Aircraft Empennage Structural Detail Design

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AE421/03/Bravo
Lead Engineer: Greg Meholic
Team Members: Ronda Brown
Melissa Hall
Robert Harvey
Michael Singer
Gustavo Tella

Submitted To:
Dr. J.G. Ladesic
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1. Project Summary

1.1 Design Goals

This project involved the detailed design of the aft fuselage and empennage structure, vertical stabilizer, rudder, horizontal stabilizer, and elevator for the Triton primary flight trainer. The main design goals under consideration were to illustrate the integration of the control systems devices used in the tail surfaces and their necessary structural supports as well as the elevator trim, navigational lighting system, electrical systems, tail-located ground tie, and fuselage/cabin interface structure. Accommodations for maintenance, lubrication, adjustment, and repairability were devised. Weight, fabrication, and (sub)assembly goals were addressed. All designs were in accordance with the FAR Part 23 stipulations for a normal category aircraft.

1.2 Statement of Work Requirements

The Statement of Work for the cockpit set forth several design requirements. The structural designs are required to sustain anticipated loading conditions which can occur in normal flight training operations. Easy removal or re-installation of the tail portions from/to the fuselage must be provided by means of interface structures which safely carry the applicable loads defined in FAR Part 23 and the Statement of Work. Adverse effects due to varying environmental conditions must be addressed, including sand or dust, rain, ice or snow, and salt or fog. The aircraft must have a service life of 20 years. The critical design components are required to provide a safe life of $10^7$ load cycles and 10,000 operational mission cycles.

Table 1 Summary of Critical Detail Parts

<table>
<thead>
<tr>
<th>Part No. (Spec #)</th>
<th>Title</th>
<th>Load(s) (psf)</th>
<th>Load Source</th>
<th>M.S.Type (part)</th>
<th>Page No.</th>
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<tr>
<td>1 (02)</td>
<td>Front Spar (HS)</td>
<td>29.511</td>
<td>Cruise Fit</td>
<td>0.017/fatigue</td>
<td>19-20</td>
</tr>
<tr>
<td>2 (02)</td>
<td>Rear Spar (HS)</td>
<td>29.189</td>
<td>Maneu. Fit</td>
<td>0.028/fatigue</td>
<td></td>
</tr>
<tr>
<td>5 (04)</td>
<td>Front Fuselage Interface (VS)</td>
<td>30.800</td>
<td>Cruise Fit</td>
<td>0.006/fatigue</td>
<td>24</td>
</tr>
<tr>
<td>6 (04)</td>
<td>Rear Fuselage Interface (VS)</td>
<td>29.800</td>
<td>Maneu. Fit</td>
<td>0.039/fatigue</td>
<td>24</td>
</tr>
<tr>
<td>1 (06)</td>
<td>Skin Panel E (Tail Cone)</td>
<td>3,564</td>
<td>Torsion</td>
<td>0.075/U twists</td>
<td>26-27</td>
</tr>
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2. Description of Design

2.1 Horizontal Stabilizer

Both the maneuvering and cruise load cases were used to size this surface. The front spar is located at 6.25" from the leading edge. There are three reasons for this placement: it is near the position of maximum thickness of the airfoil, it divides the skin panels into approximately equal sizes, and it is near the location of the cruise loading. The thickness of the front spar is 0.072" of which 0.04" is a doubler that spans to the 37.0" butt line. The width of the flanges is 0.9". To meet fatigue requirements, two L-shaped triplers which fit inside the C-channel with thicknesses of 0.04" and 0.8" legs are included up to the 22.0" butt line along the half-span (Refer to Formal Drawing 421S9303B202). The rear spar is located at the trailing edge (15.5" with respect to the leading edge) of the horizontal stabilizer, as is customary. The rear spar has characteristics identical to the front spar except that the L-shaped triplers are only 0.032" thick. Five ribs are situated along the half-span at intervals of 15.0" with the first spar located at 7.0" from the centerline of the aircraft. All ribs as well as the skin have a thickness of 0.02". There are three hinges on which the elevator rotates. These are located at (with respect to the center line of the empennage) the 5.0", 36.0", and 67.0" butt lines (See Figure 01 for a detail view of a hinge). The fuselage interface structures are C-channels which are riveted to the

![Figure 01 Horizontal Stabilizer Hinge Detail](image-url)
inside of either spar and bolted to a former inside the aft fuselage (See Figure 02). The front interface channel has a height of 2.536", flange width of 1.00", and thickness of 0.082". The rear interface channel has the same properties as the front interface with the exception that the height is 2.046". Both interface structures are joined to a former in the aft empennage by eight 0.25" bolts, four in the flanges and four in the web.

![NUT, BOLT, WASHER (8 EACH)](image)

**Figure 02 Horizontal Stabilizer Front Interface Bracket Detail**

### 2.2 Elevator

The maneuvering load distribution was used to size the elevator. The front spar is located at the leading edge of the elevator as is historically done for attachment of the hinges. The C-channel front spar has a thickness of 0.02" with a flange width of 0.5". The rear spar is positioned 7.0" behind the front spar and serves to prevent buckling of the skin panels. It has the same thickness and flange width as the front spar. The elevator has three ribs behind the front spar on each half span: one at both the root and tip and the third in the middle (Refer to Formal Drawing 421S9303B203). There are four additional ribs in the leading edge portion of the elevator, two at each position previously stated surround the hinges whose locations are described in Section 2.1 (See Figure 03 on the next page). The thickness of the ribs and skin is 0.02" thick.
2.3 Vertical Stabilizer

The design criterion for this surface was the cruise loading case. The C-channel front spar of the vertical stabilizer is positioned just behind the quarter chord at the root and angles forward until it is directly in front of the quarter chord at the tip. It has a thickness of 0.05" and a flange width of 0.7". Two L-shaped doublers are placed opposite the flanges for a length of 23.0" in order to meet fatigue requirements. These doublers have legs which are 0.5" long and 0.032" thick (Refer to Formal Drawing 421S9303B204). The rear spar is located in the standard position at the trailing edge of the stabilizer. It has characteristics which are identical to the front spar with the exception that it does not require doublers. Stringers are located between the front and rear spars to prevent buckling of the skin; this stringer extends 34.0" from the root of the stabilizer. There are four ribs: one at the root chord, one at the tip chord, and the other two at 8.0" and 28.0" from the root. Due to the sweep of the vertical stabilizer, each rib runs in two directions (See Figure 04 on next page). The ribs ahead of the front spar are perpendicular to
the spar. Both the ribs and the skin are 0.02" in thickness. There are three hinges about which the rudder rotates; these are located at the root and tip chord and at 24.0" from the stabilizer root chord. The fuselage interface structures are C-channels which attach to the either the front or rear spar by means of rivets. Eight 0.3125" bolts attach each interface structure to formers inside the aft empennage. The front interface extends 5.0" into the fuselage, is 4.1" wide and has a flange width of 1.50". The flange widths of the spars are gradually increased to meet those of the interface pieces (See Figure 05). The rear interface piece is identical to the front with the exception that it has a thinner width of 2.7". There are two navigational lights at the top of the stabilizer. The lightening holes provide space to run the electrical systems for these lights.

Figure 05 Vertical Stabilizer Front Interface Bracket Detail
2.4 Rudder

The maneuvering case was used to size this control surface. The front spar of the rudder is located at the leading edge of the rudder for attachment of the hinges. Both the front and rear spars are C-channels with a thickness of 0.02" and flange width of 0.5". The rear spar is located 7.0" behind the front spar and facilitates the prevention of buckling in the skin panels (Refer to Formal Drawing 421S9303B205). Because the rudder extends below the vertical stabilizer, its rib positions are different. There is a rib at the bottom of the rudder to provide a partially closed cavity. The other three ribs are at the hinge locations; the ribs in the leading edge portion are configured similarly to those described in the Section 2.2. The thickness of the ribs and skin is 0.02".

2.5 Tail Cone

The empennage is designed to transmit the loads and moments imposed by the control surfaces into the fuselage structure. The structure is a slightly elliptical cone constructed of stringers and formers and wrapped in a 0.025" aluminum skin. All of the torsional loads are absorbed by the skin; while the bending loads are transmitted through four stringers each at 45° to the aircraft's longitudinal axis (See Figure 06). Stringer buckling is prevented by internal

![Empennage Stringer Diagram](image_url)
formers spaces every 20 inches. The formers at the control surface interface locations are modified since they are the first components inside the empennage to receive the loads. These formers are designed to transmit the control surface loads through rivet shear from attachment brackets riveted to the former and bolted to the interface structures (See Figure 07). Except for the interface brackets, each component of the structure is made from 0.025" sheet and assembled with rivets.

![Interface Former Diagram](image)

**Figure 07** Interface Former Diagram

3. **Loads and Loading**

3.1 **Horizontal Stabilizer**

Bending in the horizontal stabilizer is carried by the spars. The bending induced during cruise mainly affects the flanges in the C-channel, front spar due to the cruise load's peaked distribution near the leading edge. The rear spar caps carry the majority of the bending caused by the maneuvering load distribution which peaks at the leading edge of the elevator. The skin carries the shear flow and is sized by the maneuvering shear force because of its greater moment arm with respect to the spars. The largest loads are carried by the front and rear spars at the root of the stabilizer. These loads are 29,508 psi and 29,169 psi of bending stress induced during cruise flight on the front and rear spars, respectively.
3.2 Elevator

The leading edge front spar of the elevator carries bending. Due to the locations of the resultant loads, only a small amount of torsion exists in the structure. The rear spar is auxiliary and serves only the purpose of preventing the skin panels from buckling. The loads are small enough that only a unit cell analysis was necessary to size the skin thickness.

3.3 Calculations on the Horizontal Stabilizer and Elevator

Loading on the Horizontal Tail Surfaces Due to Maneuvering

\( w \) is the load according to FAR Part 23 App. A, Figure A5 and Table 2 Figure (A) See Appendix 2

\( c \) is the mean aerodynamic chord of the horizontal stabilizer and elevator

**Note**
The only portion of the elevator loading which will be used in this section is that which determines the hinge loads on the horizontal stabilizer.

Loading on the Horizontal Stabilizer Due to Elevator Deflection

\( \text{mac}_{hs} \) is the mean aerodynamic chord of the horizontal stabilizer

\( w_{hs} \) is the chordwise load on the horizontal stabilizer per FAR Appendix A

\( F_r \) is the resultant force of the chordwise triangular distributed load shown below

\[
F_r = 0.5 \cdot \text{mac}_{hs} \cdot w_{hs} = 3.07 \cdot \text{lb} \cdot \text{in}^{-1}
\]

The resultant force, \( F_r \), is shown in phantom.

When this resultant force is applied spanwise across the horizontal stabilizer, it can be represented by the following free body diagram.
Loading on the Elevator Due to Deflection

\( w_e \) is the chordwise load on the elevator per FAR Part 23 Appendix A
\( d_hl \) is the distance from the leading edge of the elevator to its hinge line
\( d_a \) is the distance from the hinge line to the trailing edge of the elevator
\( F_r \) is the resultant force of the distributed load
\( x \) is the position of the resultant force with respect to the leading edge of the elevator

The chordwise loading at the root chord is shown in the following free body diagram.

![Chordwise Maneuvering Load On Elevator At Root](image)

The resultant of the chordwise force at the root is:

\[
F_r = w_e d_{hl} + 0.5 w_e d_a \quad \text{(Equation 1)}
\]

\[
F_r = 3.57 \text{ lb/in}^2
\]

The position of this resultant force is located:

\[
x = \frac{w_e d_{hl}^2}{F_r} + 0.5 w_e d_a \left( \frac{d_a}{3} + d_{hl} \right)
\]

\[
x = 5.36 \text{ in} \quad \text{(Equation 2)}
\]

The same analysis was used at the mean aerodynamic chord and tip chord. The results of the calculations are: From Equations 1 and 2 above,

**M.A.C.:** \( F_r = 2.66 \text{ lb/in}^2 \quad x = 3.85 \text{ in} \)

**Tip Chord:** \( F_r = 1.31 \text{ lb/in}^2 \quad x = 1.71 \text{ in} \)

Therefore, the spanwise load distribution on the elevator can be shown by the following free body diagram. The resultant force is shown in phantom.

![Spanwise Maneuvering Load On The Elevator](image)

The Reactions on the Horizontal Stabilizer Due to the Hinges

![Diagram of Elevator Hinge Placements](image)
The above sketch can be represented by the following free body diagram.
The middle hinge can be removed to simplify the initial analysis and later replaced.

\[ \sum F_y = R_1 + R_3 + 167.15 \text{ lb} = 0 \]
\[ \sum M_A = 167.15 \text{ lb} \cdot 22.5 \text{ in} + R_3 \cdot 68.5 \text{ in} = 0 \]
\[ R_3 = -54.9 \text{ lb} \]
\[ R_1 = -112.5 \text{ lb} \]

Free Body Diagram of the Elevator With Unknown Hinge Load

After replacing the middle hinge, the reactions due to the three hinges can be estimated as:
\[ R_1 = -56.13 \text{ lb} \]
\[ R_2 = -83.58 \text{ lb} \]
\[ R_3 = -27.45 \text{ lb} \]

By combining the original load distribution on the horizontal stabilizer due to elevator deflection with the point loads due to the hinge reactions, the final load distribution can be determined.

The following free body diagram represents this load distribution.

Using static equilibrium, the reaction forces can be calculated.

Free Body Diagram of the Stabilizer With Hinge Point Loads

Now the equations of the shear and bending moment diagrams can be determined. The following method will be used in determining all subsequent shear and bending moment diagrams. However, the mechanics of this method will not be presented again.

The first cut is made from \(0 < x < 5\) inches
\[ M = -12019.3 \text{ lb} \cdot \text{in} \]
\[ R = 372.85 \text{ lb} \]
\[ \sum F_y : V = 3.07 \text{ lb} \cdot \text{in}^{-1} \cdot x - 372.85 \text{ lb} \]
\[ \sum M_c : M_c = 1.535 \text{ lb} \cdot \text{in}^{-1} \cdot x^2 - 372.85 \text{ lb} \cdot x + 12019.3 \text{ lb} \cdot \text{in} \]
First Cut of the Stabilizer Free Body Diagram

The second cut is made from \(5 < x < 36\) inches
\[ M = -12019.3 \text{ lb} \cdot \text{in} \]
\[ R = 372.85 \text{ lb} \]
\[ \sum F_y : V = 3.07 \text{ lb} \cdot \text{in}^{-1} \cdot x - 316.72 \text{ lb} \]
\[ \sum M_c : M_c = 1.535 \text{ lb} \cdot \text{in}^{-1} \cdot x^2 - 316.72 \text{ lb} \cdot x + 12019.3 \text{ lb} \cdot \text{in} \]
Second Cut of the Stabilizer Free Body Diagram
The third cut is made from 36 < x < 67 inches

\[ M = -12019.3 \text{ lb}\cdot\text{in} \]

Third Cut of the Stabilizer Free Body Diagram

\[ \sum F_y : V = 3.07 \cdot \text{lb}\cdot\text{in}^2 \cdot x - 233.14 \cdot \text{lb} \]

\[ \sum M_c : M_c = \left( 1.535 \cdot \text{lb}\cdot\text{in}^{-1} \cdot x^2 - 233.14 \cdot \text{lb} \cdot x \right) + 8449.05 \cdot \text{lb}\cdot\text{in} \]

The shear and bending moment diagrams appear as follows:

**Shear Diagram of Horizontal Stabilizer Due to Elevator Deflection**

**Bending Moment Diagram for Horizontal Stabilizer with Elevator Deflected**

**Loading at Level Cruise**

w is the aerodynamic load at cruise condition from FAR Appendix A, Figure 5A. See Appendix 2

The resultant force, \( F_r \), of the chordwise cruise load on the horizontal stabilizer can be represented by the sketch on the left. When this resultant force is applied spanwise across the horizontal stabilizer it can be represented by the free body diagram on the right.
Elevator Hinge Loads Acting on Horizontal Stabilizer at Cruise

From the aerodynamic loading given in FAR part 23 Appendix A, the chordwise load distributions on the elevator are shown in the sketches below on the left. The spanwise load distribution on the elevator is shown by the free body diagram on the right. The resultant force is shown in phantom.

The hinge loads were determined using the same analysis shown for the maneuvering load distribution. The hinge loads were determined to be the following:

\[ R_1 = 24.5 \text{ lb} \quad R_2 = 30.84 \text{ lb} \quad R_3 = 7.16 \text{ lb} \]

By combining the original load distribution on the horizontal stabilizer during cruise with the point loads due to the hinge reactions, the final load distribution can be determined. The following free body diagram represents this load distribution.

Using static equilibrium, the reaction forces can be calculated:

\[ \sum F_y : \quad R = 400.0 \text{ lb} \]

\[ \sum M_c : \quad M = 13002 \text{ lb} \cdot \text{in} \]

The shear and bending moment diagrams appear as follows:
Loading Due to Snow

The statement of work requires that the horizontal stabilizer be able to withstand 8.0 inches of wet snow. The following is a determination of the loading that occurs due to this accumulation of snow.

- $\rho_s$ is the weight density of wet snow
- $d_s$ is the depth of the accumulated snow
- $A_{hs}$ is the area of half of the horizontal stabilizer (not including the elevator)
- $W_s$ is the weight of the snow
- $d_{ls}$ is the distributed loading due to the snow
- $s$ is the half span of the horizontal stabilizer including the fairing

Therefore, the distributed loading appears as the following sketch:

\[
\begin{align*}
\rho_s &= 12 \text{ lb-ft}^{-3} \quad A_{hs} = 7.34 \text{ ft}^2 \quad d_s = 0.667 \text{ ft} \\
W_s &= A_{hs} d_s \rho_s \quad W_s = 58.72 \text{ lb} \quad s = 73.5 \text{ in} \\
d_{ls} &= \frac{W_s}{s} \quad d_{ls} = 0.799 \text{ lb-in}^{-1}
\end{align*}
\]

Free Body Diagram of the Spanwise Snow Load Distribution

Using the equations of static equilibrium, the reaction loads can be calculated.

\[
\sum F_y : R = 53.53 \text{ lb} \quad \sum M : M = -1793.4 \text{ lb-in}
\]

Because this was not a critical design case, the shear and moment diagrams will not be shown.

3.4 Vertical Stabilizer

Bending in the vertical stabilizer is carried principally by the front spar. The cruise loading condition was used to size both the front and rear spars of the vertical stabilizer. The skin carries the shear flow. The largest loads produced by the vertical stabilizer will be carried by the fuselage interface structures which are attached to the spars. These loads are 30,800 psi and 29,800 psi on the front and rear interface brackets, respectively.

3.5 Rudder

The leading edge front spar of the rudder was designed to carry bending induced during maneuvers. The rear spar facilitates the prevention of buckling in the skin panels but is otherwise auxiliary. As with the elevator, only a unit cell analysis was required to size the skin thickness.
3.6 CALCULATIONS ON VERTICAL STABILIZER AND RUDDER

Loading on Stabilizer Due to Rudder Deflection
(Case A in FAR Part 23 Appendix A)

$\bar{c}_{mac}$ is the mean aerodynamic chord of the vertical stabilizer (minus the rudder)
$w_{mac}$ is the chordwise load on the vertical stabilizer per FAR Appendix A
$F_r$ is the resultant force of the triangular distributed load shown below on the left

$\bar{c}_{mac} = 3.33$ ft $w_{mac} = 51.18$ lb/ft$^2$
$F_r = 0.5\bar{c}_{mac}w_{mac} = 56.55$ lb/ft

The chordwise distribution as well as the magnitude and location of $F_r$ are shown in the sketch on the left. When this resultant force is applied spanwise across the vertical stabilizer, it can be represented by the diagram to the right.

Loading on Rudder Due to Deflection

*Note: the rudder is hinged at the leading edge
$c_r$ is the rudder chord at the mac of the vertical stabilizer
$w_r$ is the chordwise load on the rudder per FAR Appendix A
$F_r$ is the resultant force of the distributed load

$c_r = 1.12$ ft $w_r = 51.18$ lb/ft$^2$
$F_r = 0.5c_rw_r = 28.66$ lb/ft

The chordwise distribution as well as the magnitude and location of $F_r$ are shown in the sketch on the left. When this resultant force is applied spanwise across the vertical stabilizer, it can be represented by the diagram to the right.
The loading on rudder is transferred to the stabilizer through the three hinges. Therefore, the loads from the rudder are resolved into three point loads and added to the loading on the stabilizer.

From this distribution, the shear and moment diagrams for the vertical stabilizer due to deflection of the rudder can be drawn.

The shear and moment diagrams for the rudder are shown below.

**Unsymmetrical Loading on Stabilizer**

(Case B in FAR Part 23 Appendix A)

\[
P_r = 0.5(z_	ext{mc})x + w_{mc}(x) + 0.5(z_	ext{mc})(\text{mac}_{mc}-x) - 0.5(c_r)(y_r)
\]

\[
P_r = 78.77 \text{ lb/ft}
\]
When the resultant force is applied spanwise across the stabilizer, it can be represented by the diagram to the left.

**Unsymmetrical Loading on Rudder**

$c_r$ is the chordwise load at the leading edge of the rudder

$y_r$ is the resultant force of the distributed load

$c_r = 1.12 \text{ ft} \quad y_r = 11.5 \text{ lb/ft}^2$

$F_r = 0.5c_r y_r = 6.44 \text{ lb/ft}$

As shown in the previous analysis, the loading on the rudder is transferred to the stabilizer through the three hinges. The loads on the rudder are resolved into three point loads and added to the loading on the stabilizer as shown.

From this distribution, the shear and moment diagrams for the stabilizer in the unsymmetrical loading case can be drawn.

Analysis shows that the critical case for stabilizer design is Case B while the critical case for rudder design is Case A.
3.7 Tail Cone

The loads transmitted into the aft fuselage section of the aircraft (tail cone) are generated by the control surfaces, the tie down force, and snow blanketing. The worst case loads from the control surfaces were the highest of the three conditions and were used for the structural design of the empennage.

These loads satisfied the FAR23 required limit loads for the surfaces. Design of the empennage accounted for the torsion developed by both rudder deflection and unsymmetrical lift on the horizontal stabilizer resulting from maneuvering flight conditions. The bending and shear stresses were calculated using the resultant force created from both the stabilizer and fin. This resultant was treated as a point load at STA 269 at 60° to the horizontal. STA 269 was chosen as a mean distance between the fin and stabilizer loads. The loading conditions are shown below.

3.8 Calculations on the Tail Cone

\[
\begin{align*}
V &= 931 \text{ lb} \\
\text{vert. fin} &\quad 476 \text{ lb} \\
\text{horiz. stab.} &\quad 900 \text{ lb} \\
\end{align*}
\]

\[
\begin{align*}
\text{Shear (lbs)} &\quad 931 \\
0 &\quad 219 \quad 269 \quad 301 \\
\end{align*}
\]
Snow Load

The snow load was determined through geometrical analysis of the empennage and the surfaces:

Volume of empennage top surface area + Volume of horiz. stabilizer area:

\[(8 \text{ in snow}) (0.5 (23 \text{ in} + 9 \text{ in}) (132 \text{ in}) + (3600 \text{ in}^2)) = 45696 \text{ in}^3 = 26.44 \text{ ft}^3\]

Density of wet snow = 12 lbs/ft³, so:

\[(26.44 \text{ ft}^3) (12 \text{ lbs/ft}^3) = 317.33 \text{ lbs snow}\]

The tie down force was determined by calculating the lift on the horizontal tail per the conditions in the statement of work. The standard lift equation was used in conjunction with the 3-D lift curve slope equation as shown below:

\[L = \left[ \frac{a_0}{1 + \frac{57.3 \alpha}{\sigma}} \right] \alpha \frac{1}{2} p V^2 S = 755 \text{ lbs}\]

Where:  
\(a_0 = 0.11 \text{ per } ^\circ\) (2-D lift curve slope of NACA 0009)  
\(r = 0.85\) (from Perkins and Hage for the horiz. stabilizer)  
\(\alpha = 10^\circ\) (angle of attack at +10° tie down angle)  
\(p = \text{density at sea level}\)  
\(V = 176 \text{ fps}\) (gust velocity equal to 120 mph)  
\(S = 25 \text{ ft}^2\) (horizontal tail area)

In short, the skin was designed to absorb torsion while the stringers and formers were sized to transmit bending. Fatigue analysis is analyzed throughout the structural substantiation section for cyclically loaded components.
4. Structural Substantiation
4.1 Sizing the Horizontal Stabilizer

Sizing the Front and Rear Spars of the Horizontal Stabilizer

Note: The material chosen for all structures in the horizontal stabilizer was 2024-T3 aluminum. This material has a axial yield strength of 42 ksi, ultimate shear strength of 39 ksi, and a bearing yield strength of 88 ksi. The values were taken from MIL-HDBK-5E (See Appendix 3).

Front Spar: The front spar was placed at 6.25 inches from the leading edge of the stabilizer. This was done for three reasons: it is near the location of the cruise loading at 7.0 inches as well as the maximum thickness of the airfoil and separates the skin into evenly sized panels. From the Theory of Wing Sections, the height of the front spar along the span is known (See Appendix 4).

The front spar was sized at 5.0 inches from the centerline of the airplane, the initial position of the first rib. This rib was later moved to 7.0" inches due to an error in estimating the width of the fairing. The lever rule was used to determine the portion of the moments carried by the spar due to each type of loading, cruise and maneuvering. The moment produced during cruise is carried mainly (91.9%) by the front spar. The torsional stresses caused by the cruise and maneuvering loads were negligible when compared to the magnitude of the bending stress produced during cruise. Therefore, this cruise moment is the design criterion.

\[ h = 2.68 \text{- in} \quad A = 0.90 \text{- in}^2 \quad d = 1.34 \text{- in} \]

\[ I_1 = \frac{1}{12} \cdot h^3 + 2 \cdot A \cdot d^2 \quad I_1 = 5.13 \text{- in}^3 \]

\[ M = 11063 \text{- lb} \cdot \text{in} \quad c = 1.34 \text{- in} \quad \sigma_y = 42000 \text{- lb} \cdot \text{in}^{-2} \]

\[ \sigma_y = \frac{919 \cdot M \cdot c}{I_1} \quad t = 0.0632 \text{- in} \]

Therefore, a 0.032" sheet was chosen with a 0.04" doubler.

After performing this same analysis at several points down the span, it was determined that the doubler could be removed at 37.0 inches outboard on the span.

Fatigue Analysis: In order to meet the design specification of 10^7 cycles, the maximum loading could not exceed a thirty thousand psi limit load at a mean stress of ten thousand psi (See Appendix 5). Therefore, the added moment of inertia needed to produce these results was determined.

\[ \sigma_f = 30000 \text{- lb} \cdot \text{in}^{-2} \quad \sigma_f = \frac{919 \cdot M \cdot c}{I_2} \quad I_2 = 0.4541 \text{- in}^4 \]

\[ I_{\text{add}} = I_2 - I_1 \quad I_{\text{add}} = 0.1358 \text{- in}^4 \quad \text{MS fatigue} = 0.017 \]

To increase the moment of inertia, two L- shaped pieces were added to the center of the C channel as shown. These produce a moment of inertia of 0.1434 in^4. Similar fatigue analyses were performed along the span to determine where the additions could be removed. It was determined that they were only necessary up to 22.0 inches along the span.

Rear Spar: The rear spar was analyzed in an identical manner. The design criterion for this spar was the maneuvering moment of which 69.7% is carried by the rear spar.

\[ h = 2.19 \text{- in} \quad A = 0.90 \text{- in}^2 \quad d = 1.095 \text{- in} \quad I_3 = 3.04 \text{- in}^3 \]

\[ M = 10193 \text{- lb} \cdot \text{in} \quad c = 1.095 \text{- in} \quad \sigma_y = \frac{697 \cdot M \cdot c}{I_3} \quad t = 0.0609 \text{- in} \]

Therefore, with a thickness of 0.0609 inches, a sheet of 0.032 inches and doubler of 0.040 inches were chosen as with the front spar. Again, this doubler ends at 37.0 inches outboard along the spar.
Fatigue Analysis:

\[
\sigma_f = \frac{0.697 \cdot M \cdot c}{I_4} \quad l_4 = 0.2593 \cdot \text{in}^4 \quad I_{\text{add}} = l_4 - l_3 \\
I_{\text{add}} = 0.0626 \quad MS \text{ fatigue} = 0.028
\]

The L-shaped pieces which were added give an added moment of inertia of 0.070 \text{in}^4. These pieces will extend to 22.0 inches along the spar where they are no longer needed.

Sizing the Skin Thickness On The Horizontal Stabilizer

A multi-cell analysis was used to size this portion of the horizontal stabilizer. After determining the shear flow on the skin, the shear stress was set equal to the equation derived from the sheet buckling criterion to determine the proper skin thickness. Each flange area was determined by summing the flange area from the C channel, the area of the web one characteristic distance from the flange, and the area of the skin one characteristic distance in each direction from the flange.

\[
A_f = 0.1602 \cdot \text{in}^2 \quad V_y = -273.0 \cdot \text{lb} \quad I_z = \sum A_i y_1^2 = 0.7667 \text{in}^4 \\
P_f = \frac{V_y \cdot y \cdot A_f}{I_z} \quad P_f = -49.1 \cdot \text{lb} \cdot \text{in}^{-1} \quad P_1 = 65.3 \cdot \text{lb} \cdot \text{in}^{-1} \\
P_2 = 59.3 \cdot \text{lb} \cdot \text{in}^{-1} \quad P_4 = -P_1 \quad P_3 = -P_2
\]

By inspection, \(q_3\) is equal to \(q_1\).

\[
\sum M_1 : q_o = \frac{2019 \cdot \text{lb} \cdot \text{in} - 40.42 \cdot \text{in}^2 : q_4}{67.9 \cdot \text{in}^2}
\]

\[
\phi_1 = \phi_2 : q_o = \frac{4049.5 \cdot \text{lb} \cdot \text{in}^{-1} - 18.5 \cdot \text{in}}{5.3 \cdot \text{in} + 32.5}
\]

If a sheet thickness of 0.02 inches is tested, the results are as follows.

\[
q_o = -13.83 \cdot \text{lb} \cdot \text{in}^{-1} \quad q_1 = -7.53 \cdot \text{lb} \cdot \text{in}^{-1} \quad q_2 = -66.83 \cdot \text{lb} \cdot \text{in}^{-1} \quad q_3 = -7.53 \cdot \text{lb} \cdot \text{in}^{-1} \quad q_4 = 71.6 \cdot \text{lb} \cdot \text{in}^{-1}
\]

\(K_s = 10.2\) (this is for a 15.0" by 9.25" sheet - see Appendix 6 for graph) \(E = 10 \cdot 10^6 \cdot \text{lb} \cdot \text{in}^{-2}\)

\[
f_{\text{crit}} = K_s \cdot E \left(\frac{t}{b}\right)^2 \cdot f_{\text{crit}} = 476.8 \cdot \text{lb} \cdot \text{in}^{-2} \quad f_{q_1} = 376.5 \cdot \text{psi} \quad MS = 0.266
\]

Therefore, 0.02" is the skin and rib thickness since all panels are the same size.

Sizing the Lightening Holes in the Spars and Ribs

For the front spar at 7.0 inches along the half-span:

\[
q = q_4 \quad t = 0.072 \cdot \text{in} \quad h = 2.68 \cdot \text{in} \quad f_s = \frac{q}{t} \quad f_s = 994.4 \cdot \text{psi} \quad F_o = 24000 \cdot \text{psi}
\]

\(K_1 = \frac{f_s}{F_o} \quad K_1 = 0.1\) Using the optimum \(K_1\) curve: \(d = 0.76 \cdot \text{h}\)

\[
F = \frac{d}{0.75} \quad b = 2.67 \cdot \text{in}
\]

For the front spar at 37.0 inches along the half-span.

\[
q = q_4 \quad t = 0.032 \cdot \text{in} \quad h = 1.98 \cdot \text{in} \quad f_s = 2237.5 \cdot \text{psi} \quad F_o = 18500 \cdot \text{psi} \quad K_1 = 0.121
\]

\(d = 0.76 \cdot \text{h}\) \(d = 1.50 \cdot \text{in} \quad b = \frac{d}{0.75} \quad b = 1.88 \cdot \text{in}\)
The same analysis was used for the rear spar and the ribs. The results of these analyses are shown in the table below. (Note these are the allowed values and not necessarily the values actually used.)

<table>
<thead>
<tr>
<th>Structure</th>
<th>Length Affected</th>
<th>Hole Diameter</th>
<th>Spacing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front Spar</td>
<td>7.0&quot; - 22.0&quot;</td>
<td>2.00&quot;</td>
<td>2.67&quot;</td>
</tr>
<tr>
<td>Front Spar</td>
<td>22.0&quot; - 52.0&quot;</td>
<td>1.50&quot;</td>
<td>1.88&quot;</td>
</tr>
<tr>
<td>Front Spar</td>
<td>67.0&quot;</td>
<td>1.25&quot;</td>
<td>1.53&quot;</td>
</tr>
<tr>
<td>Rear Spar</td>
<td>7.0&quot; - 22.0&quot;</td>
<td>1.50&quot;</td>
<td>2.00&quot;</td>
</tr>
<tr>
<td>Rear Spar</td>
<td>22.0&quot; - 37.0&quot;</td>
<td>1.20&quot;</td>
<td>1.60&quot;</td>
</tr>
<tr>
<td>Rear Spar</td>
<td>37.0&quot; - 52.0&quot;</td>
<td>1.00&quot;</td>
<td>1.40&quot;</td>
</tr>
<tr>
<td>Rear Spar</td>
<td>67.0&quot;</td>
<td>0.50&quot;</td>
<td>0.87&quot;</td>
</tr>
<tr>
<td>Ribs</td>
<td>7.0&quot; - 22.0&quot;</td>
<td>1.50&quot;</td>
<td>1.88&quot;</td>
</tr>
<tr>
<td>Ribs</td>
<td>22.0&quot; - 37.0&quot;</td>
<td>1.35&quot;</td>
<td>2.00&quot;</td>
</tr>
<tr>
<td>Ribs</td>
<td>37.0&quot; - 52.0&quot;</td>
<td>1.00&quot;</td>
<td>1.40&quot;</td>
</tr>
<tr>
<td>Ribs</td>
<td>67.0&quot;</td>
<td>0.60&quot;</td>
<td>0.90&quot;</td>
</tr>
</tbody>
</table>

**The lengths are with respect to the center line of the aircraft**

**Sizing the Fuselage Interface**

**Front Spar:**  \[ R_1 = 1.64 \text{ in} \quad R_2 = 1.95 \text{ in} \]

\[
M = 11947 \text{ lb-in} \quad R_1 = 0.841 \cdot R_2 \quad P_1 = 0.841 \cdot P_2
\]

\[
\sum P_i R_i = M \quad P_1 = 754.0 \text{ lb} \quad P_2 = 897 \text{ lb} \quad FF = 1.5
\]

\[
f_{brgy} = 88000 \text{ psi} \quad P = 717.0 \text{ lb} \quad f_{brg} = \frac{P \cdot FF}{t \cdot d}
\]

If a bolt of 0.25" in diameter is selected, and \( f_{brg} \) equals \( f_{brgy} \), then the channel thickness is required to be 0.049". However, after fatigue analysis, it was determined that 0.25" bolts must be used with a plate thickness of 0.082" in addition to the spar.

\[
MS_{fatigue} = 0.074
\]

**Rear Spar:** The same analysis was used for the fuselage interface at the rear spar. Before fatigue analysis, the channel thickness required if 0.25" diameter bolts were used was 0.049". However, after fatigue analysis, it was determined that the rear interface structure have 0.25" bolts in a thickness of 0.082" sheet in addition to the rear spar.
Sizing the Hinges On the Horizontal Stabilizer

\[ P = 84.0 \text{ lb (maximum hinge load)} \quad FF = 2.0 \]

\[ f_{brg} = 11000 \text{ psi} \quad f_{brg} = \frac{P \cdot FF}{A_{\text{bolt}}} \]

Therefore, the minimum required area of the bolt is 0.0153 in². If a 0.25" bolt is chosen, then the minimum required length, \( L \), is:

\[ L = \frac{A_{\text{bolt}}}{d} \quad L = 0.0612 \text{ in} \]

Therefore, a length of 0.1" was chosen.

\[ f_{\text{tearout}} = 26000 \text{ psi} \quad f_{\text{tearout}} = \frac{P \cdot FF}{2 \cdot x \cdot L} \quad x = 0.032 \text{ in} \]

Therefore, an \( x \) value of 0.1" was chosen.

\[ f_{\text{tension}} = 47000 \text{ psi} \quad f_{\text{tension}} = \frac{P \cdot FF}{x \cdot L} \quad f_{\text{tension}} = 16800 \text{ psi} \quad MS = 1.798 \]

4.2 SIZING THE VERTICAL STABILIZER

The first step in stabilizer design was sizing of the front spar. 2024-T3 aluminum with a yield strength of 42 ksi was chosen for the entire fin structure. The worst case bending for the spar was evaluated to determine the necessary moment of inertia, which then determined the required flange areas for the spar as shown.

Judging from what other aircraft manufacturers used, a thickness of 0.05" was chosen for the spars. The flange length is then usually about twenty thicknesses, which would be 0.4". However, 0.7" was used for the flange length to allow ample space for riveting. Skin thickness was chosen as 0.02" to be within the 3:1 spar to skin ratio. The flange area was then estimated according to the diagram on the above right. The shear flows in the skin and the webs of the spars were then calculated by the same method shown in the section titled Sizing the skin thickness on the horizontal stabilizer. To obtain
representative values for the stabilizer, the shear flows were calculated at water lines 46, 59.5, and 77, with the results given in the following table in units of pounds per inch.

<table>
<thead>
<tr>
<th></th>
<th>W.L. 46</th>
<th>W.L. 59.5</th>
<th>W.L. 77</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29.57</td>
<td>16.90</td>
<td>5.72</td>
</tr>
<tr>
<td>2</td>
<td>61.13</td>
<td>36.90</td>
<td>14.60</td>
</tr>
<tr>
<td>3</td>
<td>14.92</td>
<td>8.10</td>
<td>2.33</td>
</tr>
<tr>
<td>4</td>
<td>37.40</td>
<td>24.20</td>
<td>10.91</td>
</tr>
</tbody>
</table>

Each of the shear panels and spars was then checked for buckling using the equations below. The table on the right shows the critical stress, the stress in each panel, as well as the margin of safety.

\[
K_s = 75
\]

\[
E = 10^{6} \text{lb/inch}^2
\]

\[
t = 0.02 \text{inches}
\]

\[
b = 12 \text{inches}
\]

\[
q = 29.6 \text{lb/inch}
\]

\[
F_{cr} = K_s E \left( \frac{t}{b} \right)^2 = 2083 \text{lb/inch}^2
\]

\[
F_{\text{panel}} = \frac{q}{t} = 1480 \text{lb/inch}^2
\]

<table>
<thead>
<tr>
<th>Section</th>
<th>F\text{panel} \text{ psi}</th>
<th>F_{cr} \text{ psi}</th>
<th>M.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Root nose</td>
<td>1480</td>
<td>2083</td>
<td>0.41</td>
</tr>
<tr>
<td>Root main</td>
<td>745</td>
<td>586</td>
<td>-0.21</td>
</tr>
<tr>
<td>Mid nose</td>
<td>845</td>
<td>1920</td>
<td>1.27</td>
</tr>
<tr>
<td>Mid main</td>
<td>405</td>
<td>360</td>
<td>-0.11</td>
</tr>
<tr>
<td>Tip nose</td>
<td>286</td>
<td>3109</td>
<td>9.87</td>
</tr>
<tr>
<td>Tip main</td>
<td>119</td>
<td>718</td>
<td>5.03</td>
</tr>
<tr>
<td>Spar root</td>
<td>1678</td>
<td>12755</td>
<td>6.60</td>
</tr>
<tr>
<td>Spar mid</td>
<td>895</td>
<td>14168</td>
<td>14.8</td>
</tr>
<tr>
<td>Spar tip</td>
<td>333</td>
<td>21852</td>
<td>64.6</td>
</tr>
</tbody>
</table>

As the table shows, the panel stress exceeds the critical stress in the main panels of the root and mid-section. A stringer that runs through the middle of the main panels from the root to the mac was added to prevent this buckling. Because the spars undergo considerable cyclic loading, calculations were performed to confirm the required 10^7 cycles. The values for f_{max} and f_{mean} were checked against the fatigue graph (Appendix 5) to insure the proper life. The spar as previously designed failed before the required number.
of cycles. Therefore, two L-doublers were added to the top and bottom of the main spar. With the doublers, the limit load on the spar is 27470 psi which gives a margin of safety for cyclic loading of 0.13. The L-doublers run approximately 26 inches along the spar starting at the fuselage. The stabilizer-fuselage interface pieces were designed to rivet to bottom portion of each spar and extend several inches down into the fuselage where each would be bolted to a former.

\[ f_{\text{limit}} = 27.47 \text{ksi} \]
\[ f_{\text{max}} = \frac{f_{\text{limit}}}{2} = 13.74 \text{ksi} \]
\[ f_{\text{mean}} = \frac{f_{\text{limit}}}{3.8} = 7.23 \text{ksi} \]

4.3 Sizing the Rudder

The sizing of the rudder front spar proceeded just as the calculations for the vertical stabilizer. The same material was used for simplicity and reduced manufacturing costs. However, 0.02" sheet with a 0.5" flange width was sufficient for the spar because of the relatively small loads. The tables below show the shear flows (lb/in) in the skin and spars, the critical stresses for buckling, and the corresponding margins of safety. Three evenly spaced ribs in combination with the main spar were

\[ \frac{P_1}{R_1} = \frac{P_2}{R_2} = \ldots \]
\[ \frac{P_1}{R_1} = 1.82 = 0.8 \]
\[ \frac{P_2}{R_2} = 2.28 \]
\[ \sum P_1 R_1 = M = 14472 \text{ inchlb} \]
\[ 4P_1 R_1 + 4P_2 R_2 = 14472 \]
\[ P_1 = 968.4 \text{ lb} \]
\[ P_2 = 774.7 \text{ lb} \]
sufficient for the rudder structure.

<table>
<thead>
<tr>
<th>Location</th>
<th>W.L.51.9</th>
<th>W.L.72.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skin</td>
<td>0.449</td>
<td>0.639</td>
</tr>
<tr>
<td>Spar</td>
<td>15.81</td>
<td>20.06</td>
</tr>
</tbody>
</table>

The rudder hinges were designed following the same procedure as that shown for the horizontal stabilizer hinges. The hinge is made of two 0.02" sheets plus two doublers (0.032" each) on the outside to give l=0.104". The x distance is 0.2" and the bolt diameter is 0.25". The critical factor in the hinge design was the bearing strength of the oilite bearing. With a load of 90 lb, the hinge with the doublers yielded a margin of safety of 0.59. The mate hinge on the stabilizer is essentially two rudder hinges side by side. However, the doublers were placed on the hinge inner walls for the smallest possible tolerance between the mating hinges.

4.4 Sizing the Elevator

Elevator sizing was similar to the rudder analysis. The same size spar was used to produce the following results for shear flows (lb/in) and buckling stresses. Cyclic loading was not critical.

<table>
<thead>
<tr>
<th>Location</th>
<th>B.L.22</th>
<th>B.L.52</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skin</td>
<td>6.6</td>
<td>5.95</td>
</tr>
<tr>
<td>Spar</td>
<td>18.14</td>
<td>41.65</td>
</tr>
</tbody>
</table>

The inboard skin panel made it necessary to add an auxiliary spar to prevent that panel from buckling. The elevator hinges have the same critical dimensions as the stabilizer hinges.
4.5 Sizing the Tail Cone

In order to simplify the analysis of the empennage, the tail cone was assumed to be a right circular cone and thus the stringer taper angles were constant in all planes. The analysis was then divided into seven areas: skin sizing for torsion, stringer design and former location, stringer fatigue, control surface interface former design, fuselage interface connection, tie down provision, and access panel location.

Skin Sizing:

The skin was chosen to be aluminum 2024-T3 to minimize weight and cost. The skin panels were chosen to be semicircular for easy manufacturing and assembly to the frame. Panel dimensions were determined by former locations based on the preliminary structural design. The initial 0.032" thick aluminum sheet was reduced to 0.025" due to sufficient critical buckling strength. Both the calculated and critical shear stresses for each section were determined as follows.

Shear flow equation for Panel A:
\[
q = \frac{T}{2A} = \frac{16920 \text{ in.lb}}{2(380.133 \text{ in}^2)} = 22.3 \text{ lb/in}
\]

Shear stress for panel:
\[
\sigma_{\text{shear}} = \frac{q}{t} = \frac{22.3}{0.025 \text{ in.}} = 892 \text{ psi}
\]

Allowable shear stress in Panel A given by:
\[
\sigma_{\text{critical}} = K_S E \left(\frac{t}{b}\right)^2 = 1831 \text{ psi}
\]

Where: \( K_S = 300 \) (panel stiffness factor from Niu Figure 5.4.8)
\( E = 10 \times 10^6 \text{ psi} \) (modulus of elasticity)
\( t = 0.025 \text{ in.} \) (skin thickness)
\( b = 32 \text{ in.} \) (panel length)
### Skin Panel Shear Stress Results

<table>
<thead>
<tr>
<th>Panel</th>
<th>Length (in)</th>
<th>Shear flow, q (lb/in)</th>
<th>Shear Stress (Ks, psi)</th>
<th>f_shr_act (psi)</th>
<th>f_shr_crit (psi)</th>
<th>Margin of Safety</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>32</td>
<td>22.3</td>
<td>300</td>
<td>892</td>
<td>1831</td>
<td>1.05</td>
</tr>
<tr>
<td>B</td>
<td>25</td>
<td>29.8</td>
<td>250</td>
<td>1192</td>
<td>2500</td>
<td>1.09</td>
</tr>
<tr>
<td>C</td>
<td>32</td>
<td>47.9</td>
<td>250</td>
<td>1916</td>
<td>2473</td>
<td>0.29</td>
</tr>
<tr>
<td>D</td>
<td>10</td>
<td>63.8</td>
<td>90</td>
<td>2552</td>
<td>5625</td>
<td>1.20</td>
</tr>
<tr>
<td>E</td>
<td>28</td>
<td>89.1</td>
<td>200</td>
<td>3564</td>
<td>3830</td>
<td>0.07</td>
</tr>
</tbody>
</table>

**Stringer Design and Former Location:**

The bending moment was first calculated at the left of each stringer length from the preliminary former locations. Since the 931 lb resultant load could be produced by four flight conditions, four stringers were used. The final configuration positioned the stringers at 45° to the aircraft axes as shown in figure below.

![Stringer Design Diagram](image)

A stringer moment of inertia, $I_{str}$, of 0.02 in$^4$ was assumed. By equating the bending stress formula, $f_{bend} = \frac{M_y}{I}$, and the critical buckling formula, $f_{crit} = \frac{(\pi^2 E I)}{(A l^2)}$, the equation below was derived to determine the maximum stringer length, $L$, using the assumed $I$ value.

$$L = \sqrt{\frac{(197 \times 10^6) (d_1 + d_2) I_{str}}{M_y}}$$

Where:  
- $d_1$ and $d_2$ = see figure above  
- $M_y$ = bending moment (in.lb)  
- $I_{str} = 0.02$ in$^4$ estimated stringer property  
- $y$ = distance to outermost stringer (same as $d_1$)

The stringer length dictated former placement. The stringer cross section was then designed using the skin thickness of 0.025" and the $I$ value. This cross section can be seen in Drawing 421S9303B206. Below are the results from the above length equation applied to the same stringer/former locations used during skin sizing.
Critical Stringer Lengths and Former Locations

<table>
<thead>
<tr>
<th>STA</th>
<th>Moment, ( \text{in/lb} )</th>
<th>( d_1 = y )</th>
<th>( d_2, \text{in} )</th>
<th>Length, ( L )</th>
<th>Former STA</th>
</tr>
</thead>
<tbody>
<tr>
<td>169</td>
<td>93100</td>
<td>11.1</td>
<td>3</td>
<td>22.4</td>
<td>191.4</td>
</tr>
<tr>
<td>191.4</td>
<td>72245</td>
<td>10.2</td>
<td>2.74</td>
<td>24.4</td>
<td>215.8</td>
</tr>
<tr>
<td>215.8</td>
<td>49530</td>
<td>9.2</td>
<td>2.4</td>
<td>28</td>
<td>243.8</td>
</tr>
</tbody>
</table>

The lengths indicated that another former must be placed between b and c of the initial design. Since the data showed that formers a, b, and the new former could be spaced 20 inches apart without affecting the critical length or skin buckling, this was done to move weight closer to the aircraft c.g.. The stringer between the new former and former c was then checked for buckling using the 20-inch former spacing.

<table>
<thead>
<tr>
<th>STA</th>
<th>Moment, ( \text{in/lb} )</th>
<th>( d_1 = y )</th>
<th>( d_2, \text{in} )</th>
<th>Length, ( L )</th>
<th>Former STA</th>
</tr>
</thead>
<tbody>
<tr>
<td>229 (new)</td>
<td>37240</td>
<td>8.2</td>
<td>2.2</td>
<td>30.5</td>
<td>259.5</td>
</tr>
</tbody>
</table>

Former c was not moved since it was initially located at STA 258 and the critical length exceeds that location. This shows that the stringer between the new former and c will not buckle.

The curved, skin panels D and E were then analyzed for their buckling resistance to compressive loads without stringer reinforcement. Niu Figure 5.4.4 provided the equation for the critical buckling stress, \( f_{\text{crit}} \). This value was compared to the actual bending stress.

For Panel D:

\[
f_{\text{crit}} = k_c \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{b}\right)^2 = 15182 \text{ psi}
\]

Where:
- \( k_c = 1300 \) (from Niu Figure 5.4.4)
- \( E = 10 \times 10^6 \text{ psi for aluminum} \)
- \( \mu = 0.301 \) (Poisson's ratio for aluminum)
- \( t = 0.025 \text{ in} \) (skin thickness)
- \( b = 22 \text{ in} \) (panel circumference)

Actual bending stress for Panel D:

\[
f_{\text{actual}} = \frac{M r_1}{\frac{1}{2}(r_1^2 - r_2^2)} = 1222.6 \text{ psi}
\]

Where:
- \( M = (931 \text{ lbs})(10 \text{ in}) = 9310 \text{ in.lb} \)
- \( r_1 = 7 \text{ in} \) (outside radius)
- \( r_2 = 6.95 \text{ in} \) (inside radius)
Curved Panel Buckling Analysis Results

<table>
<thead>
<tr>
<th>Panel</th>
<th>Length</th>
<th>b, in</th>
<th>OD rad</th>
<th>ID rad</th>
<th>Moment</th>
<th>Kc</th>
<th>fcrit, psi</th>
<th>f actual, psi</th>
<th>M.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>10</td>
<td>22</td>
<td>7</td>
<td>6.95</td>
<td>9310</td>
<td>130</td>
<td>1518</td>
<td>1222</td>
<td>11.4</td>
</tr>
<tr>
<td>E</td>
<td>28</td>
<td>20.</td>
<td>6.5</td>
<td>6.45</td>
<td>10472</td>
<td>195</td>
<td>2648</td>
<td>1596</td>
<td>15.5</td>
</tr>
</tbody>
</table>

These results show that no stringers are necessary after STA 258.

After the sizing calculations, the new empennage configuration was developed and is shown in Drawing 421S9303B206. A simplified version used in future calculations is shown in the figure below.

Stringer Fatigue Life:

Fatigue analysis for each stringer section was determined per the requirements in the statement of work. This was done by finding the limit bending stress, \( f_{\text{bend LL}} \), the maximum cyclic stress, \( f_{\text{cycmax}} \), and the mean stress, \( f_{\text{mean}} \). These values were then compared in Niu Figure 15.4.5 at 10^7 cycles.

For Section A:

\[
I = 2A(d_1^2 + d_2^2) = 26.44 \text{ in}^4
\]

Where: \( A = 0.1 \text{ in}^2 \) (stringer area - constant in all sections)
\( d_1 = 11.1 \text{ in} \) and \( d_2 = 2.97 \text{ in} \) (see Figure xxxxxxxxx)

I was used to find the limit bending stress for Section A:

\( f_{\text{bend LL}} = \frac{Mr}{I} = 40490 \text{ psi} \)

Where: \( M = (931 \text{ lbs}) (100 \text{ in}) = 93100 \text{ in.lb} \)
\( r = 11.5 \text{ in} \) (section radius)

Maximum cyclic stress was then:

\( f_{\text{cycmax}} = \frac{f_{\text{bend LL}}}{2} = 20245 \text{ psi} \)

Mean stress calculated from:

\( f_{\text{mean}} = \frac{f_{\text{bend LL}}}{n} = 10655 \text{ psi} \)

Where: \( n = 3.8 \) (aircraft load factor)

Niu Figure 15.4.5 indicates that 10^7 cycles is acceptable for this stringer.

The fatigue stresses for the other sections are listed below.
Stringer Fatigue Analysis and Stresses

<table>
<thead>
<tr>
<th>Section</th>
<th>d1, in</th>
<th>d2, in</th>
<th>r, in</th>
<th>l, in^4</th>
<th>M, in-lb</th>
<th>f, psi</th>
<th>bend LL, psi</th>
<th>cycmax</th>
<th>mean, psi</th>
<th>M.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10.</td>
<td>2.8</td>
<td>11.</td>
<td>26.4</td>
<td>9310</td>
<td>40490</td>
<td>20245</td>
<td>10655</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>7</td>
<td>5</td>
<td>4</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>10.</td>
<td>2.8</td>
<td>11.</td>
<td>24.2</td>
<td>7448</td>
<td>33854</td>
<td>16930</td>
<td>8908</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>10.</td>
<td>2.7</td>
<td>10.</td>
<td>22.0</td>
<td>5586</td>
<td>26600</td>
<td>13300</td>
<td>7000</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>8.2</td>
<td>2.2</td>
<td>8.5</td>
<td>14.4</td>
<td>3724</td>
<td>21900</td>
<td>10952</td>
<td>5764</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

All stringers are capable of $10^7$ life cycles.

Interface Former Design:

The formers at STA 258, 268, 278, and 301 (Drawing 421S9303B206) are located where the C-channel spars of the control surfaces mate with empennage. The forces in the bolts of the interface connections for both the stabilizer and vertical fin were designed according to the control surfaces loads. Each interface has eight bolts, two at each spar flange and four in the web. The flange bolts attach to an L-bracket riveted to the former, while the center bolts pass through the former. Since the former is made of 0.025" aluminum sheet, a doubler plate behind the spar interface was necessary to provide sufficient material through which to rivet the L-brackets.

The rivets mentioned above were sized to withstand the bolt shear transferred to the L-bracket, the vertical shear imposed by the lift of the control surface, and the resulting bearing stress developed under cyclic loading. The L-brackets and doubler plates were sized according to rivet requirements for material thickness. After the analysis of all four interface formers, the size of the rivets, L-brackets, and doubler plates did not change from those of the first. The analysis of the bolt loads at the first interface former (STA 258) is given on the next page.
Resultant load of the $S$ shears: 

$$R = 2 (897 \text{ lbs}) \cos (104 / 2) = 1104.5 \text{ lbs}$$

Min. number of $1/8''$ dia. rivets in L-bracket: 

$$1104.5 \text{ lbs} / (368 \text{ lbs / rivet}) = 3$$

rivets.

The L-bracket is 0.1'' aluminum. Using 7, $1/8''$ dia. rivets spaced between 0.75 and 1 inches, rivet bearing stress under cyclic loading is as follows:

Bearing stress per rivet:

$$f_{bng} = \frac{(11045 \text{ lbs / 7 rivets})}{(0.1 \text{ in})(0.125 \text{ in})} = 12662.8 \text{ psi}$$

Maximum cyclic stress: 

$$f_{cycmax} = f_{bng} / 2 = 6311.4 \text{ psi}$$

Mean stress: 

$$f_{mean} = f_{bng} / 3.8 = 3321.8 \text{ psi}$$

From Niu Figure 15.4.5 these rivets will not fail under 107 cycles.

The maximum rivet spacing distance was computed by finding the shear flow in half the former. The 400-lb vertical shear from one side of the horizontal stabilizer was used along with geometric information from the former.

Shear flow:

$$q = \frac{VQ}{I} = 63.61 \text{ lb/in}$$

Where: \( V = 400 \text{ lb} \)

\( Q = 15.46 \text{ in}^3 \) (at the first row of rivets in the L-bracket)

\( I = 97.21 \text{ in}^4 \) (for the area above the first rivet row)

Maximum rivet spacing:

$$d = \frac{(368 \text{ lb/rivet})}{q} = 5.78 \text{ inches}$$

Since this distance is much greater than the actual spacing, the rivets, L-brackets, and former will handle all forces imposed by the control surface. The last column in the table below is the number of $1/8''$ dia. rivets per L-bracket.
### Interface Former Statistics

<table>
<thead>
<tr>
<th>STA</th>
<th>Bolt Ld. (lb)</th>
<th>Reslnt (lb)</th>
<th>g (lb/in)</th>
<th>fbng (psi)</th>
<th>fcycmx (psi)</th>
<th>fmean (psi)</th>
<th>V (lb)</th>
<th>M.S.</th>
<th># Rivets</th>
</tr>
</thead>
<tbody>
<tr>
<td>258</td>
<td>897</td>
<td>1104</td>
<td>63.6</td>
<td>1262</td>
<td>6311</td>
<td>3321</td>
<td>400</td>
<td>1.0</td>
<td>7</td>
</tr>
<tr>
<td>268</td>
<td>629</td>
<td>774.5</td>
<td>82.0</td>
<td>8851</td>
<td>4425</td>
<td>2329</td>
<td>400</td>
<td>1.0</td>
<td>7</td>
</tr>
<tr>
<td>278</td>
<td>968</td>
<td>1747</td>
<td>46.4</td>
<td>1997</td>
<td>9985</td>
<td>5242</td>
<td>476</td>
<td>1.0</td>
<td>7</td>
</tr>
<tr>
<td>301</td>
<td>932</td>
<td>1682</td>
<td>45.7</td>
<td>1682</td>
<td>8412</td>
<td>4427</td>
<td>476</td>
<td>1.0</td>
<td>8</td>
</tr>
</tbody>
</table>

**Fuselage Interface Design:**

The empennage is attached to the aft section of the fuselage by overlapping the skins of each section and joining two identical formers (STA 169). This can be seen in Drawing 421S9303B206. The rivets joining the two formers were sized and spaced to withstand the vertical shear and torsional loads developed by the control surfaces. The external rivets joining the skin were sized according to the bending moments imposed at the outer radius of the empennage.

Ninety-six, 3/32" dia. rivets spaced 0.75 inches apart join the formers. This was calculated using circumference and maximum allowable rivet spacing. Vertical shear and torsional analysis would also determine the maximum spacing as follows.

Shear flow in interface former:  
$q = \frac{VQ}{I} = 51.63 \text{ lb/in}$

Where:  
$V = 931 \text{ lbs (worst case resultant load from surfaces)}$
$q = 13.16 \text{ in}^3$ (from former geometry)
$I = 237.3 \text{ in}^4$ (from former geometry)

Maximum rivet spacing:  
$d = \frac{(206 \text{ lb/rivet})}{51.63 \text{ lb/in}} = 4 \text{ in.}$

The torsional moment about the former was divided by the number of rivets and the section radius yielding the shear force in each rivet in pounds:

Shear force = $(16900 \text{ in.lb}) / [(96 \text{ rivets}) (11 \text{ in.})] = 16.0 \text{ lb/rivet}$

Both the shear flow and the shear force were combined to determine the loading for the former rivets.

$(16.0 \text{ lb/rivet}) + [(0.75 \text{ in/rivet}) (51.63 \text{ lb/in})] = 54.72 \text{ lb/rivet}$
Since this loading is much less than the allowable shear force for a 3/32" dia. rivet (206 lb), one row of 96 rivets joining the interface formers is acceptable (M.S. = 2.76).

The external skin rivets were then analyzed in bending. The maximum bending moment, \( M \), of 93100 in.lb (acting at STA 169) is also developed from the worst case control surface loads. This moment was broken into a couple with a shearing force of 4047 lbs on the rivets. Dividing this force by the max. allowable rivet shear of 206 lbs determined that 20 rivets were required per rivet group. This was made into 4 rows of 96 rivets spaced at 0.75 inches circumscribing the interface. Any group of 20 rivets is capable of withstanding the moment.

Tie Down Provision:

The tie down provision (Drawing 421S9303B206) is a 7/16" dia., forged, eye bolt that is connected to the STA 268 former and protrudes from the bottom of the empennage. The tie down force of 775 lbs produces 1288.8 psi of pure tensile stress which is far below the allowable 9800 psi for this type of bolt (M.S. = 6.6). Since the tie down force is less than the 931 lb resultant and is applied at the same location, the stringers and/or skin will not deform. A skin doubler plate at the bottom of the former provides sufficient material through which to fasten the bolt.

5. Manufacturing and Maintenance

5.1 General Assembly

The aft empennage tail surfaces will be assembled in the following order. The elevator will be attached to the horizontal stabilizer. The horizontal tail will then be attached to the empennage. The vertical stabilizer will be mounted to the empennage, and, finally, the rudder to it. (See Appendix 1)
5.2 Horizontal Stabilizer and Elevator

The C-channel spars and L-shaped doublers are brake formed. The lightening holes are cut from the sheets used to make the spars before they are brake formed. The interface structures are cast from aluminum, and then machined for bolt holes and trim by-pass holes (See Figure 02). The ribs are sheared and hydropressed. The patterns for the lightening holes in the ribs are made into the blank. The skin panels are sheared. The hinges are made of two sheared sheets which are riveted together and press fitted with oilite bearings. (See Figure 08).

![Diagram of Elevator Hinge Detail]

Figure 08 Elevator Hinge Detail

There is an oval access panel which is centered 13.8" from the center line of the empennage to allow for maintenance of the trim screwjack and pulley in the horizontal stabilizer. The elevator also has an access panel for maintenance of the trim tab. There are numerous water drains in each half span of the tail surfaces. In the stabilizer, these drains are located near the 30 percent chord which is the location of maximum thickness, allowing water to pool. The elevator drains are located forward and aft of the front spar.
5.3 **Vertical Stabilizer and Rudder**

The manufacturing processes for these tail surfaces are identical to those used for the horizontal stabilizer and elevator. There are no drains specifically manufactured for these surfaces. Drains are automatically provided by the lightening holes in the bottoms of the stabilizer and rudder. These lightening holes also provide space to house the electrical systems which supply power to the navigational lights on top of the tail surface.

5.4 **Tail Cone**

The empennage skin is flat-wrapped aluminum sheet consisting of two, semicircular panels that cover the structure up to the former at the first interface (STA 258). One edge of the skin panel is break formed to construct the stringer. The panels are attached to the formers by riveting the bent edge of one panel to the clean edge of the other, forming the complete stringer (Refer to Formal Drawing 421S9303B206). The other two stringers are break formed C-channels which are riveted to the inside of the skin. The formers are stamped from blanks of aluminum sheet and rubber block pressed into a C-channel cross section. The skin along the top of the interface formers is a single panel cut to form around the surfaces and attached to the interface formers before the surfaces are connected.

There are three removable panels on the empennage. Two, round inspection panels are located on the sides of the empennage just aft of the STA 229 former, and the third panel is located just behind the last horizontal stabilizer interface (STA 268) on the underside of the empennage. Two smaller panels serve to provide inspection and maintenance of the control system fairlead assembly. A skin doubler plate around the hole inside the airframe provides panel reinforcement and a base for the fasteners. The large panel allows access to the control cables, elevator torque tube, torque tube bearings, and interface connections. The hole for this panel is reinforced by continuing the two bottom stringers to the last former. Each panel is fastened to the skin using several Camloc™ fasteners (Refer to Formal Drawing 421S9303B206).
6. Weight Summary

Table 2: Estimated Weight Summary

<table>
<thead>
<tr>
<th>Part No. (Dwg. #)</th>
<th>Title</th>
<th>Number Used</th>
<th>Weight per Part (lb)</th>
<th>Total Weight (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (02)</td>
<td>Front Spar (HS)</td>
<td>2</td>
<td>1.45</td>
<td>2.90</td>
</tr>
<tr>
<td>2 (02)</td>
<td>Rear Spar (HS)</td>
<td>2</td>
<td>1.13</td>
<td>2.26</td>
</tr>
<tr>
<td>5-9 (02)</td>
<td>Ribs (HS)</td>
<td>10</td>
<td>0.06</td>
<td>0.60</td>
</tr>
<tr>
<td>13 (02)</td>
<td>Skin (HS)</td>
<td>N/A</td>
<td>7.94</td>
<td>7.94</td>
</tr>
<tr>
<td>3 (02)</td>
<td>Front Fus. Inter. (HS)</td>
<td>2</td>
<td>1.81</td>
<td>3.62</td>
</tr>
<tr>
<td>4 (02)</td>
<td>Rear Fus. Inter. (HS)</td>
<td>2</td>
<td>1.60</td>
<td>3.20</td>
</tr>
<tr>
<td>1 (03)</td>
<td>Front Spar (Elev)</td>
<td>2</td>
<td>0.34</td>
<td>0.68</td>
</tr>
<tr>
<td>2 (03)</td>
<td>Rear Spar (Elev)</td>
<td>2</td>
<td>0.20</td>
<td>0.40</td>
</tr>
<tr>
<td>3-8 (03)</td>
<td>Ribs (Elev)</td>
<td>6</td>
<td>0.33</td>
<td>0.18</td>
</tr>
<tr>
<td>16 (03)</td>
<td>Skin (Elev)</td>
<td>N/A</td>
<td>5.06</td>
<td>5.06</td>
</tr>
<tr>
<td>1 (04)</td>
<td>Front Spar (VS)</td>
<td>1</td>
<td>1.11</td>
<td>1.11</td>
</tr>
<tr>
<td>2 (04)</td>
<td>Rear Spar (VS)</td>
<td>1</td>
<td>0.77</td>
<td>0.77</td>
</tr>
<tr>
<td>7-12 (04)</td>
<td>Ribs (VS)</td>
<td>4</td>
<td>0.22</td>
<td>0.88</td>
</tr>
<tr>
<td>25 (04)</td>
<td>Skin (VS)</td>
<td>N/A</td>
<td>6.25</td>
<td>6.25</td>
</tr>
<tr>
<td>5 (04)</td>
<td>Front Fuselage Interface (VS)</td>
<td>1</td>
<td>1.03</td>
<td>1.03</td>
</tr>
<tr>
<td>6 (04)</td>
<td>Rear Fuselage Interface (VS)</td>
<td>1</td>
<td>1.03</td>
<td>1.03</td>
</tr>
<tr>
<td>1 (05)</td>
<td>Front Spar (Rudder)</td>
<td>1</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>2 (05)</td>
<td>Rear Spar (Rudder)</td>
<td>1</td>
<td>0.18</td>
<td>0.18</td>
</tr>
<tr>
<td>3-7 (05)</td>
<td>Ribs (Rudder)</td>
<td>4</td>
<td>0.04</td>
<td>0.19</td>
</tr>
<tr>
<td>14 (05)</td>
<td>Skin (Rudder)</td>
<td>N/A</td>
<td>3.35</td>
<td>3.35</td>
</tr>
<tr>
<td>7 (03), 9 (05)</td>
<td>Con. Surf. Hinges (Female)</td>
<td>6</td>
<td>0.08</td>
<td>0.50</td>
</tr>
<tr>
<td>10 (02), 13 (03)</td>
<td>Con. Surf. Hinges (Male)</td>
<td>6</td>
<td>0.06</td>
<td>0.39</td>
</tr>
<tr>
<td>1 (06)</td>
<td>Stringers</td>
<td>4</td>
<td>0.62</td>
<td>2.48</td>
</tr>
<tr>
<td>12 (06)</td>
<td>Formers</td>
<td>8</td>
<td>0.40</td>
<td>3.20</td>
</tr>
<tr>
<td>1 (06)</td>
<td>Skin</td>
<td>N/A</td>
<td>27.08</td>
<td>27.08</td>
</tr>
<tr>
<td>N/A</td>
<td>Fasteners</td>
<td>N/A</td>
<td>6.00</td>
<td>6.00</td>
</tr>
</tbody>
</table>

Table 3: Target Weight Comparison

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Target Value</th>
<th>Actual Value</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Tail Weight</td>
<td>28.0 lb</td>
<td>27.29 lb</td>
<td>-2.54%</td>
</tr>
<tr>
<td>Vertical Tail Weight</td>
<td>14.0 lb</td>
<td>15.57 lb</td>
<td>11.21%</td>
</tr>
<tr>
<td>Aft Fuselage Weight</td>
<td>Less Than 130.0 lb</td>
<td>41.02 lb</td>
<td>N/A</td>
</tr>
<tr>
<td>Moment About Zero Station</td>
<td>Less Than 35,000 lb\cdot\text{in}</td>
<td>Less than 28,184 lb\cdot\text{in}</td>
<td>N/A</td>
</tr>
</tbody>
</table>

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7. Conclusions

The design presented illustrates the detailed design of the aft empennage, vertical stabilizer and rudder, and horizontal stabilizer and elevator as well as integration of the subsystems. All requirements set forth in the statement of work are met by the final design.

7.1 Horizontal Stabilizer

The ribs were first sized by assuming that they carried all of the applied moments. However, this caused the sizes to be unacceptably large. Therefore, the portion of the moments carried by each spar was determined. This resulted in acceptable spar sizes. After performing a fatigue analysis, L-shaped triplers had to be integrated into the design. The skin was first sized using a single cell, i.e. the front spar was removed for the analysis. This resulted in an abnormally thick skin. When the front spar was replaced and multi-cell analysis performed, the skin thickness was significantly reduced. The sizes of both the front and rear interfaces had to be increased once the fatigue analysis was applied. Increasing the length of the interfaces from 2.0" to 4.0" resulted in decreased bolt loads and, in turn, a smaller required thickness.

7.2 Vertical Stabilizer

After performing a fatigue analysis on the front spar, it was determined that a larger value for the moment of inertia was required. Therefore, L-shaped doublers were added. In the initial analysis, the buckling forces in the skin panels exceeded the critical value. In order to solve this problem, stringers were added between the front and rear spars. In order to integrate the rear interface structure, the bottom hinge had to be moved further away from the root of the stabilizer.

7.3 Tall Cone

The empennage assembly design was only modified once from the original concept. This modification required changing the former spacing to every 20 inches.
Triton Primary Flight Trainer
Structural Decomposition

Appendix 1
### Surface

<table>
<thead>
<tr>
<th>ORIZONTAL TAIL I</th>
<th>DIRECTION OF LOADING</th>
<th>MAGNITUDE OF LOADING</th>
<th>CHORDWISE DISTRIBUTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Up and Down</td>
<td>Figure A5 Curve (2)</td>
<td>100% ( W ) on one side airplane ( C ) for normal and utility categories. 69% ( W ) on other side airplane ( C ). For acrobatic category see A23.11(C)</td>
<td></td>
</tr>
<tr>
<td>b) Unsymmetrical loading (Up and Down)</td>
<td>100% ( W ) on one side airplane ( C ) for normal and utility categories. 69% ( W ) on other side airplane ( C ). For acrobatic category see A23.11(C)</td>
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<table>
<thead>
<tr>
<th>VERTICAL TAIL II</th>
<th>DIRECTION OF LOADING</th>
<th>MAGNITUDE OF LOADING</th>
<th>CHORDWISE DISTRIBUTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Right and Left</td>
<td>Figure A5 Curve (1)</td>
<td>Same as (A) above</td>
<td></td>
</tr>
<tr>
<td>b) Right and Left</td>
<td>Figure A5 Curve (1)</td>
<td>Same as (B) above</td>
<td></td>
</tr>
</tbody>
</table>

### Figure A5—Average limit control surface loading.

#### (1) VERTICAL TAIL

- \( \varphi = 3.66 \left( n_1 \frac{W}{S} \right)^{\frac{1}{2}} \) for \( n_1 \frac{W}{S} < 47 \) and \( \text{AR} \leq 2.0 \)
- \( \varphi = 0.534 \left( n_1 \frac{W}{S} \right) \) for \( n_1 \frac{W}{S} > 47 \)

#### (2) HORIZONTAL TAIL (UP & DOWN LOADS)

- \( \varphi = 4.8 + 0.534 \left( n_1 \frac{W}{S} \right) \)

#### Design Maneuvering Wing Loading \( n_1 \frac{W}{S} \) Pounds/Sq. Ft.

- \( \varphi = 0.78 \left( n_1 \frac{W}{S} \right) \left( C_n / 1.80 \right) \)
- \( \varphi = 0.64 \left( n_1 \frac{W}{S} \right) \left( C_n / 1.6 \right) \)
- \( \varphi = 0.466 \left( n_1 \frac{W}{S} \right) \)

#### Plots

- (3) Tab
- (4) Flap
### Table 3.2.3.0(b).

**Design Mechanical and Physical Properties of 2024 Aluminum Alloy Sheet and Plate**

<table>
<thead>
<tr>
<th>Specification</th>
<th>Sheet</th>
<th>Plate</th>
<th>Sheet</th>
<th>Plate</th>
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</thead>
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<tr>
<td>Form</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temper</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thickness, in.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basis</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Mechanical properties:

- $F_{th} \text{ ksi}$:
  - L
  - LT
  - ST

- $F_{tp} \text{ ksi}$:
  - L
  - LT
  - ST

- $F_{tu} \text{ ksi}$:
  - L
  - LT
  - ST

- $F_{tuv} \text{ ksi}$:
  - (c/D = 1.5)
  - (c/D = 2.0)

- $F_{tuv} \text{ ksi}$:
  - (c/D = 1.5)
  - (c/D = 2.0)

- e, percent (S-basis):
  - LT

- $E$, 10^3 ksi:
  - 10.7

- $E_c$, 10^3 ksi:
  - 10.7

- $G$, 10^3 ksi:
  - 4.0

- $\mu$:
  - 0.33

#### Physical properties:

- $\omega$, lb/in.:
  - 0.101

- See Figure 3.2.3.0

*See Table 3.1.2.1.1. Bearing values are "dry pin" values per Section 1.4.7.1.

*See Table 3.2.3.0(c).

*10% for 0.500 inch.
Appendix 4

THEORY OF WING SECTIONS

\[ (\psi)' \]

<table>
<thead>
<tr>
<th>( x / (\text{per cent} c) )</th>
<th>( y / (\text{per cent} c) )</th>
<th>( (x/V)^2 )</th>
<th>( u/V )</th>
<th>( \Delta p_a / V )</th>
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</thead>
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<td>0</td>
<td>0</td>
<td>0.585</td>
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<td>1.109</td>
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<td>1.145</td>
<td>0.560</td>
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<td>1.119</td>
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<td>4.502</td>
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<td>1.100</td>
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<td>80</td>
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<td>90</td>
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<td>0.982</td>
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<td>0.605</td>
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<td>0.966</td>
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<td>100</td>
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</table>

I.E. radius: 0.89 per cent c

NACA 0009 Basic Thickness Form

---

APPENDIX I

\[ (\psi)' \]

<table>
<thead>
<tr>
<th>( x / (\text{per cent} c) )</th>
<th>( y / (\text{per cent} c) )</th>
<th>( (x/V)^2 )</th>
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<td>100</td>
<td>0.105</td>
<td>0.853</td>
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</table>

I.E. radius: 1.10 per cent c

NACA 0010 Basic Thickness
Appendix 5

Fig. 5.4.5 Compression buckling coefficients $K_c$ (circular cylinders).

Fig. 5.4.6 Shear buckling coefficients $K_s$ (circular cylinders).
Fig. 5.4.7 Shear buckling coefficients $K_v$ (curved plates, $b$ at curved side).

Fig. 5.4.8 Shear buckling coefficients $K_v$ (curved plates, $b$ at straight side).

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Appendix 6

Fig. 15.4.4 Plots of the max. and min. stresses vs. $\Sigma$ cycles (flights).

Fig. 15.4.5 Best-fit S-N curves for notched, $K = 4.0$ of 2024-T3 aluminum alloy sheet, longitudinal direction (Ref. 4.1).

552 Airframe Structural Design

ORIGINAL PAGE IS OF POOR QUALITY
6.2 Lightly Loaded Beams

The ideal construction for most shear-carrying beams is a tension field (or diagonal tension beam per Ref. 6.8). However, in some cases it is advantageous, and in other cases necessary, to incorporate circular, flanged holes in the beam webs. These cases come under two main categories:
- Lightly loaded or very shallow beams. In such cases it may not be practical to construct an efficiently designed tension field beam because of minimum gage considerations and other restrictions due to the small size of the parts involved. It may then be advantageous from a weight standpoint to omit web stiffeners and, instead, introduce a series of standard flanged lightening holes, as shown in Fig. 6.2.1.
- Moderately loaded beams with access holes. Where it is necessary to introduce access holes into the web of a shear-carrying beam, a light, low cost construction is obtained by using a flanged hole with web stiffeners between the holes.

Lightly Loaded or Very Shallow Beams

The following two types of beam construction are considered. The standard flanged lightening holes as shown in Fig. 6.2.2 are centered and equally spaced.
- The limiting conditions for the design curves is given in Fig. 6.2.3.

<table>
<thead>
<tr>
<th>D (inch)</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
<th>4.5</th>
<th>5.0</th>
<th>6.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>f (inch)</td>
<td>25</td>
<td>.3</td>
<td>.4</td>
<td>.45</td>
<td>.5</td>
<td>.5</td>
<td>.55</td>
<td></td>
</tr>
</tbody>
</table>

(a) Lightening holes of typical flanged holes (45° flanged)

<table>
<thead>
<tr>
<th>D₀ (Inch)</th>
<th>D (Inch)</th>
<th>α (Inch)</th>
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</thead>
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<tr>
<td>1.7</td>
<td>0.8</td>
<td>0.2</td>
</tr>
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<td>1.95</td>
<td>1.05</td>
<td>0.2</td>
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<td>2.65</td>
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<td>0.25</td>
</tr>
<tr>
<td>3.0</td>
<td>2.05</td>
<td>0.25</td>
</tr>
<tr>
<td>3.65</td>
<td>2.7</td>
<td>0.25</td>
</tr>
<tr>
<td>3.9</td>
<td>2.95</td>
<td>0.25</td>
</tr>
<tr>
<td>4.95</td>
<td>3.8</td>
<td>0.4</td>
</tr>
<tr>
<td>5.95</td>
<td>4.8</td>
<td>0.4</td>
</tr>
<tr>
<td>6.95</td>
<td>5.8</td>
<td>0.4</td>
</tr>
<tr>
<td>7.44</td>
<td>6.3</td>
<td>0.4</td>
</tr>
<tr>
<td>7.95 (min)</td>
<td>6.8</td>
<td>0.4</td>
</tr>
<tr>
<td>8.95 (min)</td>
<td>7.8</td>
<td>0.4</td>
</tr>
<tr>
<td>9.45 (min)</td>
<td>8.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>

(b) Lightening holes with beaded flanged

Fig. 6.2.1 Common flanged lightening holes. (t = 0.032 in - 0.125 in)

Fig. 6.2.2 Lightly loaded or very shallow beams.

Fig. 6.2.3 Ultimate allowable gross shear stress for aluminum alloy webs with flanged holes as shown in Fig. 6.2.1(a).
RIB SPECIFICATION WITH RESPECT TO BUTTLINE

<table>
<thead>
<tr>
<th>BL</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>W</th>
<th>V</th>
<th>M</th>
<th>N</th>
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<td>7.0</td>
<td>2.68</td>
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<td>2</td>
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<td>22.0</td>
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<td>2.02</td>
<td>1.35</td>
<td>0.4</td>
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<td>4</td>
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<tr>
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<td>1.58</td>
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<td>52.0</td>
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<td>1.00</td>
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<td>1.5</td>
<td>0.38</td>
<td>4</td>
</tr>
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1.5 DIA 4 HOLES EQUALLY SPACED 1.8
(BL 22.0 - BL 37.0)

2.0 DIA 3 HOLES EQUALLY SPACED 2.25
(BL 7.0 - BL 22.0)

1.2 DIA 4 HOLES EQUALLY SPACED 2.0
(BL 22.0 - BL 37.0)

1.5 DIA 4 HOLES EQUALLY SPACED 1.8
(BL 7.0 - BL 22.0)
VIEW L-L
NOTE: ALL FORMERS 0.025' 2024-T3

SECTION B-B
STA 189

SECTION D-D
STA 229

EMBRY-RIDDLE AERONAUTICAL UNIVERSITY
DAYTONA BEACH FLORIDA

SIZE DATE SCALE DRAWN BY
D 6/19/93 1/40 G. TELLA

NOTE

± 1/2°
NOTE A: SECTIONS F-F AND G-G HAVE THE SAME SPAR DIMENSIONS
SECTION J-J
CHANNEL-FORMED STRINGER

SECTION K-K
SKIN-FORMED STRINGER
NOTE: ALL MATERIAL IS 0.025" 2024-T3 SHEET.