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# LRAT: LIGHTNING RADIATIVE TRANSFER


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## I. INTRODUCTION

In this report, we extend to cloud physics the work done in (5–9) for single and multiple scattering of electromagnetic waves. We consider the scattering of light (visible or infrared) by a spherical cloud represented by a statistically homogeneous ensemble of configurations of N identical spherical water droplets whose centers are uniformly distributed in its volume V. The ensemble is specified as in (8), by the average number $\rho$ of scatterers in unit volume, and by $\rho f(\mathbf{R})$ with $f(\mathbf{R})$ as the distribution function for separation $\mathbf{R}$ of pairs. The incident light, $\vec{\phi}_0 = \hat{\mathbf{a}}_0 e^{i\vec{k}_0 \cdot \mathbf{r}}$, a plane electromagnetic wave with harmonic time dependence, is from outside the cloud. The propagation parameter $k_0$ and the index of refraction $\eta_0$ determine physically the medium outside the distribution of scatterers.

We solve the interior problem separately to obtain the bulk parameters for the scatterer equivalent to the ensemble of spherical droplets (2–5). With the interior solution or the equivalent medium approach, the multiple scattering problem is reduced to that of an equivalent single scatterer excited from outside illumination. A dispersion relation which determines the bulk propagation parameter K and the bulk index of refraction $\eta$ of the cloud is given in terms of the vector equivalent scattering amplitude $\vec{\mathcal{G}}$ and the dyadic scattering amplitude $\tilde{\mathbf{g}}$ of the single object in isolation.

Based on this transfer model we will have the ability to consider clouds composed of inhomogeneous distribution of water and/or ice particles and we will able to take into account particle size distributions within the cloud. We will also be able to study the effects of cloud composition (i.e., particle shape, size, composition, orientation, location) on the polarization of the single or the multiple scattered waves. Finally, this study will provide a new starting point for studying the problem of lightning radiative transfer (3–4).

In general, we work in spherical coordinates. We use bold face or an arrow to denote a vector or a vector operator. A circumflex indicates a vector of unit magnitude. A tilde on the top of a letter denotes a dyadic (second rank tensor). For brevity, we use [5:4] for equation 4 of Ref. ( 5) etc.

## II. MATHEMATICAL MODELING/SOLUTION INSIDE THE CLOUD FOR OUTSIDE INCIDENCE

The solution inside the cloud for outside illumination corresponds to the multiple scattering of a plane electromagnetic wave by an ensemble of configurations of N identical spherical water droplets. To obtain the solution inside the cloud, we consider first the single scatterer in isolation, second a fixed configuration of N identical scatterers, and third an ensemble of the above-mentioned configurations.

For an incident plane electromagnetic wave $\vec{\phi} = \hat{\mathbf{a}}_1 e^{i\vec{\kappa}_1 \cdot \mathbf{r}}$, $\kappa_1 = k_0\eta_1$, and $\eta_1$ being the complex relative index of refraction for the host medium inside the cloud but outside each droplet, the total outside solution for the single scatterer in isolation (outside the single water droplet but still inside the host medium) $\vec{\psi} = \vec{\phi} + \mathbf{u}$ satisfies the following differential equation obtained from Maxwell's equations after suppressing the harmonic time dependence factor $e^{-i\omega t}$

$$\left[\vec{\nabla}\times\vec{\nabla}\times+\kappa_1^2\right]\vec{\psi}=0, \vec{\nabla}\cdot\vec{\psi}=0. \qquad [1]$$

The solution inside the single spherical water droplet in isolation $\vec{\psi}_{in}$ satisfies [1] with $\kappa_1$ replaced by $\kappa_2$. Here, $\kappa_2 = \kappa_1\eta_2 = k\eta_1\eta_2$, with $\eta_2$ being the complex relative index of refraction for the medium inside the spherical water droplet. The propagation parameters $\kappa_1$ and $\kappa_2$ correspond (within the distribution of identical spheres) to the media outside and inside the water droplet respectively.

Similar to Twersky (7), we have

$$\vec{\psi}=\hat{\mathbf{a}}_1 e^{i\vec{\kappa}_1\cdot\mathbf{r}}+\left\{\tilde{h}(\kappa_1|\mathbf{r}-\mathbf{r}'|),\mathbf{u}(\mathbf{r}')\right\}, u(r)=\left\{\tilde{h},\ \mathbf{u}\right\}\equiv$$
$$-\frac{\kappa_1}{i4\pi}\int\left[\left(\tilde{h}\times\hat{\mathbf{n}}\right)\cdot\left(\vec{\nabla}\times\mathbf{u}\right)-\left(\vec{\nabla}\times\tilde{h}\right)\cdot(\hat{\mathbf{n}}\times\mathbf{u})\right]dS(\mathbf{r}'). \qquad [2]$$

Here, $\tilde{h}=\left(\tilde{\mathbf{I}}+\frac{\vec{\nabla}\vec{\nabla}}{\kappa_1^2}\right)h(\kappa_1|\mathbf{r}-\mathbf{r}'|)$, $h(x)=\frac{e^{ix}}{ix}$, and $\tilde{\mathbf{I}}$ being the identity dyadic. It is important to note that $\mathbf{r}$, and $\mathbf{r}'$ denote the observation point and a point on the surface $S$ or in the volume $v$ of the water droplet respectively.

Asymptotically $(\kappa_1 r >> 1)$ we can write

$$\mathbf{u}(\mathbf{r})=h(\kappa_1 r)\mathbf{g}(\hat{\mathbf{r}},\hat{\kappa}_1:\hat{\mathbf{a}}_1),\ \ \mathbf{g}(\hat{\mathbf{r}})=\tilde{\mathbf{I}}_*\cdot\mathbf{g}(\hat{\mathbf{r}}). \qquad [3]$$

Here, $\tilde{\mathbf{I}}_*=\left(\tilde{\mathbf{I}}-\hat{\mathbf{r}}\hat{\mathbf{r}}\right)$ is the transverse identity dyadic and $\hat{\mathbf{a}}_1\cdot\vec{\kappa}_1=0$. The spectral representation of $\mathbf{u}$ is

$$\mathbf{u}(\mathbf{r})=\frac{1}{2\pi}\int_c e^{i\vec{\kappa}_{1c}\cdot\mathbf{r}'}\mathbf{g}(\hat{\mathbf{r}})d\Omega(\theta_c,\varphi_c),\ \ r>\left(\hat{\mathbf{r}}\cdot\mathbf{r}'\right),\ \vec{\kappa}_{1c}=\kappa_{1c}\hat{\mathbf{r}}_c(\theta_c,\varphi_c), \qquad [4]$$

and the single scattering amplitude $\mathbf{g}(\hat{\mathbf{r}},\hat{\kappa}_1:\hat{\mathbf{a}}_1)=\left\{\tilde{\mathbf{I}}_t e^{-i\vec{\kappa}_1\cdot\mathbf{r}'},\mathbf{u}(\mathbf{r}')\right\}$ can also be evaluated from Mie scattering theory.

Now, we consider a fixed configuration of N identical scatterers with centers located by $\mathbf{r}_{m(m=1,2,3,\ldots,N)}$. The total outside field

$$\mathbf{\Psi}(\mathbf{r})=\vec{\phi}(\mathbf{r})+\sum_{m=1}^{N}\mathbf{U}_m(\mathbf{r}-\mathbf{r}_m),\ \mathbf{U}_m(\mathbf{r}-\mathbf{r}_m)\sim h(\kappa_1|\mathbf{r}-\mathbf{r}_m|)\mathbf{G}_m,\ |\mathbf{r}-\mathbf{r}_m|\to\infty. \qquad [5]$$

Equivalently, for the scatterer located at $\mathbf{r}_t$, we use the self-consistent approach of (6–7) to obtain the total outside configurational field

$$\mathbf{\Psi}_t(\mathbf{r})=\vec{\phi}(\mathbf{r})+\sum{}'\mathbf{U}_m(\mathbf{r}-\mathbf{r}_m)+\mathbf{U}_t(\mathbf{r}-\mathbf{r}_t),\ \sum{}'=\sum_{m=1,\,m\neq t}. \qquad [6]$$

Using [6] and the general reciprocity relation $\left\{\boldsymbol{\Psi},\ \vec{\psi}_{\hat{\mathbf{a}}_1}\right\}_t = 0$ for any arbitrary direction of incidence and polarization $\hat{\mathbf{a}}_1$, we derive as in (2) the self-consistent integral equation for the multiple configurational scattering amplitude

$$\mathbf{G}_t(\hat{\mathbf{r}}) = \tilde{\mathbf{g}}_t(\hat{\mathbf{r}},\hat{\kappa}_1)\cdot\hat{\mathbf{a}}_1 e^{i\vec{\kappa}\cdot\mathbf{r}_t} + \sum{}' \int_c \tilde{\mathbf{g}}_t(\hat{\mathbf{r}},\hat{\mathbf{r}}_c)\cdot\mathbf{G}_m(\hat{\mathbf{r}}_c) e^{i\vec{\kappa}_{1c}\cdot\mathbf{R}_{tm}}, \qquad [7]$$

with $\mathbf{R}_{tm} = \mathbf{r}_t - \mathbf{r}_m$, $\int_c = \frac{1}{2\pi}\int d\Omega_c$, $\tilde{\mathbf{g}}(\hat{\mathbf{r}},\hat{\kappa}_1)\cdot\hat{\mathbf{a}}_1 = \mathbf{g}(\hat{\mathbf{r}},\widehat{\kappa}_1 : \hat{\mathbf{a}}_1)$, and the magnitude of the separation distance $|\mathbf{R}_{tm}|$ is bounded above by the diameter D of the cloud.

We take the ensemble average of [7], use the quasi-crystalline approximation of Lax (2), the equivalent medium approach and Green's theorems, to obtain (7) the dispersion relation determining the bulk parameters

$$\vec{\mathcal{G}}\left(\vec{\kappa}_1 | \vec{\mathrm{K}}\right) = -\frac{\rho}{c_o(\mathrm{K}^2-\kappa_1^2)}\left\{\left[e^{-i\vec{\mathrm{K}}\cdot\mathbf{R}},\ \vec{\mathrm{U}}\right]\right\} + \rho\int_{V_{\mathrm{D}}-v}[f(\mathbf{R})-1]e^{-i\vec{\mathrm{K}}\cdot\mathbf{R}}\vec{\mathrm{U}}d\mathbf{R}, \qquad [8]$$

where $d\mathbf{R}$ denotes volume integration over $(V_{\mathrm{D}}-v)$. Here, $\vec{\mathcal{G}}$ is the equivalent scattering amplitude and $\vec{\mathrm{U}}$ is a radiative function defined by $\vec{\mathrm{U}} = \int_c \tilde{\mathbf{g}}(\hat{\mathbf{r}},\hat{\mathbf{r}}_c)\cdot\vec{\mathcal{G}}\left(\vec{\kappa}_{1c}|\ \vec{\mathrm{K}}\right)e^{i\vec{\kappa}_{1c}\cdot\mathbf{R}}$, and $c_o = \kappa_1/4\pi i$. The bulk propagation parameter $\mathrm{K} = \kappa_1\eta$ with $\eta$ being the bulk index of refraction, and $\{[f,g]\} = \int_S [f\partial_n g - g\partial_n f]dS$ is the Green surface operator with outward unit normal from $v$. Equation (16) solves formally the interior problem for the cloud with outside illumination.

## III. BULK PARAMETERS AND LEADING TERM APPROXIMATIONS

To simplify [8], we force the model to neglect all phase transition effects (1), and to take only into account pair interaction due to central forces. If the inter-droplet potential is negligeable, we can choose $f(\mathbf{R})$ to be always equal to unity. Hence, [8] is reduced to

$$\left[(\mathrm{K}^2-\kappa_1^2)\tilde{\mathbf{I}}_* + \left(\frac{\rho}{c_o}\right)\tilde{\mathbf{g}}\left(\hat{\mathbf{r}},\hat{\mathbf{K}}\right)\right]\cdot\vec{\mathcal{G}}\left(\vec{\kappa}_1|\ \vec{\mathrm{K}}\right) = \mathbf{0}. \qquad [9]$$

In [9], let $\hat{\mathbf{r}} = \hat{\mathbf{K}}$. In addition, because optical scattering from a cloud is highly forward peaked (7), we neglect back scattering and reduce [9] to

$$\left[(\mathrm{K}^2-\kappa_1^2)\tilde{\mathbf{I}}_*\left(\hat{\mathbf{K}}\right) + \left(\frac{\rho}{c_o}\right)\tilde{\mathbf{g}}\left(\hat{\mathbf{K}},\hat{\mathbf{K}}\right)\right]\cdot\vec{\mathcal{G}}\left(\vec{\mathbf{K}}|\ \vec{\mathrm{K}}\right) = \mathbf{0}. \qquad [10]$$

If the scatterers preserve the incident polarization (7:68), we have from [10]

$$(\mathrm{K}^2-\kappa_1^2) = -\left(\frac{\rho 4\pi i}{\kappa_1}\right)\tilde{\mathbf{g}}\left(\hat{\mathbf{K}},\hat{\mathbf{K}}\right),\ \ \eta^2 - 1 = -\left(\frac{\rho 4\pi i}{\kappa_1^3}\right)\tilde{\mathbf{g}}\left(\hat{\mathbf{K}},\hat{\mathbf{K}}\right). \qquad [11]$$

Equation [11] determines the bulk propagation parameter K and the bulk index of refraction $\eta$ of the equivalent medium for the bounded distribution of the spherical water droplets.

## IV. CONCLUSION

The multiple scattering problem has been reduced to that of a single equivalent scatterer in isolation. Formulae are given for the bulk propagation parameter K and the bulk index of refraction $\eta$ of the equivalent medium. The results are quite general in nature and can be extended to non-spherical geometries. Also, they can be applied immediately to the problem of pulsating optical point sources arbitrarily distributed throughout a scattering medium. When $f(\mathbf{R}) - 1 \neq 0$, [8] can be approximated or solved numerically.


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