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RACK INSERTION END EFFECTOR (RIEE) AUTOMATION

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## ABSTRACT

NASA is developing a mechanism to manipulate and insert Racks into the Space Station Logistic modules. The mechanism consists of a base with three motorized degrees of freedom, a 3 section motorized boom that goes from 15 to 44 feet in length, and a Rack Insertion End Effector (RIEE) with 5 hand wheels for precise alignment. The robotics section has been tasked with the automation of the RIEE unit.

In this report, for the automation of the RIEE unit, application of the Perceptics Vision System has been conceptually developed to determine the position and orientation of the RIEE relative to the logistic module and a MathCad program is written to display the needed displacements for precise alignment and final insertion of the Rack.

The uniqueness of this report is that the whole report is in fact a MathCad program including text, derivations and executable equations with example inputs and outputs.

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# RACK INSERTION END-EFFECTOR (RIEE) AUTOMATION

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## 1. INTRODUCTION

Towards the automation of the RACK INSERTION END-EFFECTOR (RIEE) Mechanism, Application of the Perceptics Vision System has been conceptually developed to determine the position and orientation of the RIEE relative to the logistic module and a MathCad program is written to compute and display the displacements required to

- 1) Align the RIEE relative to the logistic module to prepare for final insertion of the Rack, and
- 2) Translate the Rack Rotation axis and rotate the Rack about it for the Final Insertion.

The above displacements are used for the actuation of motors to be incorporated in the Improved RIEE Mechanism.

The following sections describe concepts, derivations of formulae and the MathCad program with sample input and output. With the help of this report a scaled prototype of the Improved RIEE Mechanism with an integrated Vision System can be quickly developed and tested.

## 2. THE RIEE MECHANISM

The RIEE is a space mechanism with 5 degrees of freedom. It has 12 links and 15 joints. It is used to align the axis of rotation of the Rack and perform the final insertion of the Rack into the logistic module. The boom, carrying the RIEE, has 4 degrees of freedom. Fig.1 and Fig.2 show the RIEE in retracted and installed positions as per drawing set # 82K03931.

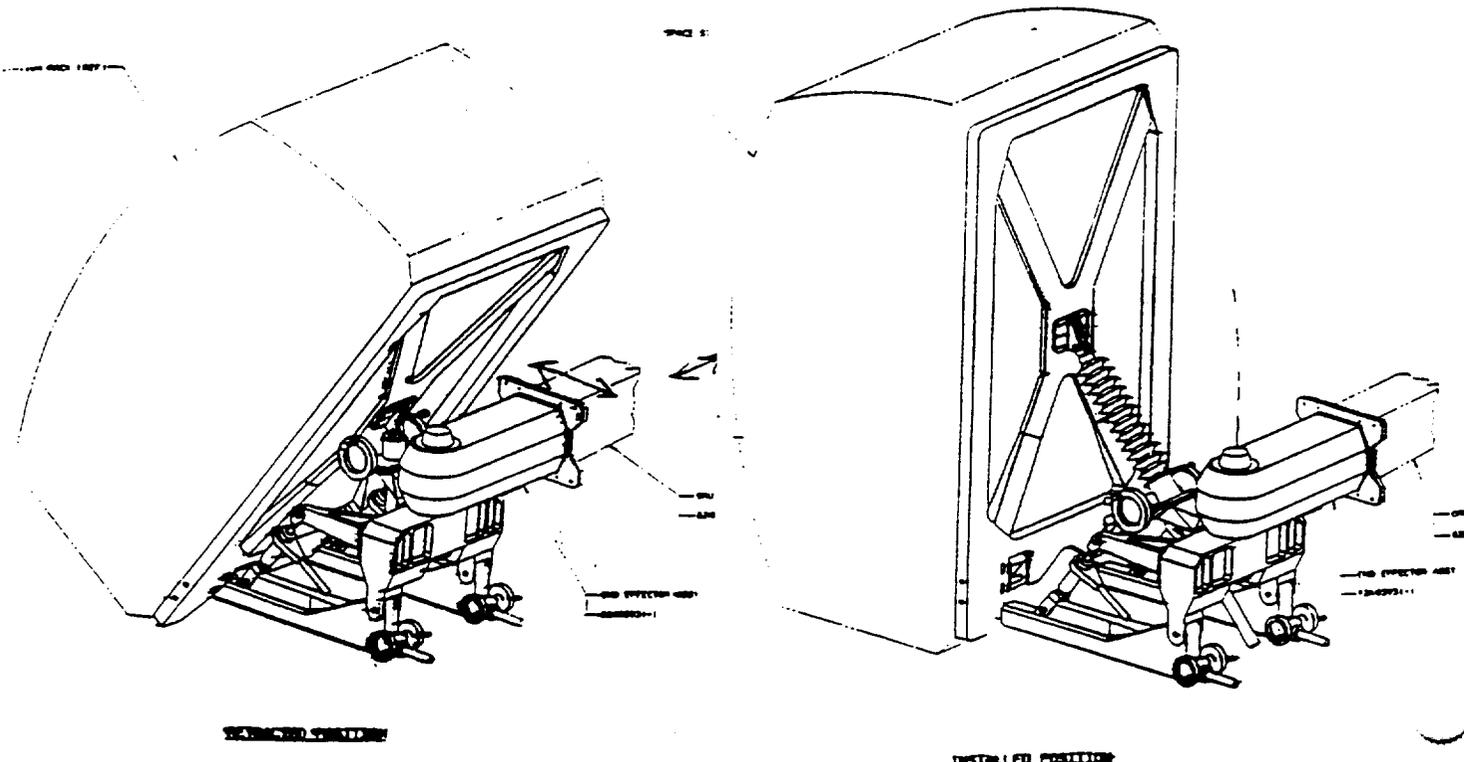


Fig. 1

Fig. 2

### 3. DETERMINATION OF THE COORDINATES OF THE TARGETS IN THE CAMERA FRAME

To insert the rack, we have to know the positions of the rack attach points on the module, shown as lower support points (S) in the Fig. 3 below. In the following analysis, we derive the expressions for the coordinates of the left and right lower support points by getting the images of the spherical targets suspended vertically from the supports employing a set of two cameras for each target. The figure shows the relationship of the cameras to the targets in the y-z plane of the boom coordinate system carrying the RIEE mechanism. The RIEE mechanism has 2 end effector points on the left and right sides shown by letter E, and a top end effector point P. Different views of this mechanism can be studied in the set of drawings # 89K03931. The points shown in the figure carry prefixes "R" for "Right" and "L" for "Left" in the analysis that follows.

## RIEE MECHANISM SCHEMATIC

(Not to scale)

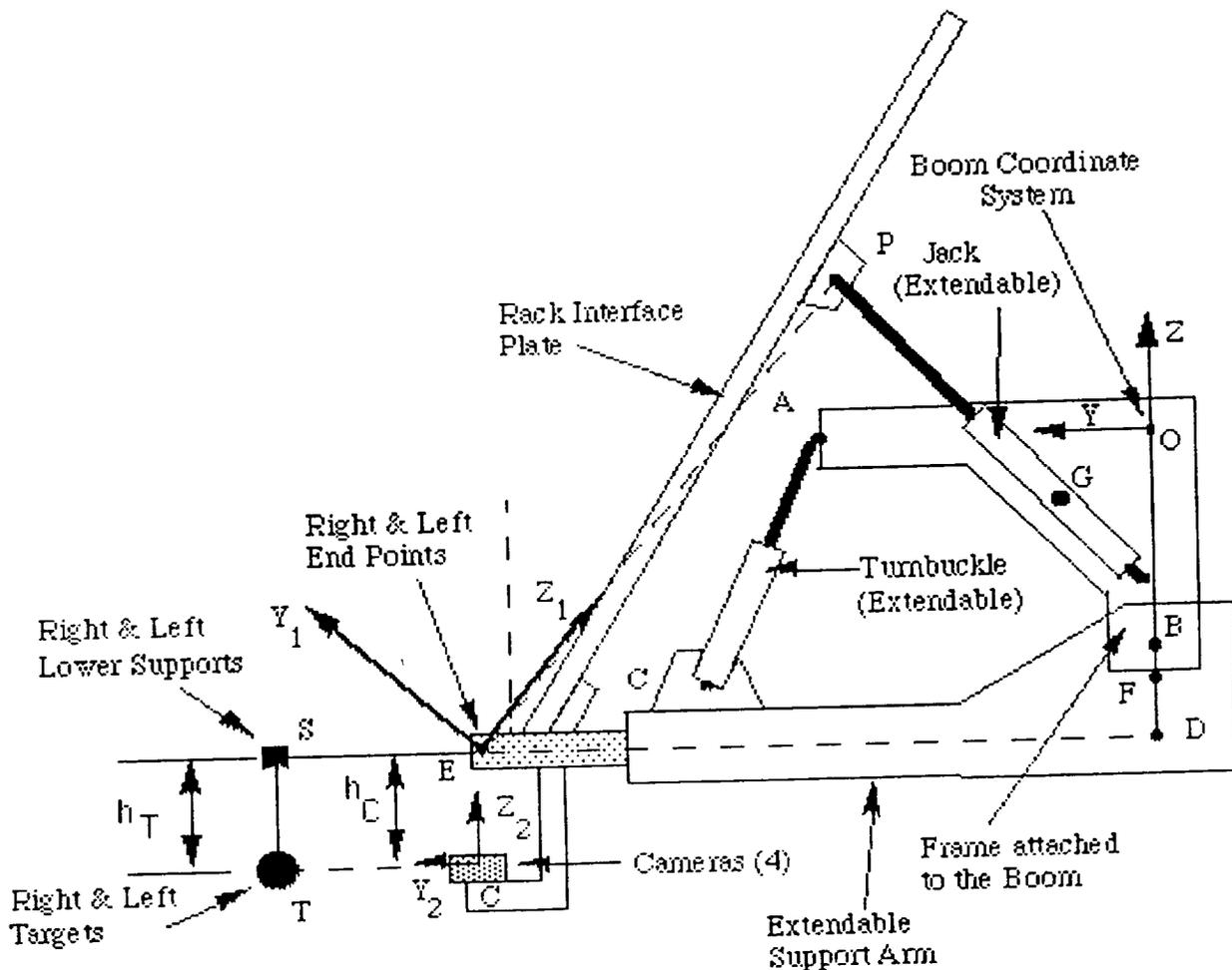


Figure 3

# TARGET AND CAMERA ARRANGEMENT SCHEMATIC

(Not to scale)

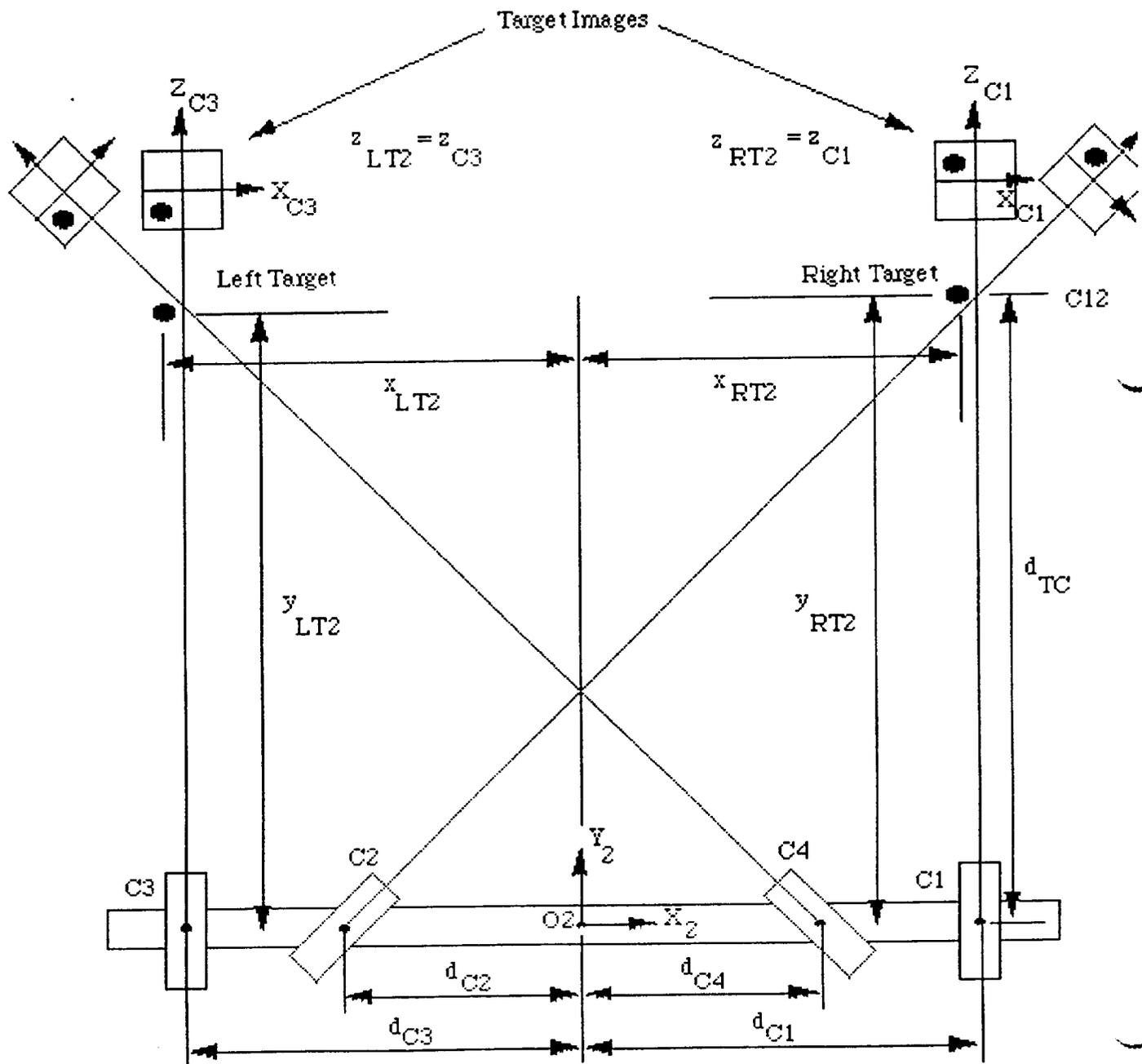


Figure 4

In Figure 4 the frame that carries the cameras of the Vision System is called the Camera frame. A coordinate system  $O_2X_2Y_2Z_2$  is established at  $O_2$ .  $X_2Y_2$  plane is the plane containing the axes of the all 4 cameras. It should be made sure that all the camera axes are co-planar with proper calibration. The camera frame is called frame 2.

In Figure 4, camera 1 and camera 2 are used to get the coordinates of the right target. The coordinates obtained by camera 1 are  $x_{C1}$  and  $z_{C1}$  which are perpendicular to the axis of the camera. Let the distance of right target from the camera 1 be  $y_{C1}$  which cannot be measured. The coordinates obtained by camera 2 are  $x_{C2}$  and  $z_{C2}$ . Let the distance from the camera 2 to the right target be  $y_{C2}$ , which cannot be measured. In the following derivation we determine  $y_{C1}$  by eliminating  $y_{C2}$ .

$z_{C1} := 1$	z coordinate of Right Target image centroid from Camera 1
$x_{C1} := -2$	x coordinate of Right Target image centroid from Camera 1
$x_{C2} := -.707$	x coordinate of Right Target image centroid from Camera 2
$d_{TC} := 20$	Nominal Distance of the Target from the line of Cameras
	Distance of Target from Camera frame
$d_{C1} := 10$	Distance of Camera 1 from origin of frame 2 (camera frame)
$d_{C2} := 10$	Distance of Camera 2 from origin of frame 2 (camera frame)
$d_{C12} := d_{C1} + d_{C2}$	Distance between camera 1 and camera 2
$\alpha_{C12} := \text{atan}\left(\frac{d_{C12}}{d_{TC}}\right)$	Angle between axes of camera 1 and camera 2

The coordinates of the right target in frame #2 situated as shown in Fig. 3 are given by:

Using the output of camera 1

$$x_{RT2} = x_{C1} + d_{C1} \quad [1]$$

$$y_{RT2} = y_{C1} \quad [3]$$

$$z_{RT2} = z_{C1} \quad [5]$$

Using the output of camera 2

$$x_{RT2} = x_{C2} * \cos(\alpha_{12}) + y_{C2} * \sin(\alpha_{12}) - d_{C2} \quad [2]$$

$$y_{RT2} = -x_{C2} * \sin(\alpha_{12}) + y_{C2} * \cos(\alpha_{12}) \quad [4]$$

$$z_{RT2} = z_{C2} \quad [6]$$

Equating the right hand sides of equations [1] and [2], and rearranging, we have:

$$y_{C2} = \frac{x_{C1} - x_{C2} * \cos(\alpha_{C12}) + d_{C1} + d_{C2}}{\sin(\alpha_{C12})}$$

or

$$y_{C2} = \frac{(x_{C1} - x_{C2} \cdot \cos(\alpha_{C12})) + d_{C12}}{\sin(\alpha_{C12})}$$

Substituting the above expression for  $y_{C2}$  into equation [4], we have

$$y_{RT2} = -x_{C2} \cdot \sin(\alpha_{C12}) + \frac{(x_{C1} - x_{C2} \cdot \cos(\alpha_{C12})) + d_{C12}}{\sin(\alpha_{C12})} \cdot \cos(\alpha_{C12})$$

$$y_{RT2} = \frac{-x_{C2} \cdot \sin(\alpha_{C12})^2 + x_{C1} \cdot \cos(\alpha_{C12}) - x_{C2} \cdot \cos(\alpha_{C12})^2 + d_{C12} \cdot \cos(\alpha_{C12})}{\sin(\alpha_{C12})}$$

$$y_{RT2} = \frac{-x_{C2} + x_{C1} \cdot \cos(\alpha_{C12}) + d_{C12} \cdot \cos(\alpha_{C12})}{\sin(\alpha_{C12})}$$

$$y_{RT2} = \frac{(d_{C12} + x_{C1}) \cdot \cos(\alpha_{C12}) - x_{C2}}{\sin(\alpha_{C12})}$$

Coordinates of Right Target in frame 2 are:

$$\begin{pmatrix} x_{RT2} \\ y_{RT2} \\ z_{RT2} \end{pmatrix} := \begin{bmatrix} x_{C1} + d_{C1} \\ \frac{(d_{C12} + x_{C1}) \cdot \cos(\alpha_{C12}) - x_{C2}}{\sin(\alpha_{C12})} \\ z_{C1} \end{bmatrix} \quad \begin{pmatrix} x_{RT2} \\ y_{RT2} \\ z_{RT2} \end{pmatrix} = \begin{pmatrix} 8 \\ 18.9998 \\ 1 \end{pmatrix}$$

In Figure 4, camera 3 and camera 4 are used to get the coordinates of the left target. The coordinates obtained by camera 3 are  $x_{C3}$  and  $z_{C3}$  which are perpendicular to the axis of the camera. Let the distance of left target from the camera 3 be  $y_{C3}$  which cannot be measured. The coordinates obtained by camera 4 are  $x_{C4}$  and  $z_{C4}$ . Let the distance from the camera 4 to the left target be  $y_{C4}$ , which cannot be measured.  $y_{C3}$  is determined by eliminating  $y_{C4}$  in the same way we did as for the right target. Following similar substitutions and simplifications, we get the coordinates of the left target in frame 2.

$z_{C3} := -1$	$z$ coordinate of Left Target image centroid from Camera 3
$x_{C3} := -2$	$x$ coordinate of Left Target image centroid from Camera 3
$x_{C4} := -2.121$	$x$ coordinate of Left Target image centroid from Camera 4
$d_{C3} := 10$	Distance of Camera 3 from origin of frame 2 (camera frame)
$d_{C4} := 10$	Distance of Camera 4 from origin of frame 2 (camera frame)
$d_{C34} := d_{C3} + d_{C4}$	Distance between camera 3 and camera 4
$\alpha_{C34} := \text{atan}\left(\frac{d_{C34}}{d_{TC}}\right)$	Angle between the axes of camera 3 and camera 4

The coordinates of the left target in frame #2 situated as shown in Figure 4 are given by:

Using the output of camera 3

$$\begin{aligned} x_{LT2} &= x_{C3} - d_{C3} \\ y_{LT2} &= y_{C3} \\ z_{LT2} &= z_{C3} \end{aligned}$$

Using the output of camera 4

$$\begin{aligned} x_{LT2} &= x_{C4} \cdot \cos(\alpha_{C34}) - y_{C4} \cdot \sin(\alpha_{C34}) + d_{C4} \\ y_{LT2} &= x_{C4} \cdot \sin(\alpha_{C34}) + y_{C4} \cdot \cos(\alpha_{C34}) \\ z_{LT2} &= z_{C4} \end{aligned}$$

$y_{C4}$  in the above is eliminated by following the procedure adopted for the right target.

Coordinates of Left Target in frame 2 are:

$$\begin{pmatrix} x_{LT2} \\ y_{LT2} \\ z_{LT2} \end{pmatrix} := \begin{pmatrix} x_{C3} - d_{C3} \\ \frac{(d_{C34} - x_{C3}) \cdot \cos(\alpha_{C34}) + x_{C4}}{\sin(\alpha_{C34})} \\ z_{C3} \end{pmatrix} \quad \begin{pmatrix} x_{LT2} \\ y_{LT2} \\ z_{LT2} \end{pmatrix} = \begin{pmatrix} -12 \\ 19.0005 \\ -1 \end{pmatrix}$$

#### 4. DETERMINATION OF COORDINATE SYSTEM ATTACHED TO THE RACK INTERFACE PLATE ROTATION AXIS.

Derivation of the equations for determining the frame of reference 1 attached to the Rack Interface Plate rotation axis at a special point (CE) on the axis joining the left and right end points (LE and RE) of the end effector. This special point is the foot of the perpendicular from the top joint (P) on the rack onto the axis of the end effector endpoints. See Figure 3 and Figure 6 for details.

- $m := 10$                       Distance between CE and CR
- $h := 30$                       Distance between CE and P
- $g := \sqrt{m^2 + h^2}$               Distance between CR and P                       $g = 31.6228$
- $r_{GP} := 15$                       Distance between G and P

G is a reference point on the boom where the end effector mechanism is connected, as shown in Figure 3 & Figure 6.

Coordinates of point G:

$$\begin{pmatrix} x_G \\ y_G \\ z_G \end{pmatrix} := \begin{pmatrix} 90 \\ 0 \\ 0 \end{pmatrix}$$

In the following,  $r_{AB}$  indicates the magnitude of vector AB, and  $x_{AB}$ ,  $y_{AB}$ , and  $z_{AB}$  indicate the x,y,z coordinates of the vector AB. The points that join the vectors are shown in Fig. 5.

Further, the "R" and "L" in the subscripts always means "Right" and the "Left" respectively. For example,  $x_{RB}$  indicates the x coordinate of Right B (point B on the right side).

- $r_{RBD} := 5.25$                $r_{RBF} := 3.125$                $r_{RFC} := 23.25$
- $r_{LBD} := 5.25$                $r_{LBF} := 3.125$                $r_{LFC} := 23.25$

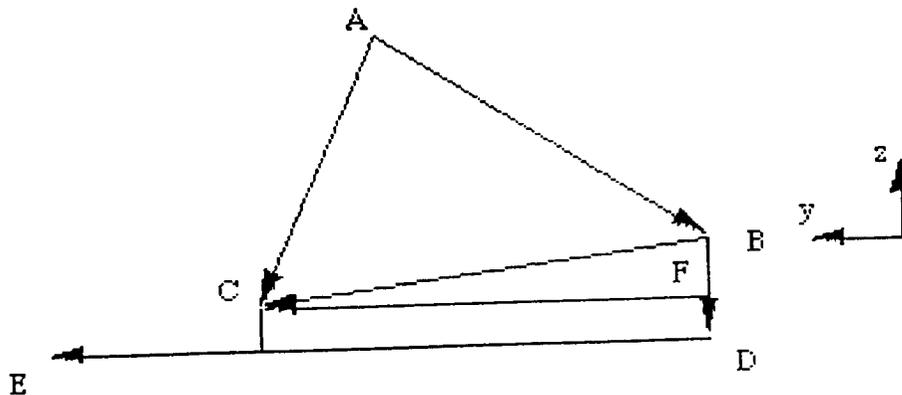


Figure 5

$$r_{RBC} := \sqrt{r_{RBF}^2 + r_{RFC}^2}$$

$$r_{RBC} = 23.4591$$

$$r_{LBC} := \sqrt{r_{LBF}^2 + r_{LFC}^2}$$

$$r_{LBC} = 23.4591$$

Coordinates of Right B

$$\begin{pmatrix} x_{RB} \\ y_{RB} \\ z_{RB} \end{pmatrix} := \begin{pmatrix} 100 \\ 0 \\ 0 \end{pmatrix}$$

Coordinates of Left B

$$\begin{pmatrix} x_{LB} \\ y_{LB} \\ z_{LB} \end{pmatrix} := \begin{pmatrix} 80 \\ 0 \\ 0 \end{pmatrix}$$

Components and magnitude of vectors AB

$$y_{RAB} := -17 \quad z_{RAB} := -11$$

$$y_{LAB} := -17 \quad z_{LAB} := -11$$

$$r_{RAB} := \sqrt{y_{RAB}^2 + z_{RAB}^2}$$

$$r_{RAB} = 20.2485$$

$$r_{LAB} := \sqrt{y_{LAB}^2 + z_{LAB}^2}$$

$$r_{LAB} = 20.2485$$

Nominal lengths of AC and DE

$$r_{RACnom} := 14.75 \quad r_{RDEnom} := 34.5$$

$$r_{LACnom} := 14.75 \quad r_{LDEnom} := 34.5$$

Increments in AC nominal and DE nominal (right and left)

$$\Delta_{RAC} := 0$$

$$\Delta_{LAC} := 0$$

$$\Delta_{RDE} := 0$$

$$\Delta_{LDE} := 0$$

$$r_{RAC} := r_{RACnom} + \Delta_{RAC}$$

$$r_{RAC} = 14.75$$

$$r_{LAC} := r_{LACnom} + \Delta_{LAC}$$

$$r_{LAC} = 14.75$$

$$r_{RDE} := r_{RDEnom} + \Delta_{RDE}$$

$$r_{RDE} = 34.5$$

$$r_{LDE} := r_{LDEnom} + \Delta_{LDE}$$

$$r_{LDE} = 34.5$$

The sets of points A,B and C on the right and left sides of the RIEE mechanism will each form a triangle. Here we derive the equations for finding the angle of the general (common to left and right) vector BC with known coordinates of points A and B, and the length AC in the y-z plane. Recalling Figure 5, we see:

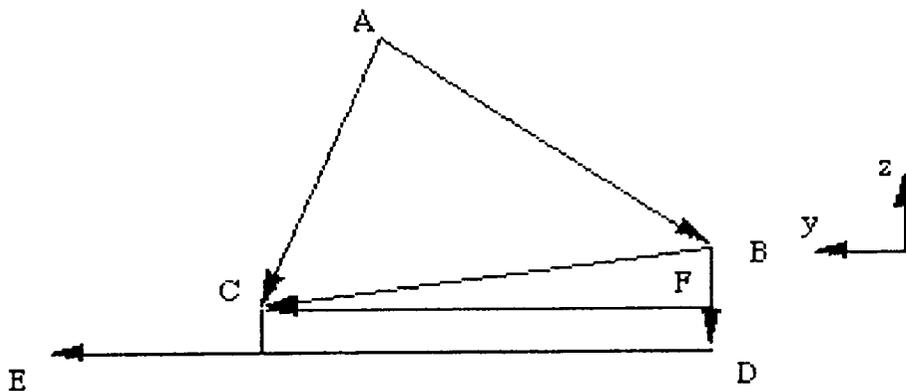


Figure 5

$$\text{Vector AC} = \text{Vector AB} + \text{Vector BC}$$

$$y_{AC} = y_{AB} + r_{BC} \cos(\theta_{BC}) \quad \text{and} \quad z_{AC} = z_{AB} + r_{BC} \sin(\theta_{BC})$$

Squaring and adding the above equations and simplifying, we have:

$$r_{AC}^2 = r_{AB}^2 + r_{BC}^2 + 2 \cdot y_{AB} \cdot r_{BC} \cos(\theta_{BC}) + 2 \cdot z_{AB} \cdot r_{BC} \sin(\theta_{BC})$$

Dividing by  $2r_{BC}$  and further rearranging, results in:

$$y_{AB} \cdot \cos(\theta_{BC}) + z_{AB} \cdot \sin(\theta_{BC}) + \frac{r_{AB}^2 + r_{BC}^2 - r_{AC}^2}{2 \cdot r_{BC}} = 0$$

$$\text{Let } c_{BC} = \frac{r_{AB}^2 + r_{BC}^2 - r_{AC}^2}{2 \cdot r_{BC}}$$

Hence  $y_{AB} \cdot \cos(\theta_{BC}) + z_{AB} \cdot \sin(\theta_{BC}) + c_{BC} = 0$

The above equation is of the form:

$$a \cdot \cos(\theta) + b \cdot \sin(\theta) + c = 0$$

Expanding the above equation using half angles, we have:

$$a \cdot \left( \cos\left(\frac{\theta}{2}\right)^2 - \sin\left(\frac{\theta}{2}\right)^2 \right) + 2 \cdot b \cdot \sin\left(\frac{\theta}{2}\right) \cdot \cos\left(\frac{\theta}{2}\right) + c \cdot \left( \cos\left(\frac{\theta}{2}\right)^2 + \sin\left(\frac{\theta}{2}\right)^2 \right) = 0$$

Dividing the above by  $\cos\left(\frac{\theta}{2}\right)^2$  and letting  $\tan\left(\frac{\theta}{2}\right) = t$  we have

$$a \cdot (1 - t^2) + 2 \cdot b \cdot t + c \cdot (1 + t^2) = 0 \quad \text{or} \quad (-a + c) \cdot t^2 + 2 \cdot b \cdot t + (a + c) = 0$$

$$\text{or} \quad (a - c) \cdot t^2 - 2 \cdot b \cdot t - (a + c) = 0$$

The above is a quadratic equation in "t" and has two solutions. The solution that is valid according to the given geometry is

$$t = \frac{2 \cdot b + \sqrt{4 \cdot b^2 + 4 \cdot (a^2 - c^2)}}{2 \cdot (a - c)} \quad \text{or} \quad t = \frac{b + \sqrt{a^2 + b^2 - c^2}}{(a - c)}$$

Therefore  $\theta = 2 \cdot \text{atan}(t)$

The trigonometric equations to solve for the angle of vector BC in the triangles formed by points A,B and C on the right and left sides of the RIEE are:

$$y_{RAB} \cdot \cos(\theta_{RBC}) + z_{RAB} \cdot \sin(\theta_{RBC}) + c_{RBC} = 0 \quad \text{and}$$

$$y_{LAB} \cdot \cos(\theta_{LBC}) + z_{LAB} \cdot \sin(\theta_{LBC}) + c_{LBC} = 0$$

Where

$$c_{RBC} := \frac{r_{RAB}^2 + r_{RBC}^2 - r_{RAC}^2}{2 \cdot r_{RBC}} \quad c_{RBC} = 15.8311$$

$$c_{LBC} := \frac{r_{LAB}^2 + r_{LBC}^2 - r_{LAC}^2}{2 \cdot r_{LBC}} \quad c_{LBC} = 15.8311$$

$$\theta_{RBC} := 2 \cdot \operatorname{atan} \left( \frac{z_{RAB} + \sqrt{r_{RAB}^2 - c_{RBC}^2}}{y_{RAB} - c_{RBC}} \right)$$

$$\theta_{RBC} = -5.6652 \cdot \text{deg}$$

$$\theta_{LBC} := 2 \cdot \operatorname{atan} \left( \frac{z_{LAB} + \sqrt{r_{LAB}^2 - c_{LBC}^2}}{y_{LAB} - c_{LBC}} \right)$$

$$\theta_{LBC} = -5.6652 \cdot \text{deg}$$

The points B,C and F on the left and right sides form a right triangle and vector FC is parallel to vector DE, therefore, the following relationships are derived to compute the angles of DEs (left and right) from the positive y axis in the y-z plane.

$$\lambda_R := \operatorname{atan} \left( \frac{r_{RBF}}{r_{RFC}} \right)$$

$$\lambda_R = 7.6552 \cdot \text{deg}$$

$$\lambda_L := \operatorname{atan} \left( \frac{r_{LBF}}{r_{LFC}} \right)$$

$$\lambda_L = 7.6552 \cdot \text{deg}$$

$$\theta_{RDE} := \theta_{RBC} + \lambda_R$$

$$\theta_{RDE} = 1.99 \cdot \text{deg}$$

$$\theta_{LDE} := \theta_{LBC} + \lambda_L$$

$$\theta_{LDE} = 1.99 \cdot \text{deg}$$

Vector RE = Vector RB + Vector RBD + Vector RDE

$$\theta_{RBD} = \left( \theta_{RDE} - \frac{\pi}{2} \right)$$

Therefore, the coordinates of point RE are:

$$\begin{pmatrix} x_{RE} \\ y_{RE} \\ z_{RE} \end{pmatrix} := \begin{pmatrix} x_{RB} \\ y_{RB} + r_{RBD} \cdot \cos \left( \theta_{RDE} - \frac{\pi}{2} \right) + r_{RDE} \cdot \cos \left( \theta_{RDE} \right) \\ z_{RB} + r_{RBD} \cdot \sin \left( \theta_{RDE} - \frac{\pi}{2} \right) + r_{RDE} \cdot \sin \left( \theta_{RDE} \right) \end{pmatrix} \quad \begin{pmatrix} x_{RE} \\ y_{RE} \\ z_{RE} \end{pmatrix} = \begin{pmatrix} 100 \\ 34.6615 \\ -4.0488 \end{pmatrix}$$

Vector LE = Vector LB + Vector LBD + Vector LDE

$$\theta_{LBD} = \left( \theta_{LDE} - \frac{\pi}{2} \right)$$

Therefore, the coordinates of the point LE are:

$$\begin{pmatrix} x_{LE} \\ y_{LE} \\ z_{LE} \end{pmatrix} := \begin{pmatrix} x_{LB} \\ y_{LB} + r_{LBD} \cdot \cos\left(\theta_{LDE} - \frac{\pi}{2}\right) + r_{LDE} \cdot \cos(\theta_{LDE}) \\ z_{LB} + r_{LBD} \cdot \sin\left(\theta_{LDE} - \frac{\pi}{2}\right) + r_{LDE} \cdot \sin(\theta_{LDE}) \end{pmatrix} \quad \begin{pmatrix} x_{LE} \\ y_{LE} \\ z_{LE} \end{pmatrix} = \begin{pmatrix} 80 \\ 34.6615 \\ -4.0488 \end{pmatrix}$$

Components of the vector from Left end LE to right end RE are:

$$\begin{pmatrix} x_{LRE} \\ y_{LRE} \\ z_{LRE} \end{pmatrix} := \begin{pmatrix} x_{RE} \\ y_{RE} \\ z_{RE} \end{pmatrix} - \begin{pmatrix} x_{LE} \\ y_{LE} \\ z_{LE} \end{pmatrix} \quad \begin{pmatrix} x_{LRE} \\ y_{LRE} \\ z_{LRE} \end{pmatrix} = \begin{pmatrix} 20 \\ 0 \\ 0 \end{pmatrix}$$

$$r_{LRE} := \sqrt{x_{LRE}^2 + y_{LRE}^2 + z_{LRE}^2} \quad r_{LRE} = 20$$

The Components of Unit vector U1 along the line from left end LE to right end RE

$$\begin{pmatrix} x_{U1} \\ y_{U1} \\ z_{U1} \end{pmatrix} := \begin{pmatrix} x_{LRE} \\ y_{LRE} \\ z_{LRE} \end{pmatrix} \cdot \frac{1}{r_{LRE}} \quad \begin{pmatrix} x_{U1} \\ y_{U1} \\ z_{U1} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Point CE is foot of the perpendicular from the top joint P on Rack Inserting Frame, to the line joining LE and RE. From the geometry it is m units from RE along the negative direction of the unit vector U1.

The Coordinates of the point CE are:

$$\begin{pmatrix} x_{CE} \\ y_{CE} \\ z_{CE} \end{pmatrix} := \begin{pmatrix} x_{RE} \\ y_{RE} \\ z_{RE} \end{pmatrix} - m \cdot \begin{pmatrix} x_{U1} \\ y_{U1} \\ z_{U1} \end{pmatrix} \quad \begin{pmatrix} x_{CE} \\ y_{CE} \\ z_{CE} \end{pmatrix} = \begin{pmatrix} 90 \\ 34.6615 \\ -4.0488 \end{pmatrix}$$

Our intention now is to establish a Coordinate system attached to the Rack Interface Plate with Unit vectors  $U_1, V_1$  &  $W_1$  along its  $x, y$  &  $z$  axes and Point CE as the origin..  $U_1$  is already defined above.  $V_1$  will be defined along the line from CE to P.

Our current objective is to determine the coordinates of the point P. We will find the coordinates by writing the equations defining its distance from three points in the RIEE mechanism. These three points are the reference point G, the right end point RE, and the point CE as defined above, and shown below.

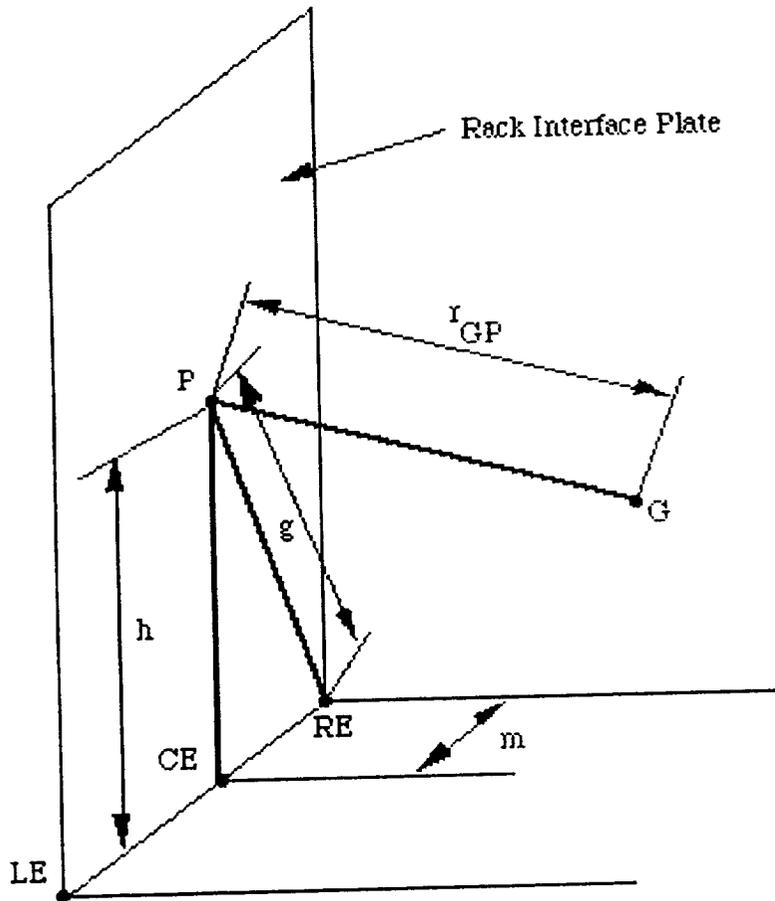


Figure 6

The equations are:

$$(x_P - x_G)^2 + (y_P - y_G)^2 + (z_P - z_G)^2 = r_{GP}^2 \quad [7]$$

$$(x_P - x_{CE})^2 + (y_P - y_{CE})^2 + (z_P - z_{CE})^2 = h^2 \quad [8]$$

$$(x_P - x_{RE})^2 + (y_P - y_{RE})^2 + (z_P - z_{RE})^2 = g^2 \quad [9]$$

Subtracting equations [8] and [9] from equation [7], we get the following equations:

$$a_1 \cdot x_P + b_1 \cdot y_P + c_1 \cdot z_P + G_2 - x_{CE}^2 - y_{CE}^2 - z_{CE}^2 = r_{GP}^2 - h^2 \quad [10]$$

$$a_2 \cdot x_P + b_2 \cdot y_P + c_2 \cdot z_P + G_2 - x_{RE}^2 - y_{RE}^2 - z_{RE}^2 = r_{GP}^2 - g^2 \quad [11]$$

where

$$a_1 := 2 \cdot (x_{CE} - x_G)$$

$$a_1 = 0$$

$$a_2 := 2 \cdot (x_{RE} - x_G)$$

$$a_2 = 20$$

$$b_1 := 2 \cdot (y_{CE} - y_G)$$

$$b_1 = 69.323$$

$$b_2 := 2 \cdot (y_{RE} - y_G)$$

$$b_2 = 69.323$$

$$c_1 := 2 \cdot (z_{CE} - z_G)$$

$$c_1 = -8.0977$$

$$c_2 := 2 \cdot (z_{RE} - z_G)$$

$$c_2 = -8.0977$$

$$G_2 := x_G^2 + y_G^2 + z_G^2$$

$$G_2 = 8.1 \cdot 10^3$$

Equations [10] and [11] can be rearranged as:

$$a_1 \cdot x_P + c_1 \cdot z_P = -b_1 \cdot y_P + k_1 \quad [12]$$

$$a_2 \cdot x_P + c_2 \cdot z_P = -b_2 \cdot y_P + k_2 \quad [13]$$

where

$$k_1 := r_{GP}^2 - h^2 + x_{CE}^2 + y_{CE}^2 + z_{CE}^2 - G_2$$

$$k_1 = 542.8125$$

$$k_2 := r_{GP}^2 - g^2 + x_{RE}^2 + y_{RE}^2 + z_{RE}^2 - G_2$$

$$k_2 = 2.3428 \cdot 10^3$$

Equations [12] and [13] can be arranged in the following matrix form:

$$\begin{pmatrix} a_1 & c_1 \\ a_2 & c_2 \end{pmatrix} \cdot \begin{pmatrix} x_P \\ z_P \end{pmatrix} = \begin{pmatrix} -b_1 \cdot y_P + k_1 \\ -b_2 \cdot y_P + k_2 \end{pmatrix} \quad [14]$$

$$D_{12} := \left| \begin{pmatrix} a_1 & c_1 \\ a_2 & c_2 \end{pmatrix} \right|$$

$$D_{12} = 161.9539$$

Using Cramer's Rule, we can solve for  $x_P$  and  $z_P$

$$x_P = \frac{\begin{pmatrix} -b_1 \cdot y_P + k_1 & c_1 \\ -b_2 \cdot y_P + k_2 & c_2 \end{pmatrix}}{D_{12}}$$

$$z_P = \frac{\begin{pmatrix} a_1 & -b_1 \cdot y_P + k_1 \\ a_2 & -b_2 \cdot y_P + k_2 \end{pmatrix}}{D_{12}}$$

After simplifying, we have

$$x_P = b_3 \cdot y_P + k_3 \quad [15]$$

$$z_P = b_4 \cdot y_P + k_4 \quad [16]$$

where

$$b_3 := \frac{-b_1 \cdot c_2 + b_2 \cdot c_1}{D_{12}}$$

$$b_4 := \frac{-a_1 \cdot b_2 + a_2 \cdot b_1}{D_{12}}$$

$$k_3 := \frac{k_1 \cdot c_2 - k_2 \cdot c_1}{D_{12}}$$

$$k_4 := \frac{a_1 \cdot k_2 - a_2 \cdot k_1}{D_{12}}$$

$$b_3 = 0$$

$$b_4 = 8.5608$$

$$k_3 = 90$$

$$k_4 = -67.033$$

Recalling equation [7] and substituting for  $x_P$  and  $z_P$  from equations [15] and [16], we have:

$$(x_P - x_G)^2 + (y_P - y_G)^2 + (z_P - z_G)^2 = r_{GP}^2 \quad [17]$$

$$(b_3 \cdot y_P + k_3 - x_G)^2 + (y_P - y_G)^2 + (b_4 \cdot y_P + k_4 - z_G)^2 = r_{GP}^2 \quad [18]$$

Expanding equation [17] and rearranging, we get:

$$A \cdot y_P^2 + B \cdot y_P + C = 0$$

where

$$A := b_3^2 + 1 + b_4^2$$

$$A = 74.2878$$

$$B := 2 \cdot [b_3 \cdot (k_3 - x_G) - y_G + b_4 \cdot (k_4 - z_G)]$$

$$B = -1.1477 \cdot 10^3$$

$$C := (k_3 - x_G)^2 + y_G^2 + (k_4 - z_G)^2 - r_{GP}^2$$

$$C = 4.2684 \cdot 10^3$$

Equation [18] is a quadratic in "y<sub>P</sub>" and has 2 solutions. The solution that is relevant to the geometry of the RIEE mechanism is:

$$y_P := \left( \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \right)$$

$$y_P = 9.2129$$

Substituting  $y_P$  back into equations [15] and [16], the coordinates of P are given by:

$$\begin{pmatrix} x_P \\ y_P \\ z_P \end{pmatrix} := \begin{pmatrix} b_3 \cdot y_P + k_3 \\ y_P \\ b_4 \cdot y_P + k_4 \end{pmatrix} \quad \begin{pmatrix} x_P \\ y_P \\ z_P \end{pmatrix} = \begin{pmatrix} 90 \\ 9.2129 \\ 11.8373 \end{pmatrix}$$

The z-axis given by unit vector  $W_1$  is along the line joining CE and P. Its components are given by:

$$\begin{pmatrix} x_{W1} \\ y_{W1} \\ z_{W1} \end{pmatrix} := \begin{pmatrix} x_P - x_{CE} \\ y_P - y_{CE} \\ z_P - z_{CE} \end{pmatrix} \cdot \frac{1}{h} \quad \begin{pmatrix} x_{W1} \\ y_{W1} \\ z_{W1} \end{pmatrix} = \begin{pmatrix} 0 \\ -0.8483 \\ 0.5295 \end{pmatrix}$$

The y-axis given by unit vector  $V_1$  is given by the cross product of vector  $W_1$  and vector  $U_1$ . Its components are given by:

$$\begin{pmatrix} x_{V1} \\ y_{V1} \\ z_{V1} \end{pmatrix} := \begin{pmatrix} x_{W1} \\ y_{W1} \\ z_{W1} \end{pmatrix} \times \begin{pmatrix} x_{U1} \\ y_{U1} \\ z_{U1} \end{pmatrix} \quad \begin{pmatrix} x_{V1} \\ y_{V1} \\ z_{V1} \end{pmatrix} = \begin{pmatrix} 0 \\ 0.5295 \\ 0.8483 \end{pmatrix}$$

Thus we have completely specified the coordinate system located at CE. We now find the relationship between this coordinate system and a coordinate system located on the camera frame.

The x-axis of the camera frame defined by  $U_2$  is taken same as the  $U_1$ , and the z-axis is defined by the unit vector  $W_2$ , which is at an angle  $\beta$  in the clockwise direction from the unit vector  $W_1$  about the x-axis.

The origin of the camera frame is situated at a distance of  $d_{21}$  along the negative direction of the unit vector  $W_2$  from CE. Therefore:

$$\beta := \text{atan} \left( \frac{y_{CE} - y_P}{z_P - z_{CE}} \right) \quad \beta = 58.0257 \cdot \text{deg}$$

$$h_C := 12$$

The components of the unit vectors of the camera frame (2) as seen from rack insertion frame (1) are given by:

$$\begin{pmatrix} x_{U21} \\ y_{U21} \\ z_{U21} \end{pmatrix} := \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} x_{V21} \\ y_{V21} \\ z_{V21} \end{pmatrix} := \begin{pmatrix} 0 \\ \cos(\beta) \\ -\sin(\beta) \end{pmatrix} \quad \begin{pmatrix} x_{W21} \\ y_{W21} \\ z_{W21} \end{pmatrix} := \begin{pmatrix} 0 \\ \sin(\beta) \\ \cos(\beta) \end{pmatrix}$$

The coordinates of the origin of the camera frame  $O_2$  as seen from Rack Interface frame (1) are given by:

$$\begin{pmatrix} x_{O21} \\ y_{O21} \\ z_{O21} \end{pmatrix} := \begin{pmatrix} 0 \\ -h_C \cdot \sin(\beta) \\ -h_C \cdot \cos(\beta) \end{pmatrix}$$

The transformation of coordinates of the right and left targets from the camera frame to the rack insertion frame is given by the following matrix equations:

Note that in the  $4 \times 4$  transformation matrix on the right hand side:  
 the first column contains the components of the unit vector  $U_2$  as seen in frame 1,  
 the second column contains the components of the unit vector  $V_2$  as seen in frame 1,  
 the third column contains the components of the unit vector  $W_2$  as seen in frame 1, and  
 the fourth column contains the coordinates of the origin of the camera frame as seen in frame 1.

$$\begin{bmatrix} x_{RT1} \\ y_{RT1} \\ z_{RT1} \\ S_{21} \end{bmatrix} := \begin{pmatrix} x_{U21} & x_{V21} & x_{W21} & x_{O21} \\ y_{U21} & y_{V21} & y_{W21} & y_{O21} \\ z_{U21} & z_{V21} & z_{W21} & z_{O21} \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_{RT2} \\ y_{RT2} \\ z_{RT2} \\ 1 \end{pmatrix} \quad \begin{bmatrix} x_{RT1} \\ y_{RT1} \\ z_{RT1} \\ S_{21} \end{bmatrix} = \begin{bmatrix} 8 \\ 0.73 \\ -21.9422 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_{LT1} \\ y_{LT1} \\ z_{LT1} \\ S_{21} \end{bmatrix} := \begin{pmatrix} x_{U21} & x_{V21} & x_{W21} & x_{O21} \\ y_{U21} & y_{V21} & y_{W21} & y_{O21} \\ z_{U21} & z_{V21} & z_{W21} & z_{O21} \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_{LT2} \\ y_{LT2} \\ z_{LT2} \\ 1 \end{pmatrix} \quad \begin{bmatrix} x_{LT1} \\ y_{LT1} \\ z_{LT1} \\ S_{21} \end{bmatrix} = \begin{bmatrix} -12 \\ -0.9662 \\ -23.0018 \\ 1 \end{bmatrix}$$

## 5. SUPPORT COORDINATES IN BOOM COORDINATE SYSTEM:

$h_T := 12$  Height of the supports above the targets as in Figure 3. Support Coordinates are:

$$\begin{bmatrix} x_{RS} \\ y_{RS} \\ z_{RS} \\ S_1 \end{bmatrix} := \begin{pmatrix} x_{U1} & x_{V1} & x_{W1} & x_{CE} \\ y_{U1} & y_{V1} & y_{W1} & y_{CE} \\ z_{U1} & z_{V1} & z_{W1} & z_{CE} + h_T \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_{RT1} \\ y_{RT1} \\ z_{RT1} \\ 1 \end{pmatrix} \quad \begin{bmatrix} x_{RS} \\ y_{RS} \\ z_{RS} \\ S_1 \end{bmatrix} = \begin{bmatrix} 98 \\ 53.6613 \\ -3.0488 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_{LS} \\ y_{LS} \\ z_{LS} \\ S_1 \end{bmatrix} := \begin{pmatrix} x_{U1} & x_{V1} & x_{W1} & x_{CE} \\ y_{U1} & y_{V1} & y_{W1} & y_{CE} \\ z_{U1} & z_{V1} & z_{W1} & z_{CE} + h_T \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_{LT1} \\ y_{LT1} \\ z_{LT1} \\ 1 \end{pmatrix} \quad \begin{bmatrix} x_{LS} \\ y_{LS} \\ z_{LS} \\ S_1 \end{bmatrix} = \begin{bmatrix} 78 \\ 53.6619 \\ -5.0488 \\ 1 \end{bmatrix}$$

## 6. DISPLACEMENTS IN THE BOOM ROBOT AND THE RIEE MECHANISM.

The center point CS on the line joining the left and right lower support points is given by

$$\begin{pmatrix} x_{CS} \\ y_{CS} \\ z_{CS} \end{pmatrix} := \begin{pmatrix} x_{RS} + x_{LS} \\ y_{RS} + y_{LS} \\ z_{RS} + z_{LS} \end{pmatrix} \cdot \frac{1}{2} \qquad \begin{pmatrix} x_{CS} \\ y_{CS} \\ z_{CS} \end{pmatrix} = \begin{pmatrix} 88 \\ 53.6616 \\ -4.0488 \end{pmatrix}$$

Having found the coordinates of the supports and the midpoint, we move the boom such that the point CE gets the displacement in the x and z direction of the boom keeping the y-distance from the targets undisturbed. This displacement is shown exaggerated in Figure 7 below.

$$\begin{pmatrix} x_{CES} \\ y_{CES} \\ z_{CES} \end{pmatrix} := \begin{pmatrix} x_{CS} \\ y_{CS} \\ z_{CS} \end{pmatrix} - \begin{pmatrix} x_{CE} \\ y_{CE} \\ z_{CE} \end{pmatrix} \qquad \begin{pmatrix} x_{CES} \\ y_{CES} \\ z_{CES} \end{pmatrix} = \begin{pmatrix} -2 \\ 19.0002 \\ -4.4409 \cdot 10^{-15} \end{pmatrix}$$

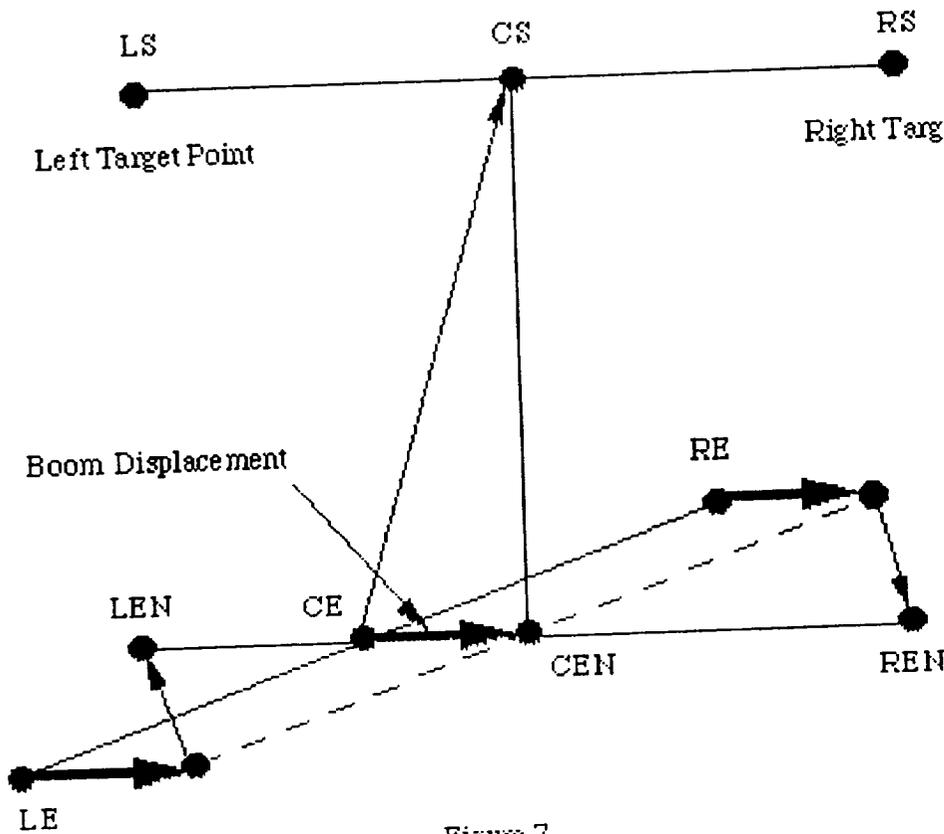


Figure 7

$$\begin{pmatrix} x_{Boom} \\ y_{Boom} \\ z_{Boom} \end{pmatrix} := \begin{pmatrix} x_{CES} \\ 0 \\ z_{CES} \end{pmatrix} \qquad \begin{pmatrix} x_{Boom} \\ y_{Boom} \\ z_{Boom} \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ -4.4409 \cdot 10^{-15} \end{pmatrix}$$

The new coordinates of CE after the boom displacement

$$\begin{pmatrix} x_{CEN} \\ y_{CEN} \\ z_{CEN} \end{pmatrix} := \begin{pmatrix} x_{CE} \\ y_{CE} \\ z_{CE} \end{pmatrix} + \begin{pmatrix} x_{Boom} \\ y_{Boom} \\ z_{Boom} \end{pmatrix}$$

$$\begin{pmatrix} x_{CEN} \\ y_{CEN} \\ z_{CEN} \end{pmatrix} = \begin{pmatrix} 88 \\ 34.6615 \\ -4.0488 \end{pmatrix}$$

The new coordinates of Right and Left "B" are:

$$\begin{pmatrix} x_{RBN} \\ y_{RBN} \\ z_{RBN} \end{pmatrix} := \begin{pmatrix} x_{RB} \\ y_{RB} \\ z_{RB} \end{pmatrix} + \begin{pmatrix} x_{Boom} \\ y_{Boom} \\ z_{Boom} \end{pmatrix}$$

$$\begin{pmatrix} x_{RBN} \\ y_{RBN} \\ z_{RBN} \end{pmatrix} = \begin{pmatrix} 98 \\ 0 \\ -4.4409 \cdot 10^{-15} \end{pmatrix}$$

$$\begin{pmatrix} x_{LBN} \\ y_{LBN} \\ z_{LBN} \end{pmatrix} := \begin{pmatrix} x_{LB} \\ y_{LB} \\ z_{LB} \end{pmatrix} + \begin{pmatrix} x_{Boom} \\ y_{Boom} \\ z_{Boom} \end{pmatrix}$$

$$\begin{pmatrix} x_{LBN} \\ y_{LBN} \\ z_{LBN} \end{pmatrix} = \begin{pmatrix} 78 \\ 0 \\ -4.4409 \cdot 10^{-15} \end{pmatrix}$$

The new coordinates of Right "E" and components of new right BE derived from Figure 5 are:

$$\begin{pmatrix} y_{REN} \\ z_{REN} \end{pmatrix} := \begin{pmatrix} y_{CEN} \\ z_{RS} \end{pmatrix}$$

$$\begin{pmatrix} y_{REN} \\ z_{REN} \end{pmatrix} = \begin{pmatrix} 34.6615 \\ -3.0488 \end{pmatrix}$$

$$\begin{pmatrix} y_{RBE} \\ z_{RBE} \end{pmatrix} := \begin{pmatrix} y_{RE} - y_{RB} \\ z_{RE} - z_{RB} \end{pmatrix}$$

$$\begin{pmatrix} y_{RBE} \\ z_{RBE} \end{pmatrix} = \begin{pmatrix} 34.6615 \\ -4.0488 \end{pmatrix}$$

$$\begin{pmatrix} y_{RBEN} \\ z_{RBEN} \end{pmatrix} := \begin{pmatrix} y_{REN} \\ z_{REN} \end{pmatrix} - \begin{pmatrix} y_{RBN} \\ z_{RBN} \end{pmatrix}$$

$$\begin{pmatrix} y_{RBEN} \\ z_{RBEN} \end{pmatrix} = \begin{pmatrix} 34.6615 \\ -3.0488 \end{pmatrix}$$

The new coordinates of Left "E" and components of left BE derived from Figure 5 are:

$$\begin{pmatrix} y_{LEN} \\ z_{LEN} \end{pmatrix} := \begin{pmatrix} y_{CEN} \\ z_{LS} \end{pmatrix}$$

$$\begin{pmatrix} y_{LEN} \\ z_{LEN} \end{pmatrix} = \begin{pmatrix} 34.6615 \\ -5.0488 \end{pmatrix}$$

$$\begin{pmatrix} y_{LBE} \\ z_{LBE} \end{pmatrix} := \begin{pmatrix} y_{LE} - y_{LB} \\ z_{LE} - z_{LB} \end{pmatrix}$$

$$\begin{pmatrix} y_{LBE} \\ z_{LBE} \end{pmatrix} = \begin{pmatrix} 34.6615 \\ -4.0488 \end{pmatrix}$$

$$\begin{pmatrix} y_{LBEN} \\ z_{LBEN} \end{pmatrix} := \begin{pmatrix} y_{LEN} \\ z_{LEN} \end{pmatrix} - \begin{pmatrix} y_{LBN} \\ z_{LBN} \end{pmatrix}$$

$$\begin{pmatrix} y_{LBEN} \\ z_{LBEN} \end{pmatrix} = \begin{pmatrix} 34.6615 \\ -5.0488 \end{pmatrix}$$

The lengths of the new BE and the new DE on the right and left sides are:

$$r_{RBEN} := \sqrt{y_{RBEN}^2 + z_{RBEN}^2}$$

$$r_{RBEN} = 34.7953$$

$$r_{LBEN} := \sqrt{y_{LBEN}^2 + z_{LBEN}^2}$$

$$r_{LBEN} = 35.0273$$

$$r_{RDEN} := \sqrt{r_{RBEN}^2 - r_{RBD}^2}$$

$$r_{RDEN} = 34.397$$

$$r_{LDEN} := \sqrt{r_{LBEN}^2 - r_{LBD}^2}$$

$$r_{LDEN} = 34.6316$$

The new angles of the vector DE on the right and left side are:

$$\theta_{RDEN} := 2 \cdot \operatorname{atan} \left( \frac{r_{RBD} + z_{RBEN}}{r_{RDEN} + y_{RBEN}} \right)$$

$$\theta_{RDEN} = 3.6512 \cdot \text{deg}$$

$$\theta_{LDEN} := 2 \cdot \operatorname{atan} \left( \frac{r_{LBD} + z_{LBEN}}{r_{LDEN} + y_{LBEN}} \right)$$

$$\theta_{LDEN} = 0.3326 \cdot \text{deg}$$

The angles for the new BC on the right and left are:

$$\theta_{RBCN} := \theta_{RDEN} - \lambda_R$$

$$\theta_{RBCN} = -4.0039 \cdot \text{deg}$$

$$\theta_{LBCN} := \theta_{LDEN} - \lambda_L$$

$$\theta_{LBCN} = -7.3225 \cdot \text{deg}$$

The new BC components are:

$$\begin{pmatrix} y_{RBCN} \\ z_{RBCN} \end{pmatrix} := \begin{pmatrix} r_{RBC} \cdot \cos(\theta_{RBCN}) \\ r_{RBC} \cdot \sin(\theta_{RBCN}) \end{pmatrix}$$

$$\begin{pmatrix} y_{RBCN} \\ z_{RBCN} \end{pmatrix} = \begin{pmatrix} 23.4018 \\ -1.638 \end{pmatrix}$$

$$\begin{pmatrix} y_{LBCN} \\ z_{LBCN} \end{pmatrix} := \begin{pmatrix} r_{LBC} \cdot \cos(\theta_{LBCN}) \\ r_{LBC} \cdot \sin(\theta_{LBCN}) \end{pmatrix}$$

$$\begin{pmatrix} y_{LBCN} \\ z_{LBCN} \end{pmatrix} = \begin{pmatrix} 23.2678 \\ -2.99 \end{pmatrix}$$

The new AC components are:

$$\begin{pmatrix} y_{RACN} \\ z_{RACN} \end{pmatrix} := \begin{pmatrix} y_{RAB} \\ z_{RAB} \end{pmatrix} + \begin{pmatrix} y_{RBCN} \\ z_{RBCN} \end{pmatrix}$$

$$\begin{pmatrix} y_{RACN} \\ z_{RACN} \end{pmatrix} = \begin{pmatrix} 6.4018 \\ -12.638 \end{pmatrix}$$

$$\begin{pmatrix} y_{LACN} \\ z_{LACN} \end{pmatrix} := \begin{pmatrix} y_{LAB} \\ z_{LAB} \end{pmatrix} + \begin{pmatrix} y_{LBCN} \\ z_{LBCN} \end{pmatrix}$$

$$\begin{pmatrix} y_{LACN} \\ z_{LACN} \end{pmatrix} = \begin{pmatrix} 6.2678 \\ -13.99 \end{pmatrix}$$

The new AC lengths are:

$$r_{RACN} := \sqrt{y_{RACN}^2 + z_{RACN}^2}$$

$$r_{RACN} = 14.167$$

$$r_{LACN} := \sqrt{y_{LACN}^2 + z_{LACN}^2}$$

$$r_{LACN} = 15.3298$$

The new increments are:

$$\Delta_{RDEN} := r_{RDEN} - r_{RDE}$$

$$\Delta_{RDEN} = -0.103$$

$$\Delta_{RACN} := r_{RACN} - r_{RAC}$$

$$\Delta_{RACN} = -0.583$$

$$\Delta_{LDEN} := r_{LDEN} - r_{LDE}$$

$$\Delta_{LDEN} = 0.1316$$

$$\Delta_{LACN} := r_{LACN} - r_{LAC}$$

$$\Delta_{LACN} = 0.5798$$

$$\eta := 0.1$$

The new rotations required are:

$$\Psi_{RDE} := \frac{\Delta_{RDEN}}{\eta} \cdot 2 \cdot \pi$$

$$\Psi_{RDE} = -370.8683 \cdot \text{deg}$$

$$\Psi_{RAC} := \frac{\Delta_{RACN}}{\eta} \cdot 2 \cdot \pi$$

$$\Psi_{RAC} = -2.0989 \cdot 10^3 \cdot \text{deg}$$

$$\Psi_{LDE} := \frac{\Delta_{LDEN}}{\eta} \cdot 2 \cdot \pi$$

$$\Psi_{LDE} = 473.7588 \cdot \text{deg}$$

$$\Psi_{LAC} := \frac{\Delta_{LACN}}{\eta} \cdot 2 \cdot \pi$$

$$\Psi_{LAC} = 2.0874 \cdot 10^3 \cdot \text{deg}$$

$$\begin{pmatrix} x_{LREN} \\ y_{LREN} \\ z_{LREN} \end{pmatrix} := \begin{pmatrix} x_{RBN} \\ y_{REN} \\ z_{REN} \end{pmatrix} - \begin{pmatrix} x_{LBN} \\ y_{LEN} \\ z_{LEN} \end{pmatrix}$$

$$\begin{pmatrix} x_{LREN} \\ y_{LREN} \\ z_{LREN} \end{pmatrix} = \begin{pmatrix} 20 \\ 0 \\ 2 \end{pmatrix}$$

$$r_{LREN} := \sqrt{x_{LREN}^2 + y_{LREN}^2 + z_{LREN}^2}$$

$$r_{LREN} = 20.0998$$

The new unit vector joining the left and right "E"s is:

$$\begin{pmatrix} x_{U1N} \\ y_{U1N} \\ z_{U1N} \end{pmatrix} := \begin{pmatrix} x_{LREN} \\ y_{LREN} \\ z_{LREN} \end{pmatrix} \cdot \frac{1}{r_{LREN}}$$

$$\begin{pmatrix} x_{U1N} \\ y_{U1N} \\ z_{U1N} \end{pmatrix} = \begin{pmatrix} 0.995 \\ 0 \\ 0.0995 \end{pmatrix}$$

$$r_{LRS} := \sqrt{(x_{RS} - x_{LS})^2 + (y_{RS} - y_{LS})^2 + (z_{RS} - z_{LS})^2}$$

$$r_{LRS} = 20.0998$$

The unit vector joining the left and right lower supports is:

$$\begin{pmatrix} x_{US} \\ y_{US} \\ z_{US} \end{pmatrix} := \begin{pmatrix} x_{RS} - x_{LS} \\ y_{RS} - y_{LS} \\ z_{RS} - z_{LS} \end{pmatrix} \cdot \frac{1}{r_{LRS}}$$

$$\begin{pmatrix} x_{US} \\ y_{US} \\ z_{US} \end{pmatrix} = \begin{pmatrix} 0.995 \\ -3.0052 \cdot 10^{-5} \\ 0.0995 \end{pmatrix}$$

Because these two unit vectors are now the same, they are parallel.

Let the final coordinates of Right and Left "E" be

$$\begin{pmatrix} y_{REF} \\ z_{REF} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} y_{LEF} \\ z_{LEF} \end{pmatrix}$$

In the forgoing equations, the above are used in place of "new" coordinates for RE and LE

$$\begin{pmatrix} y_{REN} \\ z_{REN} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} y_{LEN} \\ z_{LEN} \end{pmatrix}$$

and calculations are repeated to obtain the final displacements for the wheels of the Right and Left BE and AC.

## 7. CONCLUSIONS AND RECOMMENDATIONS:

In this report the procedure to obtain the necessary displacements to align the RIEE mechanism for the final insertion has been accomplished with example calculations. The MathCad software has allowed us to integrate the Text, Figures, Derivations and Calculations, saving time and space for a clear understanding of the procedures and calculations.

It is recommended that an experiment be set up to obtain the images of prototype targets and process the images to get the input data required for this software. The results of the software can then be verified be verified against the actual locations of the prototype targets.

## 8. REFERENCES:

- A. Drawing set #82K03931 and associated drawings