Optical Model Analyses of Galactic Cosmic Ray Fragmentation in Hydrogen Targets

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Symbols

\(A\)  
nuclear mass number

\(\binom{A}{B}\)  
binomial coefficient

\(B(e)\)  
average slope parameter of nucleon-nucleon scattering amplitude, \(\text{fm}^2\)

\(b\)  
projectile impact parameter vector, \(\text{fm}\)

\(E\)  
energy, \(\text{GeV}\) or \(\text{MeV}\)

\(e\)  
two-nucleon kinetic energy in their center-of-mass frame, \(\text{GeV}\)

\(I(b)\)  
defined by equation (3)

\(I_p(b)\)  
defined by equation (7)

\(N\)  
total number of projectile nucleus neutrons

\(n\)  
number of abraded neutrons

\(P_{esc}\)  
probability that an abraded nucleon escapes without further interaction

\(T(b)\)  
probability for not removing single nucleon by abrasion

\(y\)  
two-nucleon relative position vector, \(\text{fm}\)

\(Z\)  
total number of projectile-nucleus protons

\(z\)  
number of abraded protons

\(\mathbf{z}_0\)  
position vector of projectile along beam direction, \(\text{fm}\)

\(\xi_T\)  
collection of constituent relative coordinates for target, \(\text{fm}\)

\(\rho\)  
nuclear single-particle density, \(\text{fm}^{-3}\)

\(\sigma\)  
cross section, \(\text{fm}^2\) or \(\text{mb}\)

\(\lambda\)  
mean-free path, \(\text{fm}\)

Subscripts:

abr  
abraded

exc  
prefragment excitation

FSI  
frictional spectator interaction

NN  
nucleon-nucleon

nuc  
nuclear

\(P\)  
projectile

PF  
prefragment

\(T\)  
target
Abstract

Quantum-mechanical optical model methods for calculating cross sections for the fragmentation of galactic cosmic ray nuclei by hydrogen targets are presented. The fragmentation cross sections are calculated with an abrasion-ablation collision formalism. Elemental and isotopic cross sections are estimated and compared with measured values for neon, sulfur, and calcium ions at incident energies between 400A MeV and 910A MeV. Good agreement between theory and experiment is obtained.

Introduction

The fragmentation of galactic cosmic ray (GCR) nuclei in hydrogen targets is an important physical process in several areas of space radiation physics research. In astrophysics, it is crucial to understanding cosmic ray propagation and source abundances (ref. 1) because interstellar hydrogen is the major type of material encountered by GCR nuclei traveling through the universe. In studies of spacecraft shielding for interplanetary missions (ref. 2), hydrogen has been found to be the most effective GCR shield material per unit mass. In addition, hydrogen is a major constituent of human tissue. Therefore, accurate cross sections are needed for properly estimating GCR radiation exposures to critical body organs (ref. 3).

Previously, cross-section predictions used in these studies have been obtained from semi-empirical formulations (refs. 4 to 7). The most commonly used formulation is the one by Silberberg and collaborators (ref. 5). The most accurate formulation appears to be a recent one by Webber and collaborators (ref. 6). None are based upon fundamental physics. All have numerous parameters that are adjusted as necessary to fit existing measurements.

The production of fragments in peripheral, relativistic heavy ion collisions has been the subject of numerous theoretical and experimental investigations for about 2 decades. Many of these investigations were summarized in reviews published during this period (refs. 7 to 10). Early attempts to explain fragmentation used statistical models (refs. 11 and 12). These were followed by a two-step abrasion-ablation model (ref. 13), which was based upon earlier work by Serber in high-energy, inelastic nuclear collisions (ref. 14).

The main shortcoming associated with the use of early abrasion-ablation models for nuclear fragmentation on hydrogen targets is the unrealistically large proton radius needed for the prefragment excitation energy estimate. This radius is dictated by the reliance on excess surface energy of the misshapen liquid drop as the only source of prefragment excitation.

This shortcoming in the model can be rectified by considering an abrasion-ablation frictional-spectator-interaction (FSI) model where the abrasion stage is described by a quantum-mechanical optical model formalism and the ablation stage is modeled with cascade-evaporation techniques. There is no excess surface area energy. Instead, the prefragment excitation energy is assumed to be provided by FSI contributions from the abraded nucleons. This fragmentation model is proposed in this report.

Abrasion-Ablation Models

In an abrasion-ablation model, the projectile nuclei, moving at relativistic speeds, collide with stationary target nuclei. In the abrasion step (particle knockout), those portions of the nuclear volumes that overlap are sheared away by the collision. The remaining projectile piece, called a prefragment, continues its trajectory with essentially its precollision velocity. Because of the dynamics of the abrasion process, the prefragment is highly excited and subsequently decays.
by the emission of gamma radiation or nuclear particles. This step is the ablation stage. The resultant isotope is the nuclear fragment whose cross section is measured. The abrasion step is often formulated with methods obtained from quantum scattering theory (refs. 15 and 16) or with classical geometry arguments (refs. 13 and 17). The ablation step is typically modeled with compound nucleus decay (refs. 13 and 18) or combined cascade-evaporation (ref. 19) methods. Other approaches based upon nuclear Weizsäcker-Williams methods (ref. 20) and nucleon-nucleon cascade plus statistical decay models (ref. 21) have also been proposed.

Although abrasion-ablation fragmentation models have been quite successful in predicting fragment production cross sections, their predictive accuracy is hampered by the need to estimate the (unknown) prefragment excitation energy. Various models have been developed for this purpose (refs. 13, 15, 18, and 22). The most widely used excitation energy formalism (ref. 13) treats the fragmenting nucleus as a misshapen liquid drop whose excitation is given by the excess surface energy resulting from the abrasion step. Although this method worked fairly well for nucleus-nucleus fragmentations, its use in nucleus-hydrogen collisions, among other difficulties, required an artificially large proton radius (ref. 13).

When it was recognized that additional excitation energy was required to improve the agreement between theory and experiment for nucleus-nucleus collisions, the concept of FSI energy was introduced (ref. 22). This concept is based upon the assumption that some abraded nucleons are scattered into rather than away from the prefragment, thereby depositing additional excitation energy. This concept significantly improved the agreement between theory and experiment.

Over the past 10 years, we have formulated an optical model abrasion-ablation-FSI description of fragmentation in relativistic nucleus-nucleus collisions that is used to predict fragment production cross sections (refs. 16 and 23 to 42) and momentum distributions of the emitted fragments (refs. 43 through 47). In the present work, this fragmentation model is modified to make it applicable to nucleus-nucleon collisions. As previously discussed, the main shortcoming associated with the use of early abrasion-ablation models for nuclear fragmentation on hydrogen targets is the unrealistically large proton radius needed for the prefragment excitation energy estimate. This radius is dictated by the reliance on excess surface energy of the misshapen liquid drop as the only source of prefragment excitation.

This shortcoming in the model can be rectified by considering the physics of the fragmentation process. For instance, a picture of overlapping nuclear volumes being sheared off may be reasonable for heavier nuclei colliding with each other, but it is not reasonable for a single nucleon striking another nucleus. Instead, a more reasonable physical picture involves individual collisions between the projectile constituents and the target proton. Some struck projectile nucleons exit the fragmenting nucleus without further interaction, and some interact one or more times with the remaining constituents before departing. The remaining nucleus (prefragment), in an excited state because of the energy deposited during the collision, then deexcites by particle- or gamma-emission processes. This picture is easily described by an abrasion-ablation-FSI model where the abrasion stage is described by a quantum-mechanical optical model formalism and the ablation stage is modeled with cascade-evaporation techniques. There is no excess surface area energy. Instead, the prefragment excitation energy is assumed to be provided by FSI contributions from the abraded nucleons. This fragmentation model is proposed in this report.

Theory

In the nucleus-nucleus optical potential formalism (ref. 29), the cross section for producing, by abrasion, a prefragment of charge \( Z_{PF} \) and mass \( A_{PF} \) is given by

\[
\sigma_{abr}(Z_{PF}, A_{PF}) = \left( \begin{array}{c}
N \\
n \end{array} \right) \left( \begin{array}{c}
Z \\
n \end{array} \right) \int d^2b [1 - T(b)]^{n+z} [T(b)]^{A_{PF}}
\]

(1)
where

\[ T(b) = \exp[-A_T \sigma_{NN}(e)I(b)] \]  

and

\[ I(b) = [2\pi B(e)]^{-3/2} \int dz_0 \int d^3 \xi_T \rho_T(\xi_T) \int d^3 y \rho_P(b + z_0 + y + \xi_T) \exp[-y^2/2B(e)] \]  

(3)

The nuclear number densities \( \rho_i (i = P \text{ or } T) \) are obtained from the appropriate charge densities by an unfolding procedure (ref. 16). The constituent-averaged nucleon-nucleon cross sections \( \sigma_{NN}(e) \) are given in reference 48. Values for the diffractive nucleon-nucleon scattering slope parameter \( B(e) \) are obtained from the parameterization in reference 49.

In equation (1) a hypergeometric charge dispersion model is chosen to describe the distribution of abraded nucleons. The model assumes that \( z \) out of \( Z \) projectile protons and \( n \) out of \( N \) projectile neutrons are abraded where

\[ N + Z = A_p \]  

(4)

\[ A_F = A_p - n - z \]  

(5)

and \( \binom{A}{B} \) denotes the usual binomial coefficient expression from probability theory.

For nuclear collisions with hydrogen (proton) targets, the appropriate target number density to use is given by the Dirac delta function

\[ \rho_T(\xi_T) = \delta(\xi_T) \]  

(6)

Inserting equation (6) into equation (3) yields

\[ I_p(b) = [2\pi B(e)]^{-3/2} \int dz_0 \int d^3 y \rho_P(b + z_0 + y) \exp[-y^2/2B(e)] \]  

(7)

With \( A_T = 1 \), equation (2) becomes

\[ T(b) = \exp[-\sigma_{NN}(e)I_p(b)] \]  

(8)

The nucleus-hydrogen abrasion cross sections are calculated with equations (1), (7), and (8).

Prefragment excitation energies are estimated from the FSI energy contribution

\[ E_{exc} = E_{FSI} \]  

(9)

which is calculated with the model of Rasmussen (ref. 22). With this model, the rate of energy transfer to the prefragment is

\[ \frac{dE}{dx} = \frac{E}{4\lambda} \]  

(10)

where

\[ \lambda = \frac{1}{\rho \sigma_{NN}} \left( \sigma_{NN} \approx \frac{300}{E} \right) \]  

(11)
yields
\[ \frac{dE}{dx} = -12.75 \text{ MeV/fm} \]  
(12)

If a spherical nucleus of uniform density is assumed, the average energy deposited per interaction is
\[ (E_{FSI}) \approx 10.2A^{1/3} \text{ MeV} \]  
(13)

Therefore, the abrasion cross section for a prefragment species \( (Z_{PF}, A_{PF}) \) which has undergone \( q \) frictional spectator interactions is
\[
\sigma_{abr}(Z_{PF}, A_{PF}, q) = \left( \frac{n + z}{q} \right) (1 - P_{esc})^q (P_{esc})^{n+z-q} \sigma_{abr}(Z_{PF}, A_{PF})
\]  
(14)

where \( 0 \leq q \leq n + z \), and \( P_{esc} \) is the probability that an abraded nucleon escapes without undergoing any frictional spectator interactions (ref. 34). In this report, the choice of \( P_{esc} = 0.5 \) follows from the original work of Rasmussen (ref. 22). Such a value assumes that the nuclear surface has no curvature, and this value should be reasonably correct for heavy nuclei. For lighter nuclei, the surface can exhibit significant curvature such that the value of \( P_{esc} \) can be larger than 0.5. Methods for estimating \( P_{esc} \) when nuclear surface curvature is considered have been formulated by Vary and collaborators (ref. 50).

Depending upon the magnitude of its excitation energy, the prefragment will decay by emitting nucleons, composites, and gamma rays. The probability \( \alpha_{ij}(q) \) that a prefragment species \( j \), which has undergone \( q \) frictional spectator interactions, deexcites to produce a particular final fragment of type \( i \) is obtained with the EVA-3 Monte Carlo cascade-evaporation computer code (ref. 19). Therefore, the final hadronic cross section for production of the type \( i \) isotope is obtained from
\[
\sigma_{nuc}(Z_i, A_i) = \sum_j \sum_{q=0}^{n+z} \alpha_{ij}(q) \sigma_{abr}(Z_j, A_j, q)
\]  
(15)

where the summation over \( j \) accounts for contributions from different prefragment isotopes \( j \), and the summation over \( q \) accounts for the effects of different FSI excitation energies. Finally, the elemental production cross sections are obtained by summing all isotopes of a given element according to
\[
\sigma_{nuc}(Z_i) = \sum_{A_i} \sigma_{nuc}(Z_i, A_i)
\]  
(16)

Results

Figures 1 and 2 show isotope production cross sections obtained with equation (15) for \( ^{32}S \) beams at 400A MeV fragmenting in hydrogen targets. The figures also show recently reported experimental results (ref. 51). For clarity, the experimental error bars are not plotted. The \( ^{32}S \) nuclear density used in the calculation was a Woods-Saxon form with skin thickness and half-density radius obtained from reference 48. The agreement between theory and experiment is quite good, especially considering that no arbitrary parameters are in the theory. Quantitatively, a distribution analysis of the cross-section differences between theory and experiment finds that 32 percent agree within the experimental uncertainties, 50 percent agree within a 25-percent difference, nearly 75 percent agree within a 50-percent difference, and over 82 percent agree within a factor of 2.

Elemental production cross-section predictions obtained from equation (16) are displayed in figures 3 to 8 for \( ^{20}Ne \) beams at 400A MeV and 910A MeV and for \( ^{32}S \) and \( ^{40}Ca \) beams at
400A MeV and 800A MeV incident kinetic energies colliding with hydrogen targets. The nuclear densities used in the calculations were Woods-Saxon forms with skin thicknesses and half-density radii again obtained from reference 48. These experimental data were taken from reference 51. Overall, the agreement between theory and experiment is good, although the theory tends to predict values that are slightly larger than the reported measurements.

![Graph 1](image1.png)

Figure 1. Isotope production cross sections for 400A MeV $^{32}$S fragmentation in hydrogen targets for isotopes of P, Al, Na, and F fragments.

![Graph 2](image2.png)

Figure 2. Isotope production cross sections for 400A MeV $^{32}$S fragmentation in hydrogen targets for isotopes of Si, Mg, Ne, and O fragments.

![Graph 3](image3.png)

Figure 3. Element production cross sections for 400A MeV $^{20}$Ne fragmentation in hydrogen targets.
Figure 4. Element production cross sections for 910A MeV $^{20}$Ne fragmentation in hydrogen targets.

Figure 5. Element production cross sections for 400A MeV $^{32}$S fragmentation in hydrogen targets.

Figure 6. Element production cross sections for 800A MeV $^{32}$S fragmentation in hydrogen targets.
Figure 7. Element production cross sections for 400A MeV $^{40}$Ca fragmentation in hydrogen targets.

Figure 8. Element production cross sections for 800A MeV $^{40}$Ca fragmentation in hydrogen targets.

Concluding Remarks

A simple, yet accurate, optical potential abrasion-ablation fragmentation model has been developed for use in studies of galactic cosmic ray breakup on hydrogen targets. The model has no arbitrarily adjusted parameters. Model predictions have good agreement with recent laboratory measurements of elemental and isotopic production cross sections for the fragmenting of neon, sulfur, and calcium beams on hydrogen targets.

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References


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