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## **Time-Dependent Reliability Analysis of Ceramic Engine Components**

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1954

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## ABSTRACT

The computer program CARES/LIFE calculates the time-dependent reliability of monolithic ceramic components subjected to thermomechanical and/or proof test loading. This program is an extension of the CARES (Ceramics Analysis and Reliability Evaluation of Structures) computer program. CARES/LIFE accounts for the phenomenon of subcritical crack growth (SCG) by utilizing either the power or Paris law relations. The two-parameter Weibull cumulative distribution function is used to characterize the variation in component strength. The effects of multiaxial stresses are modeled using either the principle of independent action (PIA), the Weibull normal stress averaging method (NSA), or the Batdorf theory. Inert strength and fatigue parameters are estimated from rupture strength data of naturally flawed specimens loaded in static, dynamic, or cyclic fatigue. Two example problems demonstrating proof testing and fatigue parameter estimation are given.

## OBJECTIVE AND OUTLINE

Designing with ceramics requires a new approach involving statistics. Inherent to this method is the realization that any component will have a finite failure probability; that is, no design is failsafe. Methods of quantifying this failure probability as a function of time and loading have been investigated and refined. These theories have been programmed into the CARES/LIFE integrated design computer program. The accuracy of the FORTRAN coding and the mathematical modeling has been verified by analytical and the available experimental data in the open literature. Using CARES/LIFE, a design engineer can easily calculate the change in reliability due to a design change. This can lead to more efficient material utilization and system efficiency.

### Objective

- Develop probabilistic based integrated design programs for the life analysis of brittle material engine components

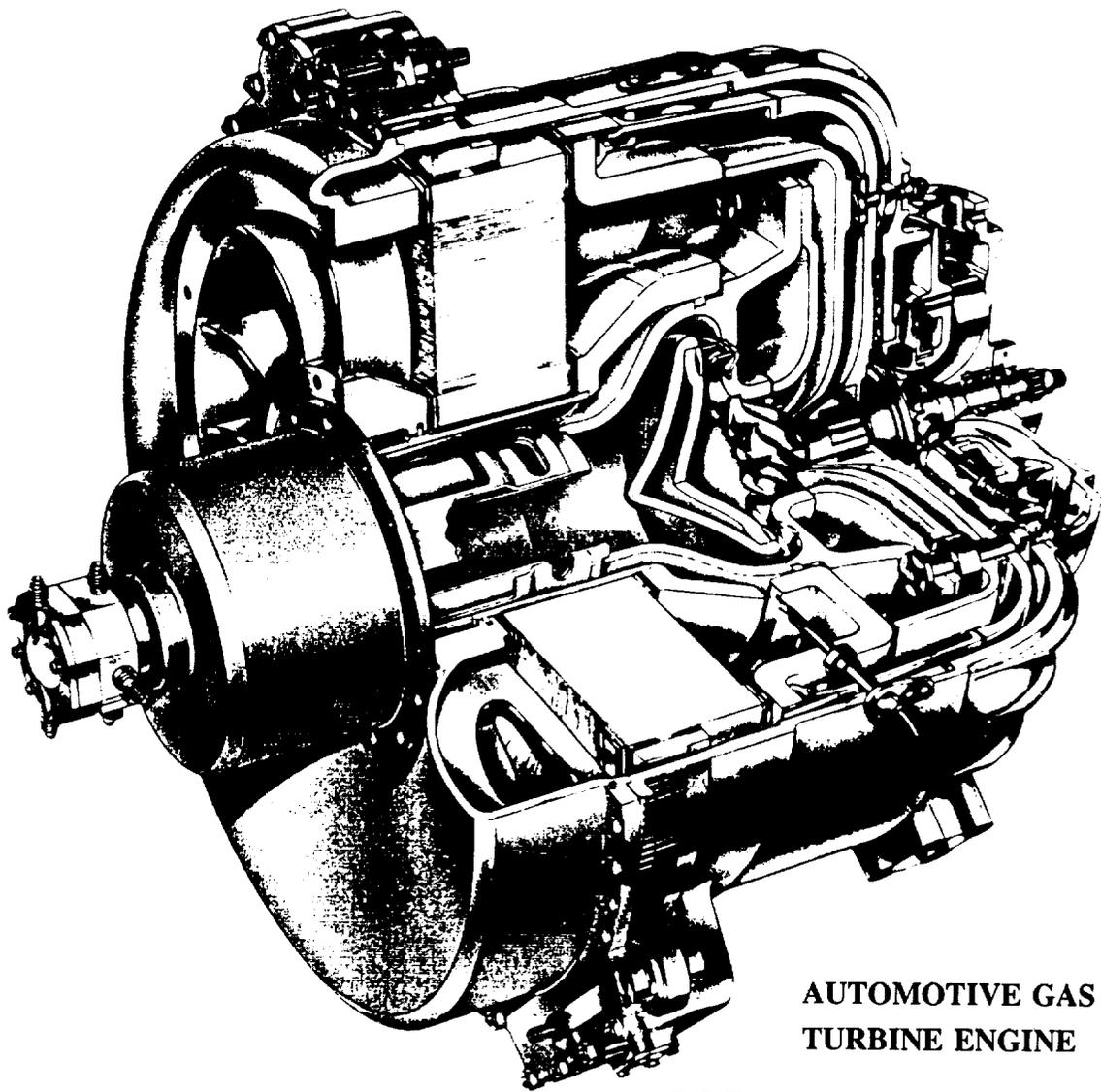
### Outline

- Introduction
- CARES/LIFE program capability
- Time-dependent reliability models
- Fatigue parameter estimation techniques
- Examples
- Conclusions

Figures 1 and 2

## CERAMICS FOR ENGINES

Structural ceramics have been utilized for various test engine components since the early 1970's. The high-temperature strength, environmental resistance, and low density of these materials can result in large benefits in system efficiency and performance. However, the brittle nature of ceramics causes a high sensitivity to microscopic flaws and often leads to catastrophic fracture. These undesirable properties are being overcome through material toughening strategies, improvements in processing to reduce the severity and number of flaws, and component designs that reduce susceptibility to foreign object damage. Ultimately, ceramics are envisioned to operate in small- and medium-sized automotive gas turbines operating with uncooled parts at temperatures as high as 1400 degrees centigrade.



**AUTOMOTIVE GAS  
TURBINE ENGINE**

**AGT101  
GARRETT TURBINE ENGINE COMPANY**

Figure 3

## CERAMIC TURBOCHARGER ROTORS

The first major commercial breakthrough for structural ceramics is the automotive turbocharger rotor. Over one half million vehicles in Japan incorporate this part. The reduced rotational inertia of the silicon nitride ceramic compared to a metallic rotor significantly enhances the turbocharger performance and efficiency. In the United States, the Garrett Automotive Division of the Allied Signal Aerospace Company is incorporating a ceramic turbocharger rotor in industrial diesel trucks.

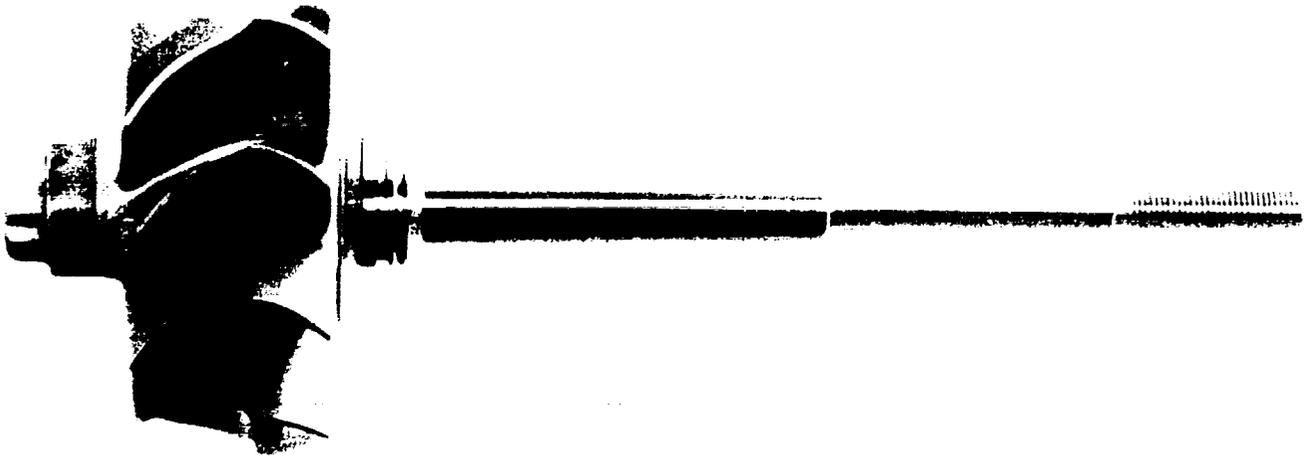


Figure 4

## BRITTLE MATERIAL DESIGN

The design of ceramics differs from that of ductile metals in that ceramic materials are unable to redistribute high local stresses induced by inherent flaws. Random flaw size and orientation require a probabilistic analysis, since the ceramic material cannot be described by a single unique strength. The weakest link theory, which analogizes the component as a series of links in a chain, accurately describes the strength response. This theory is incorporated in Weibull (1939) and Batdorf and Crose (1974) stress-volume or stress-area integrals to predict the material failure response due to thermomechanical loads. Probabilistic design is not necessarily governed by the most highly stressed location, but by the entire stress field in a component.

### CERAMICS REQUIRE PROBABILISTIC DESIGN ANALYSIS

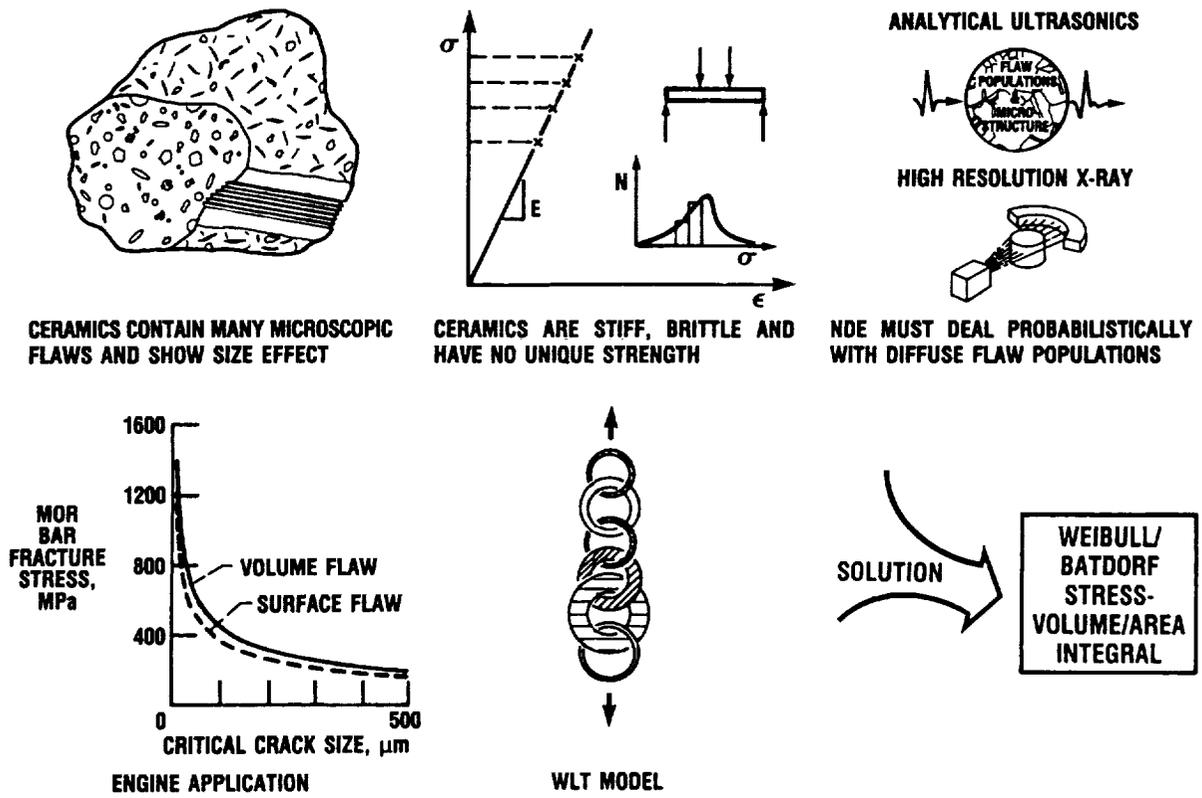


Figure 5

## STATISTICAL FRACTURE THEORIES

A common aspect of any weakest link theory is that the component volume and/or surface area of a stressed material will affect its strength, whereby larger components result in lower average strengths. This observation led Weibull (1939) to propose a phenomenological model to describe the scatter in brittle material fracture strengths in fast-fracture. To predict material fast-fracture response under multiaxial stresses, Weibull suggested averaging the tensile normal stress in all directions. As this approach is arbitrary and involves tedious numerical integration, other approaches have been subsequently introduced. The most simplistic is the Principle of Independent Action (PIA) model (Barnett (1967), and Freudenthal (1968)). The PIA theory assumes that each tensile principal stress contributes to the failure probability as if no other stress were present. Recognizing that brittle fracture is governed by linear elastic fracture mechanics (LEFM), Batdorf and Crose (1974) proposed that reliability predictions should be based on a combination of the weakest link theory and fracture mechanics. Conventional fracture mechanics dictates that both the size of the critical crack and its orientation relative to the applied loads determine the fracture stress. However, with ceramics the small critical flaw size and the large number of flaws prevent determination of the critical flaw, let alone its size and orientation. Instead, the combined probability of the critical flaw being within a certain size range and being oriented so that it may cause fracture is calculated. This model was extended to account for mixed-mode fracture by Batdorf and Heinisch (1978).

WEAKEST LINK FRACTURE MODEL	SIZE EFFECT	COMPUTATIONAL SIMPLICITY	STRESS STATE EFFECTS	THEORETICAL BASIS
WEIBULL (1939)	YES	SIMPLE	UNIAXIAL	PHENOMENOLOGICAL
NORMAL STRESS AVERAGING (1939)	YES	COMPLEX	MULTIAXIAL	PHENOMENOLOGICAL
PRINCIPLE OF INDEPENDENT ACTION (1967)	YES	SIMPLE	MULTIAXIAL	MAXIMUM PRINCIPAL STRESS THEORY
BATDORF (SHEAR-INSENSITIVE, 1974) (SHEAR-SENSITIVE, 1978)	YES	COMPLEX	MULTIAXIAL	LINEAR ELASTIC FRACTURE MECHANICS

Figure 6

## FRACTURE MAP OF A HOT PRESSED SILICON NITRIDE

Creep and subcritical crack growth (SCG) are two mechanisms which cause the average strength (per unit volume or area) of ceramic materials to degrade over time. Creep is associated with high temperatures and low stress levels. Creep is due to the formation and coalescence of voids at the glassy grain boundaries of the material. SCG is associated with elevated temperatures, moderate stress levels, chemically active environments, or mechanically (cyclically) induced damage. SCG initiates at a pre-existing flaw and continues until a critical length is reached causing catastrophic propagation.

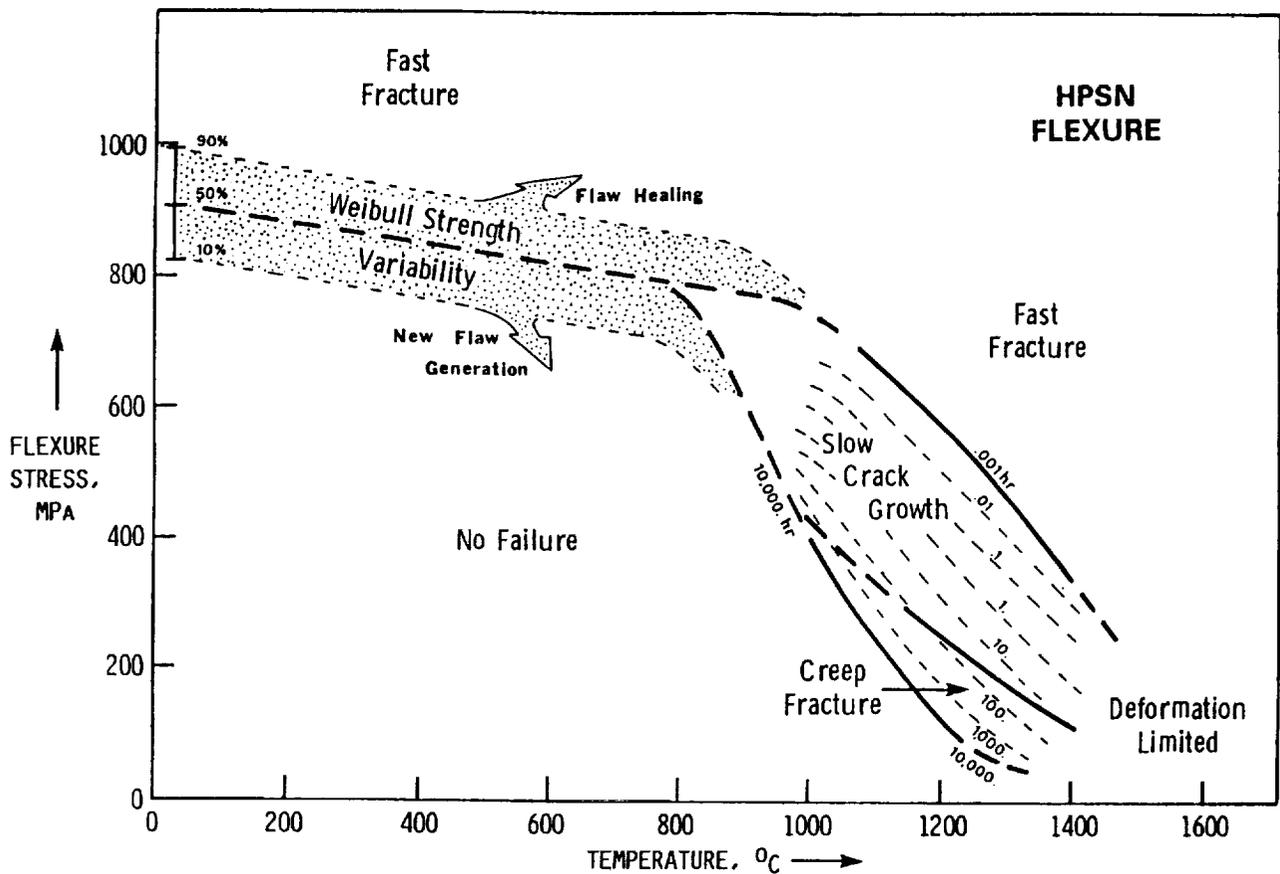


Figure 7

## THE CARES/LIFE COMPUTER PROGRAM

The CARES/LIFE program is an extension of the CARES (Ceramics Analysis and Reliability Evaluation of Structures) computer program that predicts the fast-fracture reliability of monolithic ceramic components under thermomechanical loads (Nemeth, Manderscheid and Gyekenyesi (1990), and Powers, Starlinger and Gyekenyesi (1992)). CARES/LIFE predicts the probability of failure of a component versus its service life for the SCG failure mechanism. SCG operates on the pre-existing flaws in the material and therefore requires using fast-fracture statistical theories as a basis to predict the time-dependent reliability. CARES/LIFE is coupled to widely used commercial finite element analysis programs and is a public domain program.

### Ceramics Analysis and Reliability Evaluation of Structures LIFE Prediction Integrated Design Program

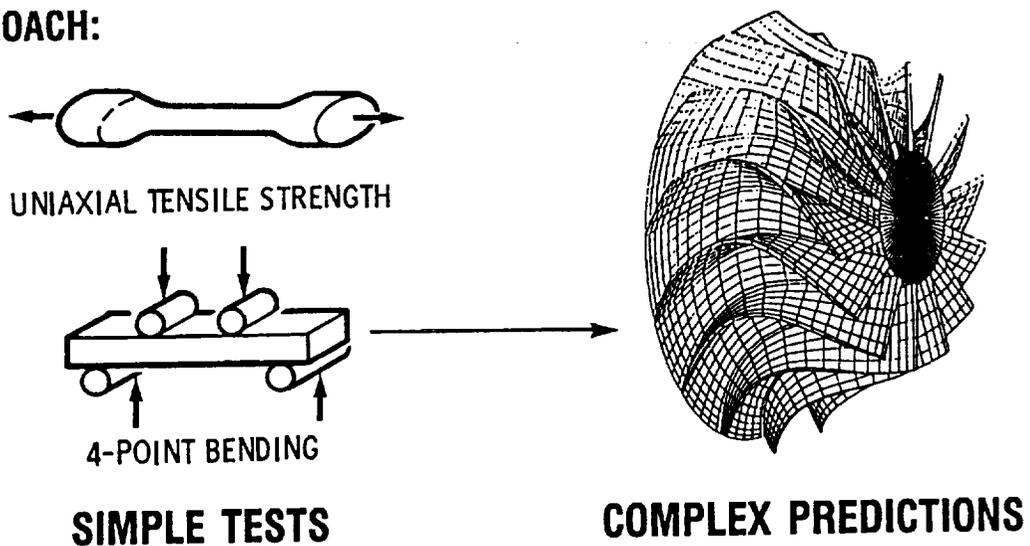
- Predicts the probability of a monolithic ceramic component's failure versus its service life
- CARES/LIFE couples commercially available finite element programs to probabilistic design methodologies

Figure 8

## CARES/LIFE DESIGN PROCEDURE

The first step of a probabilistic design methodology is the determination of a temperature-dependent and time-dependent fracture strength distribution from flexural or tensile test specimens. CARES/LIFE will estimate the fatigue and statistical parameters from the rupture data of nominally identical specimens. Typically this involves small specimens of simple geometry loaded in uniaxial flexure or tension. The specimens are usually cut from the component. Using these parameters the reliability of the component is calculated by integrating the stress distribution throughout the volume and area of the component. The stresses throughout the component are obtained from finite element analysis. Appropriate changes to the component geometry and imposed loads are made until an acceptable failure probability is achieved.

- CERAMICS ARE BRITTLE AND HAVE MANY FLAWS
- RANDOM FLAW SIZE AND ORIENTATION REQUIRE PROBABILISTIC METHOD
- APPROACH:



- REQUIRES ENTIRE STRESS FIELD, NOT MAXIMUM STRESS POINT

Figure 9

## PROGRAM CAPABILITY - COMPONENT RELIABILITY EVALUATION

CARES/LIFE computes component reliability due to fast-fracture and subcritical crack growth. The SCG failure mechanism is load-induced over time. It can also be a function of chemical reaction with the environment, debris wedging near the crack tip, the progressive deterioration of bridging ligaments, etc. Because of this complexity, the models that have been developed tend to be semi-empirical and approximate the phenomenological behavior of subcritical crack growth. The CARES/LIFE code contains modeling to account for static, dynamic and cyclic loads. Component reliability can be predicted for static (nonvarying over time) and monotonic cyclic loads. In addition, for static loading, the effects of proof testing the component prior to service can be computed.

### The CARES/LIFE Computer Program

- Component Reliability Evaluation
  - Fast-fracture
  - Time-dependent subcritical crack growth
    - a) Static Fatigue
    - b) Cyclic Fatigue
  - Proof-testing
  - Multiaxial stress states, volume flaws and surface flaws

Figure 10

## PROGRAM CAPABILITY - PROOF TESTING

CARES/LIFE incorporates proof testing methodology into the PIA, Weibull normal stress averaging, and Batdorf theories. The proof test and service loads are assumed static. The duration of the proof test and service loads are considered in the analysis. The proof test and service loads are not required to be identical. With the Batdorf theory, the two loads are allowed to be misaligned or to represent different multiaxial stress states applied in different directions. The proof test and service statistical and fatigue parameters may also be different from one another.

### CARES/LIFE Proof Testing Design Methodology

- Proof test models developed for PIA and Batdorf theories
  - time-dependent
  - proof test loads need not duplicate service loads
  - off-axis (misaligned) and multiaxial loads allowed with Batdorf theory

Figure 11

## PROGRAM CAPABILITY - PROOF TESTING

In practice it is often difficult, expensive or impossible for the proof test to exactly duplicate the service condition on the component. CARES/LIFE can analyze this situation where the two loading conditions are different using two finite element analysis results files representing the stress and temperature distribution of the proof test and service condition, respectively. A typical application of this technology is predicting the attenuated reliability distribution of a turbine rotor that was proof tested with a rotational load at ambient conditions and subsequently placed into service in the hot section of a heat engine.

### CARES/LIFE Can Predict Reliability When Proof Testing Does Not Duplicate The Stresses in Service

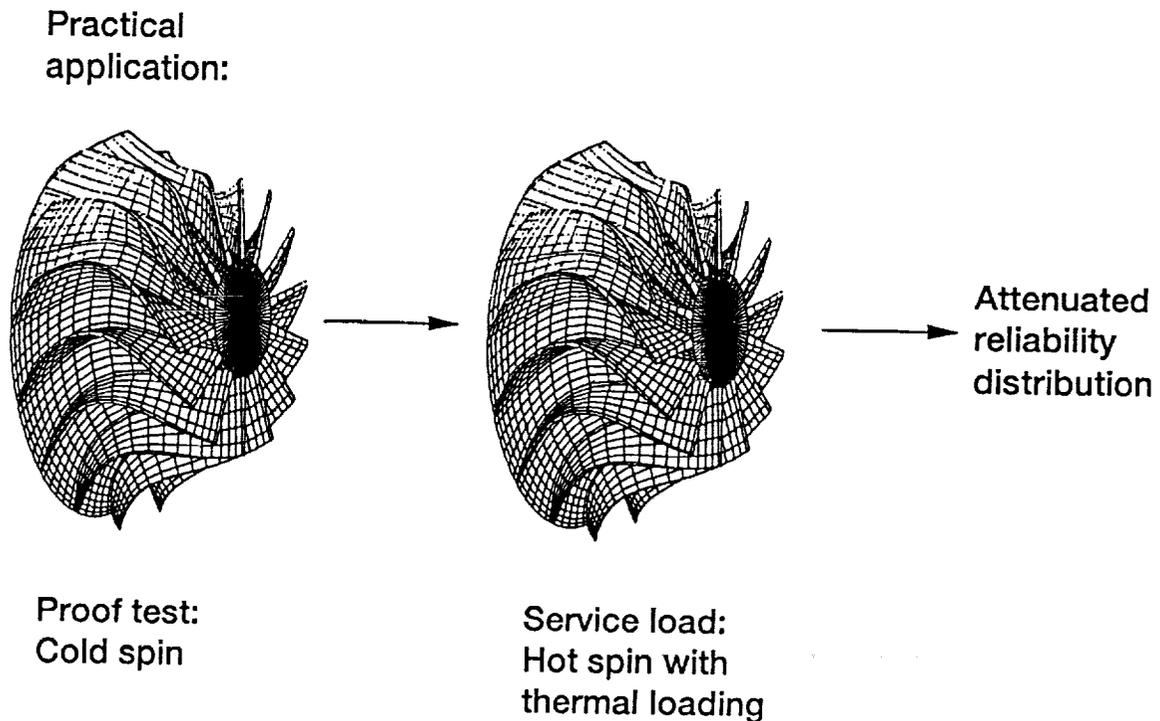


Figure 12

## PROGRAM CAPABILITY - PARAMETER EVALUATION

CARES/LIFE estimates statistical and fatigue parameters from naturally flawed specimens. These parameters must be determined under conditions representative of the service environment. When determining the fatigue parameters from rupture data of naturally flawed specimens, the statistical effects of the flaw distribution must be considered along with the strength degradation effects of subcritical crack growth. CARES/LIFE is developed on the basis that fatigue parameters are most accurately obtained from naturally flawed specimens. Batdorf and Weibull statistical material parameters are obtained from fast-fracture of nominally identical specimens under isothermal conditions. Typically, these are 3- or 4-point bend bar specimens or uniaxial tensile specimens. Fatigue parameters are also measured from these same specimen geometries. CARES/LIFE can measure fatigue parameters from static fatigue, dynamic fatigue, and cyclic fatigue experiments. In addition, information regarding the statistical distribution of the flaw population is optionally obtained from the fatigue data.

### The CARES/LIFE Computer Program

- Material parameter estimation from naturally flawed specimens
  - Weibull and Batdorf statistical material parameters
  - Fatigue parameters
    - a) Static
    - b) Dynamic (constant stress rate)
    - c) Cyclic

Figure 13

## PROGRAM CAPABILITY - PARAMETER EVALUATION

CARES/LIFE has three techniques available for estimating fatigue parameters. The median value technique is based on regression of the median data points for the various discrete load levels or stress rates. The least squares technique is based on least squares regression on all the individual data points. The modified trivariant technique is based on the minimization of the median deviation of the distribution (the trivariant technique is discussed in Jakus, Coyne and Ritter (1978)). The median value technique is the least powerful estimator of the three choices; it is included in CARES/LIFE because it is a commonly used procedure.

### Evaluation of Time Dependent Parameters

- Estimation methods to obtain fatigue parameters
  - median value
  - least squares
  - modified trivariant

Figure 14

## PROGRAM STRUCTURE

CARES/LIFE is coupled to widely used finite element analysis programs such as ABAQUS and MSC/NASTRAN. An interface code to ANSYS is being prepared. CARES/LIFE is structured into separately executable modules. These modules create a neutral file database from finite element analysis results, estimate statistical and fatigue parameters, and evaluate component reliability. CARES/LIFE uses a subelement technique to improve the accuracy of the reliability solution. The subelement technique computes reliability at the element Gaussian integration points. CARES/LIFE creates a PATRAN compatible file containing risk-of-rupture intensities (a local measure of reliability) for graphical rendering of critical regions of a component.

### CARES/LIFE Program Features

- Finite element interface program
    - Completed for MSC/NASTRAN and ABAQUS
    - Work in progress for ANSYS
  - Reliability evaluation program
    - Modular structure
    - Subelement technique
    - Postprocessor interface (PATRAN)
- 49

Figure 15

## PIA FRACTURE THEORY

Subcritical crack growth modeling is incorporated into the PIA and Batdorf theories. The PIA theory operates on the principal stresses throughout the component. CARES/LIFE includes both the semi-empirical power law (Evans and Wiederhorn (1974)), (Wiederhorn (1974)), and the Paris Law (Paris and Erdogan (1963)), (Dauskardt, Marshall and Ritchie (1990)), (Dauskardt, et al (1992)) to describe the SCG phenomenon. The power law describes the crack growth as a function of time,  $t$ , and implies that the crack growth is due to stress corrosion. The Paris law describes the crack growth as a function of the number of load cycles,  $n$ , and implies that the fatigue is a mechanical damage process. Both models require two material/environmental fatigue parameters,  $N$  and  $B$ , that describe the strength degradation.  $N$  is the fatigue crack growth exponent and  $B$  is the fatigue constant. The degree of scatter of the fracture strengths is characterized by the Weibull modulus,  $m$ . The Weibull scale parameter,  $\sigma_o$ , represents a unit volume (or area) strength where 63 percent of specimens fail. Integration is performed over the volume,  $V$  (or area  $A$ ) of the component.  $\sigma_i$  is a given principal stress component. For cyclic loading, the maximum and minimum cycle stresses, represented by the subscripts max and min, respectively, are used. For the power law, a constant called the  $g$ -factor (Mencik (1984)) can be computed such that cyclic loading can be expressed as an equivalent static load over a period,  $T$ . The fatigue constant,  $B$ , is a function of the mode I stress intensity factor,  $K_{IC}$ , the fatigue exponent,  $N$ , the crack geometry factor,  $Y$ , and the experimentally determined material/environmental constant,  $A$ .

### Component Reliability Prediction (PIA Model) ( Based on Principal Stress Distribution)

$$P_{fv}(t_f) = 1 - \exp \left[ - \int_v \sum_{i=1}^3 \left( \frac{\sigma_{i,0}(x,y,z)}{\sigma_{ov}} \right)^{m_v} dv \right]$$

Power Law :

$$\sigma_{i,0}(x,y,z) = \sigma_i \left( 1 + \frac{\sigma_i^2 g t_f}{B} \right)^{\frac{1}{N-2}}$$

equivalent static time =  $g t_f$

$$g = \frac{1}{T} \int_0^T \left( \frac{\sigma(t)}{\sigma_{max}} \right)^N dt$$

$$B = \frac{2}{A Y^2 (N-2) K_{IC}^{N-2}}$$

Paris Law:

$$\sigma_{i,0}(x,y,z) = \sigma_{i,max} \left( 1 + \frac{\sigma_{i,max}^2 (1-R)^N n_f}{B} \right)$$

$$R = \frac{\sigma_{i,min}}{\sigma_{i,max}}$$

$$B = \frac{2}{A Y^2 (N-2) K_{IC}^{N-2}}$$

Figure 16

## BATDORF FRACTURE THEORY

With the Batdorf theory fracture depends not only on the existence of a crack with a certain critical strength, but also on the crack shape, the far-field stress state, and the crack orientation described by the azimuthal angles  $\alpha$  and  $\beta$ . A collection of random crack orientations can be described by a sphere of unit radius for volume-distributed flaws (analogously, a unit radius circle describes a collection of surface flaws). For a given flaw orientation, an effective stress,  $\sigma_e$ , which is a function of crack geometry, mixed-mode fracture criterion, and the far-field stress state, describes the equivalent mode I stress intensity factor on the crack. Fracture occurs when the equivalent mode I stress intensity factor exceeds the critical stress intensity factor of the crack.

### Component Reliability Prediction (Batdorf)

- Requires integration of the projected equivalent stress over a unit radius sphere (all possible flaw orientations)

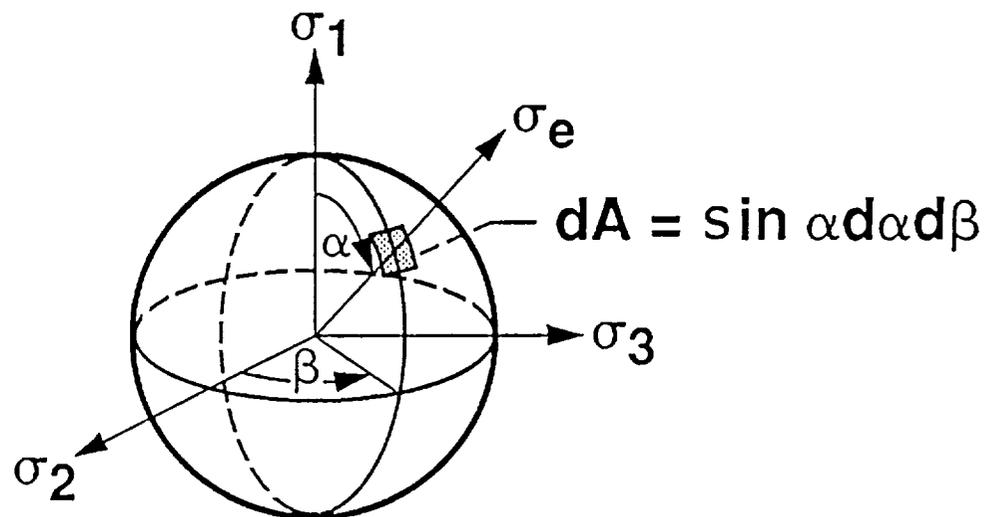


Figure 17

## BATDORF FRACTURE THEORY

The Batdorf theory combines LEFM with the weakest link mechanism to achieve a mechanistic description of the rupture process. Fracture is described as a process of crack growth from randomly oriented pre-existing flaws. CARES/LIFE models SCG with the Batdorf methodology by incorporating the power law and the Paris law to account for material degradation from stress corrosion and mechanical effects such as microcracking, degeneration of bridging ligaments, and debris wedging.

### Component Reliability Prediction (Batdorf Model)

$$P_{fv}(t_f) = 1 - \exp \left[ -\frac{2}{\pi} \bar{k}_{Bv} \int_v \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \left( \frac{\sigma_{e,0}(\Psi)}{\sigma_{ov}} \right)^{m_v} \sin \alpha \, d\alpha \, d\beta \, dV \right]$$

Power law:

$$\sigma_{e,0}(\Psi) = \sigma_e(\Psi) \left( 1 + \frac{\sigma_e^2(\Psi) g t_f}{B} \right)^{\frac{1}{N-2}}$$

Paris law:

$$\sigma_{e,0}(\Psi) = \sigma_e(\Psi) \left( 1 + \frac{\sigma_{e,max} (1-R)^N n}{B} \right)^{\frac{1}{N-2}}$$

$$R = \frac{\sigma_{e,min}}{\sigma_{e,max}}$$

$\sigma_e$  is a function of the mixed-mode fracture criteria and crack configuration

$\Psi$  is a function of  $(x, y, z, \alpha, \beta)$

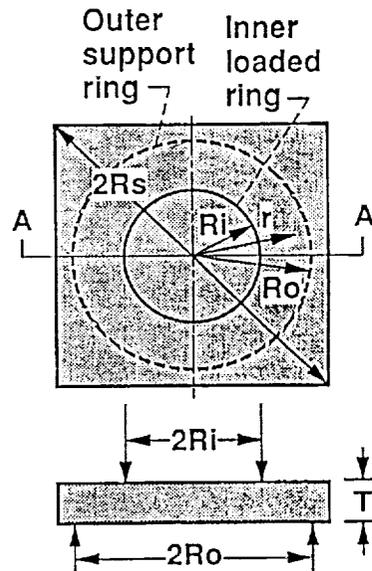
Figure 18

## EXAMPLE PROBLEM - FATIGUE PARAMETER EVALUATION

Two example problems have been selected to demonstrate various features of the CARES/LIFE code: (1) estimation of fatigue parameters from multiaxially stressed specimens, and (2) examination of the effect of misalignment between a uniaxial proof test load and a uniaxial service load. In the first example, the fatigue and statistical parameters are estimated using rupture data from 121 ring-on-ring loaded square plate specimens made of soda-lime glass. The specimens were fractured under dynamic fatigue load conditions in a distilled water environment. In addition, the fast-fracture strengths of 30- specimens were measured in an inert environment of silicon oil.

### Evaluation of Time-Dependent Parameters

#### EXAMPLE: Ring-On-Ring Loaded Square Plate Specimen



EXPERIMENT: Dynamic fatigue testing in water environment .

MATERIAL: Soda-Lime-Silica Glass

DIMENSIONS:  $R_i = 5.02 \text{ mm}$   
 $R_o = 16.09 \text{ mm}$   
 $R_s = 35.92 \text{ mm}$   
 $1.42 \text{ mm} \leq T \leq 1.58 \text{ mm}$

Figure 19

## EXAMPLE PROBLEM - FATIGUE PARAMETER EVALUATION

This figure shows the fracture strengths,  $\sigma_f$ , of the 121 soda-lime glass specimens loaded at various stressing rates,  $\dot{\sigma}$ , and the fast-fracture strengths of the 30 specimens measured in a chemically inert environment of silicon oil. The maximum fracture stress occurs at the center of the specimen. All failures occurred at the specimen surface (surface flaws).

### Dynamic Fatigue of Soda-Lime-Silica Glass

Ring-On-Ring Loaded Square Plate Specimen  
121 fatigue data points and 30 inert strength data points

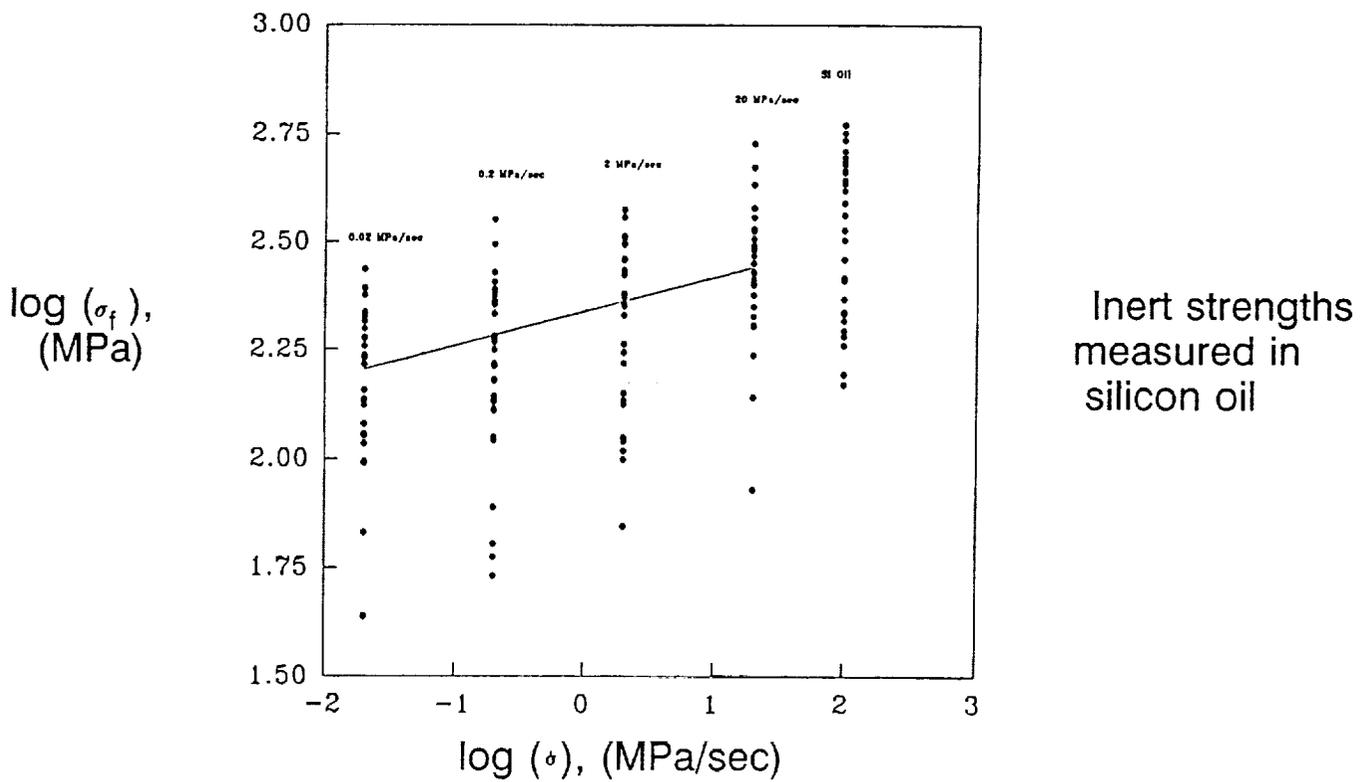
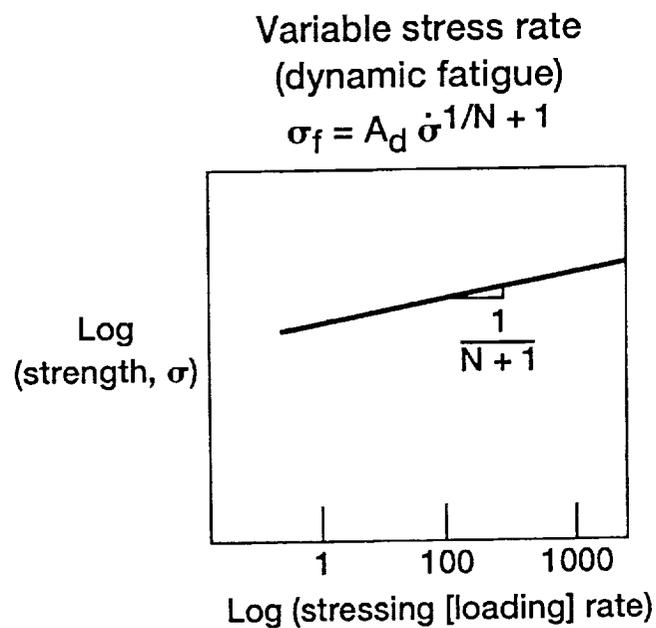


Figure 20

## EXAMPLE PROBLEM - FATIGUE PARAMETER EVALUATION

The median failure stress of the specimens versus the stress rate can be linearized by plotting the log of the median strength  $\sigma_f$  versus the log of the stress rate  $\dot{\sigma}$ . The slope of the line establishes the fatigue constant,  $N$ , and the intercept of the line,  $A_d$ , is used to determine the fatigue constant,  $B$ . Evaluation of the stress-area integral,  $A_{ef}$  (or the stress volume integral  $V_{ef}$  for volume flaws) is also required for the calculation of  $B$ . CARES/LIFE employs the various regression techniques previously mentioned to estimate these parameters.

### Evaluation of Time-Dependent Parameters: Power Law



$$B = \frac{A_d^{N+1}}{(N+1)\sigma_{ov}^{N-2}} \left[ \frac{V_{ef}}{\ln\left(\frac{1}{1-P_{fv}}\right)} \right]^{1/m_v}$$

Figure 21

## EXAMPLE PROBLEM - FATIGUE PARAMETER EVALUATION

CARES/LIFE measures the intrinsic scatter in the fatigue data. The Weibull modulus,  $m$ , and a characteristic strength,  $\sigma_0$ , representing a level of strength where 63 percent of specimens fail are estimated from the fatigue data. In this case, the characteristic strength is determined for a time to failure of one second. With these parameters the fatigue data can be transformed to an equivalent fast-fracture strength distribution. The following Weibull plot shows the ranked data failure probability versus the fast-fracture strength. The fast-fracture strengths that were measured in silicon oil are shown for comparison. As can be observed, the Weibull slope of the fatigue data derived strengths correlate well with the Weibull slope of the fast-fracture strengths measured in silicon oil.

Comparison of Inert Strengths Determined from Fatigue Data Measured in Water to Inert Strengths Measured in Silicon Oil

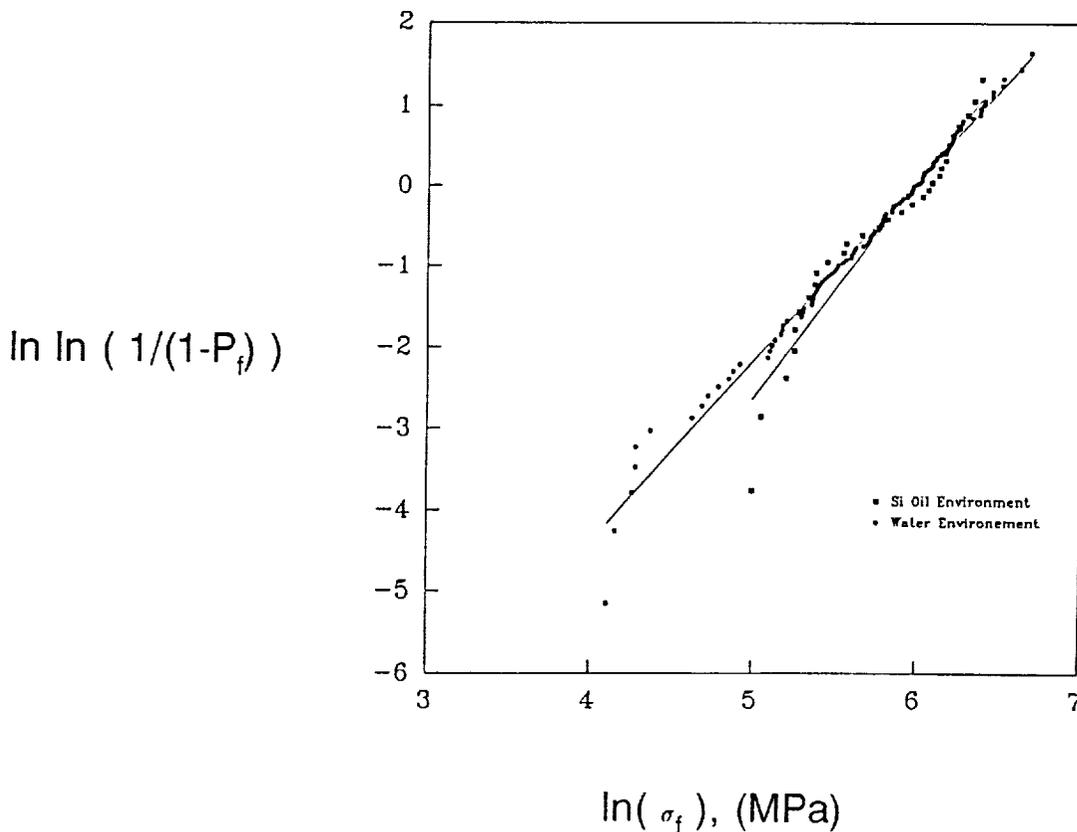


Figure 22

## EXAMPLE PROBLEM - PARAMETER EVALUATION

The fatigue parameters for the soda-lime glass are estimated using the median value, least squares, and the modified trivariate techniques. For this table all parameters are obtained solely from the fatigue data. The fatigue constant B uses the subscript B to indicate that the value was obtained using the Batdorf technique.

Parameters Estimated From Soda-Lime Glass Fatigue Data				
Method	Fatigue exponent, $N_s$	Fatigue constant, $B_{BS}$  $\text{MPa}^2 \cdot s$	Weibull modulus, $m'_s$	Scale parameter, $\sigma'_{oS}$  $\text{MPa} \cdot \text{mm}^{2/m_s}$
Median value	12.60	4 445	2.279	5 904
Least squares	11.24	6 337	2.208	6 707
Modified trivariate	11.88	5 982	2.344	5 443

Figure 23

## EXAMPLE PROBLEM - PLOTTING CRITICAL REGIONS OF FAILURE

CARES/LIFE outputs the risk-of-rupture intensities of the elements into a data file formatted such that it can be read by PATRAN. The risk-of-rupture intensity is a volume or area independent measure of the potential of failure. This information enables a component to be graphically rendered such that regions on the component where failure is most likely to occur are shown. This example shows the ring-on-ring square plate specimen where the biaxially loaded central portion of the specimen shows the highest likelihood of failure.

### Finite Element Model of Ring-On-Ring Specimen (1/4 Symmetry) Risk-of-Rupture Intensities

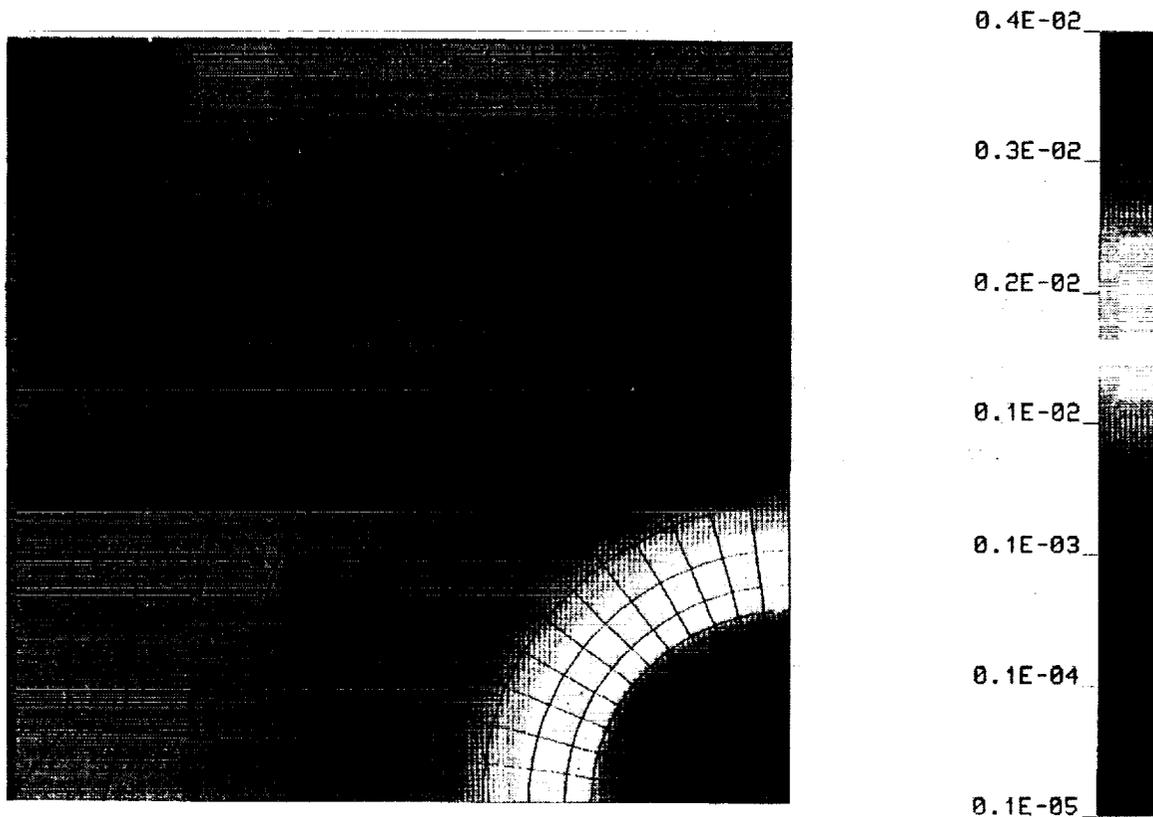
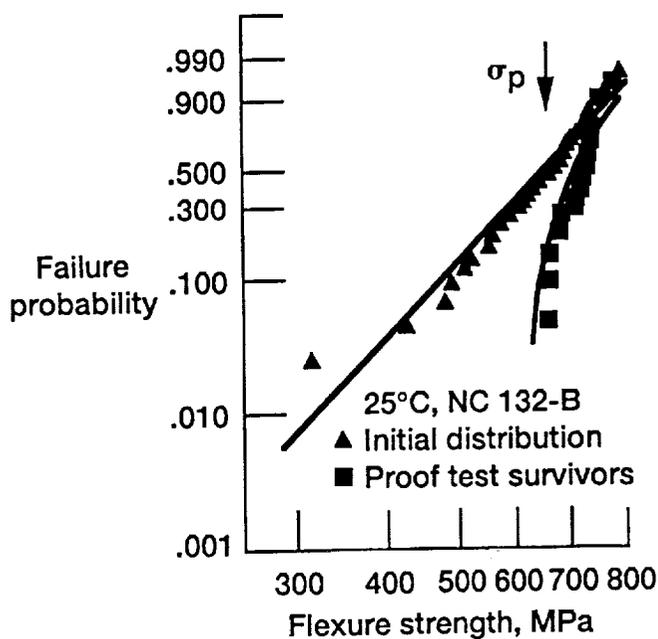


Figure 24

## EXAMPLE PROBLEM - PROOF TESTING WITH MISALIGNED LOADS

Proof testing screens out weak components so that the remaining components are less likely to fail in service. This attenuated failure probability,  $P_{fa}$ , is a function of the component failure probability,  $P_{fp}$ , from the proof test and the component probability of failure,  $P_{fi}$ , from the combined application of the proof test and the service load.  $P_{fp}$  is computed from the proof stress,  $\sigma_p$ , applied over time interval,  $t_p$ .  $P_{fi}$  is computed from the combined application of the proof stress,  $\sigma_p$ , over time  $t_p$  and the service stress,  $\sigma_i$ , applied over time,  $t_q$ . The attenuated failure probability displays a threshold stress,  $\sigma_u$ , and a threshold time,  $t_{min}$ , where the failure probability is zero.

### Proof Testing at 650 MPa (94ksi) Truncates The Weibull Strength Distribution For Hot Pressed Silicon Nitride and Eliminates Low-Strength Specimens



Attenuated failure probability:

$$P_{fa} = \frac{P_{fi} - P_{fp}}{1 - P_{fp}}$$

Attenuated reliability:

$$P_{sa} = \frac{P_{si}}{P_{sp}} = \exp[-(B_i - B_p)]$$

i = initial distribution  
 p = proof test

Figure 25

## EXAMPLE PROBLEM - PROOF TESTING WITH MISALIGNED LOADS

Ideally the proof test loading exactly duplicates the service loading except that the proof load is greater in magnitude than the service load. In this example the effect of misalignment between a uniaxial proof test and a uniaxial service load is explored using the Batdorf technique with a shear sensitive and shear insensitive fracture criterion. The proof test is such that 50 percent of the components break prior to placing them in service. The failure probability is given per unit volume of material.

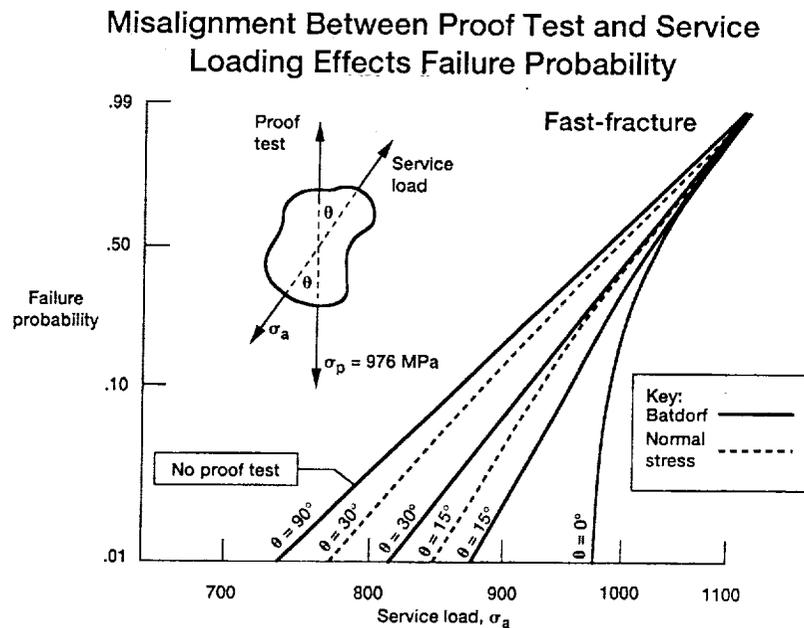
### CARES/LIFE Proof Testing Design Methodology

EXAMPLE: Effect of misalignment between uniaxial proof test and service load

- Magnitude of proof test load,  $\sigma_p$ , eliminates 1 of every 2 components

Weibull parameters:  $m_v = 15.0$ ,  $\sigma_{ov} = 1000$ . Mpa mm<sup>3/10</sup>

Fatigue parameters:  $N = 40.0$ ,  $B_w = 2000$ . MPa<sup>2</sup> second



Figures 26

## EXAMPLE PROBLEM - PROOF TESTING WITH MISALIGNED LOADS

For fast-fracture the effect of misalignment between the proof test and service load as a function of the angle of misalignment and the magnitude of the service load is shown. For aligned loads the classical truncated distribution is observed with threshold stress behavior. As misalignment increases, the failure distribution approaches the unattenuated (the original) distribution shown by the straight line. The effect of the fracture criterion is shown with the solid line indicating the coplanar strain energy release criteria with a Griffith crack and the dotted line showing a shear insensitive crack criteria. The effect of misalignment is more severe for a shear insensitive criteria. These same trends are observed with time-dependent loading.

### Misalignment Between Proof Test and Service Loading Effects Failure Probability

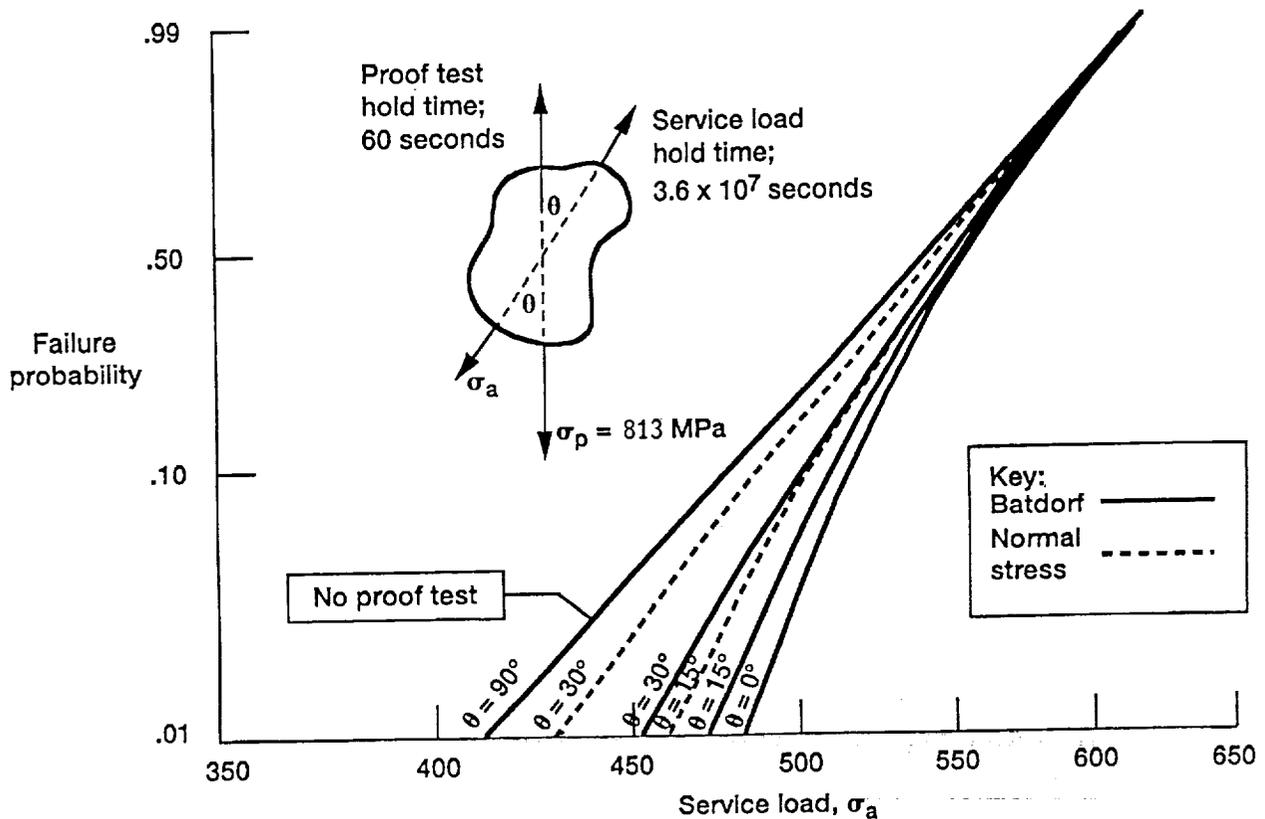


Figure 27

## SUMMARY

The use of structural ceramics for high-temperature applications depends on the strength, toughness and reliability of these materials. Ceramic components can be designed for service if the factors that cause material failure are accounted for. This design methodology must combine the statistical nature of strength controlling flaws with fracture mechanics to allow for multiaxial stress states, concurrent flaw populations, and subcritical crack growth. This has been accomplished with the CARES/LIFE public domain computer program for predicting the time-dependent reliability of monolithic structural ceramic components. Potential enhancements to the code include the capability for transient analysis, three-parameter Weibull statistics, creep and oxidation modeling, flaw anisotropy, threshold stress behavior, and parameter regression for multiple specimen sizes.

- Power law and Paris law have been incorporated into the Batdorf and PIA reliability models
- Component reliability can be predicted for static, cyclic and proof test loadings
- Fatigue parameters and Weibull parameters can be estimated from static, dynamic and cyclic loading experiments on naturally flawed specimens

Figure 28

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