ESTIMATION OF THE RAIN SIGNAL
IN THE PRESENCE OF LARGE SURFACE CLUTTER

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ABSTRACT

The principal limitation for the use of a spaceborne imaging SAR as a rain radar is the surface-clutter problem. Signals may be estimated in the presence of noise by averaging large numbers of independent samples. We applied this method here to obtain an estimate of the rain echo by averaging a set of $N_c$ samples of the clutter in a separate measurement and subtracting the clutter estimate from the combined estimate.

The number of samples required for successful estimation (within 10 - 20 %) for off-vertical angles of incidence appears to be prohibitively large. However, by appropriately degrading the resolution in both range and azimuth, we can obtain the required number of samples. For vertical incidence, the number of samples required for successful estimation is reasonable.

In estimating the clutter we assume that the surface echo is the same outside the rain volume as it is within the rain volume. This may be true for the forest echo, but for convective storms over the ocean the surface echo outside the rain volume is very different from that within. It is suggested that the experiment be performed with vertical incidence over forest to overcome this limitation.

1.0 INTRODUCTION

A rain-measurement experiment was accepted for the SIR-C/X-SAR mission. Previous studies show that there is adequate signal-to-clutter ratio (SCR) for the experiment to be feasible only at high rain-rates that have a small probability of occurrence. However, signals from smaller rain-rates that are more common are swamped by the surface clutter. To calculate the SCR we used the ocean-surface model developed by Soofi (1978). This model closely matches the AAFE Radscat, SEASAT and SKYLAB data (Schroeder, L.C., et al., 1985). It accounts for
the dependence of the surface returns upon the surface wind speed $W$, the frequency of operation $f$, the aspect angle $\phi$ and the angle of incidence $\theta$. The model is described by the equation

$$\sigma^\circ_{\text{dB}}(W, f, \theta, \phi) = A + B \cos\phi + C \cos2\phi$$

(1)

where the coefficients $A, B$ and $C$ take various values depending on polarization used, angle of incidence, frequency, surface wind speeds and aspect angle. Fig. 1, which has a plot of SCR vs rain-rate for off-vertical angles of incidence, shows that, at lower rain-rates, the SCR (in dB) is negative. This is also the case for vertical incidence, as seen from Fig. 2. This situation is analogous to the estimation of a low signal in a noisy environment. It is possible to estimate the signal even when the SNR (in dB) is negative, if many independent samples are available. We extended this concept to estimating the signal with negative SCR.

2.0 MEASUREMENT IN THE PRESENCE OF CLUTTER

A radiometer operates with a signal-to-noise ratio (SNR) that is much less than unity, and radars with a reasonable number of independent samples of fading can operate with an SNR of the order of unity or even less. Two samples may be considered as independent if they are far enough apart so that the correlation between them is essentially zero, which usually occurs when the spacing between the samples is large compared with the reciprocal of the bandwidth. Each pixel in a side-looking airborne radar (SLAR) image usually represents an average of several independent samples, although the number is small. Each synthetic-aperture radar (SAR) pixel could represent only one sample, but it usually contains 3 or 4 samples (multilook).

At the input of the receiver the received voltage is the sum of the mean values of the signal $v_s'$ and the noise $v_n$. 

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Fig. 1 SCR for given wind speeds at 30° and 60°, generated using the model described by (1).
Fig. 2 Plot of the SCR at vertical incidence for a rain cell at the given height and over ocean and forest.
\[ v_f = v_s' + v_n \]  \hspace{1cm} (2)

The mean value of the received power is the sum of the mean values of the signal and noise power because the random phase causes cancellation of the cross terms in the mean, giving

\[ P_z = P_s' + P_n \]  \hspace{1cm} (3)

where \( P_n \) is the noise power.

Now, the signal power may be broken up into two components \( P_s' = P_s + P_c \), where \( P_s \) is the power returned from the rain and \( P_c \) is the power returned from the surface. We may express the mean received power in terms of the SNR \( S_n \) and SCR \( S_c \) as

\[ P_z = P_s \left( 1 + \frac{1}{S_n} + \frac{1}{S_c} \right) \]  \hspace{1cm} (4)

When \( N_r \) independent square-law-detected samples of the received signal are averaged, the standard deviation of the average is

\[ \sigma_{\text{cdv}} = \frac{P_s}{\sqrt{N_r}} \left( 1 + \frac{1}{S_n} + \frac{1}{S_c} \right) \]  \hspace{1cm} (5)

When the SNR is high enough the estimate of clutter alone can be obtained by averaging a set of \( N_c \) samples of clutter in a separate measurement from measurements where signal and clutter are combined. The standard deviation of the clutter alone is

\[ \sigma_{\text{cdv}} = \frac{P_c}{\sqrt{N_c}} \]  \hspace{1cm} (6)

An estimate of received signal can be obtained by taking the difference between estimates of combined signal and clutter and clutter alone.
\[ Ps = P_{rd} - P_{cd} \]  \hspace{1cm} (7)

The variance of either a sum or difference of two independent random variables is the sum of the variances of the individual components, hence the variance of signal alone is

\[ \sigma_s^2 = \sigma_{rN}^2 + \sigma_{cN}^2 \]  \hspace{1cm} (8)

Now if we assume that compared to the SCR, the SNR is very high, and substitute from (5) and (6) we obtain

\[ \sigma_s^2 = \frac{P_s^2}{N_r} \left( 1 + \frac{1}{S_c} \right) + \frac{P_c^2}{N_c} \]  \hspace{1cm} (9)

Rearranging

\[ \frac{\sigma_s^2}{P_s} = \sqrt{\frac{(1 + \frac{1}{S_c})^2 + (\frac{1}{S_c})^2 K}{N_r}} \]  \hspace{1cm} (10)

where \( K = \frac{N_r}{N_c} \). When \( N_r = N_c \), which is the usual case, \( K \) is 1. We may use (10) as a measure of the estimate of signal in presence of clutter. Solving for the number of independent samples \( N_r \), for \( K = 1 \), we have

\[ N_r = \frac{(1 + \frac{1}{S_c})^2 + (\frac{1}{S_c})^2}{(\frac{\sigma_s^2}{P_s})^2} \]  \hspace{1cm} (11)

At lower SCR the number of samples required for successful estimation increases rapidly. For example, (11) shows that a change in SCR of 3 dB from -10 dB to -13 dB gives a fourfold increase in the number of samples required for successful signal estimation. Fig. 3(a) and 3(b) have plots made from (11) for different ocean-surface wind speeds at angles of incidence, 30° and 60°. The plots in the figures give us an idea of the number of independent samples required.
Fig. 3(a) Number of samples required to estimate the clutter at 30° for the given bound.
Fig. 3(b)  Number of samples required to estimate the clutter at $60^\circ$ for the given bound.
Fig. 4 Number of samples required to estimate the clutter within 20% at vertical incidence.
to estimate the signal to within 10% to 20% accuracy at off-vertical angles of incidence. Here, for rain-rates less than 10 mm/hr, the number of samples required is very large. This is because the SCR at these rain-rates decreases to small values and the numerator of (11) increases rapidly.

Fig. 4 has plots of the number of independent samples required at the vertical angle of incidence. As in the off-vertical case the number of samples required for successful estimation for rain-rates > 5 mm/hr is very large, but feasible.

3.0 SAR RESOLUTION

For a SAR, the finest along-track resolution $r_{ap}$ is possible if the entire potential aperture length $L_p$ is used, and at a range $R$ it is given by (Ulaby et al., 1982)

$$ r_{ap} = \frac{\lambda R}{2 L_p} \tag{12} $$

Writing this in terms of the length $L$ of the real aperture,

$$ r_{ap} \approx \frac{L}{2} \tag{13} $$

For a SAR observing precipitation, coherent integration can be maintained only for a short time. This is because of the Doppler spread of rain drops due to their random motion. A large Doppler spread of the target results in a short usable coherent-integration time. This means a small synthetic aperture and a large resolution cell (Atlas and Moore, 1987). We may write the bandwidth of precipitation Doppler shifts as

$$ B_{df} = \frac{4 \sigma_v}{\lambda} \tag{14} $$

where $\sigma_v$ is the standard deviation of the precipitation velocity. For a SAR with a beamwidth $\beta_h$ and platform velocity $U$, the bandwidth between the isodops is
\[ B_{dr} = \frac{2 \ U \ \beta_h}{\lambda} \quad (15) \]

From (13) and (14) we write the azimuth resolution as

\[ \Delta \alpha = \frac{2 \ \sigma_v \ R}{U} \quad (16) \]

where \( \beta_h \) is assumed to be \( \frac{\alpha_a}{R} \). Values of \( \sigma_v \) may go up to 6 m/s. At 60° the azimuth resolution \( \alpha_a \) works out to about 688 meters, and at vertical it is 344 meters.

### 4.0 POSSIBLE NUMBER OF INDEPENDENT SAMPLES

Two approaches may be used to obtain independent samples in a SAR. In one approach the finest possible resolution \( \alpha_a \) is obtained and the resulting fine resolution image has these pixels averaged together. In another approach several looks are taken from slightly different directions, each with the final resolution \( \alpha_a \), and these pixels are averaged together. In terms of the observation time we may write the number of independent samples as

\[ N_r = \Delta f_D \ T = \frac{2 \ \beta_h^2 \ R}{\lambda} \quad (17) \]

which reduces to the simple relation

\[ N_r = \frac{\alpha_a}{\alpha_p} \quad (18) \]

### 4.1 OFF-VERTICAL ANGLES OF INCIDENCE

For the X-SAR the dimensions of the antenna are 12.1 x 0.3 m. The potential resolution is 6.05 meters. The actual resolution, for an altitude of 215 km, \( \alpha_a \) is 688 meters at 60°, assuming 6 m/s turbulence. The number of samples available in azimuth is about 113. If we obtain \( N_p \) independent samples in the range by degrading the range resolution, the total number
of independent samples available is $113N_R$. The effect of degradation of resolution on the SCR is minimal because any change in the surface area is offset by a corresponding change in the rain volume.

From Fig. 3(b), for the angle of incidence $60^\circ$ and rain-rate 10 mm/hr, the number of samples required to estimate the clutter within 20% is 2000 for a surface wind speed of 24 m/s. We have 113 samples in the azimuth and need about 18 in range. This number in range can be easily obtained by degrading the range resolution to 270 m from the possible 15 m. For lower wind speeds the required number of independent samples is smaller.

At $30^\circ$ for wind speeds < 12 m/s (see Fig. 3(a)), using similar calculations, it is seen that it is possible to obtain enough independent samples to estimate rain-rates > 10 mm/hr.

4.2 VERTICAL INCIDENCE

The potential resolution of the X-SAR is 6.05 meters. The actual resolution, for an altitude of 215 km, $r_{aa}$ is 344 meters at vertical. The number of samples available in azimuth is about 57. By degrading the range resolution, the total number of independent samples available is $57 N_R$. However, since the number of rain cells available are limited in this case compared to off-vertical angles of incidence, the range resolution cannot be degraded to an arbitrarily large value.

Fig. 4 shows that, for a rain-rate of 5 mm/hr, estimation within 20% and a rain cell 3 km high, the number of samples required is 323. We have 57 samples in the azimuth and need about 6 in range. This number in range can be easily obtained by degrading the range resolution to 90 m from the possible 15 m. For a resolution cell 2 km high the corresponding numbers are 831 samples required with 15 in range and a degraded range resolution of 225 m. For a
resolution cell 1 km high, however, the required number of samples required is very large, and it is not possible to obtain a reasonable estimate for resolution cells below this height.

4.3 ESTIMATION OF THE SIGNAL

To obtain an estimate of the signal using this method, first a measurement of the signal and clutter is made from the rain volume. Then, with the same resolution, measurement of the clutter is made outside the rain volume. To obtain adequate number of independent samples, the degradation of the range resolution may be performed during post-processing by averaging $N_R$ samples.

The accuracy of the estimation heavily depends on the knowledge of the radar backscatter cross section in the presence and the absence of the rain. The ratio of the surface backscatter radar cross section in the presence of rain $\sigma^o$ to that in the absence of rain $\sigma^o_1$ is assumed to be unity. This, however, is a questionable assumption since the cross section is modified due to the impinging rain drops (Hansen, 1986). In order to overcome this limitation it is suggested that the estimation of the clutter be made in the same rain volume but with the synthetic aperture. When the synthetic aperture is used, because of the narrow Doppler bandwidth, the interference due to rain is minimal. However, the rain appears as a noise component to the clutter estimate. This noisy clutter estimate is assumed to give better results than that obtained using the clutter estimate made outside the rain volume.

5.0 CONCLUSION

The number of independent samples required to estimate the clutter to within 20% is very large. However, this number can be easily obtained by degrading the resolution in both the
range and the azimuth and by using multiple looks. Nevertheless, the resolution cannot be increased to an arbitrarily large value. As the resolution increases so does the size of the cell volume. Since the observable rain volume is fixed, the number of cells available for processing decreases. Further, as the cell size is increased, the probability of a cell being partially rain filled increases. Partially rain filled cells have smaller returned signal power, which decreases the SCR. Thus, it is concluded that the minimum detectable rain-rate is fixed to be > 10 mm at 60°. But for vertical angle of incidence rain-rates > 5 mm can be detected.

In obtaining an estimate of the clutter, which involves taking a separate measurements in absence of rain, we assume that surface echo modification due to impinging rain drops is negligible. Further we assume that the ocean surface wind speeds are the same in the rain volume as they are outside it. Sometimes these assumptions, as in the case of convective storms, are not true and these are the limitations of this approach. In order to overcome this limitation it is suggested that the estimation of the clutter be made in the same rain volume but with the synthetic aperture. When the synthetic aperture is used, because of the narrow Doppler bandwidth used for the surface echo, the interference due to rain is minimal. Thus, the effect of enhancement of the surface returns due to the impinging rain-drops is the limitation of this approach. It is suggested that an efficient surface reference algorithm be developed for accurate estimation of the rain rate.
REFERENCES


