3-D UNSTRUCTURED MESH GENERATION USING LOCAL TRANSFORMATIONS

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3-D Combinatorial Edge Swapping

- Convex sets of \( n+2 \) sites in \( \mathbb{R}^n \) can be configured in at most 2 ways

\[\text{2-D} \quad \text{3-D}\]

- This **local transformation** based on a Boolean decision serves as mechanism for local optimization

3-D Incremental Triangulation via Local Transformations

- Joe (1989) and Rajan (1991) showed that 3-D Delaunay triangulations can be constructed using local transformations based on the Boolean circumsphere test

\[\text{2-D Example of Incremental Insertion and Optimization}\]

- We have constructed triangulation algorithms in 3-D which locally optimize other mesh qualities: max-min dihedral angles, min-max dihedral angles, etc.
Motivations

• Develop a mesh generation capability suitable for generating highly stretched meshes required for viscous flow computations at high Reynolds numbers

• Experience has shown that existing triangulation methods such as Delaunay triangulation are not suitable for the generation of highly stretched meshes

• Investigate triangulation algorithms which accommodate mesh generation and adaptation while maintaining high robustness

Randomized Algorithms Based on Local Transformations

• Worst case optimal complexity can be achieved by randomizing the order in which sites are introduced into the triangulation (Guibas, Knuth, Sharir, 1992)

  • $n \log (n)$ expected performance in 2-D
  • $n^2$ expected worst case performance in 3-D

• Suggests a new "continuous" data structure which encodes a family of triangulations (coarsest to finest)

  • 2-D randomized theory predicts $O(n)$ size of this structure
  • We have exploited this construction to produce a novel multigrid scheme and theory for solving differential eqns
A New Approach to Multigrid for Unstructured Meshes

- Solution of Burgers' equation using continuous data structure

Coarsest Mesh

Finest Mesh

Convergence History

Solution Contours

Surface Mesh Generation Using Local Transforms

- Exploring new techniques capable of generation isotropic or stretched elements on tensor product spline patches

- Method supports adaptation based on geometrical or soln error

- Extension to manifold B-rep objects is being carried out by Code RFG (Maksymiuk, Chou)

Mesh with isotropic and stretched elements
Volume Triangulations

(1) Initial Triangulation of Surface Data
(2) Constrained/Conforming Triangulation to Preserve Body Integrity
(3) Incremental Insertion and Optimization of Specified Sites
Why Some Standard Triangulation Methods Fail

- Delaunay triangulation has a well known characterization that it maximizes the minimum angle for triangle pairs.
- Theoretical and practical considerations indicate that it may be more beneficial to minimize the maximum angle for triangle pairs.
- Incremental insertion and local optimization can be used to produce locally optimal Min-Max triangulations.

![Delaunay triangulation near an airfoil trailing edge](Delaunay.png)
![Extreme closeup of DT in trailing edge region](Closeup.png)
![Min-Max triangulation obtained by local optimization](LocalOpt.png)

Viscous Mesh Generation

- Automatic generation of viscous meshes by adaptive placement of sites on level sets followed by Min-Max triangulation.

![Point Selection (AR >>1)](PointSel.png)
![Point Selection and Adaptation (AR ≈ 1)](Adaptation.png)

![Distance Function](DistFunc.png)
![Min-Max Triangulation](Triang.png)
![Closeup in Flap Region](Closeup.png)
Future Directions

- Continue investigating optimization criteria for tetrahedral meshes

- Develop new strategies for site placement
  - Level set strategies
  - Steiner point strategies

- Solution adaptation based on *a priori* error estimates