3-D UNSTRUCTURED MESH GENERATION USING LOCAL TRANSFORMATIONS

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3-D Combinatorial Edge Swapping

- Convex sets of \( n+2 \) sites in \( \mathbb{R}^n \) can be configured in at most 2 ways

- This local transformation based on a Boolean decision serves as mechanism for local optimization

\[ \begin{align*}
2-D & \quad 3-D
\end{align*} \]

3-D Incremental Triangulation via Local Transformations

- Joe (1989) and Rajan (1991) showed that 3-D Delaunay triangulations can be constructed using local transformations based on the Boolean circumsphere test

\[ \begin{align*}
2-D & \quad 3-D
\end{align*} \] 2-D Example of Incremental Insertion and Optimization

- We have constructed triangulation algorithms in 3-D which locally optimize other mesh qualities: max–min dihedral angles, min–max dihedral angles, etc.
Motivations

- Develop a mesh generation capability suitable for generating highly stretched meshes required for viscous flow computations at high Reynolds numbers
- Experience has shown that existing triangulation methods such as Delaunay triangulation are not suitable for the generation of highly stretched meshes
- Investigate triangulation algorithms which accommodate mesh generation and adaptation while maintaining high robustness

Randomized Algorithms Based on Local Transformations

- Worst case optimal complexity can be achieved by randomizing the order in which sites are introduced into the triangulation (Guibas, Knuth, Sharir, 1992)
  - $n \log (n)$ expected performance in 2-D
  - $n^2$ expected worst case performance in 3-D
- Suggests a new "continuous" data structure which encodes a family of triangulations (coarsest to finest)
  - 2-D randomized theory predicts $O(n)$ size of this structure
- We have exploited this construction to produce a novel multigrid scheme and theory for solving differential eqns
A New Approach to Multigrid for Unstructured Meshes
- Solution of Burgers' equation using continuous data structure

Surface Mesh Generation Using Local Transforms
- Exploring new techniques capable of generation isotropic or stretched elements on tensor product spline patches
- Method supports adaptation based on geometrical or soln error
- Extension to manifold B–rep objects is being carried out by Code RFG (Maksymiuk, Chou)
Volume Triangulations

(1) Initial Triangulation of Surface Data
(2) Constrained/Conforming Triangulation to Preserve Body Integrity
(3) Incremental Insertion and Optimization of Specified Sites

Surface Triangulation Constrained/Conforming Triangulation of Boundary Final Volume Triangulation
Why Some Standard Triangulation Methods Fail

- Delaunay triangulation has a well known characterization that it maximizes the minimum angle for triangle pairs.

- Theoretical and practical considerations indicate that it may be more beneficial to minimize the maximum angle for triangle pairs.

- Incremental insertion and local optimization can be used to produce locally optimal Min–Max triangulations.

Viscous Mesh Generation

- Automatic generation of viscous meshes by adaptive placement of sites on level sets followed by Min–Max triangulation.

  - Distance Function
  - Min–Max Triangulation
  - Closeup in Flap Region
Future Directions

- Continue investigating optimization criteria for tetrahedral meshes
- Develop new strategies for site placement
  - Level set strategies
  - Steiner point strategies
- Solution adaptation based on \textit{a priori} error estimates