pi}{4} & \cos \frac{\pi}{4} \end{pmatrix} \begin{pmatrix} -\sin \theta & \cos \theta \end{pmatrix} E_0 e^{i\omega t}
\]

\[
= \frac{1}{\sqrt{2}} \begin{pmatrix} -\sin \theta - \cos \theta \\ -\sin \theta + \cos \theta \end{pmatrix} E_0 e^{i\omega t}
\]

and

\[
T'' = 2 \sin \theta \cos \theta = \sin 2\theta
\]

With the analyzer in this rotated position, large changes in \( T'' \) result from small changes in \( \theta \) and the system is more sensitive to small currents.

D. Effect of Birefringence on Faraday Measurement

In most realistic systems, there exists a finite amount of linear birefringence and the optical system does not act as an ideal rotator. There will instead be a superposition of circular and linear birefringence. Jones calculus gives the resultant polarization modification for a Faraday rotation angle, \( \alpha \), and a linear retardation, \( \beta \), as:

\[
E' = \begin{pmatrix} A & -B \\ B & A^* \end{pmatrix} E
\]

where
For a linear input state as before:

\[ E = \begin{bmatrix} 0 \\ 1 \end{bmatrix} E_0 e^{i\omega t} \]

\( E' \) becomes:

\[ E' = \begin{bmatrix} A & -B \\ B & A^* \end{bmatrix} E = \begin{bmatrix} -B \\ A^* \end{bmatrix} E_0 e^{i\omega t} \]

Then, with the analyzer in front of the photodetectors rotated by 45° or the \( \lambda/2 \) wave plate rotated by 22.5°, and the measured quantity \( T'' \) becomes:

\[ T'' = -\frac{|E_1|^2}{|E_1|^2 + |E_2|^2} = \frac{-4|A|B}{2B^2 + 2|A|^2} = -2|A|B \]

Substituting for \( A \) and \( B \):

\[ T'' = -2|A|B = -2(\cos^2 \frac{\phi}{2} + \cos^2 \tau \sin^2 \frac{\phi}{2}) \frac{1}{2} (\sin \tau \sin \frac{\phi}{2}) \]

If the fiber is untwisted and the Faraday effect is small:

\[ T'' \approx -2 \sin \tau \sin \frac{\beta}{2} \quad \text{for} \quad \frac{\beta}{2} \gg \alpha \]

The Faraday effect is effectively quenched by the linear birefringence. The extraction methods to detect the Faraday rotation become extremely complex.

If the fiber is twisted so that the twist induced birefringence, \( \alpha_r \), plus the Faraday induced birefringence, \( \alpha_f \), overwhelms the bend induced linear birefringence, \( \beta_b \), plus the intrinsic linear birefringence of the fiber, \( \beta_i \), then:

\[ T'' \approx -\sin 2\alpha \quad \text{for} \quad \frac{\beta}{2} \ll \alpha \quad (12) \]

and detection of the Faraday rotation is greatly simplified. This technique enables the use of fibers with considerably high linear birefringence, but, as shown later, for long fibers on the order of 100 meters, the twist birefringence value, \( \alpha_r \), drifts with temperature.
E. Environmental Factors Influencing Sensitivity

The relation \( T = \sin 2\alpha \) holds under the assumption that the input state of the light remains linearly polarized and the system ellipticity, \( P \), remains close to 1 during the measurement. For a bent fiber with intrinsic linear birefringence, there are two orthogonal linear SOPs at the input for which ellipticity is 1. Ideally, the input state should be maintained along one of these axes and the birefringence of the fiber must remain constant. This "quadrature" point of stability is also required for the output. The SOP out of the fiber must be held at 45° to the axes of the analyzer and the Faraday rotation measured from this zero point. Temperature and vibration induced birefringence variations adjust this quadrature point and reduce or quench the sensitivity of the system to small Faraday rotations.

1. Thermal Effects

Differences in the thermal expansivities between materials comprising the fiber core and cladding cause stresses which influence the propagation characteristics and affect the sensitivity of the fiber to temperature changes\(^88\). Even when thermal expansivities are matched, thermal sensitivities are observable in fibers designed with high internal stress level, with high dopant levels or in those with geometrical asymmetries, i.e. non-circularity of the fiber core. The temperature coefficient of twist induced circular birefringence, \( d\alpha/dT \), has been found to be greater for fibers with high intrinsic birefringence than for those with low intrinsic birefringence. Fibers with very low intrinsic birefringence have been obtained \(^88\), \(^105\) by tightening the circularity tolerances in the manufacturing processes and by decreasing the thermal expansivity differences between the core and cladding of single-mode fibers. These fibers are consequently less sensitive to thermal drifts.

For bare, twisted fibers that have not been treated specially to have low intrinsic birefringence, the twist-induced circular birefringence is given by\(^133\):

\[
\alpha_t = g2\pi\tau
\]

where

- \( g = -\frac{n_0^2}{2(p_{11}-p_{12})^2}2\pi\tau = n^2 p_{44} 2\pi\tau \), as in equation 4,
- \( p_{44} \) is the strain-optic tensor for fused silica and equals 0.075,
- \( n \) is the refractive index, and
- \( \tau \) is the number of 360° twists per unit length.

The temperature variation can be obtained by differentiating:

\[
\frac{d\alpha}{dT} = \frac{dg}{dT}2\pi\tau
\]

where

\[
\frac{dg}{dT} = 2n_0 p_{44} \frac{dn_0}{dT} + 2n_0 \frac{dp_{44}}{dT} \Rightarrow 2n_0 p_{44} \frac{dn_0}{dT}
\]
From 117, $\frac{dn}{dT} = 10^{-5} \, ^oK^{-1}$ and $\frac{d\alpha}{dT}$ becomes:

$$\frac{d\alpha}{dT} = 2(1.46)(0.075)(10^{-5})2\pi \tau$$

$$= 1.38 \times 10^{-5} \tau \, \text{rad/}^o\text{K} \cdot \text{m}$$

This is in good agreement with data117.

For long lengths of fibers $\approx 100$ m or more, this sensitivity to thermal instabilities can be disastrous, i.e. 413 meters, twisted 2.1 times per meter yields $\approx 12$ mrad per °K fluctuation. The Faraday rotation expected in an aurora is on the order of 0.2 $\mu$rad. Thermal effects can be alleviated by constructing the fiber coil in two sections, wound consecutively, but twisted in opposite senses98. If the temperature sensitivity in the two sections is equal, the effect should cancel. The temperature effects can also be cancelled by end-silvering the fiber and reflecting the light back through the fiber.

2. Vibration Effects

Single-mode optical fibers are sensitive to externally applied pressure. For modest pressures, light continues to propagate but, in general, the degeneracy of the two orthogonally polarized modes is lifted. In optical fiber current measurement systems, this pressure-induced linear retardation changes the device sensitivity and leads to inaccuracies.

When a line force $F$ is applied transverse to a cylinder, stress is developed within the cylinder. On the axis of the cylinder there is a compressive stress in the direction of the applied force of magnitude $\frac{6F}{\pi d}$ and a tensile stress of magnitude $\frac{2F}{\pi d}$ in the orthogonal transverse direction, where $d$ is the fiber core diameter118. The magnitude of both stresses remains within 96% of the axial value in the region near the axis which is bounded by a circle of diameter $\frac{d}{10}$.

In a single-mode fiber with $d \sim 100$ $\mu$m, light propagates typically within the central 10 $\mu$m around the axis where the two stresses are essentially uniform. Using the strain-optic coefficients, $p_{ij}$, for an isotropic material, the lateral force induced retardation is given by (See B.):

$$\beta_i = \frac{2}{r\pi}kn^3(1 + \nu)(p_{12} - p_{11})\frac{F}{E}$$

where the force $F$ is in Newtons/meter. Inserting values for fused silica, i.e. bare fiber, and a wavelength of 830 nm,

$$\beta_i = 7.31 \times 10^{-5}\frac{F}{d} \, \text{rad/meter}$$

For 413 meters of 125 $\mu$m fiber, with vibrational forces on the order of $1 \times 10^{-5}$ N/m, this is a retardation of about 3 mrad, much larger than the expected 0.2 $\mu$rad Faraday rotation.
According to Smith118, coating the bare fiber with a silicone rubber buffer reduces the sensitivity by more than 50% and placing the fiber in a loose-fitting tube removes the effect entirely for pressures up to 90 N/m. However, he suggests that although his tests of a current measurement system with the tubed fiber showed no evidence of pressure-induced retardation, this solution may not be suitable for applications where long lengths of fiber are required.
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