3-D UNSTRUCTURED MESH GENERATION USING LOCAL TRANSFORMATIONS

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3-D Combinatorial Edge Swapping

- Convex sets of \( n+2 \) sites in \( \mathbb{R}^n \) can be configured in at most 2 ways

2-D

3-D

- This local transformation based on a Boolean decision serves as mechanism for local optimization

3-D Incremental Triangulation via Local Transformations

- Joe (1989) and Rajan (1991) showed that 3-D Delaunay triangulations can be constructed using local transformations based on the Boolean circumsphere test

2-D Example of Incremental Insertion and Optimization

- We have constructed triangulation algorithms in 3-D which locally optimize other mesh qualities: max–min dihedral angles, min–max dihedral angles, etc.
Motivations

- Develop a mesh generation capability suitable for generating highly stretched meshes required for viscous flow computations at high Reynolds numbers.

- Experience has shown that existing triangulation methods such as Delaunay triangulation are not suitable for the generation of highly stretched meshes.

- Investigate triangulation algorithms which accommodate mesh generation and adaptation while maintaining high robustness.

Randomized \( \triangle \) Algorithms Based on Local Transformations

- Worst case optimal complexity can be achieved by randomizing the order in which sites are introduced into the triangulation (Guibas, Knuth, Sharir, 1992).

  - \( n \log(n) \) expected performance in 2-D.
  - \( n^2 \) expected worst case performance in 3-D.

- Suggests a new "continuous" data structure which encodes a family of triangulations (coarsest to finest).

  - 2-D randomized theory predicts \( O(n) \) size of this structure.

- We have exploited this construction to produce a novel multigrid scheme and theory for solving differential eqns.
A New Approach to Multigrid for Unstructured Meshes

- Solution of Burgers’ equation using continuous data structure

![Coarsest Mesh](image1)

![Finest Mesh](image2)

Convergence History

Solution Contours

Surface Mesh Generation Using Local Transforms

- Exploring new techniques capable of generation isotropic or stretched elements on tensor product spline patches
- Method supports adaptation based on geometrical or soln error
- Extension to manifold B–rep objects is being carried out by Code RFG (Maksymiuk, Chou)

Mesh with isotropic and stretched elements
Volume Triangulations

(1) Initial Triangulation of Surface Data

(2) Constrained/Conforming Triangulation to Preserve Body Integrity

(3) Incremental Insertion and Optimization of Specified Sites
Why Some Standard Triangulation Methods Fail

- Delaunay triangulation has a well known characterization that it maximizes the minimum angle for triangle pairs.
- Theoretical and practical considerations indicate that it may be more beneficial to minimize the maximum angle for triangle pairs.
- Incremental insertion and local optimization can be used to produce locally optimal Min–Max triangulations.

Viscous Mesh Generation

- Automatic generation of viscous meshes by adaptive placement of sites on level sets followed by Min–Max triangulation.

Distance Function  Min–Max Triangulation  Closeup in Flap Region
Future Directions

- Continue investigating optimization criteria for tetrahedral meshes
- Develop new strategies for site placement
  - Level set strategies
  - Steiner point strategies
- Solution adaptation based on *a priori* error estimates