

1993

**NASA/ASEE SUMMER FACULTY FELLOWSHIP PROGRAM**

**MARSHALL SPACE FLIGHT CENTER  
THE OHIO UNIVERSITY**

**A MINIMUM COST TOLERANCE ALLOCATION METHOD  
FOR ROCKET ENGINES**

**and**

**Robust Rocket Engine Design**

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## MINIMUM COST TOLERANCE ALLOCATION

Rocket engine design follows three phases: systems design, parameter design, and tolerance design. Systems design and parameter design are most effectively conducted in a concurrent engineering (CE) environment that utilize methods, such as Quality Function Deployment and Taguchi methods. However, tolerance allocation remains an art driven by experience, handbooks, and rules of thumb.

It was desirable to develop an optimization approach to tolerancing. The case study engine was the STME gas generator cycle. The design of the major components had been completed and the functional relationship between the component tolerances and system performance had been computed using the Generic Power Balance model. The system performance nominals (thrust, MR, and Isp) and tolerances were already specified, as were an initial set of component tolerances. However, the question was whether there existed an optimal combination of tolerances that would result in the minimum cost without any degradation in system performance.

The optimization model seeks to minimize the total system cost as determined by component tolerances subject to constraints on the tolerances:

$$MIN[total\ cost] = MIN\left[\sum_i^n C(tol_i)\right] \quad [1]$$

subject to

$$T_k^2 \geq \sum_{i=1}^n G_{ik}^2 \cdot tol_i^2 \quad (i = 1, K, n)$$

$$toll_i \leq tol_i \leq tolu_i \quad [2]$$

where:

- C(tol<sub>i</sub>)      Cost of producing tol<sub>i</sub>;
- tol<sub>i</sub>            Tolerance of the i<sup>th</sup> component performance variable;
- toll<sub>i</sub>, tolu<sub>i</sub>    Lower and upper limit of tol<sub>i</sub>;
- T<sub>k</sub>             The k<sup>th</sup> system performance tolerance;
- G<sub>ik</sub>            Is the gain of the i<sup>th</sup> component performance variable to the k<sup>th</sup> system performance variable.

Equation [2] is a statistical tolerancing equation that models non-linear systems through a first order Taylor expansion where the gains G<sub>ik</sub> are the first order partial derivatives. The linear Taylor approximation is generally valid for tolerance allocation problems since tolerances typically vary only by a small amount. The gains matrix was obtained from the generic power balance model mentioned above.

The greatest problem was determining the cost tolerance relationships, C(tol<sub>i</sub>). There are numerous models for cost tolerance equations, the most common of which are the reciprocal or inverse, reciprocal squared, and the negative exponential. However, these models have always been applied to specific manufacturing processes where the cause effect relationships between the process and tolerance were conceptually well understood. The conceptual difficulty at the high level of design in the STME study involved imagining how to tighten or loosen a component's performance, e.g., efficiency and how much such a change would cost. It is much easier to conceptualize changing the tolerance on a specific component element, such as the turbine blades, or the nozzle diameter. The difficulty in part reflects the

relationship between systems designers who think of components as inputs and characterized by component performance variables, and component designers who think of component performance variables as outputs.

Two approaches were taken to relating cost and tolerances, and for lack of imagination termed the top-down and the bottom-up method. Both methods were implemented in Excel 4.0 for Windows and the optimization problem was solved using Excel's solver function.

In the top down method, the optimization model changes the component performance tolerances directly to minimize cost and satisfy the system constraints. The method is called top down because the changes in the component performance tolerances represent top level changes that are conceptually propagated down to the element level. The cost is, however, computed at the element level and proportioned out to the performance variables through a cost-contribution matrix.

The Top-Down method has several problems. First it assumes that tightening a particular component performance tolerance is achieved by tightening all the elements that affect it by the same amount. This clearly leads to contradictions when the same component affects two performance variables, one which tolerance is being tightened, and the other loosened. Thus, the top down method fails to model physical reality, namely that cost gains are achieved because tolerances are loosened on component elements which result in different component performance variations.

Second, the element-performance cost contribution matrix is likely to be difficult if not impossible to obtain. This is in part because the method does not model reality well, and in part because companies typically do not track costs in this manner. To rectify some of these problems, the Bottom-Up approach was developed.

In the bottom-up approach the solver here varies the low level component element tolerances and computes their impact on system performance through a two phase statistical stackup analysis (see eq. [2]). This requires two gains matrices: from system to component performance, and from component performance to component element tolerance.

The cost for each tolerance is determined from a family of cost-component-element-tolerance curves. The curves are computed for each element from a set of five standard cost-tolerance curves that were then scaled to match the initial design conditions. The five curves were created in conjunction with the component designers and range from a 1/4 reciprocal to a cubed reciprocal function with differing parameters. The scaling to the initial conditions involved knowing how much a particular element cost, how much of its total cost was due to creating a component of that functionality (nominal design) versus creating the same component with tighter tolerances, and the initial design tolerance. There were instances where going to tighter tolerances would require changing manufacturing processes with drastically different cost-tolerance behavior. In these cases the resulting cost tolerance curve had both a "jump" (discontinuous) as well as a change in slope.

The Bottom-Up approach appears to be the preferred method because it models reality more accurately, the data is more readily obtainable, and it is conceptually more appealing. The major difficulties are 3 fold. First, one must be able to obtain good gains matrices; second, it is imperative to have good cost estimates; and third, which is related to 2, it is necessary to better understand and estimate the standard cost tolerance curves for each element.

However, it is believed that in the tolerance design phase these estimates are typically not well known. Thus, the answer from the optimization problem will, in all likelihood, not be

the best possible answer. However, it is believed that by encouraging engineers to run the program they will have the necessary data to make informed decisions based on cost, and gain insight into the relationships between the variables at a systems level. Thus, the minimum cost tolerancing algorithm, when used by a cross functional team with other concurrent engineering tools, could have a significant impact on the cost of a design.

### ROBUST ENGINE DESIGN

The purpose of the research was to develop a method for determining the set of optimal nominal design parameters that results in a system response that is least sensitive to variations in inlet conditions and between-component variations (manufacturing variations). Should the method prove to be successful, it could be expanded to include different cycle configurations, or become a means of evaluating the relative merits of different cycles.

Data were generated from a computer simulation program called The Generic Power Balance Model developed by RocketDyne Corporation. The program was specifically designed to aid rocket engine designers determine design configurations that would optimize system performance while ensuring conservation of mass and energy.

The particular cycle chosen for this project was a gas generator (GG) cycle to be used as an upper stage space engine. The primary system response variables of interest were thrust, mixture ratio (MR), and specific impulse (Isp). The various component environments were also considered to be important to design decisions since the environments often determine the maximum design conditions (MDCs) for the components. However, they were considered secondary to the system performance variables.

The method involved generating a series of on-design hardware configurations by altering control variables according to an L8 orthogonal array. The control variables used in the study are shown in Table 1. They were selected based on engineering knowledge and do not necessarily represent the most important design variables.

	Variable	level 1	level 2
A	Chamber Pressure	800 psia	1000 psia
B	Fuel Pump Head Coef	0.55	0.60
C	LOX Pump Head Coef	0.50	0.55
D	Fuel Turbine % Admission	50%	100%
E	LOX Turbine % Admission	50%	100%
F	Fuel Turbine Blade Angle	15°	30°
G	GG Temperature	1400°R	1600°R

Table 1. Control Variables for GG Cycle Engine.

A total of 14 noise factors representing the inlet conditions and random fluctuations in component efficiencies and resistances were considered. Creating an L16 noise array, however, would require an excessive number of simulation runs (8x16=128). Since an analysis on noise effects is meaningless, they were combined in a "worst case" fashion to ensure that the expected variability in system response is captured, thereby, reducing the number of required simulation runs. However, some factors affected the response variables in a different manner. For example, a decrease in the LOX inlet pressure would result in a decrease in thrust and MR and an increase in Isp. A decrease in the fuel inlet pressure would

also decrease thrust, but increase MR and decrease Isp. The following method was devised to determine which factors could be combined to ensure that the system would be exposed to the full range of potential noise conditions.

A gains matrix obtained from the STME study (a GG cycle low cost engine) indicated the direction of system response change with an increase in each of the noise factors. The signs of the gain factors were tabulated and all noise factors which induced a similar system response were grouped into the same class. This resulted in four classes, of which one was omitted because it contained only a single variable which gain value was very small. Thus, the outer array (noise array) was an L4 matrix with 3 noise variables.

The eight on-design configurations were run under each of the noise conditions as an open-loop off-design condition resulting in  $8 \times 4 = 32$  off-design simulation runs. For each of the dependent variables the following statistics were computed and analyzed: average, variance, and signal to noise ratios. The ANOVAs showed that none of the control factors were significant ( $F=0$ ) and the error term contributes over 90% of the variation in the data. This means that the noise factors had a greater effect on system performance than any of the control factors. This was true for all of the system performance variables as well as the component environment variables: GG temperature, the fuel pump discharge pressure, LOX pump discharge pressure, and MCC pressure. The analysis of the variation also showed that it could not be substantially reduced by any of the control factors.

The conclusion drawn from the results is that calibration of the engines is necessary to reduce the impact of component variations. The impact due to inlet conditions, however, will remain. Calibration of the engine is performed by running the off-design simulation under closed loop control by specifying two control parameters, typically the GG LOX injector resistance and the LOX turbine bypass orifice resistance. The control authority for each of these two resistances is defined here to be the full range of resistance required to balance the engine at nominal thrust and MR under worst and best case conditions.

There has been some difficulty in developing a calibration method, however, because under some on-design conditions there is insufficient flow to accommodate the necessary control authority, i.e., where the resistances are already so low under the on-design case that opening of the valves completely is not sufficient to balance the engine. Since the original on-design cases did not have a pressure drop across the control points, it may be necessary to compute a nominal pressure drop and include it in the on-design runs. This could possibly be done from the off-design data and knowing the thrust and MR gain as a function of the resistances. Since the system response ranges are known from the open-loop off-design runs, it would be straightforward to compute the required control authority and nominal resistance assuming a linear relationship between resistance and system response.

In summary, it appears that it is possible to use the generic power balance model to generate a robust design. It also appears that a certain amount of iteration may be necessary to simulate engine calibration. It is believed that it may be possible to predict the required control authority from the open-loop off-design runs alone, without further iteration. If this is true, then the optimal design can be determined and the calibration simulations need only be performed on that single design, thus eliminating the need for repeated iterations.