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Center for Turbulence Research
Annual Research Briefs 1993

N94-24159

On streak spacing in wall-bounded turbulent flows

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1. Motivation and objectives

In the 1992 CTR Annual Research Briefs, Hamilton, Kim & Waleffe (1993a) presented the results of a study of the regeneration mechanisms of near-wall turbulence structures. One of the primary motivations of this study was the observation that the low- and high-speed streaks in the near-wall region have a characteristic spanwise "wavelength" of about 100 ν/u_τ ($u_\tau = \sqrt{\tau_w/\rho}$ is the friction velocity and τ_w is the shear stress at the wall). This value of 100 wall units has been reported by numerous investigators in a wide range of flows, but many attempts to develop a theory of streak spacing have been unsuccessful.

Waleffe, Kim & Hamilton (1993) examined direct resonance (Jang, Benney & Gran, 1986) and selective amplification, two of the linear mechanisms that have been proposed to explain the characteristic streak spacing, and found that these theories fail in several respects. Jiménez and Moin (1991) addressed the issue of streak spacing with a series of direct numerical simulations of a plane Poiseuille flow at moderate Reynolds number, but of very limited streamwise and spanwise extent. They found that turbulence was not sustained in computational domains narrower than about 100 ν/u_τ. In light of this result and the failings of linear theory, Waleffe *et al.* conjectured that the streak spacing depends on the *entire* process of regeneration of near-wall structures.

The present study is a continuation of the examination by Hamilton *et al.* (1993a) of the regeneration mechanisms of near-wall turbulence and an attempt to investigate the conjecture of Waleffe *et al.* The basis of this study is an extension of the "minimal channel" approach of Jiménez and Moin that emphasizes the near-wall region and reduces the complexity of the turbulent flow by considering a plane Couette flow of near minimum Reynolds number and streamwise and spanwise extent. Reduction of the flow Reynolds number to the minimum value which will allow turbulence to be sustained has the effect of reducing the ratio of the largest scales to the smallest scales or, equivalently, of causing the near-wall region to fill more of the area between the channel walls. A plane Couette flow was chosen for study since this type of flow has a mean shear of a single sign, and at low Reynolds numbers, the two wall regions are found to share a single set of structures.

Hamilton *et al.* (1993a,b) found that the near-wall structures are regenerated quasi-cyclically and that this regeneration process can be broken down into three stages: streak formation, through a simple process of advection by streamwise vortices; streak breakdown as a result of an instability mechanism; and vortex regeneration, the result of nonlinear interactions among the modes produced by streak breakdown. This last step is necessary to complete the cycle since the streamwise vortices would otherwise decay through viscous diffusion.

To examine the conjecture by Waleffe *et al.* (1993) that it is the entire regeneration process that determines the spanwise spacing of streaks, the methods developed to study the regeneration cycle can be applied to flows in which the spanwise dimension of the computational domain has been reduced below the value required to sustain turbulence. The results of this approach are discussed in the remainder of this report.

2. Accomplishments

2.1 Numerical method and flow geometry

The direct numerical simulation results presented here were obtained using the pseudo-spectral channel flow code of Kim, Moin & Moser (1987) modified to simulate plane Couette flow and using a third-order Runge-Kutta time advancement for the convective terms rather than the original Adams-Bashforth. Dealiased Fourier expansions are used in the streamwise (x) and spanwise (z) directions, and Chebychev polynomials are used in the wall-normal (y) direction. Boundary conditions are periodic in x and z, and the no-slip condition is imposed at the walls. The mean streamwise pressure gradient is zero, and the flow is driven by the motion of the walls. The flow velocities in the x, y, and z directions are u,v, and w, respectively. The Fourier transforms of the velocities are "hatted" and are functions of the streamwise wavenumber, k_x, the spanwise wavenumber, k_z, and the untransformed y-coordinate, *e.g.* $\hat{u}(k_x, y, k_z)$. The fundamental streamwise and spanwise wavenumbers are $\alpha \equiv 2\pi/L_x$ and $\beta \equiv 2\pi/L_z$. Quantities are nondimensionalized by outer variables: half the wall separation, h, and the wall velocity, U_w. In some cases, a plus superscript is used to denote quantities nondimensionalized by wall variables: kinematic viscosity, ν, and friction velocity, $u_\tau = \sqrt{\tau_w/\rho}$. The flow Reynolds number is based on outer variables: Re$= U_w h/\nu$. The computational grid is $16 \times 33 \times 16$ in x, y, and z. The resolution in wall units for all cases presented here is better than $\Delta x^+ = 13.1$, $\Delta z^+ = 9.0$, and $\Delta y^+ = .19$ near the wall, and 3.8 at the center of the channel.

2.2 Dynamics of regeneration cycle

Since periodic solutions are obtained in these simulations, Fourier decomposition is a natural tool with which to examine the details of the flow. The size of the computational domain is such that the low- and high-speed streaks extend the full length of the flow in the streamwise (x-) direction, and a single pair of streaks fill the domain in the spanwise (z-) direction. In Fourier space, this means that the dominant mode for the streaks is the ($k_x = 0, k_z = \beta$) (or $k_z = -\beta$) mode. The modal RMS velocity (the square root of the "kinetic energy") is given by

$$M(m\alpha, n\beta) \equiv \left\{ \int_{-1}^{1} \left[\hat{u}^2(m\alpha, y, n\beta) + \hat{v}^2(m\alpha, y, n\beta) + \hat{w}^2(m\alpha, y, n\beta)\right] dy \right\}^{\frac{1}{2}}, \quad (1)$$

and $M(0, \beta)$ is a useful quantity for studying the time evolution of the streaks.

For the first flow considered here, $L_x = 1.75\pi$, $L_z = 1.2\pi$ ($L_z^+ = 116.9$–143.6), and Re=400. The upper curve in Figure 1 is a plot of $M(0, \beta)$ for this flow. The

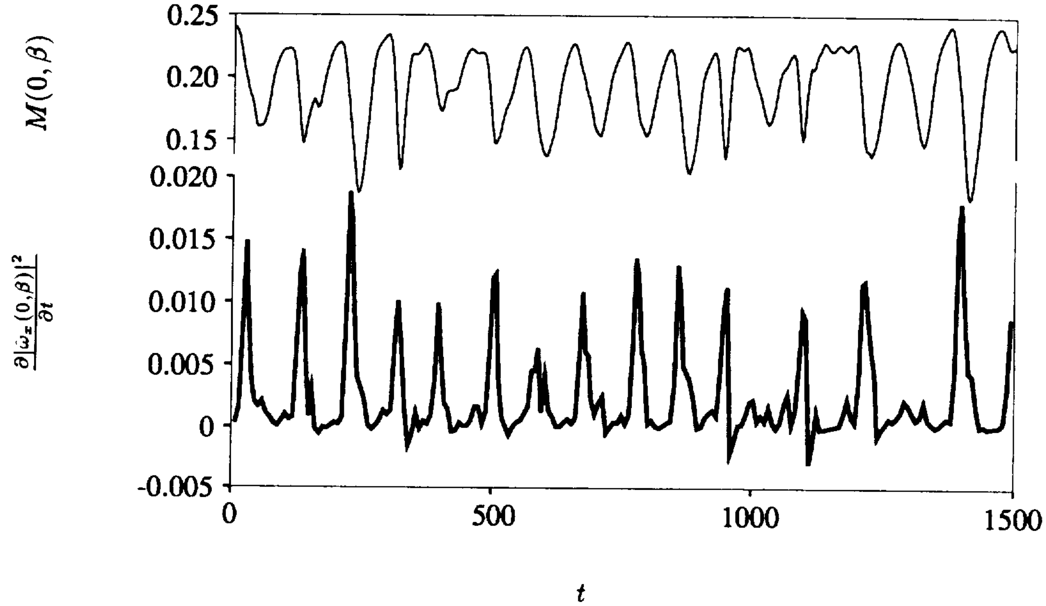

FIGURE 1. Regeneration process over many cycles. ——— , $M(0,\beta)$; ——— , $\partial|\hat{\omega}_x(0,y,\beta)|^2/\partial t$, due to nonlinear terms only, integrated in y.

quasi-cyclic nature of the turbulence in this domain is evident in this figure, and the maxima in $M(0,\beta)$ correspond to well-defined, nearly x-independent streaks, while the minima correspond to "wavy", poorly-defined, x-dependent streaks. The cycle can be broken down into two parts: streak formation where $dM(0,\beta)/dt > 0$, and streak breakdown, where $dM(0,\beta)/dt < 0$.

Streak formation has been found to be the result of a simple process of advection of streamwise momentum by the x-independent vortices, and streak breakdown is the result of an instability of the streaks (Hamilton *et al.* 1993a,b). It can be shown that the x-independent vortices responsible for streak formation will decay in the absence of any interactions among the x-dependent modes; it i.e. x-independent vortices cannot extract energy directly from the mean flow, $\hat{u}(0,y,0)$. Therefore, some form of vortex regeneration mechanism must function in order for turbulence to be sustained.

This regeneration mechanism is found to be a rather complicated set of nonlinear interactions of the $k_x = \alpha$ modes (Hamilton *et al.* 1993a,b) that produce x-independent streamwise vorticity, $\hat{\omega}_x(0,y,n\beta)$. The time evolution of vortex regeneration is most easily seen by considering the quantity

$$\frac{\partial|\hat{\omega}_x|^2}{\partial t} = \hat{\omega}_x^\dagger\frac{\partial\hat{\omega}_x}{\partial t} + \hat{\omega}_x\frac{\partial\hat{\omega}_x^\dagger}{\partial t} \tag{2}$$

(where the † superscript represents the complex conjugate), since this quantity is positive at y-locations where the existing streamwise vorticity is being augmented

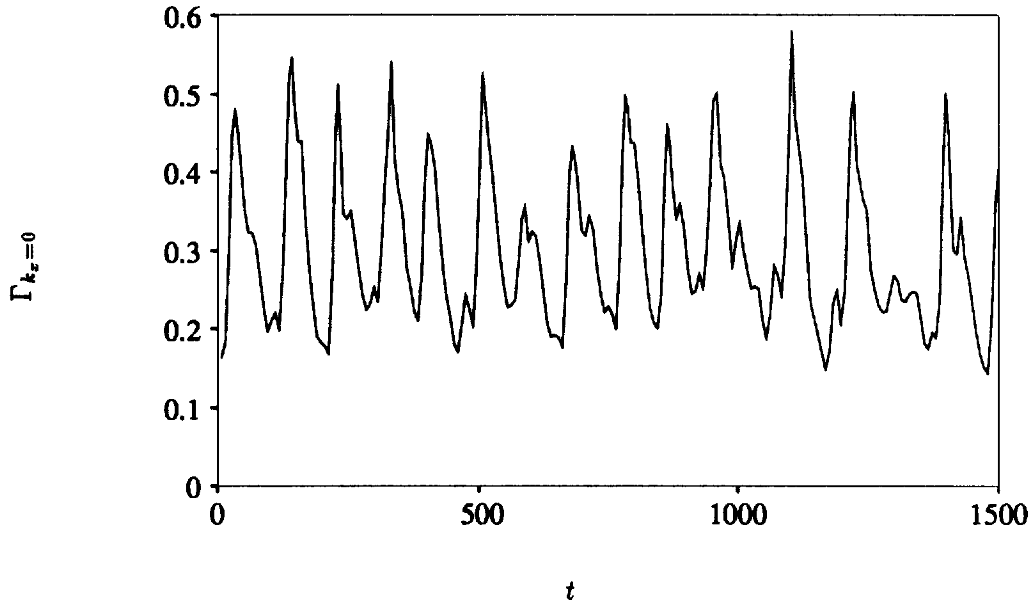

FIGURE 2. Circulation of $k_x = 0$ modes over many regeneration cycles. Circulation plotted is based on contour that gives the maximum circulation from among all rectangular contours on computational grid points in y–z plane.

and negative where the vorticity is being reduced. Only the contribution of the nonlinear terms to $\partial|\hat{\omega}_x|^2/\partial t$ is included; the effects of viscosity are ignored since viscosity acts only to diffuse $k_x = 0$ mode vorticity.

The lower curve in Figure 1 is a plot of $\partial|\hat{\omega}_x(0, y, \beta)|^2/\partial t$, integrated in y. Vortex regeneration occurs during streak breakdown, with peak amplitudes ranging from about 0.008 to nearly 0.02, except during the cycles at $t \approx 1000$ and $t \approx 1300$. These cycles produce almost no regeneration of the streamwise vortices. One measure of the strength of the streamwise vortices is the circulation of the $k_x = 0$ modes

$$\Gamma_{k_x=0} \equiv \int (\omega_x)_{k_x=0}\, dA, \tag{2}$$

and this quantity is plotted in Figure 2. Circulation was calculated for all possible rectangular contours of integration conforming to the computational grid, and the maximum values at each time, t, are plotted. Experiments by Hamilton & Abernathy (1993) showed that, in a laminar flow, streamwise vortices must have a circulation above some threshold in order to cause transition to turbulence. Analogously, near-wall streamwise vortices in a turbulent flow would be expected to require a threshold value of circulation in order to produce unstable streaks. If this is the case, regeneration of the vortices need not occur every cycle as long as vortex circulation does not decay below the threshold before subsequent cycles.

$\Gamma_{k_x=0}$ typically reaches a maximum value during streak breakdown and decays as the streak forms, reaching a minimum value at about the same time that $M(0, \beta)$

peaks. Thus, the form of the streaks at the peak in $M(0,\beta)$ is most closely tied to the minimum value of $\Gamma_{k_x=0}$ each cycle; the maximum value is relatively unimportant. Hamilton & Abernathy (1993) found that the threshold value of the circulation, using the present nondimensionalization, is about 0.15 in a steady flow. This is consistent with Figure 2 since the minimum circulations never fall much below that value, even during the two cycles which have no regeneration of the vortices.

2.3 Spanwise spacing of structures

Some of the dynamics of the regeneration process have been discussed in the previous section, and this section will focus on the question of the spanwise spacing of the streaks. To do this, the width of the computational domain is reduced so that turbulence is not sustained. It can then be established whether a single step in the regeneration process is disrupted by the constraint of the reduced spanwise dimension, or whether, as Waleffe, *et al.* (1993) conjectured, the entire process is affected. Two flows with unsustained turbulence are considered.

The modal decomposition of the first flow is plotted in the upper half of Figure 3. It is not evident in this figure, but $M(0,\beta)$ decays monotonically after $t = 1000$, and the flow eventually becomes laminar. The spanwise dimension of the flow is $L_z = 1.1\pi$, or $L_z^+ = 109.2$ to 126.1 (where L_z^+ is based on u_τ during the early part of the simulation before the turbulence begins to decay). The streamwise dimension is $L_x = 1.6\pi$, and the same Reynolds number, 400, is used. This flow was obtained by first reducing the spanwise dimension of a sustainable turbulent flow, and then reducing the streamwise dimension so as to get a well defined regeneration cycle before the turbulence decays. The simulation begins at $t = 0$, but only the last few cycles are shown.

The quasi-cyclic behavior of the streaks in the unsustained turbulent flow of Figure 3 appears similar to that of the sustained flow (Figure 1) until the final peak in $M(0,\beta)$. There is no breakdown of the flow after this peak, and without breakdown, the regeneration cycle is broken. It is found that breakdown does not occur because the streaks are too stable; *i.e.* the growth rates of small disturbances are very small or negative near the peak in $M(0,\beta)$ (cf. Hamilton *et al*, 1993a). In sustained turbulent flow, the streaks are the result of advection of momentum by streamwise vortices. Whatever changes occur in the streaks to increase their stability in the unsustained turbulent flow are likely then to be traceable to changes in the streamwise vortices. The regeneration of the streamwise vortices for the last few cycles of the unsustained turbulent flow is shown by the plot of $\partial|\hat{\omega}_x(0,\beta)|^2/\partial t$ integrated in y in the lower half of Figure 3 (heavy solid line). Note that the first two vortex regeneration events in the plot peak during streak breakdown, while the final event does not peak until the new streaks have already begun to form. Thus, even though the peak amplitude of the vortex regeneration process is nearly constant for each of the three regeneration events plotted, the final regeneration occurs late relative to the beginning of streak formation. The circulation of the vortices is plotted in Figure 4, and it can be seen that the streamwise vortices continue to decay during this delay, with the circulation falling to about 0.11 before regeneration begins. This value is lower than any observed during the sustained cycle of Figure 2.

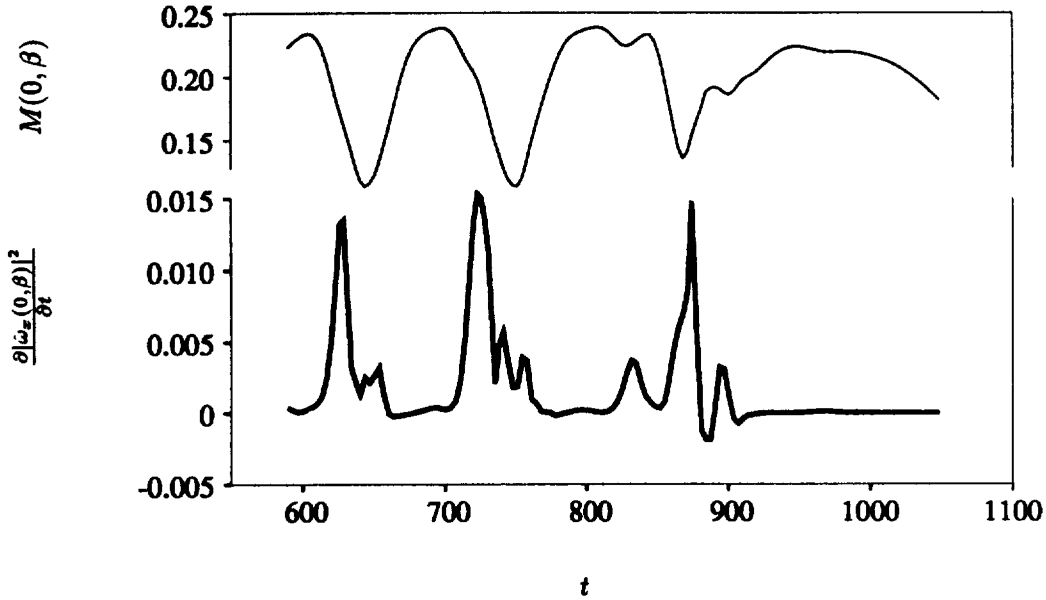

FIGURE 3. Regeneration process over last few cycles of unsustained flow. ———, $M(0,\beta)$; ———, $\partial|\hat{\omega}_x(0,y,\beta)|^2/\partial t$, due to nonlinear terms only, integrated in y.

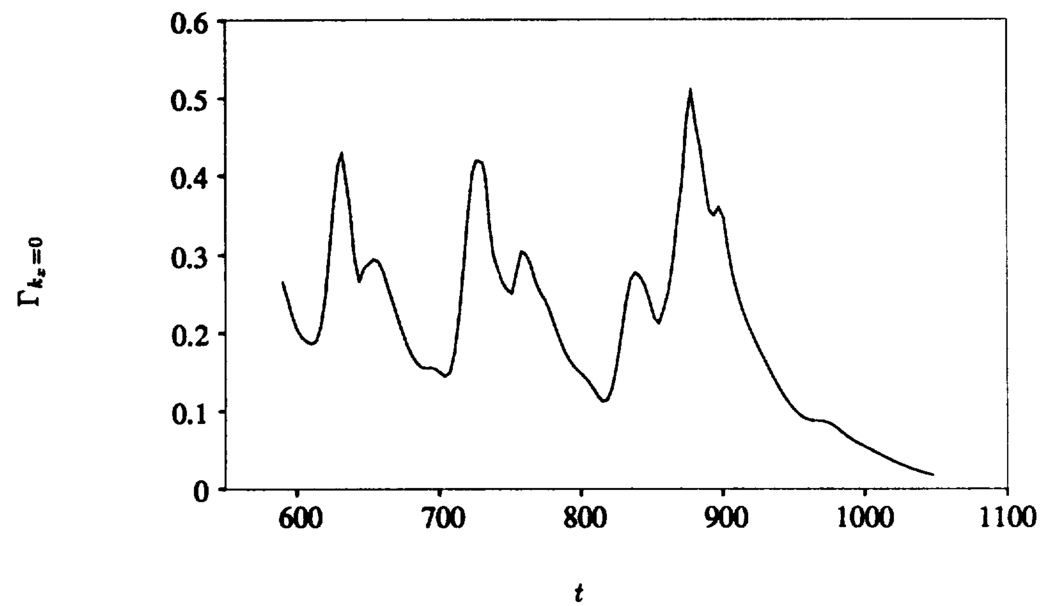

FIGURE 4. Circulation of $k_x = 0$ modes over last few cycles of flow of Figure 3.

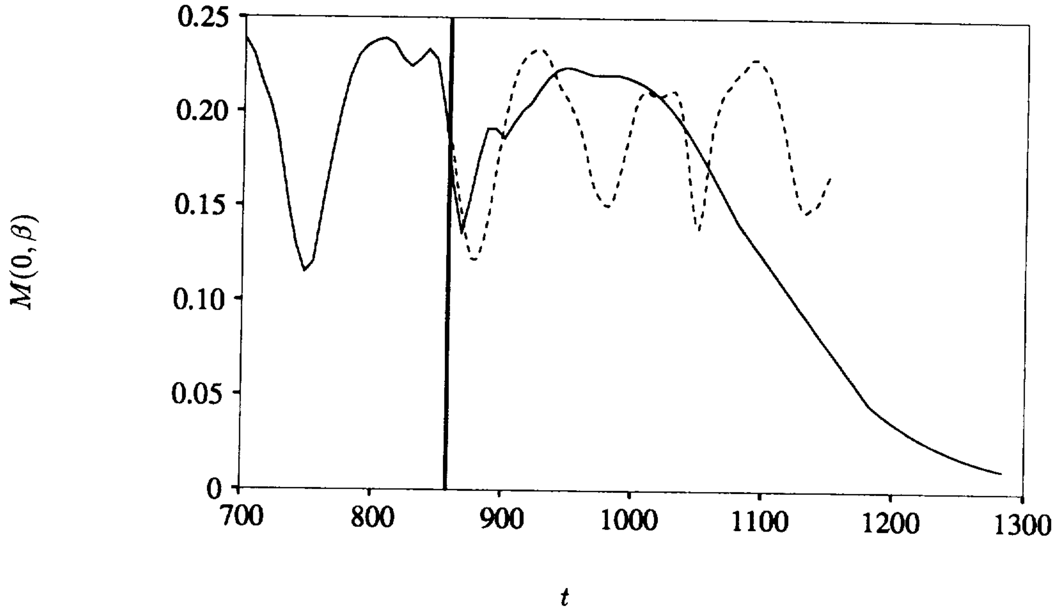

FIGURE 5. $M(0,\beta)$ for: ——— , unsustained turbulent flow of Figure 3; - - - - , same flow with $\hat{v}$, $\hat{w}(0,y,n\beta)$ $(n \neq 0)$ modes multiplied by 2.0 at $t = 858.5$ (denoted by heavy vertical line).

After regeneration finally takes place, the circulation drops to about 0.09 at the final maximum in $M(0,\beta)$, and there is no subsequent breakdown. To verify that the relative delay in vortex regeneration does indeed cause the turbulence to decay, the strength of the streamwise vortices was artificially boosted at $t = 858.5$, a time corresponding to mid-breakdown in the final full regeneration cycle of Figure 3. The result is plotted in Figure 5. The strength of the vortices was increased by multiplying the $\hat{v}(0,y,n\beta)$ and $\hat{w}(0,y,n\beta)$ $(n \neq 0)$ modes by a factor of 2.0, and all other modes were left unmodified. The effect of increasing the vortex strength is immediate, and the flow returns to the normal regeneration cycle. Note that the turbulence does not subsequently decay; the domain size is such that turbulence is marginally sustainable and can go through a large number of cycles before decaying.

A second case of unsustained turbulence ($L_z^+ = 97.0$–86.5) is presented in Figure 6. The solid line in the upper half of the plot is $M(0,\beta)$, and the associated vortex regeneration, $\partial|\hat{\omega}_x(0,\beta)|/\partial t$, is shown in the lower half. In this flow, vortex regeneration takes place at about the same point in the cycle as in the sustained cases, and the circulation, plotted in Figure 7, is increased appropriately. Thus, there is no delay in regeneration as in the previous flow. Indeed, the opposite is true, and vortex regeneration takes place too early; at the time of the final peak in $M(0,\beta)$ in Figure 6, $\Gamma_{k_x=0}$ has dropped to about 0.1. To verify this assertion, $\Gamma_{k_x=0}$ was increased by a factor of 1.5 at $t = 130.0$ (the peak in circulation in Figure 7), and $M(0,\beta)$ of the resulting flow is plotted as a dashed line in Figure 6.

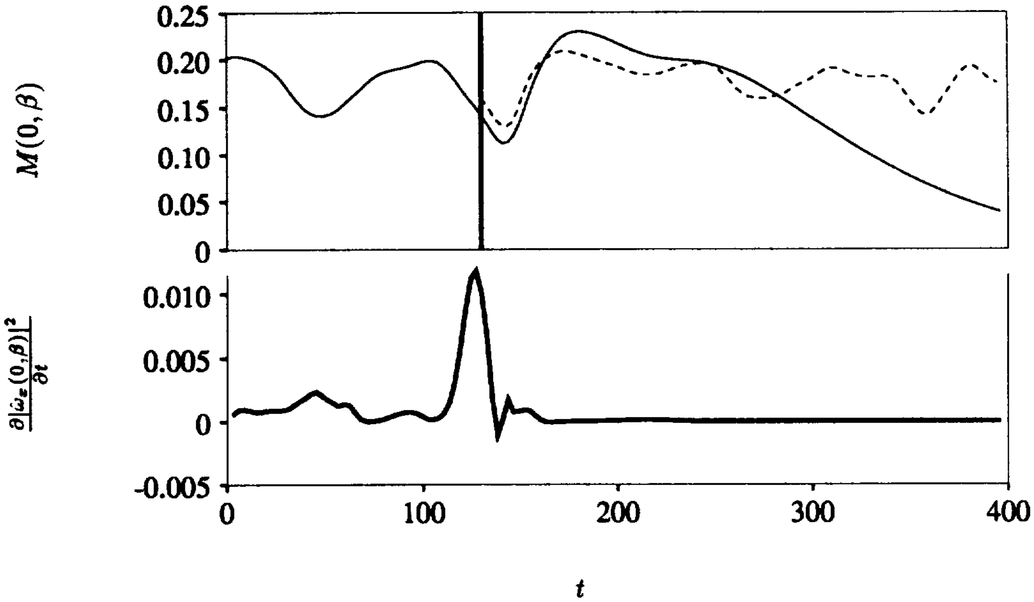

FIGURE 6. Regeneration process of unsustained flow, and modified flow. ———, $M(0,\beta)$; – – – – , same flow with $\hat{v}$, $\hat{w}(0,y,n\beta)$ ($n \neq 0$) modes multiplied by 1.5 at t = 130.0 (denoted by heavy vertical line); ——— , $\partial|\hat{\omega}_x(0,y,\beta)|^2/\partial t$, due to nonlinear terms only, integrated in y for unmodified flow.

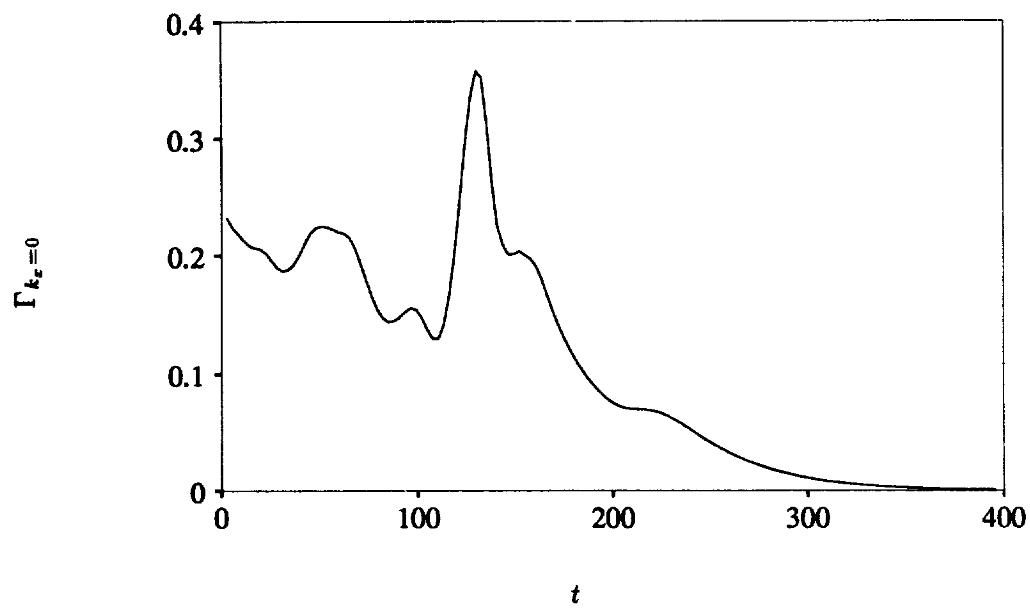

FIGURE 7. Circulation of k_x = 0 modes of unsustained (unmodified) turbulent flow of Figure 6.

The increase in circulation produces unstable streaks followed by breakdown and a return to a (rather chaotic) regeneration cycle. Since streak formation takes place during the decay of the vortex, the increase in circulation at the peak in $\Gamma_{k_x=0}$ increases circulation by a like amount at subsequent times and simulates a delay in regeneration. There are several ways to think about the effects of reducing the computational domain size below that required for sustained turbulence, but these results suggest that the most useful may be to think of the small domain as causing the flow to develop a very critical dependence on the timing of each process in the regeneration cycle. As the domain becomes smaller, the flow becomes unable to accommodate the variations in the intervals between events that naturally accompany turbulent flow. In the two cases of unsustained turbulence presented here, the regeneration of streamwise vortices occurred with full vigor, but at the wrong times. Turbulence can be sustained only when streak formation, streak breakdown, and vortex regeneration occur at the appropriate intervals.

The results of this section support the conjecture by Waleffe *et al.* (1993) that the minimum spanwise wavelength is set by the entire regeneration process, rather than any individual element of regeneration. When the computational domain is too narrow, turbulence decays because breakdown does not occur. Breakdown, in turn, depends on the creation of unstable streaks by sufficiently strong streamwise vortices. The strength of the streamwise vortices depends on vortex regeneration, and this, of course, returns us to the starting point, since regeneration depends on streak breakdown during the previous cycle.

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