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A Thermodynamic Analysis of Propagating Subcritical Cracks with Cohesive Zones

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INTRODUCTION

The results of the so-called energetic approach to fracture with particular attention to the issue of energy dissipation due to crack propagation are applied to the case of a crack with cohesive zone. The thermodynamic admissibility of subcritical crack growth (SCG) is discussed together with some hypotheses that lead to the derivation of SCG laws. A two-phase cohesive zone model for discontinuous crack growth is presented and its thermodynamics analyzed, followed by an example of its possible application.

INTRODUCTION

Subcritical crack growth (SCG), under both general and cyclic loading, is a phenomenon that has been receiving more and more attention during the last forty years. Starting with early investigations mainly on fatigue in metals (Refs. 1-9), current research covers a wide variety of materials, especially those such as polymers (Refs. 9-13), and ceramics (Ref. 14), that are becoming important in the fabrication of composites. From a theoretical standpoint, the problem is that of relating crack growth to the load history. In this sense, fundamental understanding has been provided by the energetic approach to fracture (Refs. 15-32), that showed (Refs. 15-19) how SCG is strictly related to the rate of dissipation in the vicinity of the crack front.

OBJECTIVE:

**TO RELATE CRACK GROWTH
TO THE LOAD HISTORY**

**A CRACK GROWTH LAW
AND/OR CRITERION IS NEEDED**

APPROACHES TO THE PROBLEM

Several theoretical studies in the continuum thermodynamics of fracture have shown that independently of the global or local (around the crack tip) constitutive assumptions, a sharp crack with no cohesive zone is constrained to evolve according to the Griffith criterion (Ref. 20). Unfortunately, SCG cannot be described in terms of the Griffith criterion. In the case of SCG, mainly in fatigue, a number of growth laws are available, although the great majority of them are based on phenomenological observation only.

- **GRIFFITH CRITERION (1920)**

$$l \geq 0 \quad \text{IF} \quad G \geq G_{CR}$$

ORIGINALLY FORMULATED USING AN ENERGY BALANCE APPROACH (FIRST LAW) FOR BRITTLE SYSTEMS.

- **FATIGUE GROWTH LAWS (SINCE EARLY 1950's)**
 - **CYCLIC LOADING**
 - **SUBCRITICAL CONDITIONS (GRIFFITH CRITERION DOES NOT APPLY)**
 - **MOST OF THEM ARE ONLY PHENOMENOLOGICALLY BASED**

CURRENT STATE OF RESEARCH

Modern continuum thermodynamics sees crack propagation like an internal dissipation mechanism. In this sense the propagation of fracture can be described by the evolution of a set of convenient kinematic state variables, e.g., crack length, whose driving force can be computed directly from the total free energy of the body. The application of the thermodynamics with ISV's is immediate. One of the important outcomes of such an approach is the interpretation of a moving crack tip as a moving heat source and the subsequent determination of the corresponding near crack tip temperature field.

- **ENERGETIC APPROACH AS A UNIFIED APPROACH:**
 - **FRACTURE STUDIED WITHIN THE FRAMEWORK OF CONTINUUM THERMODYNAMICS**
 - **CRACK SURFACE CONSIDERED AN INTERNAL STATE VARIABLE;**

$$G = - \frac{\partial \psi}{\partial l}$$

- **CRACK PROPAGATION IS AN INTERNAL DISSIPATION MECHANISM. IT CAN BE INCLUDED IN CONSTITUTIVE THEORIES WITH I.S.V.**
- **FORM OF TEMPERATURE SINGULARITY AT THE TIP OF A RUNNING CRACK**

IMPORTANT CONTRIBUTIONS

The present research effort employs many of the results of the modern thermodynamics approach to fracture. We therefore list some of the most important contributions of this approach.

THERMODYNAMIC APPROACH TO FRACTURE

- **GRIFFITH (1920): CRACK GROWTH CRITERION USING THE FIRST LAW OF THERMODYNAMICS**
- **CHEREPANOV, G.A. (1967): APPLICATION OF CONTINUUM MECHANICS & CONSIDERATIONS BASED ON THE SECOND LAW**
- **RICE, J.R. (1968): PATH INDEPENDENT INTEGRALS IN ELASTICITY; ENERGY RELEASE RATE AS CRACK LENGTH CONJUGATE FORCE**
- **GURTIN (1979): APPLICATION OF RATIONAL THERMODYNAMICS TO A THERMOELASTIC SYSTEM WITH A SHARP CRACK**
- **NGUYEN (1980-1985): GLOBAL THERMODYNAMIC AND DISSIPATION ANALYSIS TO FRACTURE**

**GENERALIZATION OF THE GRIFFITH CRITERION
DERIVED BY A DISSIPATION POTENTIAL
THERMOMECHANICAL SINGULARITY ANALYSIS**

MAJOR PROBLEMS WITH CURRENT METHODS

The thermodynamic approach to fracture, in the absence of a cohesive zone, derives the Griffith criterion as the only possible consequence of the second law. This result is fatigue since fatigue is an example of SCG. Another problem in the analysis of cracks with no C.Z. is the loss of weaving of the fracture parameter G for almost all material behaviors except the thermoelastic one, thus including special behaviors like that of a process zone around a sharp crack.

- **SOME RESEARCHERS HAVE DERIVED SUBCRITICAL CRACK PROPAGATION LAWS FROM THE FIRST LAW ALONE: THERMODYNAMIC ADMISSIBILITY IS DISREGARDED.**
- **WHEN THE SECOND LAW IS CONSIDERED SUBCRITICAL CRACK PROPAGATION HAS BEEN SHOWN TO BE THERMODYNAMICALLY INADMISSIBLE**
- **FOR THE RUNNING CRACK PROBLEM, SINGULARITY ANALYSES SHOW THAT G IS MEANINGLESS FOR PLASTICITY AND VISCOPLASTICITY AND FOR CERTAIN VISCOELASTIC MODELS**
- **MODELS THAT INCLUDE PROCESS ZONES AROUND SHARP CRACKS DO NOT NECESSARILY REMOVE THE THERMOMECHANICAL SINGULARITY AT THE TIP, NOR SOLVE THE ABOVE PROBLEMS.**

APPROACH USED IN THIS RESEARCH

The present research effort introduces a cohesive zone into a continuum mechanics model for SCG in order to allow for a thermodynamically consistent description of the problem. After postulating the presence of a C.Z. ahead of the crack tip, the circumstances under which SCG is thermodynamically admissible will be discussed. The assumption leading to the derivation of the traditional form of fatigue growth laws is also discussed and a similar form for discontinuous crack growth laws will be obtained.

- **CONTINUUM THERMODYNAMIC
FRAMEWORK**
 - **CLASSICAL FIELD THEORY CAN BE
USED INSTEAD OF NON-LOCAL
MODELS**

- **COHESIVE ZONE**
 - **ALL THERMOMECHANICAL
SINGULARITIES ARE REMOVED**
 - **CRACK TIP HAS A FINITE SIZE**

- **SUBCRITICAL CONDITIONS**
 - **THERMODYNAMICALLY ADMISSIBLE**
 - **UNIFIED APPROACH TO STUDY
FATIGUE AND DISCONTINUOUS CRACK
PROPAGATION.**

BASIC EQUATIONS AND DEFINITIONS

The analysis prosecuted is basically a global thermodynamic analysis. It consists of deriving global thermodynamic statements for the entire structure by interpreting the pointwise governing equations over the whole body.

POINTWISE GOVERNING EQUATIONS

$$\rho \dot{u} = \sigma_{ij} \dot{\epsilon}_{ij} - q_{i,i} + \rho r \quad (1)$$

$$\rho \dot{s} + \left(\frac{q_i}{T} \right)_{,i} - \rho \frac{r}{T} \geq 0 \quad (2)$$

$$\sigma_{j,i} + \rho f_i = 0 \quad (3)$$

$$\epsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \quad (4)$$

$$\begin{aligned} \sigma_{ij} &= \sigma_{ij}(\epsilon_{kp}, T, \alpha^n) \\ q_i &= q_i(\epsilon_{kp}, T, T_{,k}, \alpha^n) \\ u &= u(\epsilon_{kp}, T, \alpha^n) \\ s &= s(\epsilon_{kp}, T, \alpha^n) \end{aligned} \quad (5)$$

Note that the constitutive behavior is assumed to be as general as possible through the use of interval state variables (at the pointwise level) together with their correspondent solution equations.

such that

$$\sigma_{ij} = \rho \frac{\partial h}{\partial \varepsilon_{ij}} ; s = -\frac{\partial h}{\partial T} \quad (6)$$

where $h = h(\vec{x}, t)$ is the Helmholtz free energy:

$$h \equiv u - Ts \quad (7)$$

$$\dot{\alpha}^n = \Omega^n(\varepsilon_{kl}, T, \alpha^m) ; n, m = 1, \dots, N \quad (8)$$

$$q_i = -kT_{,i} \quad (9)$$

ALSO LET

$$\rho \eta_i = \rho \eta_{mic} - \frac{q_i T_{,i}}{T^2} \quad (10)$$

STRONG FORM OF THE SECOND LAW

$$\rho \eta_{mic} \geq 0 ; -\frac{q_i T_{,i}}{T^2} \geq 0 \quad (11)$$

In Figure 1 we have a schematic representation of the system analyzed. The body contains a single edge crack which terminates with a cohesive zone characterized by the points α and β .

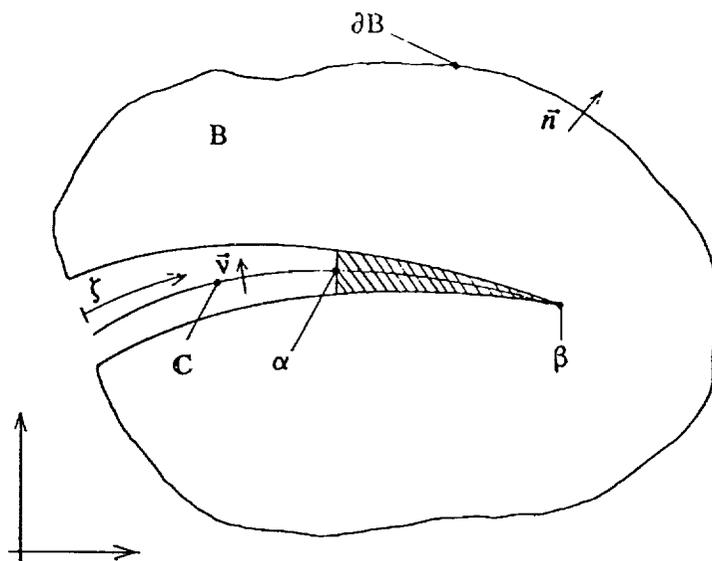


Figure 1 - Crack with a cohesive zone

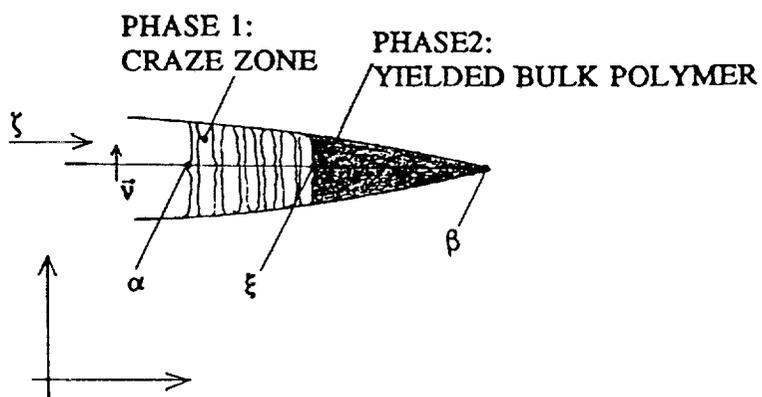


Figure 2 - Two-phase cohesive zone model

DEFINITION OF A CRACK WITH A COHESIVE ZONE

From a mathematical viewpoint a crack is represented by a line (surface) of discontinuity for the various field variables. The cohesive zone is a portion of the crack line (surface) along which a system of cohesive forces σ_i is acting, and that is also characterized by its own opening displacement δ_i . A quantity in brackets represents the jump of that quantity across the cohesive zone. At this point it is possible to derive global statements for the first and second law and for the dissipation equation.

$$\begin{aligned} \mathbb{C}(t) &= \{\bar{x}(\zeta) : 0 \leq \zeta \leq \beta(t)\} \\ \text{c.z.} &= \{\bar{x}(\zeta) : \alpha(t) \leq \zeta \leq \beta(t)\} \end{aligned} \quad (12)$$

$$\begin{aligned} \sigma_i(\zeta, t) &\equiv \sigma_{ji}^+ v_j = \sigma_{ji}^- v_j \\ \delta_i(\zeta, t) &\equiv [u_i] ; \delta_i(\beta(t), t) = 0 \end{aligned} \quad (13)$$

GLOBAL FORMS OF THE LAWS OF THERMODYNAMICS

$$\frac{d}{dt} \int_B \rho u \, dA - \int_S (\sigma_{ji} n_j \dot{u}_i - q_i n_i) \, dS = - \int_{\alpha(t)}^{\beta(t)} (\sigma_i \dot{\delta}_i - [q_i] v_i) \, d\zeta \quad (14)$$

$$\frac{d}{dt} \int_B \rho s \, dA + \int_S q_i n_i \, dS - \int_{\alpha(t)}^{\beta(t)} [q_i] v_i \, d\zeta \geq 0 \quad (15)$$

$$\int_B \rho \eta_{mic} T \, dA = \int_B \rho \dot{s} T \, dA + \int_S q_i n_i \, dS - \int_{\alpha(t)}^{\beta(t)} [q_i] v_i \, d\zeta \quad (16)$$

WHERE

$$\begin{aligned} S &\equiv \partial B \cup C^* \\ C^* &\equiv C - \text{c.z.} \end{aligned} \quad (17)$$

DEFINITION OF THE THERMODYNAMIC QUANTITIES FOR THE C.Z.

The cohesive zone is considered a thermodynamic system with its own characteristics. In order to discuss such characteristics and write the two laws of thermodynamics for the cohesive zone above, it is necessary to define the C.Z. internal energy ε , entropy φ , temperature ϑ and free energy ψ .

$$\varepsilon = \begin{cases} 2\gamma_0 = \text{const}, & 0 \leq \zeta \leq \alpha(t) \\ \varepsilon(\zeta, t), & \alpha(t) < \zeta \leq \beta(t) \\ 0, & \zeta = \beta(t) \end{cases} \quad (18)$$

$$\varphi = \begin{cases} \varphi_0 = \text{const}, & 0 \leq \zeta \leq \alpha(t) \\ \varphi(\zeta, t), & \alpha(t) < \zeta < \beta(t) \\ 0, & \zeta = \beta(t) \end{cases} \quad (19)$$

$$\begin{aligned} \vartheta &= T^+ = T^- \\ \forall \zeta: \zeta \in \text{c.z.} \end{aligned} \quad (20)$$

$$\psi = \varepsilon - \varphi \vartheta \quad (21)$$

FIRST LAW FOR THE C.Z. FROM THE GLOBAL STATEMENT AND ABOVE DEFINITIONS

$$\int_{\alpha(t)}^{\beta(t)} \dot{\varepsilon} \, d\zeta = \int_{\alpha(t)}^{\beta(t)} (\sigma_i \dot{\delta}_i - [q_i] v_i) \, d\zeta \quad (22)$$

$$\frac{d}{dt} \int_{\alpha(t)}^{\beta(t)} \varepsilon \, d\zeta + 2\gamma_0 \dot{\alpha} = \int_{\alpha(t)}^{\beta(t)} (\sigma_i \dot{\delta}_i - [q_i] v_i) \, d\zeta \quad (23)$$

LOCAL FORM

$$\dot{\varepsilon} = \sigma_i \dot{\delta}_i - [q_i] v_i \quad (24)$$

DISCUSSION ABOUT SECOND LAW AS POSTULATED BY GURTIN

The formulation of the second law for the cohesive zone is not a trivial matter. We therefore adopt the statement given by Gurtin in Ref. 26. We then impose a further restriction on the C.Z. behavior so as to restrain the C.Z. to act, as a whole, like an actual dissipative system.

GLOBAL STATEMENT (COHESIVE ZONE ALONE)

$$\int_{\alpha(t)}^{\beta(t)} \left(\dot{\phi} + \frac{[q_i]v_i}{\delta} \right) d\zeta \geq 0 \quad (25)$$

LOCAL STATEMENT

$$\dot{\lambda} = \dot{\phi} + \frac{[q_i]v_i}{\delta} \geq 0 \quad (26)$$

FURTHER RESTRICTIONS

$$\int_{\alpha(t)}^{\beta(t)} [q_i]v_i d\zeta \geq 0 \quad (27)$$

**THE ABOVE EXPRESSION IS THE DISSIPATION DUE
TO THE EVOLUTION OF THE COHESIVE ZONE.**

DISSIPATION ANALYSIS (Being α a Global..)

In order to properly discuss the dissipation associated with the C.Z. evolution, i.e., crack propagation and C.Z. deformation, the thermodynamic force (G-R) work conjugate of the global state variable α must be properly characterized.

BEING α A GLOBAL INDEPENDENT STATE VARIABLE, WE CAN WRITE:

$$\phi(x_k, t) = \phi(x_k, \alpha(t), t) \quad (28)$$

$$\dot{\phi} = \frac{\partial \phi}{\partial \alpha} \dot{\alpha} + \frac{\partial \phi}{\partial t} \Big|_{\alpha = \text{const.}} \quad (29)$$

FIRST LAW:

$$\int_{\alpha(t)}^{\beta(t)} \left(\sigma_i \frac{\partial \delta_i}{\partial t} - \frac{\partial \epsilon}{\partial t} \right) d\zeta + (G-R) \dot{\alpha} = \int_{\alpha(t)}^{\beta(t)} [q_i] v_i d\zeta \quad (30)$$

where

$$G = \int_{\alpha(t)}^{\beta(t)} \sigma_i \frac{\partial \delta_i}{\partial \alpha} d\zeta ; R = \int_{\alpha(t)}^{\beta(t)} \frac{\partial \epsilon}{\partial \alpha} d\zeta \quad (31)$$

CRACK ADVANCEMENT RESISTANCE IS A FUNCTION OF THE C.Z. THERMODYNAMIC STATE.

DISSIPATION ANALYSIS (Case 1)

We will now consider a special type of C.Z. evolution: crack growth with pure translation of the cohesive zone. The above assumption is certainly restrictive, but it yields results analogous to that obtained in the study of a crack without a cohesive zone. This leads to the following interpretation: a running crack with no cohesive zone behaves like a crack with a cohesive zone when the C.Z. is constrained to simply translate with the crack tip.

- CASE 1:**
- **PURE TRANSLATION**
 - **BARENBLATT ASSUMPTIONS**

$$\Delta = \beta(t) - \alpha(t) \quad (32)$$

$$\dot{\phi} = -\frac{\partial \phi}{\partial \zeta} \dot{\alpha} = \frac{\partial \phi}{\partial \alpha} \quad (33)$$

- **PURELY ELASTIC COHESIVE ZONE**

RESULTS:

$$(G - 2\gamma) \dot{\alpha} = \int_{\alpha(t)}^{\beta(t)} [q_i] v_i d\zeta \geq 0 \quad (34)$$

$$G = \int_{\delta(\beta)=0}^{\delta(\alpha)} \sigma_i d\delta_i \quad (35)$$

RESULTS ANALOGOUS TO THOSE FOR THE CASE OF A CRACK WITHOUT COHESIVE ZONE

DISSIPATION ANALYSIS (Case 2)

When a cohesive zone with general behavior, thus with some being of dissipation mechanism, is left to evolve without special constraints, we see from the first and second law for the C.Z. that subcritical crack propagation, that is $\dot{\alpha} > 0$ when $G < R$, is an admissible phenomenon.

CASE 2: - ELASTO-PLASTIC COHESIVE ZONE - GENERAL DEFORMATION

$$\frac{\partial \Psi}{\partial \delta_i^e} = \sigma_i ; \quad -\frac{\partial \Psi}{\partial \vartheta} = \varphi \quad (36)$$

$$\vartheta \dot{\lambda} = \sigma_i \dot{\delta}_i^p \geq 0 \quad (37)$$

$$\dot{\delta}_i^p = \frac{\partial \omega}{\partial \sigma_i} \quad (38)$$

FIRST LAW BECOMES

$$\int_{\alpha(t)}^{\beta(t)} [\tilde{q}_i] v_i d\zeta + (G-R)\dot{\alpha} = \int_{\alpha(t)}^{\beta(t)} [q_i] v_i d\zeta \geq 0 \quad (39)$$

WHERE

$$[\tilde{q}_i] v_i \equiv \sigma \frac{\partial \delta_i^p}{\partial t} - \vartheta \frac{\partial \varphi}{\partial t} = \sigma \frac{\partial \delta_i}{\partial t} - \frac{\partial \varepsilon}{\partial t} \quad (40)$$

SUBCRITICAL CRACK GROWTH ADMISSIBLE WHEN

$$\int_{\alpha(t)}^{\beta(t)} [\tilde{q}_i] v_i d\zeta > (R-G)\dot{\alpha} \quad (41)$$

WHEN $\dot{\alpha} = 0$ WE HAVE EVOLUTION OF THE C.Z. INTERNAL STATE VARIABLES

DISSIPATION ANALYSIS (Case 3)

In general, thermodynamics does not allow to derive evolution laws for the internal state variables, thus for the kinematic variables that describe crack propagation and fatigue, some special assumptions can be made that allow us to derive a crack evolution law strictly from the first law of thermodynamics. For some cases of slow crack propagation, the principle of the minimum entropy production can be evoked, thus leading to a certain form of crack growth law.

CASE 3 - SLOW CRACK GROWTH - DISSIPATIVE COHESIVE ZONE (ELASTO-PLASTIC)

ASSUME PRINCIPLE OF MINIMUM ENTROPY
PRODUCTION HOLDS

$$\int_{\alpha(t)}^{\beta(t)} [q_i] v_i d\zeta = 0 \quad (42)$$

FROM FIRST LAW:

$$\dot{\alpha} = \frac{\int_{\alpha}^{\beta} \left(\sigma_i \frac{\partial \delta_i^T}{\partial t} - \dot{\vartheta} \frac{\partial \varphi}{\partial t} \right) d\zeta}{R - G} \quad (43)$$

● CYCLIC LOADING

INTEGRATE OVER A CYCLE

$$\Delta \alpha = \frac{\Delta \Lambda - \Delta Q}{2\gamma_0 - G_M} \quad (44)$$

SIMPLIFIED FORM

$$\Delta \alpha = \frac{\Delta \Lambda}{2\gamma_0 - G_{Max}} \quad (45)$$

DISCONTINUOUS CRACK GROWTH

The analysis presented so far can be easily extended to describe discontinuous crack propagation (DCP). With reference to Fig. 2, we present a two-phase cohesive zone model inspired by the experimental work by Hertzberg, et al (Ref. 33). Proceeding as in the case of a single phase C.Z. model, assuming that the principle of minimum entropy production holds, we obtain an evolution equation for the phase separation coordinate ξ that allows to study DCP.

A VERY SIMPLE 2-PHASE MODEL (HERTZBERG, ET AL., 1979)

$$\int_{\alpha(t)}^{\beta(t)} \left(\sigma_i \frac{\partial \delta_i}{\partial t} - \frac{\partial \varepsilon}{\partial t} \right) d\zeta + (G_\alpha - R_\alpha) \dot{\alpha} + (G_\xi - R_\xi) \dot{\xi} = \int_{\alpha(t)}^{\beta(t)} [q_i] v_i d\zeta \quad (46)$$

where

$$G_\xi = \int_{\alpha(t)}^{\beta(t)} \sigma_i \frac{\partial \delta_i}{\partial \xi} d\zeta ; R_\xi = \int_{\alpha(t)}^{\beta(t)} \frac{\partial \varepsilon}{\partial \xi} d\zeta \quad (47)$$

ASSUME

$$\dot{\alpha} = 0$$

$$\dot{\xi} > 0 ; 0 < G_\xi < R_\xi \quad (48)$$

UNDER CYCLIC LOADING

$$\Delta \xi = \frac{\Delta \Lambda - \Delta Q}{R_{\xi M} - G_{\xi M}} \quad (49)$$

SIMPLIFIED

$$\Delta \xi = \frac{\Delta \Lambda}{R_{\xi M} - G_{\xi M}} \quad (50)$$

OTHER DISCONTINUOUS CRACK GROWTH MODELS

Very few models of DCP have been presented in the open literature. The ones mentioned below are those in Refs. 12 and 34. Further study of the thermodynamics of the process is certainly needed.

- **J.G. WILLIAMS, 1977**
 - **TWO PHASES**
 - **NOT THERMODYNAMICALLY BASED**

- **K. KADOTA & A. CHUDNOVSKY, 1992**
 - **SINGLE PHASE**
 - **BASED ON THE PREDICTION OF A
PROCESS ZONE CRACK RESISTANCE
DEGRADATION**

EXAMPLE

A very simple example of application of the DCP model presented is given. We have assumed that $G\xi_n = G\xi_{nmax}$ and $R\xi_n = X$ where X is the phase transformation energy per unit lengths of transformed material. The cohesive zone has been modeled as a two-phase Dugdale zone.

GEOMETRY:

- STRAIGHT CRACK IN UNBOUNDED MEDIUM

LOAD:

- UNIFORM TENSILE STRESS APPLIED AT INFINITY
- CYCLIC, FROM 0 TO T_0
CASE1: $T_0=0.1\text{MPa}$
CASE2: $T_0=0.2\text{MPa}$
- PLANE STRESS

MATERIAL SYSTEM:

- PS

PROPERTIES*:

- $E=2.2\text{GPa}$
- $\sigma_{c.z.}=18.0\text{MPa}$
- $f=2.0$
- $\chi=30.0\text{J/m}^2$

*Botsis et Al, Int. J. Fract., v.33, 1987, pp.263-276.

FIGURES 3 AND 4

In Figs. 3 and 4 we have a schematic of the geometry, load conditions and detailed view of the cohesive zone.

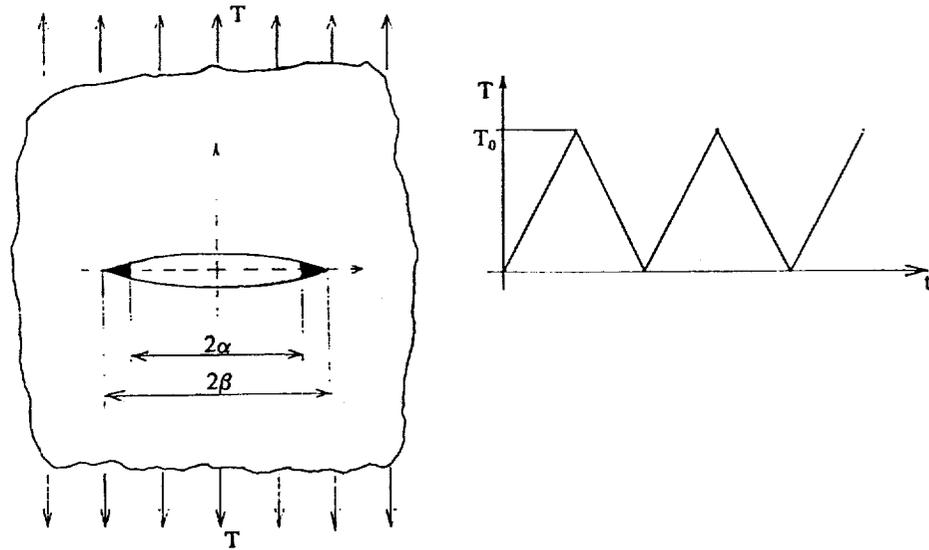


Figure 3 - Example geometry and load conditions

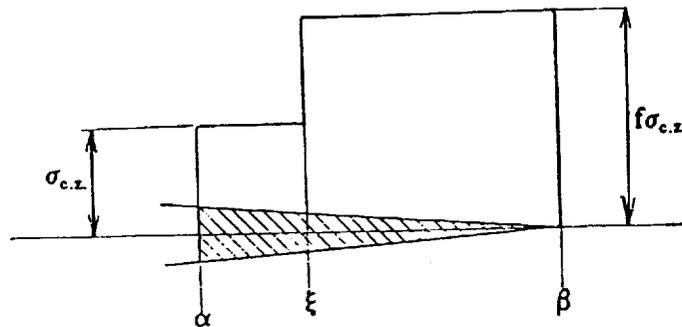


Figure 4 - Two-phase Dugdale model

FIGURE 5

The trend of C.Z. evolution obtained is shown in Fig. 5. It is easy to recognize the discontinuous crack growth pattern.

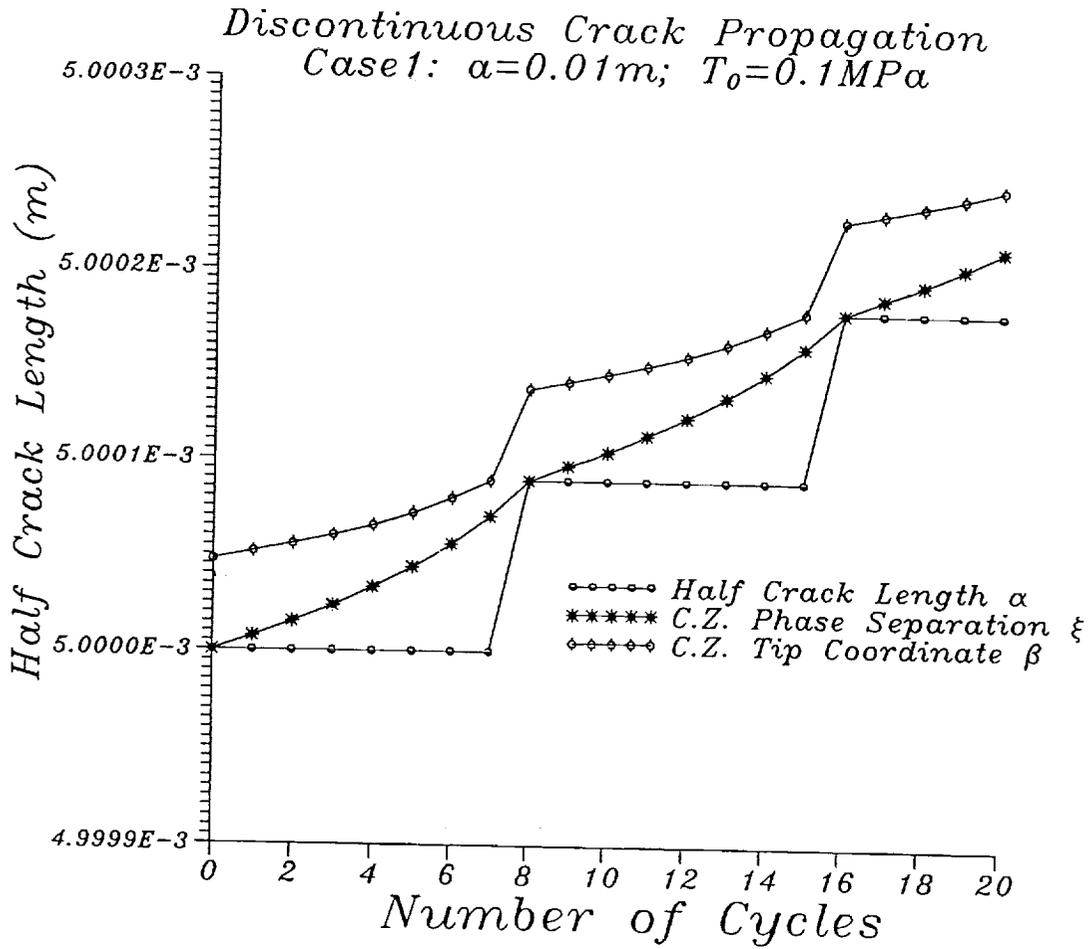


Figure 5 - C.Z. Evolution during DCP

FIGURE 6

Figure 6 shows the trend of the growth of α only.

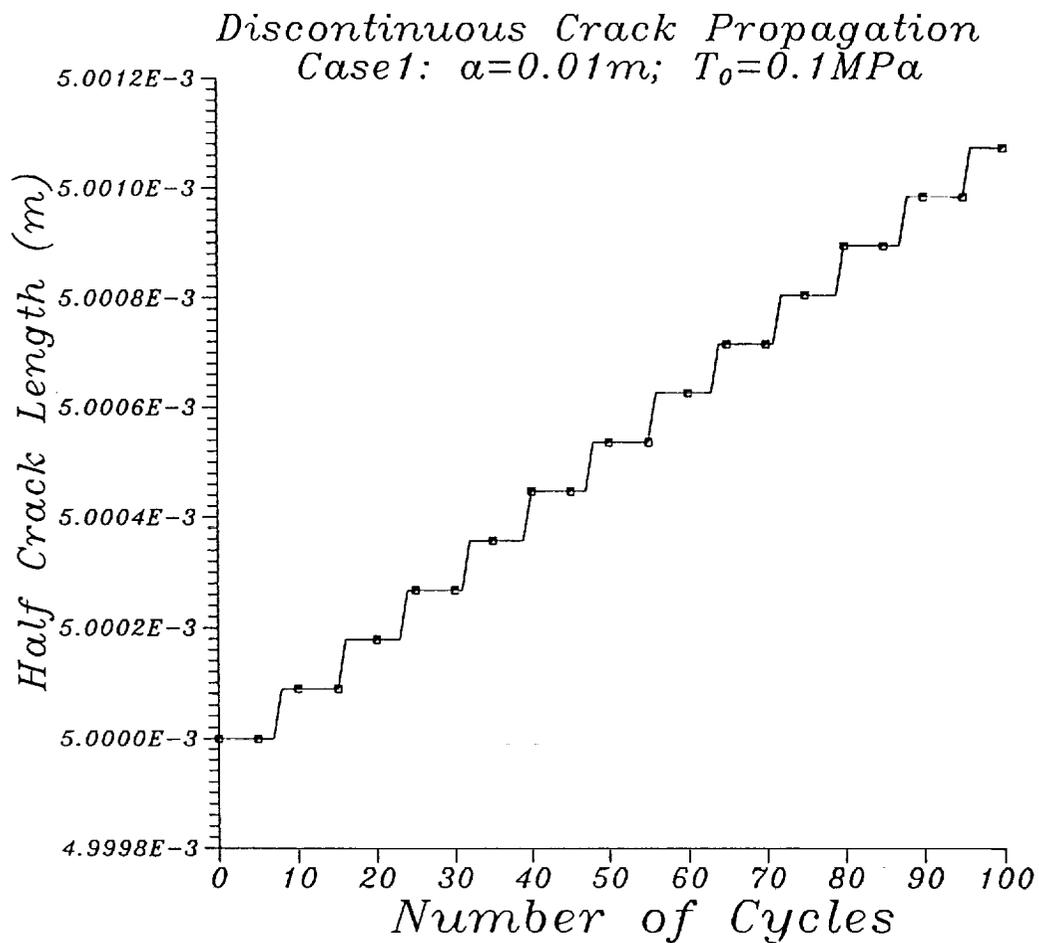


Figure 6 - Discontinuous crack propagation

CONCLUSIONS

In this work a continuum thermodynamic analysis of a crack with a cohesive zone has been presented. In particular, the issue of thermodynamic admissibility of subcritical crack growth has been addressed. The theory espoused has been applied to the study of DCP and an approximated DCP law has been obtained. An example of application of the DCP law is provided.

- **APPLICATION OF THE CONTINUUM THERMODYNAMICS APPROACH TO THE CASE OF A CRACK WITH A COHESIVE ZONE**
- **DISCUSSION OF THE ADMISSIBILITY OF SUBCRITICAL CRACK GROWTH**
- **SUBCRITICAL GROWTH LAWS OBTAINED USING THE DISSIPATION EQUATION FOR FATIGUE AND DISCONTINUOUS CRACK PROPAGATION**
- **SIMPLE EXAMPLE OF APPLICATION OF THE DISCONTINUOUS CRACK GROWTH LAW**

FUTURE WORK

Further study is certainly necessary, especially toward a better characterization of the constitutive equations for the cohesive zone. Also necessary is the coupling of the presented thermodynamic analysis with elements of the kinetic theory of fracture in order to obtain more general crack advancement laws. A more accurate analysis is needed for the study of DCP together with a stability analysis. Possible applications of the theory include the study of delamination in laminated composites, R-toughening in ceramics and problems of matrix-fiber interface degradation in fiber reinforced composites.

- **OBTAIN MORE REALISTIC C.Z. CONSTITUTIVE EQUATIONS**
- **STABILITY ANALYSIS OF THE CRACK PROPAGATION EVENT DURING DISCONTINUOUS CRACK GROWTH AND TRANSITION FROM D.C.P. TO STANDARD (SUBCRITICAL) CRACK GROWTH**
- **COMPARISON AND COUPLING OF THE PRESENT THEORY WITH THE LATEST RESULTS OF THE KINETIC THEORY OF FRACTURE (SUBCRITICAL CRACK GROWTH IN QUASI-PERFECTLY BRITTLE SYSTEMS)**
- **POSSIBLE APPLICATIONS:**
 - **FRACTURE OF POLYMERS**
 - **DELAMINATION IN LAMINATED COMPOSITES**
 - **FRACTURE OF FIBER-MATRIX INTERFACES IN MMC**

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