3-D UNSTRUCTURED MESH GENERATION USING LOCAL TRANSFORMATIONS

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3-D Combinatorial Edge Swapping

- Convex sets of \( n+2 \) sites in \( \mathbb{R}^n \) can be configured in at most 2 ways

2-D

3-D

- This local transformation based on a Boolean decision serves as mechanism for local optimization

3-D Incremental Triangulation via Local Transformations

- Joe (1989) and Rajan (1991) showed that 3-D Delaunay triangulations can be constructed using local transformations based on the Boolean circumsphere test

2-D Example of Incremental Insertion and Optimization

- We have constructed triangulation algorithms in 3-D which locally optimize other mesh qualities: max–min dihedral angles, min–max dihedral angles, etc.
Motivations

- Develop a mesh generation capability suitable for generating highly stretched meshes required for viscous flow computations at high Reynolds numbers

- Experience has shown that existing triangulation methods such as Delaunay triangulation are not suitable for the generation of highly stretched meshes

- Investigate triangulation algorithms which accommodate mesh generation and adaptation while maintaining high robustness

Randomized △ Algorithms Based on Local Transformations

- Worst case optimal complexity can be achieved by randomizing the order in which sites are introduced into the triangulation (Guibas, Knuth, Sharir, 1992)
  - \( n \log(n) \) expected performance in 2-D
  - \( n^2 \) expected worst case performance in 3-D

- Suggests a new "continuous" data structure which encodes a family of triangulations (coarsest to finest)

- 2-D randomized theory predicts \( O(n) \) size of this structure

- We have exploited this construction to produce a novel multigrid scheme and theory for solving differential eqns
A New Approach to Multigrid for Unstructured Meshes

- Solution of Burgers' equation using continuous data structure

![Coarsest Mesh](image1)
![Finest Mesh](image2)

Convergence History

Solution Contours

Surface Mesh Generation Using Local Transforms

- Exploring new techniques capable of generation isotropic or stretched elements on tensor product spline patches
- Method supports adaptation based on geometrical or soln error
- Extension to manifold B–rep objects is being carried out by Code RFG (Maksymiuk, Chou)

Mesh with isotropic and stretched elements
Volume Triangulations

(1) Initial Triangulation of Surface Data
(2) Constrained/Conforming Triangulation to Preserve Body Integrity
(3) Incremental Insertion and Optimization of Specified Sites

| Surface Triangulation | Constrained/Conforming Triangulation of Boundary | Final Volume Triangulation |

Boeing 737 with High Lift Devices Deployed
(300,000 Tetrahedra)
Why Some Standard Triangulation Methods Fail

- Delaunay triangulation has a well known characterization that it maximizes the minimum angle for triangle pairs.
- Theoretical and practical considerations indicate that it may be more beneficial to minimize the maximum angle for triangle pairs.
- Incremental insertion and local optimization can be used to produce locally optimal Min–Max triangulations.

Delaunay triangulation near an airfoil trailing edge

Extreme closeup of DT in trailing edge region

Min–Max triangulation obtained by local optimization

Viscous Mesh Generation

- Automatic generation of viscous meshes by adaptive placement of sites on level sets followed by Min–Max triangulation.

Point Selection (AR >> 1)

Point Selection and Adaptation (AR ≈ 1)

Distance Function

Min–Max Triangulation

Closeup in Flap Region
Future Directions

- Continue investigating optimization criteria for tetrahedral meshes
- Develop new strategies for site placement
  - Level set strategies
  - Steiner point strategies
- Solution adaptation based on *a priori* error estimates